COUNT&ADD IN TIME

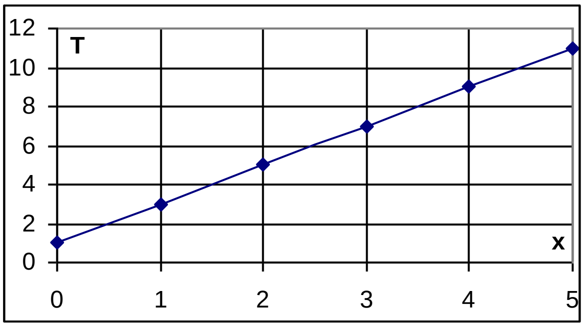
T1

Question	**Answer**
How can counting & adding be reversed?	By calculating backward moving a number to the other side reversing its calculation sign.
Counting ? 3s and adding 2 gives 14.	x*3+2 = 14 is reversed to x = (14-2)/3
Can all calculations be reversed?	Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x =b/a, x^a =b is reversed to x =a√b, a^x =b is reversed to x =logb/loga

1 REVERSED CODING

Question. How can we decode a coded number?
Answer. Use reversed calculations, also called solving equations. **x**
Example. **::: .** **x .**
Coding hides the bundle-size: T=2*3+1 -> T=2*x+1.
A table can be used to guess the Total when coded.
The table can be drawn as a graph.

x	0	1	2	3	4	5			
T = 2*x+1	1	3	5	7	9	11			



A decoding can take place in three steps:
1. First the coding x + 3 = 5 is decoded by restacking: From the 5-stack we take away 3 to a new stack leaving 5-3 = 2 in the original stack as predicted by the restack-equation T = (T-3)+3: T = 5 = (5-3)+3 = 2+3

```
          o
          o
     o    o          o
     o    o      o   o
x    o =  o ->   o   o
```

(matches above)	x + 3 = 5 = (5-3) + 3 = 2+3 x = 2	or quicker:	x + 3 = 5 x = 5-3 = 2

x + 3 = 5 = (5-3) + 3 = 2+3

So the question x+3 = 5 is answered by restacking 5 to (5-3)+3 making x = 5-3. Thus an equation x+b = T is solved by x = T-b to be found by moving the number b across the equation sign and reversing its calculation sign from + to -.
2. Next the coding 2*x = 6 is decoded by recounting: The 6 is recounted to 3 2s and overturned to 2 3s as predicted by the recount-equation T = (T/2)*2: T = 6 = (6/2)*2 = 3*2

```
                                oo
x                               oo       ooo
x   = oooooo -> oo  oo  oo ->   oo  ->   ooo
```

(matches above)	2*x = 6 = (6/2)*2= 3*2 x = 3	or quicker:	2*x = 6 x = 6/2 = 3

2*x = 6 = (6/2)*2 = 3*2 = 2*3

So the question 2*x = 6 is answered by recounting 6 to (6/2)*2 making x = 6/2. Thus an equation b*x = T is solved by x = T/b to be found by moving the number b across the equation sign and reversing its calculation sign from * to /.
3. Finally the coding 2*x+1 = 7 is decoded. First we restack 7 by taking away 1: 7 = (7-1)+1 = 6+1. Then the 6 is recounted in 2s and overturned.

```
                                                oo
x                                               oo         ooo
x  o = ooooooo -> oooooo  o -> oo  oo  oo  o -> oo  o ->   ooo  o
```

(matches above)	2*x+1 = 7 = (7-1)+1 = 6+1 2*x = 6 = (6/2)*2 = 3*2 x = 3	2*x+1 = 7 2*x = 7-1 = 6 x = 6/2 = 3

2*x+1 = 7 = (7-1)+1 = 6+1 = (6/2)*2 + 1= 3*2 + 1 = 2*3 + 1

Here the result is predicted by applying both the restack-equation and the recount-equation.
Remark. The recount-equation and the restack-equation show directly that equations are solved when moving a number to the other side of the equation sign reversing its calculation sign:

Recounting:	T	= (T/4) * 4	Restacking:	T	= (T-4) + 4
Equation	T	= x * 4	Equation	T	= x + 4
Solution	T/4	= x	Solution	T-4	= x

Exercise1. Decode 2*x+4=10, 3*x+5=17, 4*x+1=9, 5*x+2=17. First use matches, then write.
Exercise2. Decode 2*x-4=6, 3*x-5=7, 4*x-1=11, 5*x-2=18. First use matches, then write.
Exercise3. Decode x*2=6, x*2=7, x*2+1=6, x*2-1=6, x*3+2=16, x*3-4=12. First use matches, then write.

2 REVERSED VERTICAL CALCULATIONS

Question. How can a calculation be reversed vertically?
Answer. Use arrows to illustrate the forward and backward calculation steps (dance the equation).
Example. The equation 3*x + 2 = 14 is a story about two calculations that took place after each other. FIRST the number x was multiplied by 3, THEN 2 was added producing a total of 14.
This sequence can be reversed to produce x: FIRST 2 is subtracted from 14; THEN this is divided by 3.
So 3*x + 2 = 14 makes x = (14-2)/3. Finally, to check, the forward calculation can be repeated.

x	*--(*3)-->*	3*x	*--(+ 2)-->*	3*x + 2	(forward)
4	*<--(/3)--*	12	*<--(- 2)--*	14	(backward)

Exercise1. Dance 2*x+4=10, 3*x+5=17, 4*x+1=9, 5*x+2=17. First on the floor, then write.
Exercise2. Dance 2*x-4=6, 3*x-5=7, 4*x-1=11, 5*x-2=18. First on the floor, then write.
Exercise3. Dance x*2=6, x*2=7, x*2+1=6, x*2-1=6, x*3+2=16, x*3-4=12. First on the floor, then write.

3 REVERSED HORIZONTAL CALCULATIONS

Question. How can a calculation be reversed horizontally?
Answer. Use arrows to illustrate upward and downward calculation steps (climb the equation)
Example. The equation 3*x + 2 = 14 is a story about two calculations that took place after each other. FIRST, on the forward side, the calculation is built up to give a total: x is multiplied by 3, and 2 is added giving a total of 14.
THEN, on the backward side, the result is broken down to produce the initial number: 2 is subtracted from 14 and the result is divided by 3. So 3*x + 2 = 14 makes x = (14-2)/3. Finally, to check, the upward calculation can be repeated. If we leave out the arrows the move&change-method becomes visible.

Forward-side		*Backward-side*
3*x + 2	=	14
	+2 ↑↓ -2	
3*x	=	14-2 = 12
	**3 ↑↓ /3*	
x	=	12/3 = 4

Forward	*Backward*
3*x + 2	= 14
3*x	= 14-2 = 12
x	= 12/3 = 4

Forward	*Backward*
a*x + b	= c
a*x	= c-b
x	= (c-b)/a

Exercise1. Climb 2*x+4=10, 3*x+5=17, 4*x+1=9, 5*x+2=17. First from the floor, then write.
Exercise2. Climb 2*x-4=6, 3*x-5=7, 4*x-1=11, 5*x-2=18. First from the floor, then write.
Exercise3. Climb x*2=6, x*2=7, x*2+1=6, x*2-1=6, x*3+2=16, x*3-4=12. First from the floor, then write.

4 CALCULATION TABLES

Question. How can we report solving an equation?
Answer. Use a calculation-table showing both what we know and don't know and the equation to be solved.
Example. In the equation 3*x+2 = 14 the double-calculation 3*x+2 is split up into two calculations by the 'invisible' parenthesis: 3*x+2 = (3*x)+2.

Calculating numbers

x = ?	3*x + 2	= 14	
	(3*x) + 2	= 14	
	3*x	= 14-2	
	x	= 12/3	
	x	**= 4**	
Check:	*3*4 + 2*	*= 14*	
	14	*= 14*	☺

Calculating letters

x = ?	a*x + b	= c	
a = 3	(a*x) + b	= c	
b = 2	a*x	= c-b	
c = 14	x	$= \frac{(c-b)}{a}$	
	x	$= \frac{(14-2)}{3}$	
	x	**= 4**	
Check:	*3*4 + 2*	*= 14*	
	14	*= 14*	☺

Exercise1. Solve in a calculation-table 2*x+4=10, 3*x+5=17, 4*x+1=9, 5*x+2=17.
Exercise2. Solve in a calculation-table 2*x-4=6, 3*x-5=7, 4*x-1=11, 5*x-2=18.
Exercise3. Solve in a calculation-table x*2=6, x*2=7, x*2+1=6, x*2-1=6, x*3+2=16, x*3-4=12.

5 APPLYING CALCULATION TABLES WITH FORMULAS

Question. Where can we use calculation-tables?
Answer. Calculation-tables can be used with formulas.

Example1. Recounting units

3 $ for 4 pieces: 21$ for ? pieces

pieces = ?	pieces = (pieces/$)*$
pieces/$ = 4/3	pieces = 4/3*21
$ = 21	pieces = 28

3 $ for 4 pieces: ?$ for 24 pieces

$ = ?	$ = ($/pieces)*pieces
$/pieces = 3/4	$ = ¾*24
pieces = 24	$ = 18

Example2. Percentages I

30 $ = 40%, 21$ = ?%

% = ?	% = (%/$)*$
%/$ = 40/30	% = 40/30*21
$ = 21	% = 28

30 $ = 40%, ?$ = 24%

$ = ?	$ = ($/%)*%
$/% = 30/40	$ = 30/40*24
% = 24	$ = 18

Example2. Percentages II

25% af 200 $. = ? $.

A = ?	p = a/T
p = 25%	p*T = a
T = 200	25%*200 = a
	50 = a

25% af ? $. = 40 $

T = ?	p = a/T
p = 25%	p*T = a
a = 40	T = a/p
	T = 40/25%
	T = 160

?% af 200 $. = 40 $

p = ?	p = a/T
a = 40	p = 40/200
T = 200	p = 0.20
	p = 20/100
	p = 20%

Example3. Adding percentages

200 + 25% = ?

K = ?	K = Ko*(1+r)
Ko = 200	K = 200*(1+0.25)
r = 25%	**K = 250**
= 0.25	

? + 25% = 400

Ko = ?	K = Ko*(1+r)
K = 500	K/(1+r) = Ko
r = 25%	500/(1+0.25) = Ko
= 0.25	**400 = Ko**

200 + ?% = 280

r = ?	K = Ko*(1+r)
K = 280	K/Ko = 1+r
Ko = 200	(K/Ko)-1 = r
	(280/200)-1 = r
	0.40 = r
	40% = r

Example4. Per-numbers

25kg. à 4 $./kg. = ? $.

T = ?	k*p = T
k = 25	25*4 = T
p = 4	100 = T

25kg. à ? $./kg. = 200$.

p = ?	k*p = T
k = 25	p = T/k
T = 200	p = 200/25
	p = 8

?kg. à 4 $./kg. = 300$.

k = ?	k*p = T
p = 4	p = T/k
T = 300	p = 300/4
	p = 75

Exercise. Repeat the calculations with different numbers. Remember to check.

6 THE LEVER METHOD FOR NEUTRALISING

Question. Are there other ways to solve equations?

Answer. Modern mathematics introduced the lever-method to make solving equations understandable being under the false assumption that the move&change method was only accessible through rote learning.
The equation sign is considered an example of an equivalence relation. And an equation 3*x+2=14 is considered an example of an open statement expressing the equivalence between two numbers 3*x+2 and 14. The equation is solved by determining the statement's truth-set, i.e. the set of numbers that make the open statement a true statement. This is done by neutralising the numbers with their inverse numbers that has to be included on both sides of the equal sign to preserve the equivalence as indicated by the equivalent arrows.

L = {x| 3*x + 2 = 14}	L is the truth-set making the open statement 3*x + 2 = 14 true
3*x + 2 = 14	The open statement
↕ (3*x) + 2 = 14	A hidden parenthesis is added according to priority
↕ ((3*x) + 2)+(-2) = 14+(-2)	To neutralise +2 its inverse under +, –2, is added on both sides
↕ (3*x) + (2+(-2)) = 12	+ parentheses can be removed or added (the associative law)
↕ (3*x) + 0 = 12	+2 and –2 neutralise each others, and +'s neutral number is 0
↕ 3*x = 12	The definition of the neutral number says that a+0 = 0+a = a
↕ (3*x)*(1/3) = 12*(1/3)	To neutralise *3 its inverse under *, 1/3, is multiplied on both side
↕ (x*3)*(1/3) = 4	*numbers (and +numbers) may be commuted (the commutative law)
↕ x*(3*(1/3)) = 4	*parentheses can be removed or added (the associative law)
↕ x*1 = 4	*3 and 1/3 neutralise each others, and *'s neutral number is 1
↕ x = 4	The definition of the neutral number says that a*1 = 1*a = a
L = {x | 3*x + 2 = 14} = {4}	The truth-set L. Because of the arrows the check is not needed.

Exercise. Repeat the lever or neutralising method with a different equation as e.g. 3+2*x = 11.

7 BACKWARD CALCULATION WITH CODED STOCKS I

Question. What about coded stacks?

Answer. Backward calculation can also be used with coded stocks using the reversed FOIL-method.

Example1. T = (4*x+3*y)*? = 24*x^2 + 46*x*y + 21*y^2

Dividing stocks (polynomials)

(4*x + 3*y) * ? = 24*x^2 + 46*x*y + 21*y^2

4x * ? = 24*x^2   *? = 6*x*

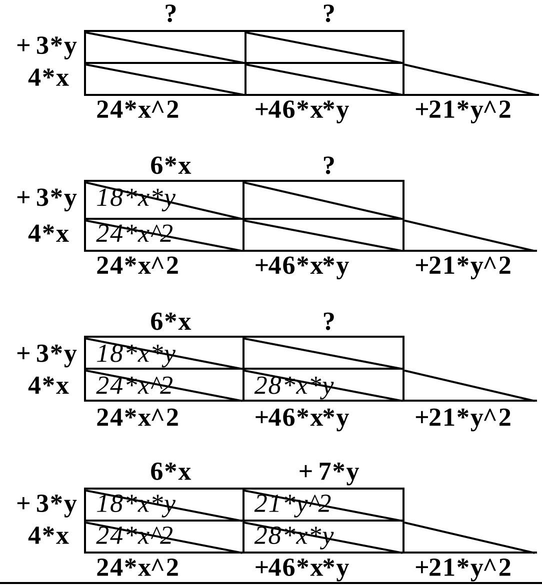
3y * 6x = 18*x*y

18*x*y + ? = 46*x*y   *? = 28*x*y*

4*x * ? = 28*x*y   *? = 7*y*

3*y * 7*y = 21*y^2

So 24*x^2 + 46*x*y + 21*y^2 = (4*x+3*y)*(6*x+7*y)



Example2. (x+a)(x-a) = x^2-a^2

Forward	*Backward*
(x + 6)(x - 6) = x^2 – 6^2 = x^2 – 36	x^2 – 36 = x^2 – 6^2 = (x + 6)(x - 6)
(x + √20)(x - √20) = x^2 – (√20)^2 = x^2 – 20	x^2 – 20 = x^2 – (√20)^2 = (x + √20)(x - √20)

Example3. (x±a)^2 = x^2+a^2±2*a*x

Forward	*Backward*
(x + 6)^2 = x^2 + 6^2 + 2*6*x = x^2 + 36 + 12x	x^2 + 36 + 12x = x^2 + 6^2 + 2*6*x = (x + 6)^2
(x - 6)^2 = x^2 + 6^2 - 2*6*x = x^2 + 36 - 12x	x^2 + 36 - 12x = x^2 + 6^2 - 2*6*x = (x - 6)^2

Example4. A second degree equation: If a*x^2 + b*x + c = 0 then x = ?

Forward	*Backward*	*Summary*
T = (x + k)^2 T = x^2 + k^2 + 2*k*x 0 = x^2 + (2*k)*x + (k^2 – T) 0 = x^2 + p*x + q p = 2*k og q = k^2 – T p/2 = k og T = k^2 – q = (p/2)^2 – q	(x + k)^2 = T x + k = ± √T x = -k ± √T ------- If x^2 + p*x + q = 0 then **x = - p/2 ± √((p/2)^2 – q)**	a*x^2 + b*x + c = 0 x^2 + (b/a)*x + (c/a) = 0 x^2 + p*x + q = 0 p = b/a og q = c/a x = - b/(2a) ± √((b/(2a))^2 – c/a) **x = (-b ± √D)/(2*a) , D = b^2 – 4*a*c**

Exercise1. Do (8x^2 + 22 xy + 15y^2):(2x+3y). Do (6x^2 + 26 xy + 24y^2):(3x+4y)

Exercise2. Factorise x^2-25, 3x^2-48, x^2+6x+8, 2x^2-4x-30. Solve x^2-8x+12=0, 4x^2-24x+20

8 A SUMMARY

Question. Is there a common principle when solving equations?

Answer. Move a number to the other side of the equation sign and change its calculation sign.

	5+3 = 5+1+1+1	5*3 = 5+5+5	5^3 = 5*5*5	
Forward	5 + 3 = ?	5 * 3 = ?	5^3 = ?	5^3 = ?
Backward	? + 3 = 8 *? = 8-3*	? * 3 = 15 *? = 15/3*	?^3 = 64 *? = 3√64*	5^? = 64 *? = log64/log5*
Definitions	8-3 is the +number that together with 3 gives 8	15/3 is the *number that together with 3 gives 15	3√64 is the base that together with the exponent 3 gives 64	Log64/log5 is the exponent that together with the base 5 gives 25
Move&change	x + 3 = 8 x = 8-3	x * 3 = 15 x = 15/3	x^3 = 64 x = 3√64	x^2 = 25 x = √25
Move-rules	+numbers <--> -numbers	*numbers <--> /numbers	^3 <--> 3√	^2 <--> √

Exercise1. Solve the following equations using both the move-method and the neutralising-method.

1	T = a+(1+b)*c	T = a+(1-b)*c	T = a+b*(1+c)	T = a+b*(1-c)
2	T = a-(1+b)*c	T = a-(1-b)*c	T = a-b*(1+c)	T = a-b*(1-c)
3	T = a+(1+b)/c	T = a+(1-b)/c	T = a+b/(1+c)	T = a+b/(1-c)
4	T = a-(1+b)/c	T = a-(1-b)/c	T = a-b/(1+c)	T = a-b/(1-c)

Exercise2. Solve the following equations using both the move-method and the neutralising-method.

1	T = a+b*c^d	T = a+(1+b)*c^d	T = a+(1-b)*c^d	T = a+b*(1-c)^d
2	T = a-b*c^d	T = a-(1+b)*c^d	T = a-(1-b)*c^d	T = a-b*(1-c)^d
3	T = a+b/c^d	T = a+(1+b)/c^d	T = a+(1-b)/c^d	T = a+b/(1-c)^d
4	T = a-b/c^d	T = a-(1+b)/c^d	T = a-(1-b)/c^d	T = a-b/(1-c)^d

Exercise3. Solve the following equations.

	*Find **a, b** and **c*** *Answer:*	**a**	**b**	**c**
1	$T = a + b\cdot c$	$T - b\cdot c$	$\frac{T-a}{c}$	$\frac{T-a}{b}$
2	$T = a - b\cdot c$	$T + b\cdot c$	$\frac{a-T}{c}$	$\frac{a-T}{b}$
3	$T = a + \frac{b}{c}$	$T - \frac{b}{c}$	$(T-a)\cdot c$	$\frac{b}{T-a}$
4	$T = a - \frac{b}{c}$	$T + \frac{b}{c}$	$(a-T)\cdot c$	$\frac{b}{a-T}$
5	$T = (a + b)\cdot c$	$\frac{T}{c} - b$	$\frac{T}{c} - a$	$\frac{T}{a+b}$
6	$T = (a - b)\cdot c$	$\frac{T}{c} + b$	$a - \frac{T}{c}$	$\frac{T}{a-b}$
7	$T = \frac{a+b}{c}$	$T\cdot c - b$	$T\cdot c - a$	$\frac{a+b}{T}$
8	$T = \frac{a-b}{c}$	$T\cdot c + b$	$a - T\cdot c$	$\frac{a-b}{T}$
9	$T = \frac{a}{b+c}$	$T\cdot(b+c)$	$\frac{a}{T} - c$	$\frac{a}{T} - b$
10	$T = \frac{a}{b-c}$	$T\cdot(b-c)$	$\frac{a}{T} + c$	$b - \frac{a}{T}$
11	$T = \frac{a}{b} + c$	$(T-c)\cdot b$	$\frac{a}{T-c}$	$T - \frac{a}{b}$
12	$T = \frac{a}{b} - c$	$(T+c)\cdot b$	$\frac{a}{T+c}$	$\frac{a}{b} - T$
13	$T = a\cdot b^{c}$	$\frac{T}{b^{c}}$	$\sqrt[c]{\frac{T}{a}}$	$\frac{\log(\frac{T}{a})}{\log b}$
14	$T = \frac{a}{b^{c}}$	$T\cdot b^{c}$	$\sqrt[c]{\frac{a}{T}}$	$\frac{\log(\frac{a}{T})}{\log b}$
15	$T = (a\cdot b)^{c}$	$\frac{\sqrt[c]{T}}{b}$	$\frac{\sqrt[c]{T}}{a}$	$\frac{\log T}{\log(a\cdot b)}$
16	$T = (\frac{a}{b})^{c}$	$\sqrt[c]{T}\cdot b$	$\frac{a}{\sqrt[c]{T}}$	$\frac{\log T}{\log(\frac{a}{b})}$
17	$T = (a + b)^{c}$	$\sqrt[c]{T} - b$	$\sqrt[c]{T} - a$	$\frac{\log T}{\log(a+b)}$
18	$T = (a - b)^{c}$	$\sqrt[c]{T} + b$	$a - \sqrt[c]{T}$	$\frac{\log T}{\log(a-b)}$
19	$T = a + b^{c}$	$T - b^{c}$	$\sqrt[c]{T-a}$	$\frac{\log(T-a)}{\log b}$
20	$T = a - b^{c}$	$T + b^{c}$	$\sqrt[c]{a-T}$	$\frac{\log(a-T)}{\log b}$
21	$T = a^{(b+c)}$	$\sqrt[(b+c)]{T}$	$\frac{\log T}{\log a} - c$	$\frac{\log T}{\log a} - b$
22	$T = a^{(b-c)}$	$\sqrt[(b-c)]{T}$	$\frac{\log T}{\log a} + c$	$b - \frac{\log T}{\log a}$