COUNT&ADD IN SPACE **S1**

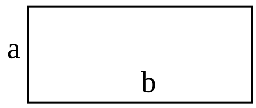
Question	Answer
How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 recount-equations: $\sin A = a/c$, $\cos A = b/c$, $\tan A = a/b$. By adding squares by the 3 Pythagoras', mini, midi & maxi.

1 COUNTING A LENGTH

Question. How can we count a one-way extension, i.e. length?
Answer. By a ruler constructed by parallel lines dividing the distances between two points in equal lengths.
Example. In a 4*8 stack on quadratic paper connect the horizontal distances 2, 4, 6 and 8 with the vertical distances 1, 2, 3 and 4. The lines are called parallel.
Exercise1. Use parallel lines to divide 10cm in 2 equal lengths. Check by measuring.
Exercise2. Use parallel lines to divide 10cm in 5 equal lengths. Check by measuring.
Exercise3. Use parallel lines to divide 12cm in 6 equal lengths. Check by measuring.
Exercise4. Use parallel lines to divide 15cm in 10 equal lengths. Check by measuring.

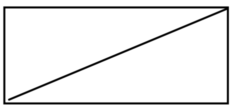
2 COUNTING A SURFACE

Question. How can we count a two-way extension, i.e. area?
Answer1. By dividing the surface in squares. Or by dividing the surface in triangles.

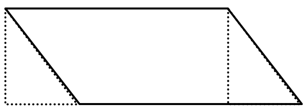


An a*b stack is called a rectangle; or a square if a = b.

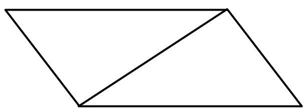
An a*b stack can be recounted as (a*b) 1s, thus having the surface-number or area $T = a*b$ = altitude*base.



A rectangle is divided in two equal right triangles by its diagonal. Thus the area of a right triangle is $T = ½*a*b$ = ½*altitude*base.



By moving the two outside triangles in an skew stack (a parallelogram) it is transformed into an a*b stack with the same area, thus having the area T = altitude*base.

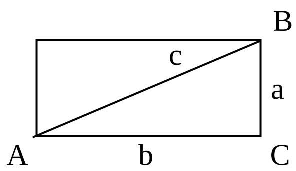


A triangle is half of a parallelogram thus having the area T = ½ * altitude*base.

Exercise1. Cut out two similar triangles and find their area by transforming them into a stack.
Exercise2. Draw a triangle and find its area by transforming it into a stack.

3 COUNTING AN ANGLE

Question. How can we count the size of an angle?
Answer1. By counting with a protractor, or by recounting one side by another.



In an a*b stack a is turned ¼ round or 90 degrees from b.
A full round is 360 (days) = (360/4)*4 = 90*4.
An acute angle is less than 90 degrees and an obtuse angle is greater than 90 degrees.
In an a*b stack the diagonal is called c. The corners or vertices are called angles A, B and C.

Recount a in bs: $a = a/b*b = \tan A*b$
Recount a in cs: $a = a/c*c = \sin A*c$
Recount b in cs: $a = b/c*c = \cos A*c$

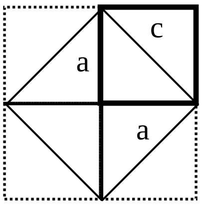
Exercise1. Draw a diagonal in a stack. Measure the angles A and B by a protractor. Predict the result by backward calculation.

tan A = a/b	(A = (tan-1)(a/b))	tan B = b/a	(B = (tan-1)(b/a)).
sin A = a/c	(A = (sin-1)(a/c))	sin B = b/c	(B = (sin-1)(b/c))
cos A = b/c	(A = (cos-1)(b/c))	cos B = a/c	(B = (cos-1)(a/c))

Exercise2. Test by calculating that B+A = 90 and tan A = (sin A)/(cos A) and (sin A)^2 + (cos A)^2 = 1

4 ADDING SQUARES

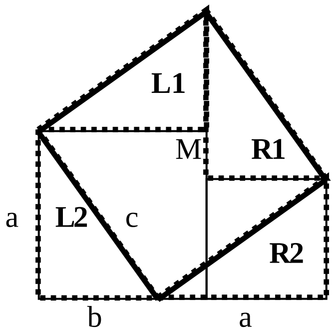
Question. How can we add squares?
Answer1. By using one of the 3 Pythagorean theorems.
Examples.



Mini-Pythagoras:

c is the diagonal in an a*a stack. Turning the stack around the diagonal's endpoints produces a c*c stack. By rearranging the triangles inside and outside the c*c stack we see that

$$\mathbf{a^2 + a^2 = c^2}$$

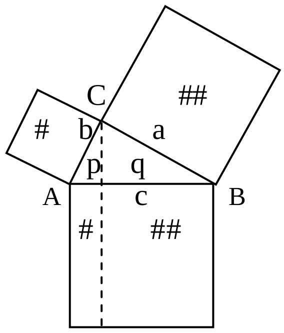


Midi-Pythagoras:

c is the diagonal cutting a paper sheet (an a*b stack) in two, L & R. Turning L and R from position 1 to 2 changes the area from c^2 to $a^2 + b^2$. So

$\mathbf{a^2 + b^2 = c^2}$ (the Pythagoras' theorem)

Remark. c can also be calculated as $c = a /(\sin(\tan^{-1}(a/b)))$



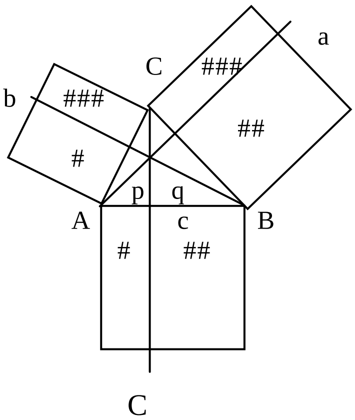
Calculating I:

$c^2 = M + 4*½*a*b$ where $M = a-b$
$= (a-b)^2 + 2*a*b$
$= a^2 + b^2 - 2*a*b + 2*a*b$
$= a^2 + b^2$

Calculating II:

b = c*cosA and p = b*cosA, so # = c*p = (b*cosA)*(b/cosA) = b^2
a = c*cosB and q = a*cosB, so ## = c*q = (a*cosA)*(a/cosA) = a^2

$c^2 = q*c + p*c = a^2 + b^2$



Maxi-Pythagoras:

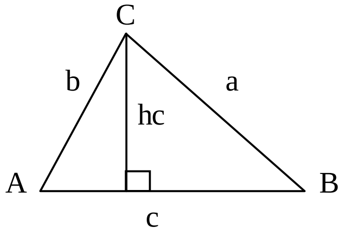
In an acute triangle the altitudes divide the outside squares in pieces corresponding pair wise:

= c*p = c*b*cosA = b*c*cosA, ## = a*c*cosB, ### = a*b*cosC

c^2 = ## + # = (a^2 - ###) + (b^2 - ###) = a^2 + b^2 – 2*###

Or, expressed as the **Cosine-relations**:

$c^2 = a^2 + b^2 - 2*a*b*\cos C$
$b^2 = a^2 + c^2 - 2*a*c*\cos B$
$a^2 = b^2 + c^2 - 2*b*c*\cos A$



The **Sine-relations** come from the altitudes:

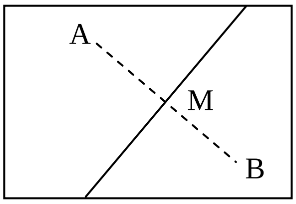
hc: a*sinB = b*sinA, so a/sinA = b/sinB
hb: a*sinC = c*sinA, so a/sinA = c/sinC
ha: b*sinC = c*sinB, so b/sinB = c/sinC

Thus $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

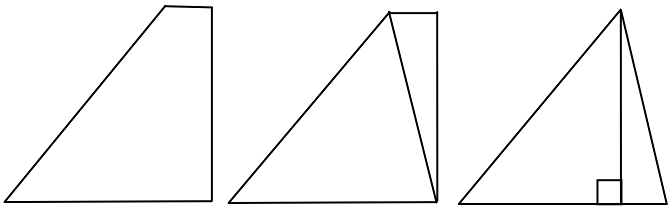
Exercise1. Predict the length of the diagonal in a stack and test by measuring.
Exercise2. Predict the length of the three unknown sides or angles in a triangle and test by measuring. Try all kinds of triangles, SideSideSide, SideSideAngel, SideAngelSide, SideAngelAngel
Exercise3. Do the sine- and cosine-relations also hold for obtuse triangles?

5 DIVIDING THE LAND

Question. How can we divide a piece of land between two persons?
Answer. By using perpendicular bisectors.



A piece of land is to be divided between two settlements A and B. They decide upon the dividing-principle ‘equal distance to a border-point’. This makes the border-points form a perpendicular bisector, i.e. a straight line perpendicular to the midpoint M of the connecting line AB.



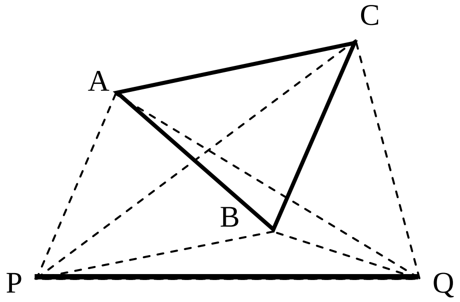
To measure it a piece of land can be divided into triangles (triangulation). And a triangle can be divided into two right triangles. Thus we are especially interested in right triangles.

Geo-metry means ‘earth-measuring’ in Greek.

Exercise1. Draw a piece of land with two settlements. Divide it so AM/AB = 1/1. Divide it so AM/AB = 1/2.
Exercise2. Draw a piece of land with three settlements. Divide it so there is equal distance to a border-point.

6 REESTABLISHING THE LAND

Question. How can we re-establish a piece of land where the fences disappeared under a flooding?
Answer. By using a baseline to measure distances.



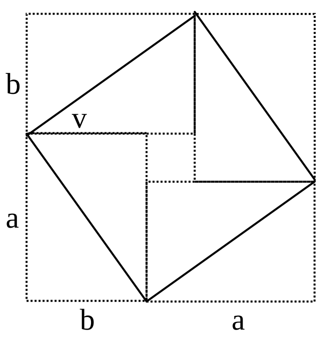
A piece of land is divided into triangles. From the endpoints P and Q on a permanent baseline the distances are measured to A B and C (triangular coordinates).

After the flood of the Nile the point A is re-established by laying out the triangle PAQ. Or by calculating the angle PAQ. Or as the intersection point between circles from P and Q having the distances as the radius.

Exercise1. Draw a triangle on the floor or in the sand. Remove and re-establish the triangle from a base line.

7 THE SLOPES OF STRAIGHT LINES

Question. How can we count the steepness of a straight line?
Answer1. By the slope counting the rise over the run.
Example.



The four diagonals are parallel pair wise.

The slope is the rise recounted in runs.

The upward diagonals have the slope b/a.

The downward diagonals have the slope -a/b.

The slopes of two perpendicular lines are reciprocal with different signs:

Linie l ⊥ linie m: slope for l * slope for m = -1

The steepness-angle v is determined by tan v = slope

Exercise1. What is the slope of a 20-degree line? What is the angle of a 10% slope?

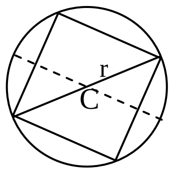
8 DIMENSIONS

Question. What is the difference between the line, the plane and the space?
Answer. The line has one direction, the plane has two and space has three directions or dimensions.
Examples.
A line segment is part of a line having 1 dimension. It’s points are identified by 1 number, the out-number x.

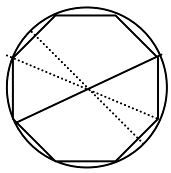
A stack is part of a plane having 2 dimensions. It's points are identified by 2 numbers, the out-number x and the up-number y.
A box is part of a space having 3 dimensions. It's points are identified by 3 numbers, the out-number x and the up-number y, and the back-number z.
Exercise1. Take a random walk at a line by throwing a dice where odd means – and even means +.
Exercise2. Take a random walk at a plane by throwing 2 dices where odd means – and even means +.
Exercise3. Take a random walk in a space by throwing 3 dices where odd means – and even means +.

9 CIRCUMFERENCE AND AREA OF A CIRCLE

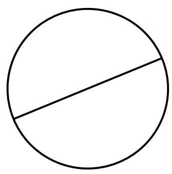
Question. How can we count the length and the area of a circle?
Answer1. By approximating the circle with a regular polygon to develop formulas.



A circle has a centre C, a width or diameter d and a radius r = d/2.
A square is wrapped in a circle with radius 1.
The length of the side (the cord) is k4 = 2* sin(180/4).
The circumference of the square is C4 = 4 * 2 sin(180/4).
The area of the square is A4 = 8*½*alt.*base = 4*sin(180/4)*cos(180/4).
Pulling the midpoints to the circle transforms the square from a regular 4sided polygon to a regular 8sided polygon.



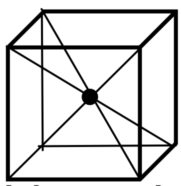
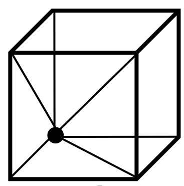
The length of the side (the cord) is k8 = 2* sin(180/8).
The circumference of the polygon is C8 = 8 * 2 * sin(180/8).
The area of the polygon is A8 = 8 * sin (180/8) * cos(180/8).
Continuing to pull the midpoints we get approximate formulas:
Circle circumference ≈ Cn = n * 2 * sin(180/n).
Circle area ≈ An = n * sin (180/n) * cos(180/n).
For n = 1000 we get:



Circle circumference = C ≈ 1000 * 2 * sin(180/1000) = 6,28317 ≈ 2*π
Circle area ≈ 1000*sin (180/1000)*cos(180/1000) = 3,14157 ≈ π
We see that the circumference and the area of a circle can be described by a special number π = 3,141592654; and that this number can be calculated as a limit-number
n*sin(180/n) -> π for n -> ∞, or $\lim_{n \to \infty} (n \cdot \sin(180/n)) = \pi$.
With the radius r the formulas become C = 2*π*r and A = π*r^2
Exercise. Predict the circumference of a bottle and test by measuring.

10 SHAPES IN SPACE

Question. How can spatial shapes be counted?
Answer. A shape has a 1dimensional extension, a 2dimensional area and a 3dimensional volume.
Example1. Bundling 5 1s gives 1 5-bundle having the length 5 cm. Stacking 4 5s gives a 4*5 stack having the breadth √(4^2+5^2) and having the area 4*5=20 cm^2. Stacking 3 4*5 stacks gives a 3*4*5 box having the breath 3√(3^2+4^2+5^2) and having the surface area 2*(3*4+3*5+4*5) = 94 cm^2; and having the volume 3*4*5 = 60 cm^3.
Example2. From the centre point an a*a box can be divided into 6 pyramids each having the volume V = 1/6*a*a*a = 1/3*(1/2a)*(a*a) = 1/3*height*base-area. If the centre point is drawn to a corner, 3 pyramids disappear giving each of the 3 remaining pyramids the volume V = 1/3*a*a*a = 1/3*height*base-area

Example3. A circle has two 1dimensional extensions, its diameter d = 2*r, and its circumference C = 2*π*r. A circle has one 2dimensional extension, its area A = π*r^2. A circular-disk is called a cylinder having two 2dimensional extensions and one 3dimensional extension, its volume V = π*r^2*altitude. Constricted in the one end a cylinder becomes a cone having the volume V = 1/3*π*r^2*altitude. Constricted in both ends a cylinder becomes a ball having the volume V = 4/3*π*r^3*and the surface S = 4*π*r^2.
Exercise1. The volume of a liquid can be measured in a measuring glass. Pour water or sand from different shapes into a measuring glass to measure the volume. Predict the result from a formula.