POSTMODERN THINKING IN MATHEMATICS EDUCATION RESEARCH

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ABSTRACT

Q: I have three questions: What is the postmodern? Is there a postmodern research paradigm? To what field of mathematics education research can postmodern research contribute, and what are examples of postmodern studies? Please be brief, we can take the details later.

R: OK. 1. The postmodern means goodbye to the modern, both as postmodernism on the discursive level and as postmodernity on the practice level. Postmodernism is poststructuralism rejecting the belief that the world has a structure to be represented and echoed by language, and warning against becoming enslaved and clientified by echo-phrasings and echo-discourses. Postmodernity is post-industry now using electrons to carry also information, to machines and to global TV's, thus creating automation and globalisation; and individualisation since the individual can no longer get an identity through echoing the local tradition, but has to construct a self-identity through self-stories. In short the postmodern means a transition from an echo-society to a dilemma-society.

2. Modern research echoing the structure of the world becomes problematic in a world without structure. Postmodern research is a search for hidden contingency, i.e. for alternatives to choice presented as nature to install false patronization and clientification. So postmodern contingency research deconstructs modern structure claims and echo-discourses by rephrasing echo-phrasings.

3. Postmodern contingency research is working within the sociological part of the social turn in mathematics education. In mathematics education 'mathematics' and 'education' are echo-phrasings that can be rephrased as 'grammar of the number language' and 'cure'. This rephrasing exposes the contemporary mathematics education tradition as treating clients in need of a number language with a grammar cure; and it provides a counter cure for, not the client, but the cure: language before grammar; a counter cure that might make mathematics a human right. The inspiration comes from the French republic and especially from Foucault (Foucault 1972).

POSTMODERNISM AND POSTSTRUCTURALISM

Q: So there are different sides to the postmodern?

R: Yes. In his book about the idea of the postmodern, Hans Bertens distinguishes between postmodernism and postmodernity. About postmodernism he writes

> Postmodernism, then, means and has meant different things to different people at different conceptual levels... If there is a common denominator to all these postmodernisms, it is that of a crisis in representations: a deeply felt loss of faith in our ability to represent the real, in the widest sense (Bertens 1995 p.10).

Later he distinguishes between two moments of postmodernism, a deconstructionist derived from Derrida and a poststructural derived from Foucault:

> Like the earlier deconstructionist postmodernism, the later poststructural postmodernism assumes a reality of textuality and signs, of representations that do not represent. Here however, the emphasis is on the workings of power, and the constitution of the subject. From the perspective of this postmodernism, knowledge, which had once seemed neutral and objective to the positivists and politically emancipatory to the left, is inevitably bound up with power and thus suspect. ... This postmodernism interrogates the power that is inherent in the discourses that surround us - and that is continually reproduced by them - and interrogates the institutions that support those discourses and are, in turn, supported by them. It attempts to expose the politics that are in work in representations and to undo institutionalised hierarchies, and it works against the hegemony of any single discursive system - which would inevitably victimize other discourses - in its advocacy of difference, pluriformity, and multiplicity (Bertens 1995 p.7).

To sum up 'under erasure' as Derrida puts it (Derrida 1991) to acknowledge the contingency of all phrasings, postmodernism means poststructuralism, i.e. a goodbye to the conviction that the world has a structure that can be represented or echoed by language and discursive knowledge that is discovered through research, transmitted through education that makes student knowledge an echo of scientific knowledge, and finally applied in industries. Phrasings are not representing, but constructing what they phrase. All phrasings could be otherwise. Unaware of the hidden contingency of phrasings the emancipated free human being becomes enslaved and clientified by ruling echo-phrasings. Also phrasing is praising, but this time the Lord is not living in castles or churches, but in schools and universities, where the hidden subjection of the apparent free and rational modern human being takes place through echo-phrasings.

Q: You don't mention nihilism. Often postmodernism is considered just negative nihilism and rejected because of that.

R: Well, the key idea of poststructuralism is that things cannot name themselves. And of course structuralism has good reasons to demonise poststructuralism by phrasing it with a negative word. As structuralism itself was demonised by what it replaced, the feudal order.

Q: So structuralism is basically postfeudalism?

R: Yes. The premodern feudal world was governed by the will of two masters, a physical and a metaphysical; and the ways of the metaphysical Lord were many, His will was unpredictable. All we could do was to go to church, believe and pray. Or go to a monastery and study the Holy Scripture. Then Newton discovered, that the will that makes things fall can be quantified and calculated, hence predicted. Calculation thus freed us from the arbitrary laws of the Lord, we had become our own master, since we could make the calculable laws serve us at will. Thus structuralism and enlightenment and democracy was born: The modern free world would be governed by the will of a parliament representing the will of the people, and by the will of metaphysical laws, that were predictable and representable in language. All we had to do was to go to school, learn and calculate. Or go to a university and study the research scriptures, and

study the laws through experiments and observations. The physical world had a structure echoed from metaphysical laws. These laws could be discovered by researchers, taught by teachers and applied by professionals. Hence a modern society should invest in, not impressive baroque monasteries and churches as in the catholic southern Europe, but in universities, schools and industry as in the protestant northern Europe. At the universities the natural sciences searches for the quantitative laws determining particle behaviour in physical force fields. Likewise human and social sciences search for the qualitative laws determining human behaviour in social force fields.

Q: And post-structuralism rejects these beliefs?

R: Yes, in a way the structuralism/poststructuralism debate is a repetition of the former Plato/Aristotle and realist/nominalist debates. To poststructuralism there is no metaphysical structure to be echoed in the world, and there is no physical structure to be echoed in language. Any description constructs what it describes. Words are not echoes from above, words are names from below. The structure of the world is not a natural consequence of metaphysical laws, but a cultural consequence of the way we have chosen to phrase the world, of ruling discourses. A branch of poststructuralism, the SSK, Sociology of Scientific Knowledge, even argues that also natural sciences are social constructions (Pickering 1995). We can only ask nature through measuring instruments, and since measuring instruments are human constructs, the answers of nature will also be human constructs. Another name for the structuralism/poststructuralism debate is 'the science war' with social constructionism on the one side writing off the enlightenment project, and the paradigm of complexity on the other trying to save it through system theory, chaos theory etc.

Q: Foucault has been rather influential to postmodernism?

R: Yes. In his inaugural speech 'Orders of discourse' Foucault says:

> The fundamental notions now imposed upon us are no longer those of consciousness and continuity (with their correlative problems of liberty and causality), nor are they those of sign and structure. They are notions, rather, of event and of series, with the group of notions linked to these (Foucault 1971, p. 23).

This is how Foucault says goodbye to the core concepts of modernism: the concepts of liberty, causality and structure, and replaces them with the core concept of postmodernism: eventuality, possibility, potentiality, otherwiseness, alternatives. Or contingency as Rorty phrases it, or re-enchantment as Bauman phrases it (Rorty 1989, Bauman 1992).

POSTMODERNITY AND POSTINDUSTRY

Q: And then there is postmodernity

R: Yes. Postmodernity is about technological and social changes. In his book 'Runaway world' Giddens talks about globalisation:

> Globalisation also influences everyday life as much as it does events happening on a world scale. ... [Globalisation] contributes to the stress and strains affecting traditional ways of life and cultures in most regions of the world. ... Fundamentalism originates from a world of crumbling traditions (Giddens 1999).

Instead of postmodernity Giddens uses other terms as radicalised, late, high, reflexive, post-traditional modernity. In his book 'Modernity and Self-Identity' he writes

> In the post-traditional order of modernity ... self-identity becomes a reflexively organised endeavour. The reflexive project of the self, which consists in the sustaining of coherent, yet continuously revised, biographical narrative, takes place in the context of multiple choice as filtered through abstract systems (Giddens 1991).

In short we can say that postmodernity means post-industry. In modernity electrons carry energy nationwide to machines producing echoes of their products in high numbers at low costs. In postmodernity electrons also carry information to machines, and to global TV's. Routine echo-knowledge is moved from human brains to artificial brains, computers, becoming integrated with machines to robots taking over the echo-production. Global TV's in local cultures produce dilemmas by echoing global counter-answers to local echo-answers. This globalisation creates individualisation: with many answers individuals can no longer obtain identity by echoing the answer, individuals now have to construct their own self-identity through biographical self-stories build upon authenticity and meaning. The postmodern individual has become a self-story builder and a self-educator.

Q: So post-modernity is post-industry, which probably also is post-something?

R: The history of technology can briefly be thought of as the gradual construction of the artificial human. First the construction of an artificial hand, a tool, enabled a transition from gather/hunter culture to agriculture. Then the construction of an artificial muscle, a motor, and the combination of a tool and a motor to a machine enabled a transition to the industrial culture, to modernity. And now the construction of an artificial brain, a computer, and the combination of a machine and a computer to a robot has made possible a transition to a postmodern information culture. This postmodern culture resembles very much the former gather/hunter culture: Then nature was producing and humans just had to gather and hunt the food. Now robots, clones and construction molecules made by the new self reproducing GNR-technologies, i.e. gene-, nano- and robot-technology, will take over production. Again most humans just have to gather the food in the supermarket and hunt for stories on the global TV. So one way of naming the postmodern culture will be 'gather/hunter culture II' (Tarp 2000 b).

Q: Does it matter how you name it? A name is just a label.

R: In postmodern thinking a name constructs what it names. Hence we witness a fierce battle of definition. As long as the modern can demonise the postmodern its institutions can survive. But if the term postmodern becomes accepted, all the institutions of modernity have to defend and re-legitimise themselves. In the SSK terminology they then stop being closures, i.e. accepted naturalized solutions, and become instead possible and potential solutions that have to fight with

other potential solutions to become the new closure. And just calling the new order ‘post’ isn’t enough, we have to find a new name in order to think and talk about it, make decisions and institute institutions.

Q: What would be gained by naming postmodernity the new gather/hunter culture.

R: Then we wouldn’t have to build a new culture from the bottom; we could learn and become inspired by former and present gather/hunter cultures. Instead of phrasing these cultures undeveloped and primitive we can phrase them exemplars to be learned from and to get inspiration from as to how life can be organised so individuals become actively producers instead of passive consumers of culture.

Q: To sum up, what is the effect of the postmodern to research and education?

R: Well, the university and the school are modern institutions that succeeded the premodern institutions of the monastery and the church. All are studying or teaching metaphysical laws, in the premodern case unpredictable laws understood through belief, in the modern case predictable laws understood through knowledge. Postmodernism cancels the metaphysical world, which of course seriously questions the relevance of research and education in a postmodern world. Can these modern institutions survive a transition to a postmodern era, or do they in a desperate ‘apres nous le deluge’ situation try to keep the illusion of modernity alive as long as possible?

Also the students have changed. In the premodern will-society the church wanted to save the lost individual by offering faith to believe in. In the modern echo-society the school wanted to cure the uneducated individual by offering knowledge to be echoed. The postmodern dilemma-society has created the individualised self-story builder shopping among numerous stories, rejecting those who do not tell them something new about what they already now, thus rejecting modern education from above.

POSTMODERN CONTINGENCY RESEARCH

Q: So does research have any meaning in a postmodern world? Is there a postmodern research paradigm? Please start with a summary.

R: In a postmodern world research has meaning as contingency research studying how things could also be otherwise. Postmodern contingency research means goodbye to modern research trying to echo the structure of the world in true phrasings. The world does not phrase itself, we do, so all phrasings are eventual, contingent, and could be otherwise. Contingency research produce counter examples to ruling discourses and to structure claims, which by claiming a status of truth automatically become echo-statements. Postmodern contingency research is a ‘re-search’ re-searching for re-phrasings, using both discovery and imagination. Discovery is used to search ruling discourses for echo-phrasings. And imagination is used to rephrase these echo-phrasings to unhide hidden alternative counter-discourses that might change our convictions, institutions and routines. Thus transforming rituals to routines. In short, postmodern contingency research is guarding the borderline between necessity and contingency by unhiding hidden contingency in claimed necessity.

Q: So postmodern research is based upon poststructuralism?

R: Yes. Again let us turn to Foucault. In his inaugural speech Foucault talks about four methodological principles: a principle of reversal, a principle of discontinuity, a principle of specificity and a principle of exteriority. As to the principle of specificity he says:

> The principle of specificity declares that a particular discourse cannot be resolved by a prior system of significations; that we should not imagine that the world present us with a legible face, leaving us merely to decipher it; it does not work hand in glove with what we already know; there is no prediscursive fate disposing the world in our favour. We must conceive discourse as a violence that we do to things, or, at all events, as a practice we impose upon them; it is in this course that the events of discourse find the principle of their regularity (Foucault 1971 p. 22).

This is Foucault's version of the post-structuralism statement. The world has no structure to be echoed, so whenever we are talking about the structure of the world, we are talking about something we impose upon the world. Thus one kind of a postmodern methodology is the one Foucault described in his inaugural speech and carried out in his studies.

Q: So basically Foucault's thinking is the foundation of postmodern research?

R: Not exactly. In my own work I have tried to make a compromise between structuralism and poststructuralism by formulating what I call a postmodern contingency research paradigm. Research is supposed to produce true knowledge claims about the world. But when we are talking about truth and describing the world we are faced with three levels: the world, the language describing the world and the meta-language describing the language. Between the two upper levels we meet the 'meta-dilemma': Is the meta-language describing the language - or is the language an application or echo of the meta-language? This dilemma is parallel to the theory-dilemma: Is theory describing the world - or is the world an echo of theory? And between the two lower levels we meet what I call the 'pencil-dilemma'.

Q: The pencil-dilemma?

R: Place the thing we call a pencil between a ruler and a dictionary, both enabling us to lift up things from the thing-level to the language-level by assigning numbers and words to them. The pencil-dilemma says that the pencil can point to its own length, but not to its own name. Hence there is a fundamental difference between number-statements and word-statements: Number-statements about a thing as 'its length is 25 cm' can be verified by asking the thing itself, word-statements as 'this is a pencil' cannot. Number-statements follow from necessity; word-statements follow from contingency, from a choice that could be otherwise.

Q: Just a moment, a cup is a cup. I mean nobody will deny, that this thing is a cup?

R: Correct. But the fact that nobody would deny it doesn't make it a cup. 'Cup' is not a necessary quality by the thing; it is a description of its use in the current culture. People from other cultures or other times might name the thing differently. In 200 years the thing we call a cup might be called e.g. a stick-container, again depending of the use. Also 'cup' can be seen as

an excellent social inclusion and exclusion technique, nobody denies it is a cup, for if they did they would be excluded. The point is that the thing cannot name itself in the same way as it can number itself e.g. by being put on a balance. Also we have different ways to decide if number- and word statements are true. Number-statement are decide through measuring, word-statements are decided in court, i.e. through a choice between two different phrasings of the same thing.

Q: But both words and numbers are constructed by humans?

R: Of course humans have to construct their own interface towards the world. But once the ruler and the dictionary have been constructed there is still a necessity in the thing that enables it to point to its own length. Modern science discovered the five necessities of nature that physics is build upon: mass, charge, extension in space, extension in time, and multiplicity. These necessities exist, but of course how we name them is dependent on social constructions in local cultures. The pencil-dilemma points to the fact, that there exists a borderline between necessity and contingency, contingency, i.e. a borderline between numbering nature and naming culture.

Q: Didn't Foucault consider numbers as contingent?

R: Yes, both Foucault's and Nietzsche's scepticism towards truth also include numbering. As a matter of fact neither structuralism nor poststructuralism accepts the borderline between necessity and contingency. Structuralism believes all is necessity and sees apparent contingency as hidden necessity to be uncovered through research. And poststructuralism believes all is contingency and sees apparent necessity as hidden contingency to be uncovered through narratives. The pencil-dilemma opens up for a moderate postmodernism paying tribute to both sides by taking on the task as a guardian of the borderline between necessity and contingency by pointing to hidden contingency in phrasings and by accepting the numbering of nature, where the truth can be controlled by asking the numbered itself through measuring instruments. To use a metaphor from mathematics: Within numbers there is a borderline between constants and variables to be guarded against parameters who hide a variable nature beneath a constant appearance.

Q: But we cannot live in a world full of variation. It would be chaotic and without order.

R: Correct, we need to have what the SSK calls closures and we use our political democratic institutions to install such closures all the time. Also different cultures show different form of closures. But still it is important to distinguish between what might be changed and what might not, between political laws and physical laws. Inspired by the concept of entropy from physics measuring the degree of order, we could talk about different degrees of contingency. And inspired by Giddens' terms system and structure (Giddens 1984) we can divide the social in two layers: culture and society. Nature would then have no contingency, culture would have a low, society a medium and individual lives a high degree of contingency.

Q: So phrasing might be problematic, but we are phrasing all the time.

R: Of course we are phrasing all the time, but we should be aware of when we phrase contingency as necessity. The ability of phrasing is what separates humans from animals.

Animals are bound by rituals, but we humans can free ourselves from rituals by using phrasing and installing to generate an RRR-cycle: Ritual-Reflection-Routine. Rituals can be lifted from the practice level to the reflection level by a phrasing. Once phrased we can think, discuss, form convictions, make decisions and take action by returning to the practice level to install and institute rational institutions and routines - but these routines might become new rituals, freezing us until we rephrase them again. Phrasing is freeing, echo-phrasing is freezing, and rephrasing is re-freeing. Routine is hot, ritual is not. Postmodern contingency research is aiming at re-freeing through re-phrasing, at transforming rituals into routines, both at a practice and at a discursive level.

Q: But how can you tell routine from ritual?

R: A routine is generated and justified by a discourse rationalising why the routine is installed. If the generating and justifying discourse has degenerated into an echo-discourse, then the generated routine has become a ritual. As e.g. if mathematics education is constantly justified by a "mathematics is, and mathematics is applied, hence mathematics must be taught" discourse.

Q: But still I have problems distinguishing between modern research and postmodern contingency research.

R: The difference is that modern human and social science produce word statements and general claims. Postmodern contingency research produces counter examples, and a counter claim is en existence claim and not a general claim. So postmodern contingency research has to provide proof of existence and not proof of generality. And a proof of existence is indirectly a number statement, for as e.g. Frege says, a confirmation of existence is a negation of zero-occurrence. So postmodern contingency research includes first a numbering, then a rephrasing. First the institutionalised and ruling traditional practise is searched for rituals by searching its justifying discourse for echo-words and excluded words, i.e. words that are never varied or absent, word with zero variation or presence. Once these zero-occurrences have been identified through discovery, through modern research, then imagination takes over to rephrase the echo-phrasing thus creating a counter discourse, that might make us change the way we see the world and might change our convictions, institutions and routines. But this rephrasing is just one example of a rephrasing serving to prove the existence of hidden contingency, and an example might always be otherwise. So the inherent contingency of an example protects postmodern contingency research from its own demand to respect the contingency of phrasings. And the example serves two purposes, to convince and to inspire: it serves to convince about the existence of a counter example and it inspires to look for other examples. In this way the postmodern contingency research unites two essential values, the convincing power of modern research and the inspiring power of postmodern narratives.

Q: Do you say that quantitative research is OK?

R: Yes and no. Numbering nature is OK if it is based upon the five necessities of nature. Numbering culture is not OK since it is based upon a previous phrasing as e.g. 'abnormal student'. The contingency of such a phrasing will be transferred to the corresponding numbering.

Q: But you say that qualitative research is problematic?

R: Yes and no. Theory based qualitative research will always contain hidden contingency, partly for poststructural reasons, partly because questionnaires and interview lists basically are interrogation methods only allowing the informants to answer to what they have been asked. However empiric based grounded research is OK as long as it only produces new words and not new theory. I.e. as long as it is 'grounded naming' and not 'grounded theory'; new names are examples of existence-statements, and they can inspire to new rephrasings. It is only with new sentences one can ask if it is fact or unintended fiction. And grounded naming is used in postmodern ethnography.

Q: Postmodern ethnography?

R: Ethnography means graphing the ethno, i.e. phrasing the other. The premodern ethnographer was the missionary send out to save the "Prodigal" natives by preaching to them. The modern ethnographer is the researcher send out to develop the "under developed" natives by teaching about them, telling innovation agents how they can be phrased. The postmodern ethnographer is the postmodern researcher dropping all claims that his phrasing of the other can be true. Instead you want to learn about hidden contingency in yourself from the other, and you can do so by exchanging gifts with the other. Instead of conquering the conversation room to preach or teach, the others are invited in to receive a gift: the prestigious role of the narrators, by being asked and allowed to tell about themselves. If the other returns the gift, the two self-stories can produce a new third mutual story and perhaps a new practice in the case of postmodern action research. So using postmodern ethnography both the learner and the other will leave the conversation room with two stories about hidden contingency: the story of the other and another story.

Q: Could you give an example?

R: In many cases the students are considered the raw material of the educational process. The processing of the students is studied, and the students are observed as if they were particles in a force field. Occasionally students are interrogated through questionnaires or interviews. In short, students are what Foucault calls excluded from the discourse. But students are human beings who are reflecting about their experiences and able to tell stories about them. So to them it is a present to be asked 'Tell me about your learning life with mathematics'. In the case of Ruth she tells about the teacher entering the room, opening the textbook and starting to teach. But what he says is precisely what is in the book. At home the students find the book difficult to understand, so next day they ask the teacher to explain the book, and once more he repeats the book. I coin the name 'echo-teaching' from this story. I handed out Ruth's story to other students, many of whom recognised the echo-teaching. From their reaction I coined two other words. 'Echo-rejection' takes place when students say 'I don't want to learn what I can't understand. If the teacher will not explain, I will not learn.' 'Echo-learning' takes place when the student echo the teacher or the textbook, e.g. by saying 'I want good marks for my future career so I cheated the system by learning by heart.' Instead of 'echo' I could have chosen names as copy, parrot, clone etc. Thus there is a fair chance that 'echo-teaching' will develop into en echo-word. By choosing 'echo' 'echo-teaching' connects to the 'echo-society' discourse, which reveals a hidden

understanding of the exodus-problem in mathematics, where more students reject mathematics based educations within science and technology (Jensen et al 1998): a postmodern dilemma-society rejects the echo-answers of a modern echo-society. So while modern students might like modern mathematics, postmodern students reject must reject it.

Another example was Africa, where I was invited to assist in an action research programme. However the wish for change only existed at a rector level, but not at department level ad hence not at teacher and student level. Staff development meetings were planned but not realised. So I changed strategy from modern action research to postmodern ethnography using gift exchange: Instead of waiting for a meeting in the seminar room or at my office, I went to the teachers' room and said 'tell & show me what YOU do'. This gift was then returned by asking me what I did. I then told them postmodern mathematics as a hidden alternative to modern mathematics, and I was asked to implement it in the classroom. In this way the students were also exposed to the hidden contingency. In the following discussion the terms 'Top-Down' and 'Bottom-Up' mathematics arose substituting modern and postmodern mathematics. Thus both parts left the conversation room with two new stories, the story of the other and another story.

Q: In your summary you said that postmodern contingency research produces counter examples to structure claims, which by claiming a status of truth automatically become echo-statements. But doesn't this counter example become a new structure claim itself that becomes a new echo-statement?. Isn't there a contradiction here?

R: Yes. It seems as if we have a problem of self-reference and infinite regress here, another example of Russell's paradox 'This sentence is false'. It is a main argument against the postmodern that it cancels itself. If all phrasings are contingent, so is yours, so how can you claim to be telling truths? And against Loytard's statement 'Simplifying to the extreme, I define *postmodern* as incredulity toward metanarratives': Doesn't his incredulity also apply to his own metanarrative?

Q: Precisely! So how can postmodern statements claim to be true? Isn't here that postmodernism becomes nihilism?

R: Apparently. But we forget that Russell solved his paradox himself in his type theory: Self-description leads to nonsense; a level can only be described from a metalevel. So a statement about a statement is a meta-statement and therefore not affected by the contingency of statements. Or in the case of Lyotard: A narrative about a metanarrative is a meta-metanarrative and hence not affected by the incredulity toward metanarratives. Or the poststructuralist version: Al phrasings are contingent - except for this metaphrasing. From this perspective postmodern contingency research becomes a meta-research, not studying the world, but studying truth descriptions about the world, assuming the role of the naughty boy in Hans Christian Andersen's fairy tale about the emperor's new clothes commenting 'but he is not wearing anything'.

Q: But isn't it destructive just to be critical?

R: Yes and no. Postmodern contingency research has a destructive critical part and a constructive emancipating part. And even the destructive part becomes constructive if it is used to destruct a

destructive practise or cure within e.g. education, as presenting mathematics so difficult that it excludes people when it has the potential to do the opposite.

Q: Is postmodern contingency research just another word for critical research?

Critical mathematics is critical towards the application of mathematics. Postmodern mathematics is critical towards the echo-phrasing of mathematics. Critical mathematics is accepting mathematics as it is, but believes humans become empowered by being critical and reflective to how mathematics is applied in society. To postmodern mathematics the current mathematics discourse is just another ruling echo-discourse that hides its hidden contingency. Postmodern thinking believes that humans become empowered by rephrasing existing echo-discourses. In this case by rephrasing mathematics, from the ruling top-down application discourse presenting mathematics as examples of more abstract concepts, to a bottom-up number language discourse presenting mathematics as tales about the social practices that created mathematics: how to divide the earth and what it produces.

POSTMODERN CONTINGENCY RESEARCH IN MATHEMATICS EDUCATION

Q: How is postmodern research relevant within education?

R: In modern thinking institutions are created to serve a public need for e.g. education. Foucault questions this, suggesting that institutions are a result of the disappointment of the enlightenment discovering that the normality of nature was not paralleled by a normality of culture. This unbalance however could be cured by modern institutions created to cure the phrased or diagnosed abnormality. The institution even delivers a circular self reference proof for the correctness of the diagnose: Abnormal are those who are not cured by the institution's cure against abnormality. Thus the institution school arises from the diagnose 'uneducated' to be cured through education. Which unfortunately is not successful in all cases. A problem the school blames on a bad funding. Instead inspired by Foucault postmodern contingency research suggests that it is the cure and not the client, that has a problem: Cure the cure, not the client.

Q: Within mathematics education, what field is postmodern research working?

R: Postmodern contingency research is working within the sociological part of the social turn in mathematics education research (Lehrman in Boaler 2000). Postmodern re-search will first search for rituals on the practise level by identifying echo-phrasings in the ritual's justifying discourses. The practices can be studied in the classroom by observing or by listening to students and teachers. The justifying discourses can be studied in textbooks and curriculum descriptions. Then imagination and inspiration is used to rephrase these echo-phrasings to unhide hidden contingency. Then curriculum-architecture is used to design alternative micro- or macro-curricula, which implementation finally is reported.

Q: Tell me about your own research.

R: I call my working theme 'mathematics as a human right'. As a part of this I have focused on the exodus problem or enrolment problem in mathematics, where more students reject

mathematics based education within science and technology (Jensen et al 98). I ask: Can there be hidden reasons behind this exodus phenomena? And here I find postmodern thinking useful suggesting we are blindfolded by ruling echo-discourses. Hence I have set up a research symphony consisting of four movements or phases asking: What is postmodern research? How do postmodern students react to modern mathematics? What is a postmodern mathematics curriculum? How do postmodern students react to postmodern mathematics? In brief the answers are: Postmodern contingency research is a re-search producing counter examples to ruling rituals by searching for and rephrasing echo-words in the ritual's justifying discourses. Postmodern students ask 'tell me something I don't know, about something I know', but modern Top-Down mathematics by phrasing abstract concepts as examples or echoes of more abstract concepts, tells the students something they don't know about something they don't know, which makes some students reject and other students echo modern mathematics. A postmodern mathematics curriculum will phrase mathematics Bottom-Up as quantitative calculation stories about two fundamental social practices: dividing the earth and what it produces. Postmodern mathematics transforms many dropouts to dropins asking for more mathematics until they meet Top-Down mathematics.

So the conclusion of this study is: A possible hidden factor behind the exodus-problem in mathematics is its ruling top-down ritual installed by a discourse having mathematics and application as echo-phrasings. This echo-phrasing is freezing the educational system, forcing it to install mathematics education from above resulting in echo-teaching and echo-learning, and preventing it to see the hidden alternative, mathematics from below. This creates no problem to modern students getting their identity through echoing; but it creates problems to postmodern students building self-identity through self-stories. Thus the exodus problem might be an 'echodus'-phenomena: The echo-tradition of modern mathematics forces postmodern students to exodus. It is however possible to install an alternative bottom-up practise making mathematics a human right, but its corresponding justifying 'mathematics is easy' discourse has to fight the ruling 'mathematics is difficult' discourse. Which parallels the classical situation of the reformation, where Luther and later Kierkegaard pointed out, that it is possible to be saved without the assistance of the saving institution: It is possible to develop a number language without the assistance of an institutionalised Top-Down mathematics cure. This conclusion makes me very sad remembering all the unhappiness that came out of the wars between Catholics and Protestants.

Q: But is it possible to make mathematics a human right. It is generally accepted, that mathematics is difficult.

R: Mathematics is not difficult by nature; mathematics has been made difficult by being phrased from above. Phrased form below my research shows that mathematics becomes easy. And I think it is important to make mathematics a human right. If not mathematics could be used to exclude the unfitted from the future labour market where the 'math-based' GNR technologies makes most jobs superfluous, except of course the 'math-based' ones. Making mathematics difficult creates a 'natural' selection technique, where the excluded only have themselves to thank for not working hard enough with the 'difficult' mathematics. Also it is problematic to say that these

technologies are math-based. It is the number-language and not its grammar, mathematics, is applied in these new technologies.

Q: This study sounds as traditional modern research.

R: Traditional research adds to the zone of necessity. This study adds to the zone of contingency. It demonstrates the existence of a counter example to an apparent necessary top down ritual and to an apparent unavoidable exodus problem. It does not offer a new truth claim, a new statement of necessity about the client that can be used to install a new cure of the client. It offers a counter claim, a statement of hidden contingency in the justifying discourse of the ruling cure. It does not prescribe a new cure; it only gives an example of how the cure could also be otherwise. It is a meta-cure curing the cure and not the client.

Q: To what kind of questions within mathematics education can postmodern research contribute?

R: With its ability to unhide hidden contingency postmodern contingency research reminds very much about architecture. So one field of application is to help to establish a new education and profession called 'curriculum-architecture'. Traditional curricula are filled with echo-phrasings, that could be rephrased. It was calculation that freed humans from feudality by changing the metaphysical laws from unpredictable to predictable thus making modern industrialised society possible. Hence we could assume, that calculation was a core subject in the curriculum of modern education. However it is not, instead mathematics is. At the curriculum level 'mathematics' is used as self referring echo-word in most object statements: The object of mathematics education is mathematics; mathematics is taught so that students can learn mathematics. There is no mentioning of outside needs or objects calling for mathematics as a means. Terms like 'multiplicity', 'number-language', 'quantity', 'calculation', 'predictability, 'change', 'per-numbers' etc. are absent. A list of mathematical qualities might be listed as positive values: its ability to further logical thinking, its widespread application in modern society, its role as a tool in further education etc. In short the object-discourse seems to say: "Mathematics is, and mathematics is applied, hence mathematics is taught". The feudal origin of this statement can be seen be substituting 'mathematics' with 'God'. This mathematics&application-discourse automatically installs education from above together with 'echo-teaching' and 'echo-learning'.

Q: How does a mathematics&application-discourse automatically install education from above?

R: The mathematics&application-discourse says: 'Modern society builds heavily upon application of mathematics, hence mathematics education must be instituted at all levels'. If we ask: 'What should be learned first, mathematics or application of mathematics?' the answer automatically becomes: 'Of course mathematics must be learned before it can be applied! Without mathematics we have nothing to apply!?'. Within a mathematics&application-discourse you cannot say 'mathematics should be learned after its applications' without being considered mad and thus excluded form the discourse. This discourse is typical for modern structuralist thinking. Up there are metaphysical mathematical laws that are echoed down here. These laws

have to be found and taught before they can be applied, creating a knowledge flow from the metaphysical down through universities and schools to applications in the industry.

Q: But mathematics is mathematics, how can mathematics be rephrased?

R: I once asked a mathematics professor what this 'mathematics' is. His answer was: 'Mathematics is what mathematicians do'. I.e. a self referring echo-answer without meaning installing mathematics as a ritual. An alternative to the structuralist top-down thinking would be a poststructuralist bottom-up thinking starting out from the zone of necessity. It is culturally given that we describe the world in words and numbers using a word-language and a number-language. And that we describe our languages in metalanguages or grammars. So one possible rephrasing of mathematics could be: Mathematics is a grammar of our number-language. Based upon the fact that 'multiplicity is, hence a number language and its grammar should be developed' this rephrasing enables us to formulate a non self referring object statement as e.g.: 'The object of mathematics education is to develop the students number-language enabling them to describe multiplicity in quantities and calculations'. This multiplicity-discourse installs education from below giving mathematics meaning and authenticity, which is what the postmodern self-story builders and learners are demanding.

In this rephrased grammar-discourse the question 'What should be learned first, mathematics or application of mathematics?' is rephrased to 'What should be learned first, the grammar of the language or the language?' Again totally convinced most people would say: 'Of course the language must be learned before the grammar of the language! Without a language, a grammar of a language has nothing to talk about!?'. The problem here of course is that now we are totally convinced about the opposite. This proves the postmodern point, that our beliefs and actions depend on the ruling phrasing and discourse, and can be changed through a rephrasing. Thus rephrasing mathematics to the grammar of the number-language frees us to think and to act otherwise.

Q: In what way?

R: We could compare the grammar of the number-language with the grammar of the word-language. The grammar of the word-language is successfully applied within the mother tongue by all, it is installed as a human right, but it is tacit knowledge developed through extensive use of the word-language. Very few are able to discursively justify why they are forming the sentences as they do. And if we make the rules discursive through teaching, many would drop out. Inspired by this analogy we can now ask: Maybe the grammar of the number-language, mathematics, can only become a human right if is developed as tacit knowledge through extensive use of the number-language? Meaning if we want mathematics to become a human right, maybe we should not teach it. In any case we should not teach it before its applications but through its applications. And since mathematics is applied to the number language and not the world, we should teach the number-language before its grammar. It is however impossible because we are frozen in a discourse, where the words as number-language, reckoning (regning in Danish, rechnung in German) and calculations are absent.

Q: But still mathematics is increasingly applied in modern society?

R: Is grammar increasingly applied in today's society? A grammar is applied to a language, which is applied to the world. It is the number-language, which is being increasingly applied, not its grammar.

Q: But isn't there a new discourse coming up talking about competences?

R: Yes, a new ruling object-discourse seems to be developing. Mogens Niss (1999) says that in order to prevent 'syllabilitis', the blindly following of content lists, we should instead describe mathematics in competence-terms. He then lists a series of mathematical competences all based on insight: Mathematical thinking, mathematical reasoning, etc. But again 'mathematics' is still used as a self-referring echo-word. And secondly, by being build upon insight 'competence' tries to unite two complementary words, qualifications and competence. Until now the biological fact that humans have both a human and a reptile brain has been reflected in language through the words 'qualifications' and 'competences'. Giddens calls it discursive and practical consciousness (Giddens 1984). Qualifications are discursive knowledge and insights resulting from discursive teaching. Competences are practical, tacit know-how. I can qualify my students, but I can't 'competence' them. To introduce a new word 'competence' by reusing the name 'competence' but giving it the meaning of 'qualification' seems deeply problematic. It becomes a meaningless echo-word impossible to rephrase. Is this an act of despair by an institution whose justification is fainting?

Q: So much about the object discourse. How about the content list of mathematics.

R: In the content list we also find an echo-tradition. Mathematics is presented from above derived from the mother concept set: number sets, expressions, equations, functions, and calculus. And the operation order is always addition, subtraction, multiplication, division etc. A rephrasing of this would unhide a 'mathematics from below' content list containing e.g.: multiplicity, quantities and qualities, number-language and word-language, differentiating degrees of many by number-names, counting by ones, counting bundles, stacking, rebundling, multiplication as division, ...

Q: Just a moment. What do you mean with multiplication as division? And how about addition?

R: Multiplicity is one of nature's five necessities. Number are names for different degrees of multiplicity being bundled in ten-bundles, although France and Denmark partly used twenty-bundles. A stack of four three-bundles can be rebundled or divided into six two-bundles or into one ten-bundle and two ones. In this way multiplication becomes rebundling into ten-bundles: Asking what is 7*4, is asking what happens, if seven four-bundles are rebundled or divided into ten-bundles; and the answer is two ten-bundles and eight ones: 7*4 = 28 = 2*10+8. If we want to rebundle in other bundle sizes than ten, we can no longer multiply, then we have to divide: 28 = ?·8, ? = 28/8 = 3 rest 4, so 28 = 3*8 + 4.

As to addition it is deeply problematic since addition is a false abstraction, in opposition to multiplication, which is a true abstraction, meaning it is true whenever you meet it. 7*4 is always

28 since multiplication is just describing a rebundling. With addition it seems a little difficult to claim that 2+3 = 5, when in most cases it is not: 2m+3cm=203cm, 2w+3d=17d, 23 is 23 and not 5 etc. It only has meaning to add if the units are the same: 2·m+3·cm = 2·100·cm+3·cm = (200+3)·cm = 203·cm. I.e. addition is only meaningful inside a parenthesis, that ensures that the units are the same. So multiplication and addition belong to different sides of the borderline between necessity and contingency.

Adding fractions suffers from the same problem as adding numbers without units. According to the principle of a common denominator 2/3+4/5 = 22/15. Adding numerators and denominators 2/3+4/5 = 6/8 is considered a meaningless mistake. However 2 cokes out of 3 cans and 4 cokes out of 5 cans total 6 cokes out of 8 cans, and not 22 cokes out of 15 cans. Now the meaningless becomes meaningful and vice versa.

So from this viewpoint multiplication is more fundamental than addition, which of course calls for a whole new curriculum design. A curriculum where the starting point is not a metaphysical concept set contaminated with syntax errors, but the social practices of bundling, stacking and totalling through counting and calculating.

Q: What do you mean with the syntax errors of set theory?

R: Geometry was successful to set up a system of axioms from which geometry could be derived. Algebra also tried, but the resulting set-theory is mixing elements and sets in its axioms thus committing the syntax error, that comes from mixing language and metalanguage: "This sentence is false", "The verb had to much to drink" etc. Both Russell and Gödel have shown, that mathematics can't prove itself. Still modern mathematics builds upon set theory making mathematics almost fundamentalism in Giddens' sense: "Fundamentalism may be understood as an assertion of formulaic truth without regard to consequences (Giddens 1994)."

In the textbooks we also find echo-definitions, echo-names and killer-equations. The definition of a function is an echo-statement: "A function is an example of a relation between two sets that assigns to each element in one set one and only one element in the other set." This phrasing is an echo of the university's definition and is echoed in other textbooks nationally and internationally. But using Foucault's genealogy going back in time to when the name was born, we find another phrasing when Euler says, that a function i a name for a calculation with a variable number, or more precisely: "A function of a variable quantity is an analytic expression composed in any way whatsoever of the variable quantity and numbers or constant quantities (Euler 1748)." The modern function concept also contains syntax errors. f(x) = x+3 means: Let f(x) denote the calculation x+3 with x as a variable number. But then f(x) = 7 means: Let f(x) denote the calculation 7 with x as a variable number. And f(2) = 5 means: Let f(2) denote the calculation 5 with 2 as a variable number. Both cases are syntax errors violating Russell's type theory by mixing language and metalanguage. Calculations belong to the language level, functions to the metalanguage level. Calculations are calculating numbers, function are labelling calculations. Historically 'function' is born just after calculus. And with good reason. Before calculus calculations had basically been used to predict rebundling results thus only operating on numbers. Calculus is used to predict change: Can we calculate the change of a calculation?

Calculus thus operates on calculations with changing quantities, and this is precisely how Euler defined a function.

By using the term ‘example of’ the modern definition of a function is relating an abstract concept upwards to something more abstract thus presenting mathematics from above as examples or echoes of the mother concept set. Such definition can only be echoed ant not be rephrased, since you can only have one mother. The meaning structure of the modern definition is ‘bublibub is an example of bablibab’, which can only be learned by heart as an echo. In short, coming from above the modern mathematics is forced to become echo-statements forcing teachers to practice echo-teaching and forcing students to learn by echo-learning. This is no problem in a modern echo-society where students get their identity by echoing. But it is a problem in a postmodern society, where students are self-story builders asking for stories that can be related to their existing story.

On the contrary by using the term ‘name for’ the Euler definition is relating an abstract concept downwards to something less abstract, thus presenting mathematics from below, allowing indefinite rephrasings by relating it to other examples of calculations with variable numbers; allowing teachers to practice example-teaching showing how mathematics is constructed through abstraction; and allowing students to extend their self-stories. The meaning structure of the Euler definition is ‘bublibub is a name for a calculation’ making the ideal type of a postmodern learner say: ‘I know what a calculation is. I didnot know that a calculation could be called a function. But I know now since you have told me something I don’t know about something I know. Now tell me something I don’t know about functions’.

Q: But in most textbooks the function concept is introduced after many practical examples. In this way the concept seems to be abstracted from below!

R: It seems so, yes. But the examples are all means, sugar around the pill: the definition, which is phrased from above. All examples can be otherwise, but the definition can’t. The examples are chosen because they are examples or applications of the concept to be taught. If the examples were the object, we have already seen, that e.g. Euler made a different abstraction than the modern one. But of course we can say that both definitions are naming something. In the case of Euler we name a socially constructed classification of concrete objects, calculations; a classification made socially important by calculus. The other definition is naming an abstract structure: a relation between two sets. So the first definition is a nominalist one naming concrete objects, and the second a structuralist one naming an abstract structure, that after a rephrasing appears to be a triviality: Of course a calculation can only give one result!

Q: You also mentioned echo-names and killer-equations.

R: Echo-names from above occur all over mathematics just waiting to be rephrased from below. Equations can be rephrased to calculation stories. Functions can be rephrased to calculations with variable quantities. Linear and exponential functions can be rephrased to change by adding and multiplying. Differential equations can be rephrased to change calculation stories. Differentiable can be rephrased to locally linear. Continuous can be rephrased to locally

constant. Limits can be rephrased to about-numbers. Numbers can be divided into unit-numbers and per-numbers. Unit-numbers are e.g. $, m, s, kg etc. Per-numbers are e.g. $/m, m/s or m/100m = %.

And equations can be divided into killer-equations and practice equations. Killer-equations are equations you never meet outside the classroom, and whose only application is to kill the interest of the students. Killer-equations come from above where an equation is an example of an equality relation between two expressions allowing e.g. arbitrary polynomial fractions to be equalled. Rephrased killer-equation become practice-equations allowing only equation to arise as calculation stories from social practices as e.g. 2x+4=10 arising from the question "2 kg at ?$/kg including a 4$ fee total 10$".

In the word-language we always use full sentences to evaluate the truth of a sentence. Instead of 'green' we say e.g. 'This table is green'. For the same reason also the number-language should use full sentences from day one, saying 'T = 3x5' instead of just '3x5' thus specifying both what is being calculated and the calculation. Standard formulations from first year mathematics as '3+5' is a third order abstraction being abstracted from reality, from the units and from the equation. Such abstractions construct mathematics as encapsulated and create serious problems to the students when they later meet wor(l)d problems.

Also solving equations contains rituals an echo-phrasings. A Top-Down approach will phrase '$2+3\cdot x = 14$' as an equation only solvable after equation theory has been introduced thus showing the relevance and applicability of modern abstract algebra in performing the neutralising 'do the same on both sides' method. Alternatively a Bottom-Up approach will phrase '$2+(3\cdot x) = 14$' as a calculation story reporting both a calculation process ($2+3\cdot x$) and a calculation product (14), thus accessible together with calculations and solvable by reversing or walking the calculations:

x	– (·3) –>	3·x	– (+2) –>	2+(3·x) = 14
x = 4	<– *(/3)* –	12	<– *(–2)* –	14

Q: To sum up, how would a postmodern mathematics curriculum look like?

R: Schools, education and curricula should meet human needs. To decide what we need we have to go back to the zone of necessity. We have holes in the head because we need food for the body and stories for the brain. So school could be rephrased as a story-house similar to other story houses as the church and the global TV. And education could be rephrased to storytelling. In modern echo-society students needed echo-stories to build echo-identity. In a postmodern dilemma-society individuals have to construct their own self-identity by building self-stories. Hence a postmodern school should provide stories from below with known concrete subjects, telling the students something they don't know about something they know, thus enabling them to extend their existing self-story.

Q: And where does mathematics and education come in?

R: Mathematics consists of Geometry and Algebra, so we could ask: what are geometry and algebra stories about? With geometry it is easy: translated from Greek it means 'earth measuring'. So Geometry can be introduces in school as stories about earth-measuring together of course with a lot of exercises in dividing and measuring earth. With algebra it is more difficult. Very few textbooks or people know that algebra means 'reunite' in Arabic. In all money based cultures there is a cultural practice called totalling: If we buy several items we don't have to pay them individually, we can ask for the total. And we don't have to pay exactly the total; we can pay more, which then is split into the price and the change. To check we reunite the total. In a way we can say that geometry and algebra are stories about how we divide the earth and what it produces.

Q: But how about calculation?

R: Algebra is about uniting and dividing the total. And the total can be united through in four different calculation types depending upon the nature of the numbers to be united. Multiplication and exponentials total constant unit- and per-numbers. Addition and integration total variable unit- and per-numbers. In reverse division and root and logarithm split totals into constant unit- and per-numbers. And subtraction and differentiation split totals into variable unit- and per-numbers (Tarp 2000 a).

CONCLUSION

Q: So to sum up, what can you say about postmodern research and mathematics education in few sentences.

R: Postmodernism accepts the contingency of all phrasings, and sees humans clientified and frozen by echo-phrasings. Postmodern contingency re-search searches for echo-words in ruling discourse to create counter discourses by re-phrasing these echo-words. In a modern industrialised echo society students get identity through echoing the tradition making echo-mathematics from above rational and efficient. In a postmodern information dilemma society the students have to build up self-identity through self-stories build upon meaning and authenticity, making mathematics from below rational and efficient. Mathematics education is frozen in an Mathematics&Application discourse that forces it to practice echo-teaching from above and prohibits it to see the hidden alternative, a Mathematics&Grammar discourse seeing Geometry and Algebra as stories about two fundamental social practices: How two divide the earth and how it produces, and how two name different degrees of multiplicity.

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