ADDING PER-NUMBERS **A2**

Questions	Answers
What is a prime-number?	Fold-numbers can be folded: 10 = 2fold5. Prime-numbers cannot: 5 = 1fold5
What is a per-number?	Per-numbers occur when counting, when pricing and when splitting
How to add per-numbers?	The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b

1 FOLD-NUMBERS AND PRIME-NUMBERS

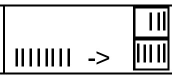
Question. What are there primary numbers?
Answer. The primary numbers are numbers that cannot be folded, prime-numbers.
Example1. The stack T = 1*4 can be recounted to another stack T = 2*2. The stack T = 1*3 cannot. Thus 4 = 2fold2 whereas 3= 1fold3. 4 is called a fold-number, and 3 is called a nonfold-number or a prime number.
Fold-numbers are built from prime numbers, e.g. by folding in tables: 2, 4, 6, etc. 3, 6, 9, etc. 5, 10, 15, etc.
Fold-numbers can be factorised in prime numbers by using a prime-number filter:

48:	48/2	24/2	12/2	6/2	3/3	1
	*2	*2	*2	*2	*3	

So 48 = 2*2*2*2*3 = 2^4*3. Likewise 72 = 2*2*2*3*3 = 2^3*3^2
Example2. A packet consists of a mix of different goods coded with different prime-numbers.
The packet with the code 42 contains the goods coded 2 and 3 and 7 but not the good coded 5.
The packets 30 and 42 have the goods coded 2 and 3 in common (the greatest common factor HCF).
HCF(30,42) = HCF(2*3*5, 2*3*7) = 2*3 = 6. The HCF can be found by Euclid's algorithm.
Exercise1. Factorise the fold-numbers 6, 16, 26, 36, 46, 56, 66, 76, 86, 96. Factorise other fold-numbers.
Exercise2. Find the HCF of 30 and 105. Find the HCF of 30 and 40, etc.

2 PER-NUMBERS IN COUNTING

Question. What does a per-number consist of?
Answer. A per-number consists of a stack-number and a bundle-number.

A total stack of 8 is counted in bundles of 5s as T= 8 = (8/5)*5 = 1 3/5*5. Thus the per-number (fraction) 8/5 consists of a stack-number (nominator) 8 and a bundle-number (denominator) 5.	

Example1. Repeated recounting creates double per-numbers:
1 = (1/2)*2 = (1/2) 2s = (1/2/3)*3 2s = (1/2/3)*3*2 = (1/2/3)*6. Since also 1 = (1/6)*6: **1/(2*3) = 1/2/3**
1 = (1/(2/3))*(2/3). Since also 1 = (1/3)*3 = (((1/2)*2)/3)*3 = (1/2*3)*2/3: **1/(2/3) = 1/2*3**
*The bracket rule: a per-bracket can be added or removed if the signs are changed from / to * and reverse.*
a is counted in bs, c is counted in ds: a=(a/b)*b & c=(c/d)*d. Thus a*c = (a/b)*b*(c/d)*d = (a/b)*(c/d)*(b*d).
a*c is counted in (b*d)s: a*c = ((a*c)/(b*d))*(b*d). Thus (a/b)*(c/d) = (a*c)/(b*d)
The multiplication rule 'stack by stack & bundle by bundle': **(a/b)*(c/d) = (a*c)/(b*d)**
The division rule 'stack by bundle & bundle by stack: **(a/b)/(c/d)** = a/b/c*d = a*d/b/c = **(a*d)/(b*c)**
Exercise. Calculate 24/(2*3), 24/2/3 and 24/(2/3) in different ways. Likewise with x^5*y^3/(x^2*y).

3 PER-NUMBERS IN PRICING AND RATING

Question. How to use per-numbers in pricing?
Answer. The price 2$ per 3kg can be written as a per-number 2$/3kg or 2/3 $/kg.
Since the price is 5*2$ for 5*3kg the price could also be written as 5*2$ per 5*3kg or (5*2)/(5*3) $/kg or 10/15 $/kg. Thus 10/15 = (5*2)/(5*3) = 2/3 in accordance with (5*2)/(5*3) = 5*2/5/3 = 5/5 * 2/3 = 2/3.
Removing common prime-factors from the stack and the bundle (the nominator and the denominator) in a per-number is called *reducing* the per-number. Adding common prime-factors to the stack and the bundle (the nominator and the denominator) in a per-number is called *extending* the per-number.
Extending a per-number: 2/3 = (2*7)/(3*7) = 14/21 in accordance with 2/3*7/7 = 2*7/7/3 = (2*7)/(7*3) = 14/21.
Reducing a per-number: 14/21= (2*7)/(3*7) = 2 * 7 / 3 / 7 = 2 / 3 * 7 / 7 = 2/3 according to the bracket rule.
Example1. Reduce the per-number 48/72.
48/72 = (2*2*2*2*3)/(2*2*2*3*3) = 2*2*2*2*3/2/2/2/3/3 = 2/2 * 2/2 * 2/2 * 2 * 3/3 /3 = 2/3
Example2. Reduce the per-number (4*x^2*y)/(6*x*y^3).
(4*x^2*y)/(6*x*y^3) = (4*x*x*y)/(6*x*y*y*y) = 2*2*x*x*y/2/3/x/y/y/y = 2*x/3/y/y = (2*x)/(3*y*y)
Example3. Comparing prices. Which is the higher price, 2/3 or 4/7? Both per-numbers are extended to 3*7:
P1 = 2/3 = 2/3 * 7/7 = (2*7)/(3*7) = 14/21. And P2 = 4/7 = 4/7 * 3/3 = (4*3)/(7*3) = 12/21.
So P1>P2, and the difference is P1-P2 = 14/21-12/21 = 2/21.

Which is the higher price, 3/4 or 5/6? Since 4=2*2 and 6=2*3 both per-numbers are extended to 2*2*3 (the lowest common multiple, LCM):
P1 = 3/4 = 3/4 * 3/3 = (3*3)/(3*4) = 9/12. And P2 = 5/6 = 5/6 * 2/2 = (5*2)/(6*2) = 10/12.
So P1<P2, and the difference is P2-P1 = 10/21-9/12 = 1/12
Example 4. Adding prices. What is the total price consisting of a basis price 11/6 + a fee of 4/9?
Since 6 = 2*3 and 9 = 3*3, the LCM(6,9) = LCM(2*3,3*3) = 2*3*3
P1 = 11/6 = 11/6 * 3/3 = (11*3)/(6*3) = 33/18. And P1 = 4/9 = 4/9 * 2/2 = (4*2)/(9*2) = 8/18
So the total is P1 + P2 = 33/18 + 8/18 = 41/18.
Exercise1. Reduce the per-number 36/64. Etc.
Exercise2. Reduce the per-number (6*x^3*y^2)/(8*x^2*y^4).
Exercise3. What are the difference and the sum of 2/5 and 3/7? 3/8 and 4/10? 23/36 and 31/48?

4 PER-NUMBERS IN SPLITTING

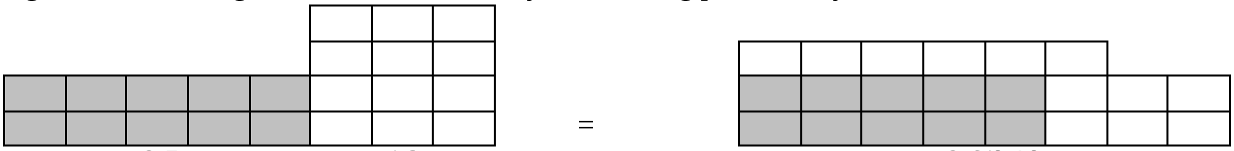
Question. How to split a winning?
Answer. By returning the stake several times. Or by receiving a proportional part of the winning.
Example1. The players A, B and C split a winning of 400$ from putting 2$, 3$ & 5$ into a pool creating 10$.
Method 1. The winning is counted in pools to get the odds: W = 400$ = (400/10)*10 = 40*10. Thus the players get their stake back 40 times: A gets 2$ 40 times, i.e. 80$, etc.
Method 2. The winning is split in the ratio 2:3:5. A gets 2 ten parts of the winning: A = 2/10*W = W/10*2.
Together A and B get 2/10 and 3/10 of W i.e. 2/10*W+3/10*W = (2/10+3/10)*W = ((2+3)/10)*W = 5/10*W.
Example2. An apartment is sold in 2 ¼-shares, 2 1/8-shares and 4 1/16-shares.
There are no shares left since 2/4*A + 2/8*A + 4/16*A = 2/(2*2)*A + 2/(2*2*2)*A + 2*2/(2*2*2*2)*A = ½*A + ¼*A + ¼*A = (½+¼+¼)*A = 1*A. Here ½+¼+¼ = ½*2/2 + ¼ + ¼ = 2/4 + ¼ + ¼ = 4/4 = 1.
B buys 1 1/8-share and 3 1/16-shares giving B a total control of 1/8*A + 3/16*A = (1/8 + 3/16)*A = (1/8*2/2 + 3/16)*A = (2/16 + 3/16)*A = 5/16*A.
Example3. B receives 2/10 of a 200$-winning and 3/5 of a 300$-winning.
The total income for B is T = 2/10*200 + 3/5*300 = 200/10*2 + 300/5*3 = 40 + 180 = 220 = (220/500)*500 = (2*2*5*11)/(2*2*5*5) = 11/25*500. So in this case 2/10+3/5=11/25, and not 20/25 as in example 1 above.
Example4. ½ of 2 cokes + 2/3 of 3 cokes = 1 coke + 2 cokes = 3 cokes = (3/5)*5 cokes = 3/5 of 5 cokes.
So in this case ½+2/3 = 3/5 and not 7/6 as in the example 1 and 2 above.
Example5. The fraction-paradox:

Inside the classroom	20/100	+	10/100	=	30/100
	=		=		=
	20%	+	10%	=	30%
Outside the classroom	20%	+	10%	=	32% in the case of compound interest
e.g. in the laboratory				or =	b% (10<b<20) in the case of the total average

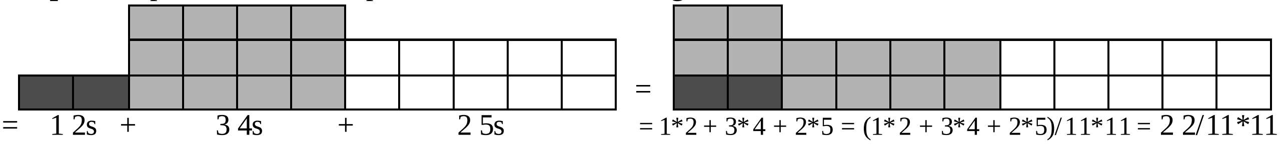
20% + 10% = 30% only when taken of the same total: 20% of 300 + 10% of 300 = 30% of 300. In all other cases the sum is different from 30%, so there is no general rule saying that 20% + 10% = 30%.
Exercise1. Split the winning 1600$ between A, B and C in the ratio 3:1:4.
Exercise2. The king gets 2 7parts of the harvest, and the bishop get 1 9part. How much is left?
Exercise3. My two investments 200$ and 500$ gave 12% and 4% yield. What is the total yield percentage?

5 ADDING PER-NUMBERS I

Question. How to add per-numbers with number units?
Answer. Per-numbers can be added upward keeping the bundle-size; or sideward extending the bundle-size.
Example1. Sideward addition of two per-numbers.
Adding two stacks as e.g. 2 5s and 4 3s is done by a recounting predicted by a recount calculation

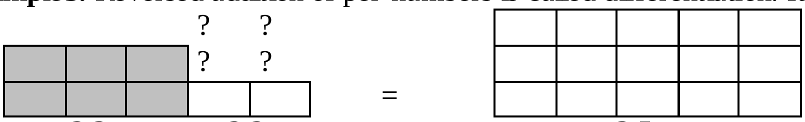


T = 2 5s + 4 3s = 2*5 + 4*3 = (2*5 + 4*3)/8*8 = 2 6/8 *8
Example2. Repeated addition of per-numbers is called integration.



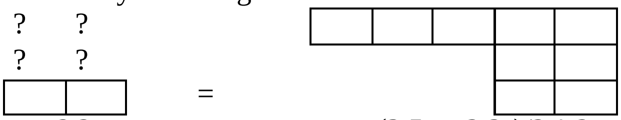
T = 1 2s + 3 4s + 2 5s = 1*2 + 3*4 + 2*5 = (1*2 + 3*4 + 2*5)/11*11 = 2 2/11*11

Example3. Reversed addition of per-numbers is called differentiation. It asks e.g. 2 3s+? 2s = 3 5s:



T = 2 3s + ? 2s = 3 5s

The answer can be obtained by removing the 2 3s from the 3 5s and then counting the remaining 9 in 2s.



? 2s = (3 5s – 2 3s)/2 * 2 = (9/2)*2 = 4 1/2 * 2

Or the answer can be obtained by a predication through a reversed calculation. In this way solving equations becomes another name for reversed calculations.

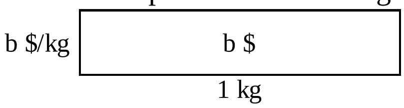
2 3s + ? 2s = 3 5s	The question	
2*3 + x*2 = 3*5	The equation	T1 + x*b = T2
x*2 = 3*5-2*3 = 9	The 2 3s are removed from the 3 5s leaving 9	x*b = T2 – T1 = ΔT
x*2 = (9/2)*2 = 4 1/2 * 2	The 9 is recounted as 2s	
x = 1 1/5	The answer	$x = \frac{\Delta T}{b} = \frac{\Delta T}{\Delta n}$

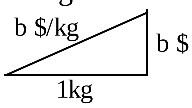
Exercise. Integrate 3 4s, 5 6s and 7 8s, etc. Differentiate 3 4s+? 5s = 6 7s, etc.

6 ADDING PER-NUMBERS II

Question. How to add per-numbers with letter units?

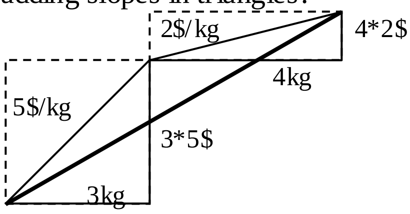
Answer. The per-number a is multiplied with the bundle-number b before added to the total stack T: T2=T1+a*b. Per-numbers occur in Renaissance trade questions as price-numbers 4 $/kg or rent-numbers 4 $/day. Again the per-number can be represented as the height of a stack, or as the slope of the diagonal in a change-triangle.





Adding per-numbers from trade takes place in a table or by adding slopes in triangles:

a kg @	*b $/kg*	*= a*b $*		
3 kg @	5 $/kg	= 3 * 5		= 15 $
4 kg @	2 $/kg	= 4 * 2		= 8 $
7 kg @	x $/kg	= 7 * x	= ∑a*b	= 23 $
	x	= 23/7		= 3 2/7 $



So per-numbers are added by their totals:

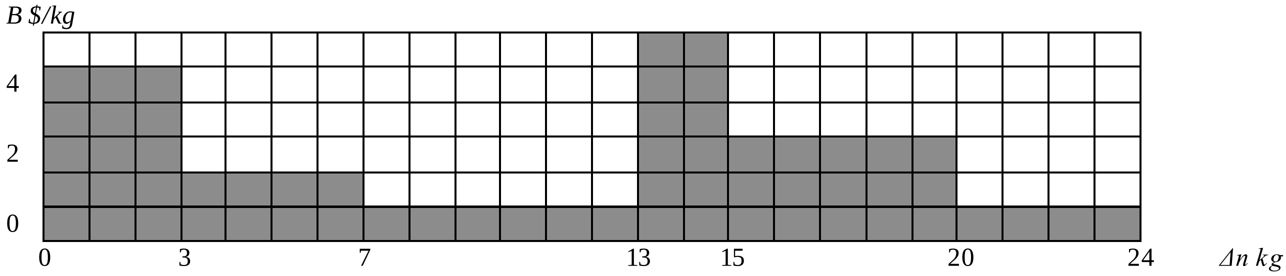
3 kg @ 5 $/ kg + 4 kg @ 2 $/ kg = (3+4) kg @ (∑a*b)/(3+4) $/ kg

The table can be supplemented with two columns showing the added values of both the kg-number Δn, and of the $-number ΔT, and of the per-number Σ b, as in this example where a teashop is adding different amounts with different prices to create a blending.

Δn kg	*b $/kg*	*Δn*b = ΔT*		*Σ Δn = Δn*	*Σ ΔT = ΔT*	*Σ b $/kg = ΔT/Δn*
3 kg @	5$/kg =	3 * 5 = 15		3	15$	15/3 = 5.0
4 kg @	2$/kg =	4 * 2 = 8		7	23$	23/7 = 3.3
6 kg @	1$/kg =	6 * 1 = 6		13	29$	29/13 = 2.2
2 kg @	6$/kg =	2 * 6 = 12		15	41$	41/15 = 2.7
5 kg @	3$/kg =	5 * 3 = 15		20	56$	56/20 = 2.8
4 kg @	1$/kg =	4 * 1 = 4		24	60$	60/24 = 2.5

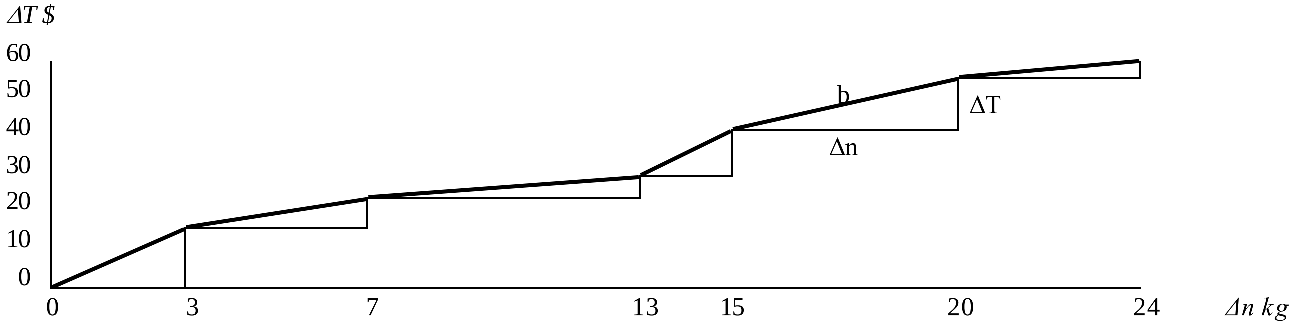
When plotting the per-number b $/kg against Δn kg in a coordinate system the total $-number is the area under the curve representing the sum of the stacks.

The Total as an area under the PerNumber curve



When plotting ΔT against Δn in a coordinate system the curve shows both the added kg-number Δn, the added total ΔT, and the single per-numbers b = ΔT/Δn as the slopes.

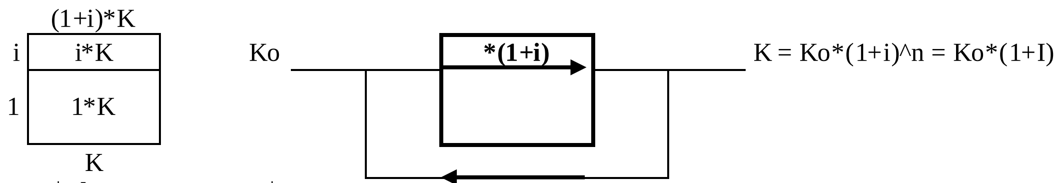
The PerNumber as the slope of the Total curve



Thus from blending tea in a shop we learn that:
The Total is the area under the PerNumber curve predicted by an integration formula: T= ∑$/kg*kg = ∑b*Δn.
The PerNumber is the slope of the Total curve predicted by a differentiation formula: b = Δ$/Δkg = ΔT/Δn.
Exercise. Travel, first 5 seconds @ 4m/s, then 3 seconds @ 6m/s, then 4 seconds @ 2m/s etc.

7 ADDING INTEREST PERCENTAGES

Question. How can we add stacks sideward?
Answer. Adding stacks sideward in time is called integration. It can be done by recounting and restacking.
Example. T = 2 3s + 4 5s = ? 8s. The result can be predicted by the recount-equation.
The interest i*K is added to a capital K giving K + i*K = 1*K + i*K = (1+i)*K. Repeated n times the terminal capital is K = Ko*(1+i)^n.



5 days @ 9 $/day = 5*9 = 45 $
5 days @ 9 %/day = 5*9% (i.e. 45 %) + CI = simple interest + compound interest, or I = n*i + CI, so
5 days @ 9 %/day = 5*9% (i.e. 45 %) + 8.9% = 53.9% since (1+9%)^5 = 1.539
Exercise. Predict 6 days @ 7%/day, 4 days @ -8%/day etc.

8 PER-NUMBERS IN WORD-PROBLEMS

Question. How do we treat per-numbers in word problems?
Answer. Also in word problems the per-number must be transformed to a stack-number before being added.
Example1: Train1 travels from A to B at 40 km/h. Two hours later train2 travels from A to B at 60 km/h. When do they meet?

Per-numbers	*Text*	*Stack-numbers*	*Prediction*		*ANSWERS*
	Hours	x = ?	40*(x+2)	= 60*x	4
40 km/h	Velocity1		40*x + 80	= 60*x	
60 km/h	Velocity2		80	= 60*x – 40*x = 20*x	
	Distance1	40*(x+2) km	80/20	= x	*240*
	Distance2	60*x km	4	= x	*240*

Example2: Train1 travels from A to B at 40 km/h. At the same time train2 travels from B to A at 60 km/h. When do they meet if the distance from A to B is 300km?

Per-numbers	*Text*	*Stack-numbers*	*Prediction*		*ANSWERS*
	Hours	x = ?	40*x + 60*x	= 300	4
40 km/h	Velocity1		100*x	= 300*x	
60 km/h	Velocity2		x	= 300/100	
	Distance1	40*x km	x	= 3	*120*
	Distance2	60*x km			*180*

Exercise1. Repeat the train problems with other numbers.
Exercise2. It takes a motor boat 2 hours downstream and 3 hours upstream to travel the same distance. The current is 5 km/h. What is the speed of the boat?
Exercise3. ? litre 40% alcohol + 3 litre 20% alcohol gives ? litre 32% alcohol
Exercise4. ? $ @ 3%/$ + ? $ @ 8%/$ = 200$/4000$
Exercise5. B can dig a ditch in 4 hours, C in 3 hours. How long time does it take if they work together?