Revision Problems Using TI-89


Problem 1. Linear model

| Equation: | $y=a x+b$ |
| :--- | :--- |
|  | $y=x+9$, found by data/matrix- <br> editor, F5, linear regression |
| Test | $y \mid x=3$ gives 12 |


| $\mathrm{y}=?$ | $\mathrm{y}=\mathrm{x}+9$ |
| :--- | :--- |
| $\mathrm{x}=10$ | $\mathrm{y}=19$ <br> found by $\mathrm{y}(10)$ |
| Test | $\mathrm{y}=19$ found by Graph F5 value |


| $x=?$ | $y=x+9$ |
| :--- | :--- |
| $y=40$ | $x=31$ <br> found by $F 2$, solve $(y 1(x)=40, x)$ |
| Test | $y \mid x=31$ gives 40 |

Exponential model

| Equation: | $\mathrm{y}=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}$ |
| :--- | :--- |
|  | $\mathrm{y}=9.671^{*} 1.075^{\wedge} \mathrm{x}$ <br> found by data/matrix-editor, $\mathrm{F5}$, <br> exponential regression |
| Test | $\mathrm{y} \mid \mathrm{x}=3$ gives 12 |


| $y=?$ | $y=9.671^{*} 1.075^{\wedge} x$ |
| :--- | :--- |
| $x=10$ | $\mathrm{y}=19.853$ <br> found by $\mathrm{y}(10)$ |
| Test | $\mathrm{y}=19.853$ found by Graph F5 value |


| $\mathrm{x}=?$ | $\mathrm{y}=9.671 * 1.075^{\wedge} \mathrm{x}$ |
| :--- | :--- |
| $\mathrm{y}=40$ | $\mathrm{x}=19.740$ <br> found by F2, solve $(\mathrm{y} 1(\mathrm{x})=40, \mathrm{x})$ |
| Test | $\mathrm{y} \mid \mathrm{x}=19.740$ gives 40 |

Doubling time $\mathrm{T}=\log 2 / \log b=\log 2 / \log 1.075=9.6$
Power model

| Equation: | $\mathrm{y}=\mathrm{a}^{*} \mathrm{x}^{\wedge} \mathrm{b}$ |
| :--- | :--- |
|  | $\mathrm{y}=8.264^{*} \mathrm{x}^{\wedge} 0.340$ <br> found by data/matrix-editor, F5, <br> power regression |
| Test | $\mathrm{y} \mid \mathrm{x}=3$ gives 12 |


| $y=?$ | $y=8.264 * x^{\wedge} 0.340$ |
| :--- | :--- |
| $x=10$ | $y=18.060$ <br> found by $y(10)$ |
| Test | $\mathrm{y}=18.060$ found by <br> Graph F5 value |

Problem 2. Quadratic model

| Equation: | $\mathrm{y}=\mathrm{a}^{*} \mathrm{x}^{\wedge} 2+\mathrm{b}^{*} \mathrm{x}+\mathrm{c}$ |
| :--- | :--- |
|  | $\mathrm{y}=-0.048 \mathrm{x}^{\wedge} \wedge+1.476 \mathrm{x}+8$ <br> found by data/matrix-editor, $\mathrm{F5}$, <br> quadratic regression |
| Test | $\mathrm{y} \mid \mathrm{x}=3$ gives 12 |


| Maximu m: | $\mathrm{y}=-0.048 \mathrm{x}^{\wedge} 2+1.476 \mathrm{x}+8$ |
| :--- | :--- |
|  | $(\mathrm{x}, \mathrm{y})=(15.500,19.140)$ <br> found by graph, F5, maximum |
| Test | Solve(dy/dx=0,x) gives 15.5 <br> $\mathrm{y} \mid \mathrm{x}=15.5$ gives 19.14 |


| Tangent | $\mathrm{y}=-0.048 \mathrm{x}^{\wedge} 2+1.476 \mathrm{x}+8$ |
| :--- | :--- |
| $\mathrm{x}=2$ | $\mathrm{y}=1.286 \mathrm{x}+8.190$ <br> found by graph, F5, tangent |
|  |  |


| Area <br> formula: | $y=-0.048 x^{\wedge} 2+1.476 x+8$ |
| :--- | :--- |
| $x=2$ | $\mathrm{ydx}=-0.016^{*} \mathrm{x}^{\wedge} 3+$ <br> $0.738^{*} \mathrm{x}^{\wedge} 2+8.000^{*} \mathrm{x}$ <br> found by F3, integrate <br> Test <br> $\mathrm{d}(\mathrm{ydx}) / \mathrm{dx}=-0.048 \mathrm{x}^{\wedge} 2+1.476 \mathrm{x}+8$ |


| $\mathrm{x}=?$ | $\mathrm{y}=8.264^{*} \mathrm{x}^{\wedge} 0.340$ |
| :--- | :--- |
| $\mathrm{y}=40$ | $\mathrm{x}=104.024$ <br> found by F2, <br> solve $\mathrm{y} 1(\mathrm{x})=40, \mathrm{x})$ |
| Test | $\mathrm{y} \mid \mathrm{x}=104.024$ gives 40 |


| $y=?$ | $y=-0.048 x^{\wedge} 2+1.476 x+8$ |
| :--- | :--- |
| $x=15$ | $y=19.429$ <br> found by $y(15)$ |
| Test | $y=19.429$ found by Graph F5 <br> value |


| $\mathrm{x}=?$ | $\mathrm{y}=-0.048 \mathrm{x}^{\wedge} 2+1.476 \mathrm{x}+8$ |
| :--- | :--- |
| $\mathrm{y}=40$ | $\mathrm{x}=1.420$ or 29.580 <br> found by F2, <br> solve $(\mathrm{y} 1(\mathrm{x})=40, \mathrm{x})$ |
| Test | $\mathrm{y} \mid \mathrm{x}=1.420$ gives 40 <br> $\mathrm{y} \mid \mathrm{x}=29.580$ gives 40 |


| Gradient <br> formula | $y=-0.048 x^{\wedge} 2+1.476 x+8$ |
| :--- | :--- |
|  | $y^{\prime}=-0.095^{*} x+1.476$, <br> found by F3, differentiate |
| Test | Jy $y^{\prime} \mathrm{d}=-0.048 x^{\wedge} 2+1.476 x$ |


| Area <br> number: | $\mathrm{y}=-0.048 \mathrm{x}^{\wedge} 2+1.476 \mathrm{x}+8$ |
| :--- | :--- |
|  | $\int_{\mathrm{ydx}}=62.421$, <br> 1 <br> found by F3, $\int(\mathrm{y} 1(\mathrm{x}), \mathrm{x}, 1,6)$ |
| Test | 62.421, found by graph, F5, <br> integrate |


| Intersection points | $y=-0.048 x^{\wedge} 2+1.476 x+8$ and $y=3+2 x$ |
| :--- | :--- |
| $\mathrm{x}=5$ | $(\mathrm{x}, \mathrm{y})=(-17.130,-31.260)$ and |
|  | $(x, y)=(6.130,15.260)$, |
|  | found by F2, solve $(\mathrm{yl}(\mathrm{x})=3+2 \mathrm{x}, \mathrm{x})$ and $\mathrm{y} 1(\mathrm{x}) \mid \mathrm{x}=-17.130$ etc. |
| Test | tested by graph, F5, intersection. |

## Problem 3. Cubic model

| Equation: | $y=a^{*} x^{\wedge} 3+b^{*} x^{\wedge} 2+c^{*} x+d$ |
| :--- | :--- |
|  | $y=0.086 x^{\wedge} 3-$ <br> $1.952 x^{\wedge} 2+13.752 x-14$ <br> found by data/matrix-editor, F5, <br> quadratic regression |
| Test | $y \mid x=3$ gives 12 |


| Maximum <br> Minimum: | $\mathrm{y}=0.086 \mathrm{x}^{\wedge} 3-1.952 \mathrm{x}^{\wedge} 2+13.752 \mathrm{x}-14$ |
| :--- | :--- |
|  | Max: $(\mathrm{x}, \mathrm{y})=(5.552,16.841)$ <br> found by graph, F5, maximum <br> Min: (x,y) $)=(9.634,13.925)$ <br> found by graph, 55, minimum |
| Test | Solve(dy/dx $=0, \mathrm{x})$ gives 5.552 and 9.634 <br> $\mathrm{y} \mid \mathrm{x}=5.552$ gives 16.841 <br> $\mathrm{ylx}=9.634$ gives 13.925 |


| $y=?$ | $y=0.086 x^{\wedge} 3-1.952 x^{\wedge} 2$ <br> $+13.752 x-14$ |
| :--- | :--- |
| $x=15$ | $y=42.286$ <br> found by $y(15)$ |
| Test | $\mathrm{y}=42.286$ found by Graph <br> F5 value |


| Tangent | $y=0.086 x^{\wedge} 3-$ <br> $1.952 x^{\wedge} 2+13.752 x-14$ |
| :--- | :--- |
| $x=2$ | $y=6.971 x-7.562$ <br> found by graph, F5, tangent |
|  |  |


| $\mathrm{x}=?$ | $\mathrm{y}=0.086 \mathrm{x}^{\wedge} 3-1.952 \mathrm{x}^{\wedge} 2$ <br> $+13.752 \mathrm{x}-14$ |
| :--- | :--- |
| $\mathrm{y}=30$ | $\mathrm{x}=13.885$ <br> found by F2, <br> solve(y1 $(\mathrm{x})=30, \mathrm{x})$ |
| Test | $\mathrm{y} \mid \mathrm{x}=13.885$ gives 30 |


| Gradient formula | $\begin{aligned} & y=0.086 x^{\wedge} 3- \\ & 1.952 x^{\wedge} 2+13.752 x-14 \end{aligned}$ |
| :---: | :---: |
|  | $\begin{aligned} & \hline y^{\prime}=0.257^{*} x^{\wedge} 2- \\ & 3.905^{*} x+13.752, \\ & \text { found by F3, differentiate } \end{aligned}$ |
| Test | $\begin{aligned} & \hline \mathrm{y} \wedge \mathrm{dx}=0.086 \mathrm{x}^{\wedge} 3- \\ & 1.952 x^{\wedge} 2+13.752 \mathrm{x} \end{aligned}$ |


| Gradient number: | $\begin{aligned} & y=0.086 x^{\wedge} 3- \\ & 1.952 x^{\wedge} 2+13.752 x-14 \end{aligned}$ | Area formula: | $\begin{aligned} & y=0.086 x^{\wedge} 3- \\ & 1.952 x^{\wedge} 2+13.752 x-14 \end{aligned}$ | Area number: | $\begin{array}{\|l} y=0.086 x^{\wedge} 3- \\ 1.952 x^{\wedge} 2+13.752 x-14 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}=5$ | $\begin{aligned} & \mathrm{y} \text { '(5) }=0.657 \\ & \text { found by graph, F5, dy/dx } \end{aligned}$ | $x=2$ | $\begin{aligned} & \operatorname{lydx}=0.021^{*} x^{\wedge} 4- \\ & 0.651^{*} x^{\wedge} 3+6.876^{*} x^{\wedge} 2+14^{*} x \\ & \text { found by F3, integrate } \end{aligned}$ |  | $\begin{aligned} & 6 \\ & \int_{\mathrm{ydx}}=58.496, \\ & 1 \\ & \text { found by F3, } \mathrm{f}(\mathrm{yl}(\mathrm{x}), \mathrm{x}, 1,6) \end{aligned}$ |
| Test | $\mathrm{y}^{\prime} \mathrm{x}=5$ gives 0.657 | Test | $\begin{aligned} & \hline \mathrm{d}(\mathrm{Jdx}) / \mathrm{dx}=0.086 \mathrm{x}^{\wedge} 3- \\ & 1.952 \mathrm{x}^{\wedge} 2+13.752 \mathrm{x}-14 \end{aligned}$ | Test | 58.496 , found by graph, F5, integrate |

Intersection points with $\mathrm{y}=3+2 \mathrm{x}:(\mathrm{x}, \mathrm{y})=(2.129,-7.259)$ and $(\mathrm{x}, \mathrm{y})=(6.657,16.315)$ and $(\mathrm{x}, \mathrm{y})=(13.991,30.981)$ found by F2, solve ( $\mathrm{y} 1(\mathrm{x})=3+2 \mathrm{x}, \mathrm{x}$ ), tested by graph, F5, intersection.

## Problem4

Solutions: $(x, y)=(3.632,1.027)$, found by F2, solve $(3 x+4 y=15$ and $5 x-6 y=12,\{x, y\})$.
Tested by $A * B=C, B=A^{\wedge}-1 * C=\binom{3.632}{1.027}$, hvor $A=\left(\begin{array}{cc}3 & 4 \\ 5 & -6\end{array}\right)$ og $B=\binom{x}{y}$ og $C=\binom{15}{12}$.
Problem5

| Midpoint: | $(\mathrm{x}, \mathrm{y})=\left(\frac{\mathrm{x} 1+\mathrm{x} 2}{2}, \frac{\mathrm{y} 1+\mathrm{y} 2}{2}\right)$ | Gradient PQ: | $\mathrm{a}=\frac{\mathrm{y} 2-\mathrm{y} 1}{\mathrm{x} 2-\mathrm{x} 1}$ | Line PQ: | $y=y 1+a^{*}(x-x 1)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{x} 1=2 \\ & \mathrm{x} 2=6 \\ & \mathrm{y} 1=4 \\ & \mathrm{y} 2=10 \end{aligned}$ | $(x, y)=(4,7)$ <br> found by $\left.\left(\frac{x 1+x 2}{2}, \frac{y 1+y 2}{2}\right) \right\rvert\, x 1=2 \text { and } x 2=6$ <br> and $y 1=4$ and $y 2=10$ | $\begin{aligned} & \mathrm{x} 1=2 \\ & \mathrm{x} 2=6 \\ & \mathrm{y} 1=4 \\ & \mathrm{y} 2=10 \end{aligned}$ | $\begin{aligned} & a=3 / 2 \\ & \text { found by } \left.\frac{y 2-y 1}{x 2-x 1} \right\rvert\, x 1=2 \text { and } \\ & x 2=6 \text { and } y 1=4 \text { and } y 2=10 \end{aligned}$ | $\begin{aligned} & \mathrm{a}=3 / 2 \\ & \mathrm{x} \mathrm{l}=2 \\ & \mathrm{y} \mathrm{l}=4 \end{aligned}$ |  | $\begin{aligned} & 1.5^{*} x+1 \\ & d \text { by } y 2+a^{*}(x- \\ & x 1=2 \text { and } y 1=4 \text { and } \end{aligned}$ |
| Test | Tested geometrically | Test | Tested geometrically | Test |  | geometrically |
| Gradient perpend.: | $c^{*} \mathrm{a}=-1 \quad$ Normal: | $y=y 1+a^{*}(x$ | -x1) | Distance PQ | $d=\sqrt{ }$ | $(2-x 1)^{2}+(y 2-y 1)^{2}$ |
| $a=3 / 2$ | $\mathrm{c}=-2 / 3$ <br> found by <br> solve $\left(c^{*} 3 / 2=-1, c\right)$  $\mathrm{x}=-2 / 3$ | $\begin{aligned} & \mathrm{y}=-2 / 3^{*} \mathrm{x}+ \\ & \text { found by } \mathrm{y} 2+\mathrm{a}^{*} \\ & \mathrm{a}=-3 / 2 \end{aligned}$ | $\begin{aligned} & 5.333 \\ & a^{*}(x-x 2) \times 1=2 \text { and } y 1=4 \text { and } \end{aligned}$ | $\begin{aligned} & \hline \mathrm{x} 1=2 \\ & \mathrm{x} 2=6 \\ & \mathrm{y} 1=4 \\ & \mathrm{y} 2=10 \end{aligned}$ | $\begin{aligned} & \hline \mathrm{d}=7.21 \\ & \text { found by } \\ & \sqrt{(x 2-x 1} \\ & x 2=6 \text { and } \end{aligned}$ | $\begin{aligned} & y^{2}+(y 2-y 1)^{2} \mid x 1=2 \text { and } \\ & y 1=4 \text { and } y 2=10 \end{aligned}$ |
| Test | Tested geometrically $\quad$ Test | Tested geometri |  | Test | Tested | metrically |
| Distance point-line | $\mathrm{d}=\frac{\left\|\mathrm{c} 2-\mathrm{a}^{*} \mathrm{c} 1-\mathrm{b}\right\|}{\sqrt{1+\mathrm{a}^{2}}}$ | Circle equation | $(x-c 1)^{2}+(y-c 2)^{2}=r^{2}$ | Inters | section | $\begin{aligned} & (x-c 1)^{2}+(y-c 2)^{2}=r^{2} \\ & \text { and } y=12-2 x \end{aligned}$ |
| $\begin{aligned} & a=1.5 \\ & b=1 \\ & x 1=8 \\ & y 1=1 \end{aligned}$ | $\begin{aligned} & \mathrm{d}=6.66 \\ & \text { found by } \left.\mathrm{d}=\frac{\mathrm{cc} 2-\mathrm{a}^{*} \mathrm{c} 1-\mathrm{b} \mid}{\sqrt{1+\mathrm{a}^{2}}} \right\rvert\, \mathrm{x} 1=8 \end{aligned}$ <br> and $\mathrm{y} 1=1$ and $\mathrm{a}=1.5$ and $\mathrm{b}=1$ | $\begin{aligned} & \mathrm{r}=1 / 2 * 7.21 \\ & \mathrm{r}=3.61 \\ & \mathrm{c} 1=4 \\ & \mathrm{c} 2=7 \end{aligned}$ | $\begin{aligned} & (x-4)^{2}+(y-7)^{2}=13.03 \\ & \text { found by } \\ & (x-c 1)^{2}+(y-c 2)^{2}=r^{2} \mid c 1=4 \text { and } \\ & c 2=7 \text { and } r=3.61 \end{aligned}$ | $\begin{aligned} \mathrm{r} & =1 / 2^{*} \\ = & 3.6 \\ \mathrm{c} 1 & =4 \\ \mathrm{c} 2 & =7 \end{aligned}$ | $\begin{aligned} & * 7.21 \\ & 61 \end{aligned}$ | $\begin{array}{\|l} \hline(\mathrm{x}, \mathrm{y})=(1.30,9.40) \text { and } \\ (4.30,3.40) \\ \text { found by } \\ \text { solve }\left((\mathrm{x}-4)^{2}+(\mathrm{y}-7)^{2}=\right. \\ 13.03, \mathrm{x}) \mid \mathrm{y}=12-2 \mathrm{x} \\ \hline \end{array}$ |
| Test | Tested geometrically | Test | $\begin{aligned} & (7.61-4)^{2}+(7-7)^{2}=13.03 \\ & 13.03=13.03 \end{aligned}$ | Test |  | Tested geometrically |

Angle: $\tan (v)=a, a=3 / 2 ; v=56.31$ found by solve $(\tan v=3 / 2, v) \mid v>0$ and $v<90$. Tested geometrically

Problem7
$\mathrm{p}(\mathrm{X}<115)=0.894$, found by normCdf( $-\infty, 115,100,12$ ) $\mathrm{p}(\mathrm{X}<89)=0.180$, found by normCdf $(-\infty, 89,100,12)$
$p(X>108)=0.253$, found by normCdf( $108, \infty, 100,12)$ $\mathrm{p}(93<\mathrm{X}<109)=0.494$, found by normCdf( $93,109,100,12)$

## Problem8

$\mathrm{p}(\mathrm{X}<70)=0.827$, found by binomCdf( $100,0.65,0,69)$
$\mathrm{p}(\mathrm{X} \leq 60)=0.172$, found by binomCdf( $100,0.65,0,60)$
$\mathrm{p}(\mathrm{X} \geq 58)=0.941$, found by binomCdf $(100,0.65,58,100)$
$\mathrm{p}(63<\mathrm{X} \leq 72)=0.571$, found by binomCdf( $100,0.65,64,72)$

Problem9

| $\mathrm{x}=?$ | $\operatorname{Sin}(3 \mathrm{x})=0.4$ |
| :--- | :--- |
|  | $\mathrm{X}=0.137$, or 0.910, or |
|  | 2.232 or 3.004 or 4.326 or |
|  | 5.099 |
|  | found by solve $(\operatorname{Sin}(3 \mathrm{x})=0.4, \mathrm{x})$ |
|  | $\mid \mathrm{x}>0$ and $\mathrm{x}<2 \pi$ |
| Test | $\operatorname{Sin}(3 \mathrm{x}) \mathrm{x}=0.137$ gives 0.4 etc. |


| $\mathrm{x}=?$ | $\cos (1 / 2 \mathrm{x})=-0.3$ |
| :--- | :--- |
|  | $\mathrm{X}=3.745$ <br> found by solve $(\cos (1 / 2 x)=-0.3, \mathrm{x}) \mid \mathrm{x}>0$ <br> and $\mathrm{x}<2 \pi$ |
| Test | $\cos (1 / 2 \mathrm{x}) \mid \mathrm{x}=3.745$ gives -0.3 |


| $\mathrm{x}=$ ? | $\tan (2 \mathrm{x})=0.7$ |
| :---: | :---: |
|  | $\mathrm{X}=0.305$, or 1.876 , or <br> 3.447 or 5.018 <br> found by solve $(\cos (1 / 2 x)=-0.3, x)$ <br> $\mid x>0$ and $x<2 \pi$ |
| Test | $\tan (2 \mathrm{x}) \mid \mathrm{x}=0.305$ gives 0.37 etc. |

## Problem 10

| $\mathrm{a}=?$ | $\tan \mathrm{~A}=\mathrm{a} / \mathrm{b}$ |
| :--- | :--- |
| $\mathrm{A}=40$ | $\mathrm{a}=5.874$ |
| $\mathrm{~b}=7$ | found by solve $(\tan 40=\mathrm{a} / 7, \mathrm{~b})$ |
| Test | $\tan 40=5.874 / 7$ |
|  | $0.839=0.839$ |


| $\mathrm{B}=?$ | $\mathrm{~A}+\mathrm{B}=90$ |
| :--- | :--- |
| $\mathrm{~A}=40$ | $\mathrm{B}=50$ <br> found by solve(40+B=90,B) |
| Test | $50+40=90$ <br> $90=90$ |

## Problem 11

| $\mathrm{b}=?$ | $\mathrm{a}^{\wedge} 2+\mathrm{b}^{\wedge} 2=\mathrm{c}^{\wedge} 2$ |
| :--- | :--- |
| $\mathrm{a}=4$ | $\mathrm{~b}=5.745$ |
| $\mathrm{c}=7$ | found by solve $\left(4^{\wedge} 2+\mathrm{b}^{\wedge} 2=7 \wedge 2, \mathrm{~b}\right)$ |
| Test | $4^{\wedge} 2+5.745^{\wedge} 2=7 \wedge 2$ <br>  <br> $49=49$ |


| $\mathrm{A}=?$ | $\sin \mathrm{~A}=\mathrm{b} / \mathrm{c}$ |
| :--- | :--- |
| $\mathrm{a}=4$ | $\mathrm{~A}=34.85$ |
| $\mathrm{c}=7$ | found by solve $(\sin \mathrm{A}=4 /, \mathrm{A})$ |
| Test | $\operatorname{Sin} 34.85=4 /$ <br>  <br> $0.571=0.571$ |


| $\mathrm{B}=?$ | $\mathrm{~A}+\mathrm{B}=90$ |
| :--- | :--- |
| $\mathrm{~A}=$ | $\mathrm{B}=55.15$ |
| 34.85 | found by solve $(34.85+\mathrm{B}=90, \mathrm{~B})$ |
| Test | $34.85+55.15=90$ <br> $90=90$ |
|  |  |

Problem 12

| $\mathrm{B}=?$ | $\mathrm{~A}+\mathrm{B}+\mathrm{C}=180$ |
| :--- | :--- |
| $\mathrm{~A}=40$ | $\mathrm{~B}=72$ |
| $\mathrm{C}=68$ | found by <br> solve( $40+\mathrm{B}+68=180, \mathrm{~B})$ |
| Test | $40+72+68=180$ <br> $180=180$ |
|  |  |


| $\mathrm{a}=?$ | $\mathrm{a} / \sin \mathrm{A}=\mathrm{b} / \sin \mathrm{B}$ |
| :--- | :--- |
| $\mathrm{A}=40$ | $\mathrm{a}=4.731$ |
| $\mathrm{~B}=72$ | found by solve $(\mathrm{a} / \sin 40=$ |
| $\mathrm{b}=7$ | $7 / \sin 72, \mathrm{a})$ |
| Test | $4.731 / \sin 40=7 / \sin 72$ |
|  | $7.360=7.360$ |


| $\mathrm{c}=?$ | $\mathrm{c} / \sin \mathrm{C}=\mathrm{b} / \sin \mathrm{B}$ |
| :--- | :--- |
| $\mathrm{C}=68$ | $\mathrm{c}=6.824$ |
| $\mathrm{~B}=72$ | found by solve $(\mathrm{c} / \sin 68=7 / \sin 72, \mathrm{c})$ |
| $\mathrm{b}=7$ |  |
| Test | $6.824 / \sin 68=7 / \sin 72$ |
|  | $7.360=7.360$ |

## Problem 13

| $\mathrm{a}=?$ | $\mathrm{a}^{2}=\mathrm{c}^{2}+\mathrm{b}^{2}-2 * \mathrm{c}^{*} \mathrm{~b}^{*} \cos \mathrm{~A}$ |
| :--- | :--- |
| $\mathrm{~A}=40$ | $\mathrm{a}=4.724$ |
| $\mathrm{c}=6.8$ | found by |
| $\mathrm{b}=7$ | solve $\left(\mathrm{a}^{2}=6.8^{2}+7^{2}-2 * 6.8 * 7 * \cos 40, \mathrm{a}\right)$ |
| Test | $4.724^{2}=6.8^{2}+7^{2}-2 * 6.8^{*} 7 * \cos 40$ |
|  | $22.316=22.316$ |


| $\mathrm{B}=?$ | $\mathrm{a} / \sin \mathrm{A}=\mathrm{b} / \sin \mathrm{B}$ |
| :--- | :--- |
| $\mathrm{A}=40$ | $\mathrm{~B}=72.3$ |
| $\mathrm{~b}=7$ | found by |
| $\mathrm{a}=4.724$ | solve $(4.724 / \sin 40=7 / \sin \mathrm{B}, \mathrm{B})$ |
| Test | $4.724 / \sin 40=7 / \sin 72.3$ |
|  | $7.348=7.348$ |


| $\mathrm{C}=?$ | $\mathrm{~A}+\mathrm{B}+\mathrm{C}=180$ |
| :--- | :--- |
| $\mathrm{~A}=40$ | $\mathrm{C}=67.7$ <br> found by <br> solve $(40+72.3+\mathrm{C}=180, \mathrm{C})$ |
| Test | $40+72.3+67.7=180$ <br> $180=180$ |

## Problem 14

| $\mathrm{B}=$ ? | $\mathrm{a} / \sin \mathrm{A}=\mathrm{b} / \mathrm{sin} \mathrm{B}$ | $\mathrm{C}=$ ? | $\mathrm{A}+\mathrm{B}+\mathrm{C}=180$ | $\mathrm{c}=$ ? | $a / \sin \mathrm{A}=\mathrm{c} / \sin \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \mathrm{A}=40 \\ & \mathrm{a}=6.2 \\ & \mathrm{~b}=7 \end{aligned}$ | $\begin{aligned} & \mathrm{B}=46.53 \text { or } \mathrm{B}=133.47 \\ & \text { found by } \\ & \text { solve( } 6.2 / \sin 40=7 / \sin \mathrm{B}, \mathrm{~B}) \mid \mathrm{B}>0 \text { and } \mathrm{B}< \\ & 180 \end{aligned}$ | $\begin{aligned} & \hline \mathrm{A}=40 \\ & \mathrm{~B}=46.53 \\ & \text { or } \\ & \mathrm{B}=133.47 \end{aligned}$ | $\begin{aligned} & \mathrm{C}=93.47 \text { or } \mathrm{C}=6.53 \\ & \text { found by } \\ & \text { solve } 40+\mathrm{B}+\mathrm{C}=180, \mathrm{C}) \end{aligned}$ | $\begin{aligned} & \hline \mathrm{A}=40 \\ & \mathrm{a}=6.2 \\ & \mathrm{C}=93.47 \\ & \text { or } \\ & \mathrm{C}=6.53 \end{aligned}$ | $\begin{aligned} & \mathrm{c}=9.628 \text { or } \mathrm{C}=1.097 \\ & \text { found by } \\ & \text { solve }(6.2 / \sin 40=c / \sin \mathrm{C}, \mathrm{c}) \end{aligned}$ |
| Test | $\begin{aligned} & 6.2 / \sin 40=7 / \sin 46.53=7 / \sin 133.47 \\ & 9.645=9.645=9.645 \end{aligned}$ | Test | $\begin{aligned} & 40+46.53+93.47=180 \\ & 180=180 \end{aligned}$ | Test | $\begin{aligned} & 6.2 / \sin 40=9.628 / \sin 93.47=9.628 / \sin 6.53 \\ & 9.645=9.645=9.645 \end{aligned}$ |

Problem 15

| A = ? | $a^{2}=c^{2}+b^{2}-2 * c^{*} b^{*} \cos A$ | $\mathrm{B}=$ ? | $\mathrm{b}^{2}=\mathrm{a}^{2}+\mathrm{c}^{2}-2 * \mathrm{a}^{*} \mathrm{c}^{*} \cos \mathrm{~B}$ | $\mathrm{C}=$ ? | $\mathrm{A}+\mathrm{B}+\mathrm{C}=180$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}=4$ | $\mathrm{A}=33.66$ | $\mathrm{a}=4$ | $\mathrm{B}=75.91$ | A $=33.66$ | $\mathrm{C}=70.43$ |
| $\mathrm{c}=6.8$ | found by solve $\left(4^{2}=6.8^{2}+7^{2}-\right.$ | $\mathrm{c}=6.8$ | found by solve $\left(7^{2}=4^{2}+6.8^{2}-\right.$ | $\mathrm{B}=75.91$ | found by |
| $\mathrm{b}=7$ | $2 * 6.8 * 7 * \cos \mathrm{~A}, \mathrm{~A})$ | $\mathrm{b}=7$ | $2 * 6.8 * 4 * \cos \mathrm{~B}, \mathrm{~B})$ |  | solve(33.66+75.91+C=180,C) |
| Test | $\begin{aligned} & 4^{2}=6.8^{2}+7^{2}-2 * 6.8 * 7 * \cos 33.66 \\ & 16=16 \end{aligned}$ | Test | $\begin{aligned} & 7^{2}=4^{2}+6.8^{2}-2 * 6.8 * 4 * \cos 75.91 \\ & 49=49 \end{aligned}$ | Test | $\begin{aligned} & 33.66+75.91+70.43=180 \\ & 180=180 \end{aligned}$ |

## Problem16

| $\mathrm{d}=$ ? | $\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}$ | $\mathrm{e}=$ ? | $\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}$ | $\mathrm{f}=$ ? | $\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}$ | $\mathrm{g}=$ ? | $\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{d}=(\mathrm{e}-\mathrm{f})^{*} \mathrm{t}-(\mathrm{e}-\mathrm{f})^{*} \mathrm{~g}$ <br> found by $\operatorname{solve}\left(\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}, \mathrm{~d}\right)$ |  | $\mathrm{e}=\frac{\mathrm{f}^{*} \mathrm{t}+\mathrm{d}-\mathrm{f}^{*} \mathrm{~g}}{\mathrm{t}-\mathrm{g}}$ <br> found by $\operatorname{solve}\left(\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}, \mathrm{e}\right)$ |  | $\mathrm{f}=\frac{\mathrm{e}^{*} \mathrm{t}-\mathrm{d}-\mathrm{e}^{*} \mathrm{~g}}{\mathrm{t}-\mathrm{g}}$ <br> found by $\operatorname{solve}\left(\mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g}, \mathrm{f}\right)$ |  | $g=\frac{(e-f)^{*} t-d}{e-f}$ <br> found by $\operatorname{solve}\left(T=\frac{d}{e-f}+g, g\right)$ |
| Test | $\begin{aligned} & \text { Solve }\left(\mathrm{d}=(\mathrm{e}-\mathrm{f}) * \mathrm{t}-(\mathrm{e}-\mathrm{f})^{*} \mathrm{~g}, \mathrm{t}\right) \\ & \text { gives } \mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g} \end{aligned}$ | Test | $\begin{aligned} & \text { Solve }\left(e=\frac{f^{*} t+d-f^{*} g}{t-g}, t\right) \\ & \text { gives } T=\frac{d}{e-f}+g \end{aligned}$ | Test | $\begin{aligned} & \text { Solve }\left(\mathrm{f}=\frac{\mathrm{e}^{*} \mathrm{t}-\mathrm{d}-\mathrm{e}^{\pi g}}{\mathrm{t}-\mathrm{g}}, \mathrm{t}\right) \\ & \text { gives } \mathrm{T}=\frac{\mathrm{d}}{\mathrm{e}-\mathrm{f}}+\mathrm{g} \end{aligned}$ | Test | $\begin{aligned} & \text { Solve }\left(g=\frac{(e-f)^{x} t-d}{e-f}, t\right) \\ & \text { gives } T=\frac{d}{e-f}+g \end{aligned}$ |

## Problems 17-19

| $\mathrm{y}=?$ | $\mathrm{y}=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}$ |
| :--- | :--- |
| $\mathrm{a}=785$ | $\mathrm{y}=896.85$ |
| $\mathrm{~b}=1.027$ | found by |
| $\mathrm{x}=5$ | solve $\left(\mathrm{y}=785^{*} 1.027^{\wedge} 5, \mathrm{y}\right)$ |
| Test | $785^{*} 1.027^{\wedge} 5$ gives 896.85 |
| $\mathrm{~T}=\ln (2) / \ln (1.027)=26.0$ |  |


| $\mathrm{x}=?$ | $\mathrm{y}=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}$ |  |
| :--- | :--- | :--- |
| $\mathrm{a}=785$ | $\mathrm{x}=8.3$ |  |
| $\mathrm{~b}=1.027$ | found by |  |
| $\mathrm{y}=980$ | solve $\left(980=785^{*} 1.027^{\wedge} \mathrm{x}, \mathrm{x}\right)$ |  |
| Test | $785^{*} 1.027^{\wedge} \mathrm{x} \mid \mathrm{x}=8.3$ gives 980 |  |
| $=\ln (2) / \ln (1.027)=26.0$ | $\mathrm{~T}=$ |  |


| $\mathrm{b}=?$ | $\mathrm{y}=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}$ |
| :--- | :--- |
| $\mathrm{a}=785$ | $\mathrm{~b}=1.045=1+4.5 \%$ |
| $\mathrm{y}=980$ | found by solve $\left(980=785^{*} \mathrm{~b}^{\wedge} 5, \mathrm{~b}\right)$ |
| $\mathrm{x}=5$ |  |
| Test | $785^{*} \mathrm{~b}^{\wedge} 5 \mid \mathrm{b}=1.045$ gives 980 |

Problems 20-22

| $\mathrm{y}=?$ | $\mathrm{y}=\mathrm{a} * \mathrm{x}+\mathrm{b}$ |
| :--- | :--- |
| $\mathrm{b}=785$ | $\mathrm{y}=798.5$ |
| $\mathrm{a}=2.7$ | found by |
| $\mathrm{x}=5$ | solve $(\mathrm{y}=2.7 * 5+785, \mathrm{y})$ |
| Test | $2.7 * 5+785$ gives 798.5 |


| $\mathrm{x}=?$ | $\mathrm{y}=\mathrm{a}^{*} \mathrm{x}+\mathrm{b}$ |
| :--- | :--- |
| $\mathrm{b}=785$ | $\mathrm{x}=72.2$ |
| $\mathrm{a}=2.7$ | found by solve $(980=2.7 * \mathrm{x}+785, \mathrm{x})$ |
| $\mathrm{y}=980$ |  |
| Test | $2.7 * \mathrm{x}+785 \mid \mathrm{x}=72.2$ gives 980 |


| $\mathrm{a}=?$ | $\mathrm{y}=\mathrm{a}^{*} \mathrm{x}+\mathrm{b}$ |
| :--- | :--- |
| $\mathrm{b}=785$ | $\mathrm{a}=39$ |
| $\mathrm{y}=980$ | found by solve(980=a*5+785,a) |
| $\mathrm{x}=5$ |  |
| Test | $2.7^{*} \mathrm{x}+785 \mid \mathrm{a}=39$ gives 980 |

