Proposal for a Research Project in Icon-counting and NextTo Addition

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”How old will you be next time?” I asked the child. “Four”, he answered and showed me four fingers. “Four, you said?” I asked and showed him four fingers held together two by two. “No, that is not four, that is two twos!” the child replied thus insisting upon the difference between four ones and two twos. Likewise, preschool children have no difficulties counting in other units than ten, even if they only learn how to count in tens. This observation motivates the following question:

What kind of mathematical learning takes places when children count in icons less than ten?

The methodology comes from the two Enlightenment republics by mixing French skepticism and American pragmatism. First postmodern contingency research will deconstruct the ruling traditions by uncovering hidden differences that might make a difference when tested by designing different micro-curricula, in this case for the last year in preschool mathematics.

When testing the design in a group of students, grounded theory resonating with Piaget assimilation/ accommodation is used to gather observations and create categories. Finally, the categories are validated or refined by tests in other student groups.

The following micro-curricula use activities with concrete material to obtain its learning goals in accordance with Piaget’s principle ‘greifen vor begrifen’ (grasp to grasp). In the first, children learn to use sticks to build the number icons up to nine, and to use strokes to draw them, thus realizing there are as many sticks and strokes in the icon as the number it represents, if written less sloppy. In the second, children learn to count a given total in icons manually, using an abacus and by using a calculator. In the third, children learn to recount a total in the same unit. In the fourth, children learn to recount a total in a different unit. In the fifth, children learn to add two icon-numbers on top of each other. In the sixth, children learn to add two icon-numbers next to each other. In the seventh, children learn to reverse on-top addition. And in the eights, children learn to reverse next-to addition

As to concrete material, anything goes in the first micro-curriculum. The others will use sticks and strokes, beads on an abacus, LEGO-like blocks and squares, and a calculator respecting the priority of the operations. Fingers, pegs on a pegboard and other concrete material might also be used.

Some of the curricula can be tested using silent education where the teacher is allowed to demonstrate and guide through actions, but not through words; or by using words form a foreign language not understood by the child.

Curricula:

Micro-curriculum 1. Creating Icon-numbers

A. Deconstructing the tradition

The tradition sees digits and letters as socially constructed symbols receiving meaning through their use. Skepticism would ask if this is indeed the case. Both describe something in the world, sounds and many-ness. However, there is no universal a-sound, b-sound etc. whereas there are many specific examples of four-ness, five-ness etc. and the digit 4 can be seen as four sticks rearranged into a four-icon thus directly showing the degree of many it represents.

So, letters are symbols that do not label something universal but receive their meaning through local use, thus being socially constructed, both as to the sound they represent and as to the symbols used. Digits, on the other hand, are icons containing as many sticks or strokes as they label if written in a special way, that might be socially constructed, but that is globally recognized and that label something that is universal and not socially constructed, different degrees of Many.

To emphasize this fundamental difference between digits and letters, the traditional digits are deconstructed and replaced by digits that more clearly show the degree of many they label; and with a similarity, which does not disturb the meaning of traditional digits. Instead, these can be seen as a quick and slightly sloppy way to write the 'natural' number digits, clearly showing their degree of Many.

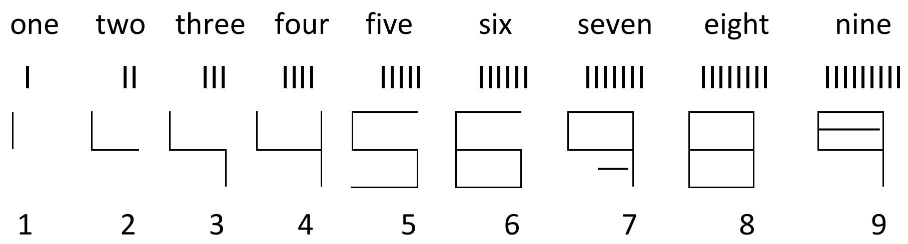


Figure 101. Creating a 5-icon out of five sticks, etc.

Also the educational tradition presenting digits as arbitrary socially constructed symbols of the same kind as letters can be deconstructed by allowing preschool children to experience themselves the construction of digits from 0 to 9 that contains as many things as they represent.

This deconstruction thus raises the following question:

What happens to children when allowed to create number-icons themselves?

B. Designing a Micro-curriculum

On the floor the children place six hula hoop rings next to each other as six different lands: empty-land, 1-land, 2-land, 3-land, 4-land and 5-land shown by the corresponding number of chopsticks on a piece of paper outside the ring. Each child is asked to find a thing to place in 1-land, and to explain why. Then they are asked to turn their thing so it has the same direction as the chopstick. Finally the group walks around the room and points out examples of 'one thing' always including the unit, e.g. 1 chair, 1 ball, etc.

In the same way each child finds a thing to place in 2-land. The instructor shows how to rearrange two chopsticks to form one 2-icon. The children are asked to pick up two sticks and do the same; and to draw many examples of the 2-icon on a paper discussing with the instructor why the 2-icon on the wall is different from the ones they draw. Now the children are asked to rearrange their 2s in 2-land so they have the same form as the 2-icon. Again the group walks around the room and points out examples of 'two things' that is also called 'one pair of things'. This is now repeated with 3-land where three things are called one triplet.

Before going on to 4-land the instructor asks the children to do the same with empty-land. Since the empty-icon cannot be made by chopsticks the instructor ask for proposals for an empty-icon hoping that one or more will suggest the form of the ring, i.e. a circle. Again the group walks around the room to try to locate examples of 'no things' or zero things.

Now, the activity is repeated with 4-land where the children are asked to suggest an icon for four made by four sticks. When summing up the teacher explains that the adults have rejected the square since it reminds too much of a zero, so the top stick is turned and placed below the square to the right. Here the children are asked to rearrange their 4s in 4-land so they have the same form as a square, and as the 4-icon. And again, the group walks around the room and points out examples of ‘four things’ that is also called ‘a double pair’.

Now the activity is repeated with 5-land. Here the instructor asks the children to suggest an icon for five made by 5 sticks. When summing up the teacher explains that the adults have decided to place the five stick in an s-form. When walking around the room to point out examples a discussion is initiated if ‘five things’ is the same as a pair plus a triplet, and as a double pair plus one. This activity can carry on to design icons for the numbers from six to nine realizing that the existing icons can be recycled if bundling in tens.

Finally, the instructor invites the children to experience that when counting in 5s we do not use the word five or the icon five since the number sequence becomes 1, 2, 3, 4, Bundle, 1B1, 1B2, 1B3, 1B4, 2B, 2B1 etc. So when counting in tens we do not need a ten-icon.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these. In this case, the different ways to construct icons allow for additional observations as to preferences and performances.

Examples of observations

When trying out the micro-curriculum, the preschool teachers are asked to look for examples of re-cognition and new cognition. One teacher noticed the confusion created by asking the children to bring things to empty-land. It disappeared when one child was asked what he had just put into the ring and answered no elephant. Now all of the children were eager to put no cars, no planes etc. into the ring. Later the teacher witnessed children discussing why the 3-icon was not a triangle, and later used the word four-angle for the square. Also this teacher noticed that some children began to use their fingers instead of the chopsticks. Under the walk around the room a fierce discussion about cheating broke out when a child suggested that clapping his hand three times was also an example of three things. Its not, another child responded. It is. No its not! Why not? Because you cannot bring it to 3-land! Let’s ask the teacher! After telling about space and time, children produced other examples as three knocks, three steps, three rounds around a table, three notes. Other children began to look at examples of threes at their own body soon finding three fingers, three parts on a finger, and three hands twice when three children stood side by side and the middle one lent out his two hands to his neighbors.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Photos from the MATHeCADEMY.net stand at the Mathematics Biennale in Sweden 2014

Notice the difference between its Piaget based approach using concrete real world materials and the neighbor stand using a Vygotsky approach with symbols.

Micro-curriculum 2. Counting in Icons

A. Deconstructing the Tradition

Traditionally, counting takes place by stacking bundles of tens where e.g. 345 means a total of 3 ten-tens and 4 tens and 5 ones. Using ten as bundle-size is based on the biological fact that we have ten fingers.

Skepticism could point out that only accepting ten as a bundle-size will give totals the same unit. This might hide learning possibilities coming from changing units when counting in different icon-bundles less than ten; and that might be problematic since changing units, also called proportionality or linearity, is a core part of mathematics. Furthermore, a calculator cannot predict the counting result since ten does not have its own icon on a calculator.

This deconstruction raises the question:

'*What learning possibilities occur if allowing preschool children in to count in icons less than ten?*

B. Designing a Micro-curriculum

It is a natural fact that we live in space and in time. To include both when counting, we can introduce two different ways of counting: counting in space, geometry-counting, and counting in time, algebra-counting.

Counting in space, we count blocks and report the result on a ten-by-ten abacus in geometry-mode, and with squares.

Counting in time, we count sticks and report the result on a ten-by-ten abacus in algebra-mode, and with strokes.

<u>Counting task 1:</u> '6 1s is how many 2s?'

Using geometry-counting, 6 blocks are placed on a table and counted in 2s by taking away a 2-bundle 3 times to be stacked on top of each other, and reported orally as '6 1s can be counted as 3 2s'. Then the stack of 3 2s is written down as T = 3 x 2 = 3 2s using the multiplication symbol as an icon showing the three times lifting up of the 2s. To use an abacus in geometry mode, again 6 beads are moved to the right on the first from below separated from the above line by a rubber band. Now we move 1 2s to the left and 1 2s to the right on the second line. Again we move 1 2s to the left and 1 2s to the right on the third line. And, again we move 1 2s to the left and 1 2s to the right on the fourth line. Again, the result is reported as '6 1s can be counted as 3 2s'. On a squared paper we draw the total as 3x2 squares.

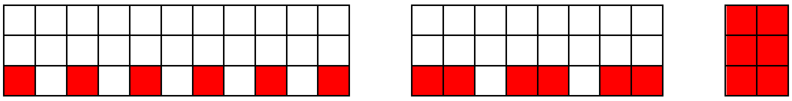


Figure 201. Counting 6 1s as 3 2s by bundling and stacking squares

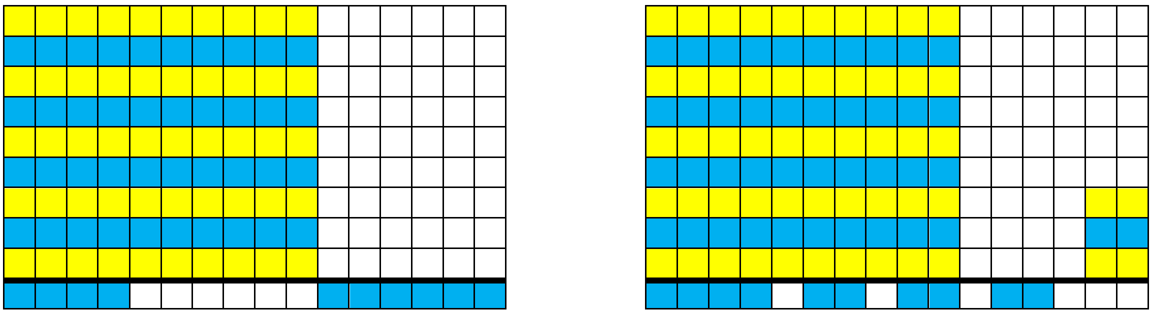


Figure 202. Counting 6 1s as 3 2s on a western ten by ten abacus in geometry mode

Using algebra-counting, 6 sticks are placed on a table and counted in 2s by taking away a 2-bundle 3 times. Orally we report this as '6 1s can be counted as 3 2s'. Now for each bundle and each single, a bead is placed in two cups, a left bundle-cup and a right single-cup. Then the result is written down using 'cup-writing' and 'decimal-writing' with a dot to separate the bundles from the unbundled, i.e. as 'T = 3)0) 2s = 3.0 2s. On an abacus in algebra-mode, 6 beads are moved to the right on the bottom line. For each 2-bundle moved to the left one bead is moved to the right on the above line, again showing that 6 1s can be counted as 3 2s.

I I I I I I → II II II → ***)) → 3) 0) 2s → 3.0 2s

Figure 203. Counting 6 1s as 3 2s by sticks, cup-writing and decimal writing

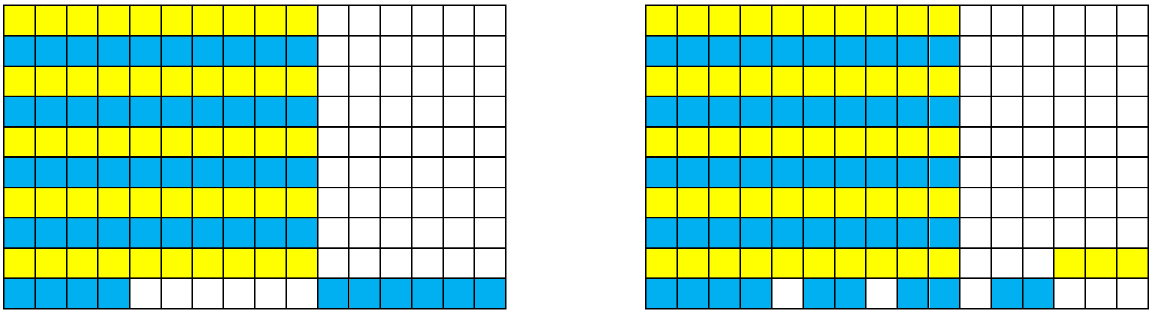


Figure 204. Counting 6 1s as 3 2s on a western ten by ten abacus in algebra mode

Finally, we use the calculator to predict the result. The calculator has two icons for taking away, subtraction showing the trace left when taking away just once, and division showing the broom wiping away several times. So entering ‘6/2’ means asking the calculator ‘from 6 we take away 2s how many times?’ The calculator gives the result ‘3’. To test the result we enter ‘6 – 3x2’ to ask the calculator ‘from 6 we take away 3 2s leaving what?’ As expected, the answer is 0.

6 / 2	3
6 – 2 x 3	0

Figure 205. Using a calculator to predict the result of counting 6 1s in 2s, and to check for singles

<u>Counting task 2:</u> ‘6 1s is how many 3s?’, is performed in a similar way, but first we ask the calculator to predict the result.

<u>Counting task 3:</u> ‘8 1s is how many 2s?’ is performed in a similar way, but first we ask the calculator to predict the result.

<u>Counting task 4:</u> ‘8 1s is how many 4s?’ is performed in a similar way, but first we ask the calculator to predict the result.

<u>Counting task 5:</u> ‘7 1s is how many 2s?’

Using geometry-counting, 7 blocks are placed on a table and counted in 2s by taking away a 2-bundle 3 times to be stacked on top of each other leaving 1 unbundled to the right, and reported orally as ‘7 1s can be counted as 3 2s and 1’. Then the stack of 3 2s is written down as T = 3.1 2s To use an abacus in geometry mode, again 7 beads are moved to the right of the bottom line. Now we move 1 2s to the left and 1 2s to the right on the first line above. Again we move 1 2s to the left and 1 2s to the right on the second line above. And, again we move 1 2s to the left and 1 2s to the right on the third line above leaving 1 unbundled on the bottom line. Again, the result is reported as ‘7 1s can be counted as 3 2s and 1’. On a squared paper, we draw the total as 3x2 squares with an additional square to the right in a different colour.

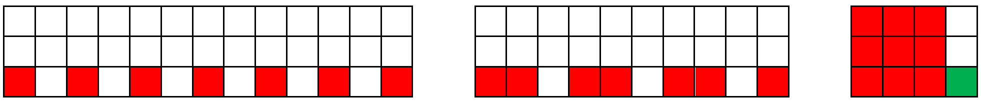


Figure 206. Counting 7 1s as 3.1 2s by bundling and stacking squares

Using algebra-counting, 7 sticks are placed on a table and counted in 2s by taking away a 2-bundle 3 times leaving 1 stick unbundled. Orally we report this as ‘7 1s can be counted as 3 2s and 1’. Now for each bundle and each single, a bead is placed in two cups, a left bundle-cup and a right single-cup. Then the result is written down using ‘cup-writing’ and ‘decimal-writing’ with a dot to separate the bundles from the unbundled, i.e. as ‘T = 3)1) 2s = 3.1 2s. On an abacus, 7 beads are moved to the right of the bottom line. For each 2-bundle moved to the left one bead is moved to the right on the above line, again showing that 7 1s can be counted as 3 2s and 1, i.e. as 3.1 2s.

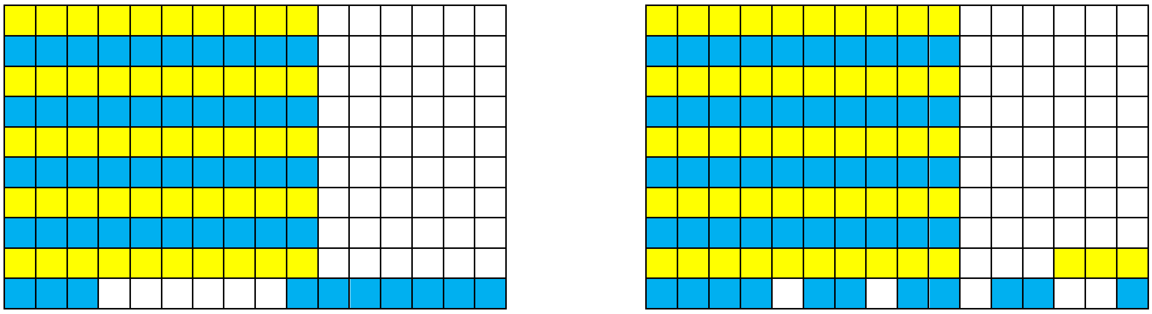


Figure 207. Counting 7 1s as 3.1 2s on a western ten by ten abacus in algebra mode

Finally, we use the calculator to predict the result. Entering '7/2' means asking the calculator 'from 7 we take away 2s how many times?' The calculator gives the result '3.some'. To see how much is left unbundled we enter '7 – 3x2' to ask the calculator 'from 7 we take away 3 2s leaving what?'. As expected, the answer is 1. A display showing that 7 - 3x2 = 1 indirectly predicts that 7 can be counted as 3 2s and 1.

7 / 2	3.some
7 – 3 x 2	1

Figure 208. Using a calculator to predict the result of counting 7 1s in 2s, and to check for singles

Counting task 6: '7 1s is how many 3s?' is performed in a similar way, but first we ask the calculator to predict the result.

Counting task 7: '7 1s is how many 5s?' is performed in a similar way, but first we ask the calculator to predict the result.

Rolling dices can inspire additional counting tasks, e.g. by adding 1 or 2 to the numbers, so that rolling a 2 and a 5 creates the counting tasks '5 is how many 2s?', '6 is how many 3s?', '7 is how many 4s?', etc.

B2. Designing an Additional Micro-curriculum

Most languages use ten as the bundle size calling 'bundle and one' for 'ten-one' and 'bundle and two' for 'ten-two', etc. English instead uses the words 'eleven' and 'twelve' meaning 'one-left' and 'two-left' before passing on to thirteen, ten-three. Inspired by this, nine and eight could be called 'less 1' and 'less 2' to be symbolized as '-1' and '-2' thus creating the following counting sequence to be practiced counting fingers and counting beads on an abacus:

1, 2, 3, 4, 5, les 4, less 3, less 2, less 1, ten.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to perform counting in icons, algebra-counting in time, geometry-counting in space and calculator prediction, allow for additional observations as to preferences and performances.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Micro-curriculum 3. Re-counting Icon-numbers in the Same Icon

A. Deconstructing the Tradition

Traditionally, recounting does not takes place when using ten as the only allowed bundle-size.

Skepticism could point out that this will provide one and only one way to count a total, which will hide the learning possibilities coming from recounting a total in the same unit to deal with overloads, created when adding and needed when subtracting totals; and that might be problematic since adding and subtracting are core part of mathematics.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to recount icon-numbers in the same icon?

B. Designing a Micro-curriculum

<u>Counting task 1:</u> 'Recount 3 4s in 4s?'

Using geometry-counting, a stack of 3 4-bundles is placed on a table. One is split into 1s and placed vertically to the right of the stack. Orally we report this as '3 4s can be recounted as 2 4s and 4'. On an abacus, 4 beads are moved to the right on the lines 2 to 4 from below reserving the bottom line for the singles. Moving the top 4-bundle to the left allows moving 4 singles to the right on the bottom single-line, again showing that 3 4s can be recounted as 2.4 2s. On a squared paper we draw the total as a stack of 2x4 squares and 4 1s to the right of the stack. Likewise, we can show that 3 4s can be recounted to 1.8 4s.

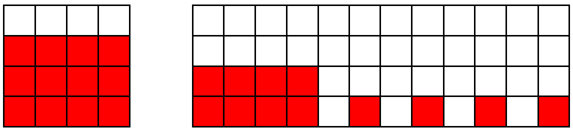


Figure 301. Recounting 3 4s as 2.4 4s by bundling and stacking squares

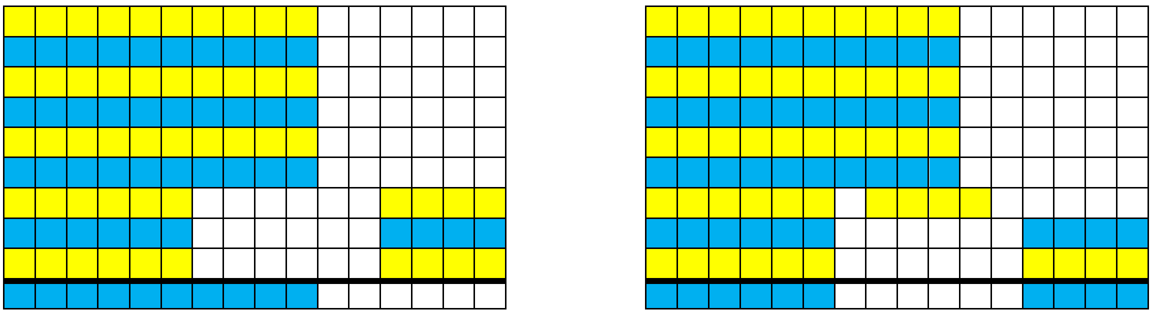


Figure 302. Recounting 3 4s as 2.4 4s on a western abacus in geometry mode

Using algebra-counting, a total of 3 4s are placed on a table. One if the 4-bundles is split into 1s. Orally we report this as '3 4s can be recounted as 2 4s and 4'. Now for each bundle and each single, a bead is placed in two cups, a left bundle-cup and a right single-cup. Then the result is written down using 'cup-writing' and 'decimal-writing' with a dot to separate the bundles from the unbundled, i.e. as 'T = 3 4s = 2)4) 4s = 2.4 4s. On an abacus, 3 bundle-beads are moved to the right on the bundle-line. Moving one bundle-bead to the left allows moving 4 beads to the right on the single line, again showing that 3 4s can be recounted as 2.4 4s. Likewise, we can show that 3 4s can be recounted to 1.8 4s.

IIII IIII IIII → IIII IIII I I I I → **)****) → 2) 4) 4s → 2.4 4s

Figure 303. Recounting 3 4s as 2.4 4s by sticks, cup-writing and decimal writing

IIII IIII I I I I → ***)****) → 3) -4) 4s → 3.-4 4s

Figure 304. Recounting 2 4s as 3 less 4 4s by sticks, cup-writing and decimal writing

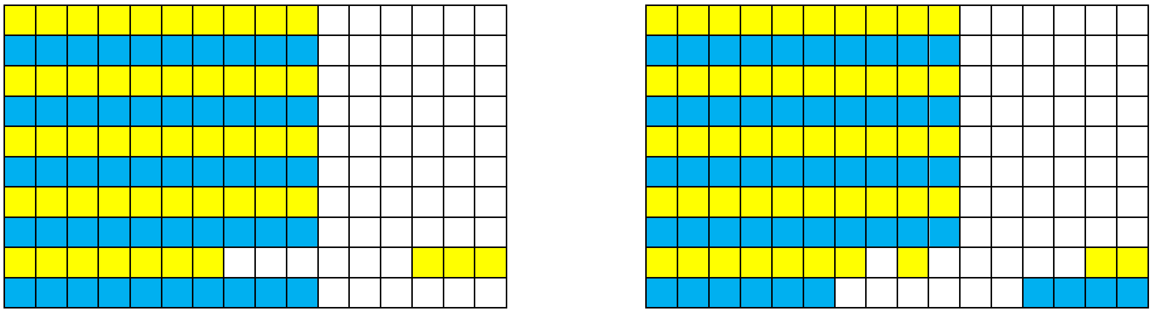


Figure 305. Recounting 3 4s as 2.4 4s on a western abacus in algebra mode

When recounting in the same unit, a calculator cannot be used to predict the result. Instead, cups or cup-writing can be used to recount in the same unit by showing directly how removing one 4s in the bundle-cup allows adding 4 1s in the single cup

3 4s: [*** |] [** | ****] [* | ********]

3 4s: 3) 0) 4s = 3-1) 0+4) 4s = 2) 4) 4s = 2.4 4s = 2-1) 4+4) 4s = 1) 8) 4s = 1.8 4s

Figure 305. Recounting 3 4s as 2.4 4s and 1.8 4s using cups and cup-writing

Vice versa, removing 4 1s from the single cup allows adding 1 bundle in the bundle-cup

1.8 4s: [* | ********] [** | ****] [*** |]

1.8 4s: 1) 8) 4s = 1+1) 8-4) 4s = 2) 4) 4s = 2.4 4s = 2+1) 4-4) 4s = 3) 0) 4s = 3 4s

Figure 306. Recounting 1.8 4s as 2.4 4s and 3 4s using cups and cup-writing

<u>Counting task 2:</u> 'Recount 2 3s in 3s?' is performed in a similar way.

Rolling dices can inspire additional counting tasks, e.g. by adding 1 or 2 to the numbers, so that rolling a 2 and a 4 creates the counting tasks 'Recount 4 2s in 2s?', 'Recount 5 3s in 3s?'etc.

Note that recounting in small units will avoid numbers above nine.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to perform counting in icons, algebra-counting in time, geometry-counting in space allow for additional observations as to preferences and performances.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Micro-curriculum 4. Re-counting Icon-numbers in a Different Icon

A. Deconstructing the Tradition

Traditionally, recounting does not takes place when using ten as the only allowed bundle-size.

Skepticism could point out that this will give all totals the same unit, which will hide the learning possibilities coming from changing units when counting in different icon-bundles less than ten; and that might be problematic since changing units, also called proportionality or linearity, is a core part of mathematics. Furthermore, where it cannot predict the counting result with no ten button, now a calculator gets a central role as a recount-predictor.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to recount icon-numbers in a different unit?

B. Designing a Micro-curriculum

Counting task 1: '3 4s is how many 5s?'

Using geometry-counting, a stack of 3 4s is placed on a table. The top bundle is changed to 1s in the single stack to the right and twice a stick is removed from there to enlarge the 4- bundles to a 5 bundle. This shows that '3 4s can be recounted as 2.2 5s.' To use an abacus in geometry mode, 3 4s are moved to the right leaving the bottom line for the singles. From the top, a 4-bundle is moved to the left to change the 4- bundles above to 5-bundles. Again, the result is reported as '3 4s can be recounted as 2.2 5s'. On a squared paper we draw the total as a 2x5 block with a 2x1 block to the right.

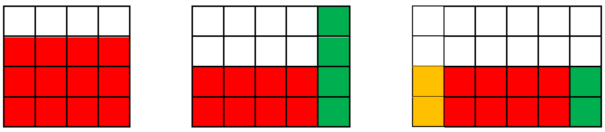


Figure 401. Recounting 3 4s as 2.2 5s by bundling and stacking squares

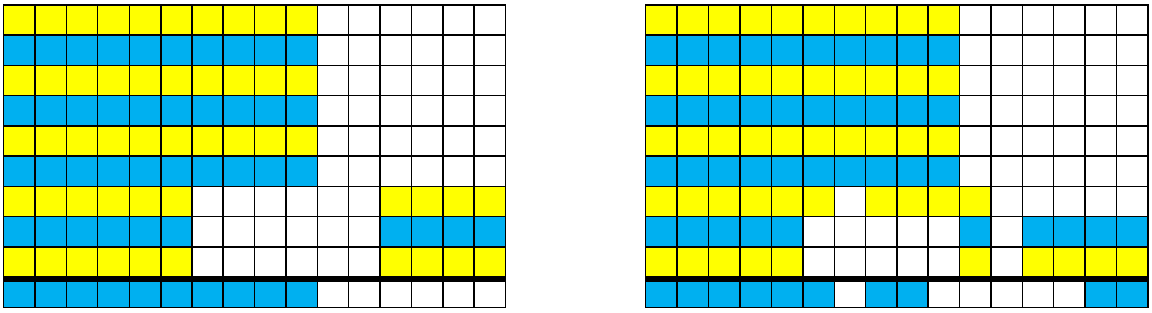


Figure 402. Recounting 3 4s as 2.2 5s on a western abacus in geometry mode

Using algebra-counting, a total of 3 4s is placed on a table and split into 1s. Now the total is counted in 5s by taking away a 5-bundle 2 times. Orally we report this as '3 4s can be counted as 2 5s and 2'. Now for each bundle and each single, a bead is placed in two cups, a left bundle-cup and a right single-cup. Then the result is written down using 'cup-writing' and 'decimal-writing' with a dot to separate the bundles from the unbundled, i.e. as 'T = 2)2) 5s = 2.2 5s. On an abacus, 3 beads are moved to the right on the bundle-line. For each bundle moved to the left, 4 beads are moved to the right on the single line. Moving 2 beads to the left on the bundle-line allows moving 8 beads to the right on the single-line. This allows moving 5 beads to the left on the single line and to move 1bead to the right on the top-line counting the 5-bundles. Moving the last 3-bundle to the left gives 7 beads on the single line, from which 5 1s can be changed to 1 5s on the top line.

Again, we see that 3 4s can be recounted as 2.2 5s.

IIII IIII IIII → IIIII IIIII I I → **)**) → 2) 2) 5s → 2.2 5s

Figure 403. Recounting 3 4s as 2.2 5s by sticks, cup-writing and decimal writing

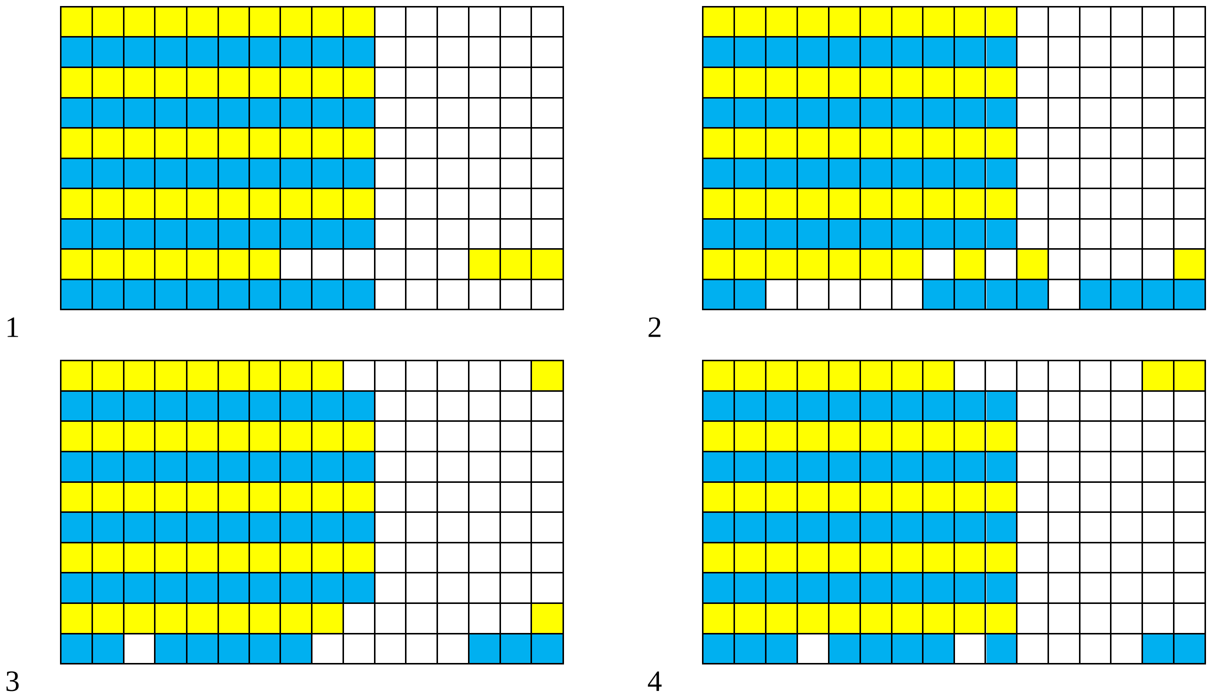


Figure 404. Recounting 3 4s as 2.2 5s on a western abacus in algebra mode

Finally, we use the calculator to predict the result. Entering ‘3x4/5’ means asking the calculator ‘from 3 4s we take away 5s how many times?’ The calculator gives the answer ‘2.some’. To see what is left we enter ‘3x4 – 2x5’ to ask the calculator ‘from 3 4s we take away 2 5s leaving what?’ As expected, the answer is 2. A display showing that 3x4 – 2x5 = 2 indirectly predicts that 3 4s can be recounted as 2 5s and 2.

3 x 4 / 5	2.some
3 x 4 – 2 x 5	2

Figure 405. Using a calculator to predict the result of recounting 3 4s in 5s, and to check for singles

<u>Counting task 2</u>, ‘4 5s is how many 6s?’, is performed in a similar way, but first we ask the calculator to predict the result.

<u>Counting task 3</u>: ‘3.2 4s is how many 5s?’ is reduced to ‘3 4s is how many 5s?’, adding 2 singles in the end: ‘Since 3 4s can be recounted to 2.2 5s, 3 4s and 2 can be recounted to 2.4 5s.’

<u>Counting task 4</u>, ‘4.3 5s is how many 6s?’ is performed in a similar way, but first we ask the calculator to predict the result.

Rolling dices can inspire additional counting tasks, e.g. by adding 1 or 2 to the numbers, so that rolling a 2 and a 4 and a 5 creates the counting tasks ‘Recount 2 4s in 5s?’, ‘Recount 3 5s in 6s?’, etc.

Note that recounting in a bigger unit will avoid numbers above nine.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these. In this case, the three ways to recount totals in a different icon, algebra-counting in time, geometry-counting in space and calculator prediction allow for additional observations as to preferences and performances.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Micro-curriculum 5. Adding Icon-numbers OnTop

A. Deconstructing the Tradition

Traditionally, we add totals on top of each other creating no problems when counted in tens except for dealing with overloads by carrying.

Skepticism could point out that this does not give a method to deal with nor experience with on-top addition of totals counted in different units, as e.g. adding 2 3s and 4 5s on top as 3s or as 5s. And that might be problematic since changing units, also called proportionality or linearity, is a core part of mathematics.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to perform on-top addition of icon-numbers.

B. Designing a Micro-curriculum

<u>Adding task 1</u>: '4 5s and 2 3s total how many 5s?'

To be added on-top, the units must be the same, so the 2 3s must be recounted in 5s giving 1.1 s that added to the 4 5s gives a grand total of 5.1 5s.

On an abacus in geometry mode, leaving the bottom line empty, a stack of 4 5s is moved to the right and a stack of 2 3s is moved to the middle. Now, the 2 3s is changed to 6 1s on the bottom line allowing one additional 5s to be moved to the top of the stack of 5s to show the grand total is 5.1 5s.

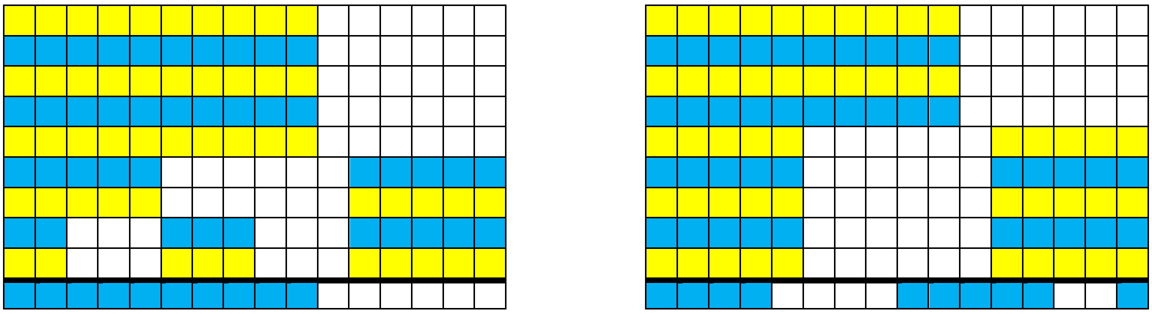


Figure 501. Adding 2 3s to 4 5s as 5.1 5s on a western abacus in geometry mode

On an abacus in algebra mode and split in the middle by a horizontal rubber band, the 4 5s are moved to the right on the bundle line below the band, and the 2 3s on the bundle line above the band. Now the 2 3s above the band is changed to 6 1s below, where moving 5 1s to the left allows moving 1 5s to the right, showing the grand total to be 5.1 5s.

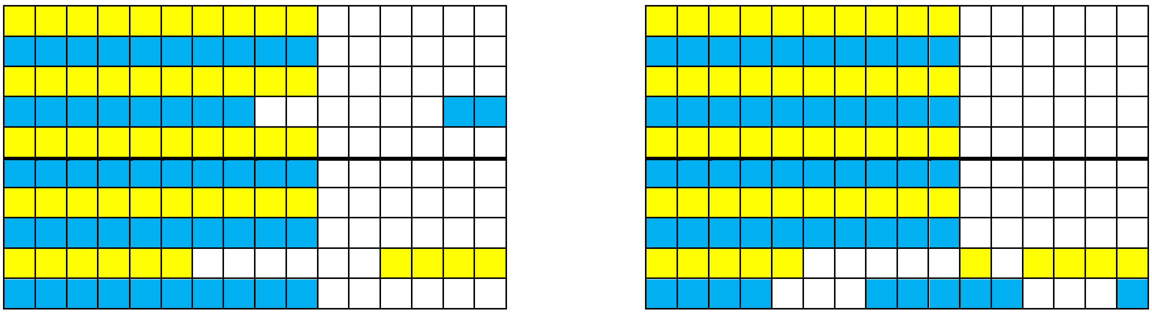


Figure 502. Adding 2 3s to 4 5s as 5.1 5s on a western abacus in algebra mode

When using a calculator to predict the result, we include the two totals in a bracket before counting in 5s: Asking '(4x5 + 2x3)/5' gives the answer 5.some. Then asking '(4x5 + 2x3) – 5x5 gives the answer 1. So the grand total is 5.1 5s. A display showing that (4x5 + 2x3) – 5x5 = 1 indirectly predicts that 2 3s can be added to 4 5s as 5.1 5s.

(2 x 3 + 4 x 5)/ 5	5.some
(2 x 3 + 4 x 5) – 5 x 5	1

Figure 503. Using a calculator to predict the result of adding 2 3s to 4 5s as 5.1 5s

<u>Adding task 2</u>: '4.4 5s and 2.2 3s total how many 5s?'

To be added on-top, the units must be the same, so 2.2 3s must be recounted in 5s giving 1.3 5s that added to the 4.4 5s gives a grand total of 5.7 5s that can be recounted to 6.2 5s to remove the overload.

On an abacus in geometry mode, a stack of 4.4 5s is moved to the right and a stack of 2.2 3s is moved to the middle. Now, the 2.2 3s is changed to 1s on the bottom line allowing 2 additional 5s to be moved to the top of the stack of 5s to show the grand total is 6.2 5s.

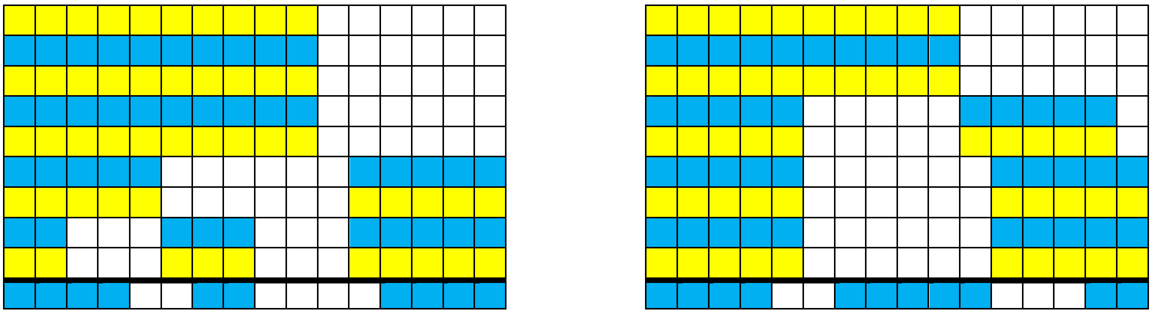
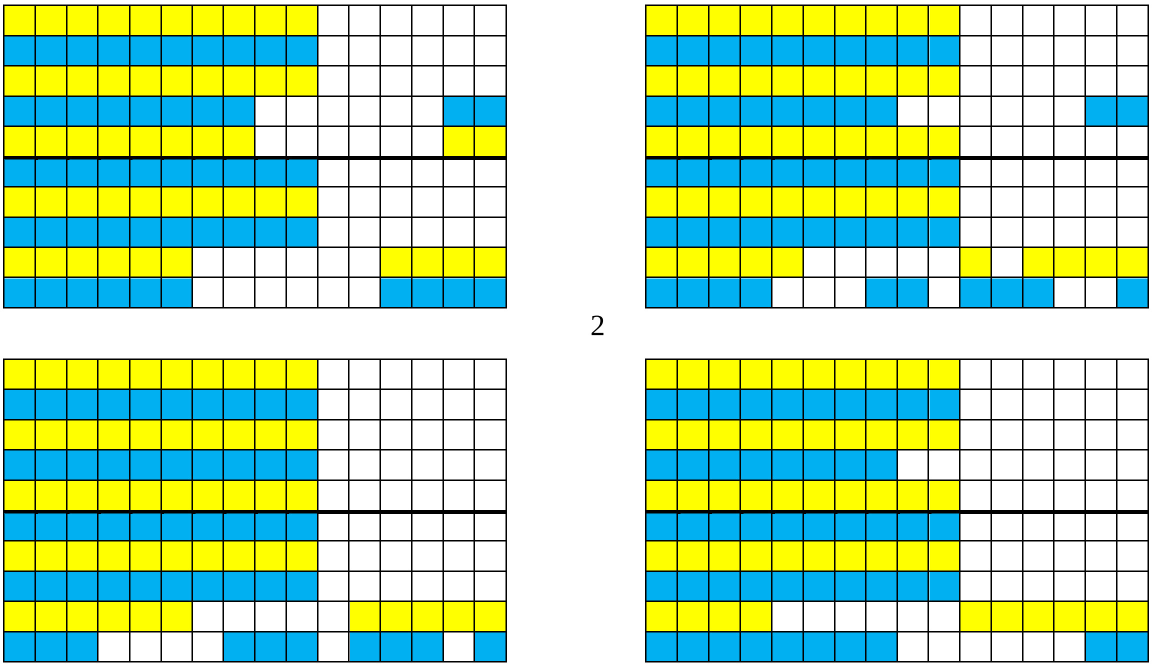


Figure 504. Adding 2.2 3s to 4.4 5s as 6.2 5s on a western abacus in geometry mode

On an abacus in algebra mode and split in the middle by a horizontal rubber band, the 4.4 5s are moved to the right below the band, and the 2.2 3s above the band. Now the 2.2 3s is changed to 1s added to the 4 1s on the single line below the band and bundled in 5s when possible to give a grand total of 6.2 5s.



1 2 3 4

Figure 505. Adding 2.2 3s to 4.4 5s as 6.2 5s on a western abacus in algebra mode

When using a calculator to predict the result, we include the two totals in a bracket before counting in 5s: Asking '(4x5+4 + 2x3+2)/5' gives the answer 6.some. Then asking '(4x5+4 + 2x3+2)– 6x5 gives the answer 2. So the grand total is 6.2 5s.

(4 x 5 + 4 + 2 x 3 + 2)/ 5	6.some
(4 x 5 + 4 + 2 x 3 + 2) – 6 x 5	2

Figure 506. Using a calculator to predict the result of adding 2.2 3s to 4.4 5s as 6.2 5s

Rolling dices can inspire additional adding tasks, e.g. by adding 1 or 2 to the numbers, so that rolling a 2 and a 3 and a 4 and a 5 creates the adding tasks 'Add 2 4s to 3 5s?', 'Add 3 5s to 4 6 s?', etc.

Note that adding in the bigger unit will avoid numbers above nine.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to add totals using geometry or algebra or a calculator for prediction allow for additional observations as to preferences and performances.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Micro-curriculum 6. Adding Icon-numbers NextTo

A. Deconstructing the Tradition

Traditionally, we add totals on top of each other creating no problems when counted in tens except for dealing with overloads by carrying. Adding next-to is called integration and introduced as the inverse operation to differentiation. Both are seen as examples of the abstract limit concept, hence postponed to gifted students at the end of high school.

Skepticism could point out that not allowing counting in icon-numbers forces the unit ten upon all numbers, which makes it irrelevant to change units or to add next-to, as e.g. adding 2 3s and 4 5s as 8s. That might be problematic since adding next-to, also called integration, is a core part of mathematics, and accepting counting in icon-numbers makes integration a natural way to add in preschool.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to perform next-to addition of icon-numbers.

B. Designing a Micro-curriculum

<u>Adding task 1</u>: '4 5s and 2 3s total how many 8s?'

Adding the two totals next-to each other as 8s is possible if recounting both in 8s to be added on top, showing that 4 5s = 2.4 8s and 2 3s = 0.6 8s, thus resulting in a grand total of 2.4 8s + 0.6 8s which can be recounted to 3.2 8s.

On an abacus in geometry mode, leaving the bottom line empty, a stack of 4 5s is moved to the right and a stack of 2 3s is moved to the middle and then to the right. Moving 3 1s to the left on the top line allows filling up the 8 on the line below. The reaming 2 1s on the top line is moved to the bottom line showing the grand total to be 3.2 8s.

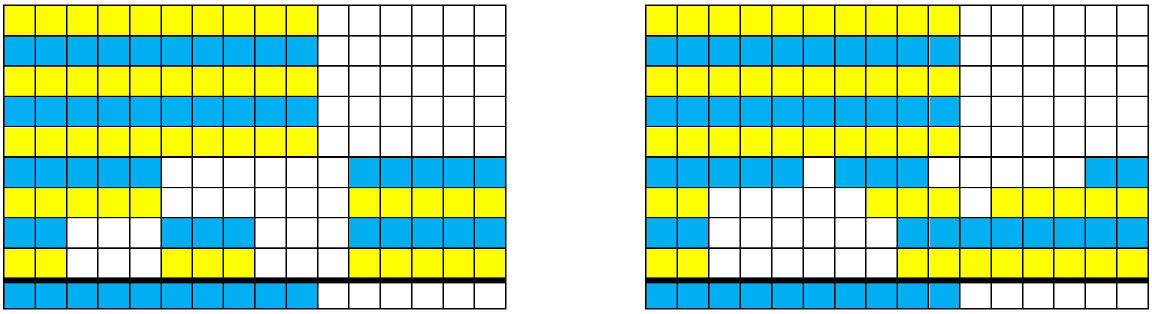


Figure 601. Adding 2 3s to 4 5s as 3.2 8s on a western abacus in geometry mode

An abacus in algebra mode can be used to recount the two totals in 8s to be added afterwards.

When using a calculator to predict the result, we include the two totals in a bracket before counting in 5s: Asking '(4x5 + 2x3)/8' gives the answer 3.some. Then asking '(4x5 + 2x3) – 3x8 gives the answer 2. So the grand total is 3.2 8s.

(4 x 5 + 2 x 3)/ 8	3.some
(4 x 5 + 3 x 2) – 3 x 8	2

Figure 602. Using a calculator to predict the result of adding 2 3s to 4 5s as 3.2 8s

<u>Adding task 2</u>: '4.4 5s and 2.2 3s total how many 8s?'

Adding the two totals next-to each other as 8s is possible if recounting both in 8s to be added on top, showing that 4.4 5s = 3 8s and 2.2 3s = 1 8s and, thus resulting in a grand total of 3 8s + 1 8s which is 4 8s.

On an abacus in geometry mode, 2 1s is moved to the right on the bottom line, and a stack of 2 3s is moved to the right above. Then 4 1s and a stack of 4 5s is moved to the middle and then to the right. Moving 3 1s to the left on the top line allows filling up the 8 on the line below. The reaming 2 1s on the top line is moved to the bottom line showing the grand total to be 4 8s.

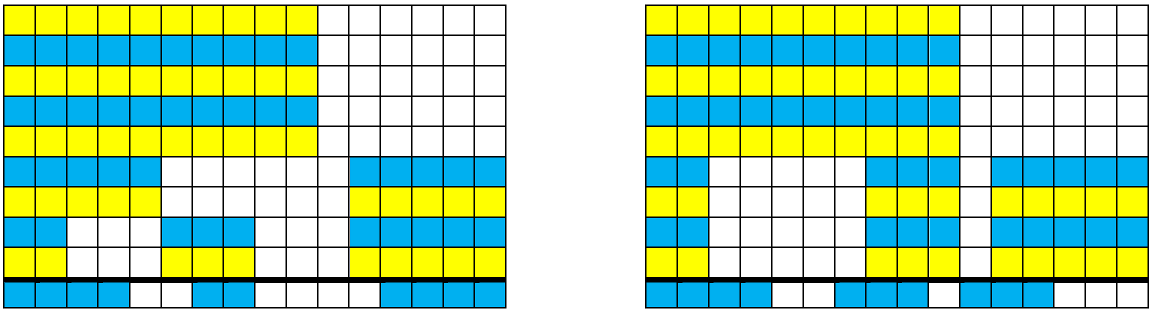


Figure 603. Adding 2.2 3s to 4.4 5s as 4 8s on a western abacus in geometry mode

An abacus in algebra mode can be used to recount the two totals in 8s and added afterwards.

When using a calculator to predict the result, we include the two totals in a bracket before counting in 5s: Asking ‘(4x5+4 + 2x3+2)/8’ gives the answer 4. So the grand total is 4 8s.

(4 x 5 + 4 + 2 x 3 + 2)/ 8	4
(4 x 5 + 4 + 3 x 2 + 2) – 4 x 8	0

Figure 604. Using a calculator to predict the result of adding 2.2 3s to 4.4 5s as 4 8s

Rolling dices can inspire additional adding tasks, e.g. by adding 1 or 2 to the numbers, so that rolling a 2 and a 3 and a 4 and a 5 creates the adding tasks ‘Add 2 4s to 3 5s?’, ‘Add 3 5s to 6 4 s?’, etc.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to add totals using geometry or algebra or a calculator for prediction allow for additional observations as to preferences and performances. Furthermore, using icon-numbers to perform integration in preschool is a new idea that has never been researched before.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Some of the micro-curricula can be tasted as ‘silent-math’ where you are not allowed to speak.

Micro-curriculum 7. Reversing Adding Icon-numbers OnTop

A. Deconstructing the Tradition

Traditionally, reversed addition as ‘2 + ? = 5’ is reformulated to ‘2 + u = 5’, called an equation and defined as an example of an equivalence relation between the two ‘number-names’ u+2 and 5. Performing identical operation to both number-names will keep the equivalence intact. By applying the laws of associativity and community as well as the definitions of a neutral and an inverse element, it is possible to change one of the number-names to a single u called the solution of the equation. Testing the solution is not necessary because of the equivalence relation. Because of the degree of abstractness, equations are traditionally postponed to secondary school.

2 + u = 5	Addition has 0 as its neutral element, and 2 has -2 as its inverse element
(2 + u) + (-2) = 5 + (-2)	Adding 2’s inverse element to both number-names
(u + 2) + (-2) = 3	Applying the commutative law to u + 2. 3 is the short number-name for 5+(-2)
u + (2 + (-2)) = 3	Applying the associative law
u + 0 = 3	Applying the definition of an inverse element
u = 3	Applying the definition of a neutral element

Figure 701. Solving the equation 2 + u = 5 the traditional algebraic way

Skepticism could point out that instead of seeing reversed equation as an example of an equation, the solution of which is seen as an example of applying abstract algebra, reversed addition could be treated as what it is, reversed addition. Furthermore equations with only one occurrence of the unknown number as is the case in most formulas in geometry, commence and physics could be treated as reversed calculations. Finally, a solution to a reversed addition should always be tested by the corresponding forward addition.

Because of the close relationship between forward and reversed addition, equations should be introduced in preschool along with forward addition.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to solve equations as reversed addition?

B. Designing a Micro-curriculum

<u>Reversing task 1</u>: ‘2.3 5s and ? 5s total 4.1 5s?’

On an abacus in geometry mode, a stack of 4.1 5s is moved to the right. Changing the top 5-bundle to 5 1s on the single line allows taking away the 2.3 5s to leave 1.3 5s

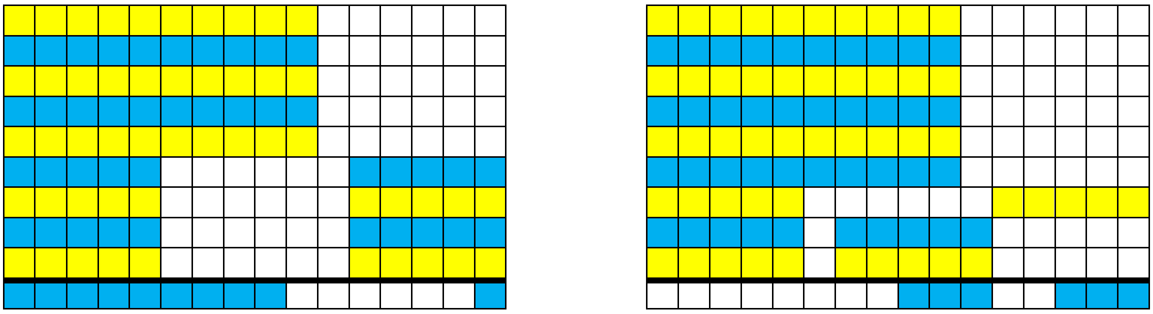


Figure 702. Solving the equation 2.3 5s + ? 5s total 4.1 5s on a western abacus in geometry mode

Using algebra-counting we recount the 4.1 5s to 3.6 5s to be able to take away what was added, the 2.3 5s, thus leaving 1.3 5s

4) 1) 5s = 4-1) 1+5) 5s = 3) 6) 5s = 3.6 5s = 2.3 5s + 1.3 5s

4) 1) 5s – 2) 3) 5s = 4-2) 1-3) 5s = 2) -2) 5s = 2-1) -2+5) 5s = 1) 3) 5s = 1.3 5s

Figure 703. Solving the equation 2.3 5s + ? 5s total 4.1 5s using cup-writing and decimal writing

On an abacus in algebra mode, the 4.1 5s are moved to the right below the band. Changing 1 5-bundle to 5 1s on the single line allows taking away the 2.3 5s to leave 1.3 5s

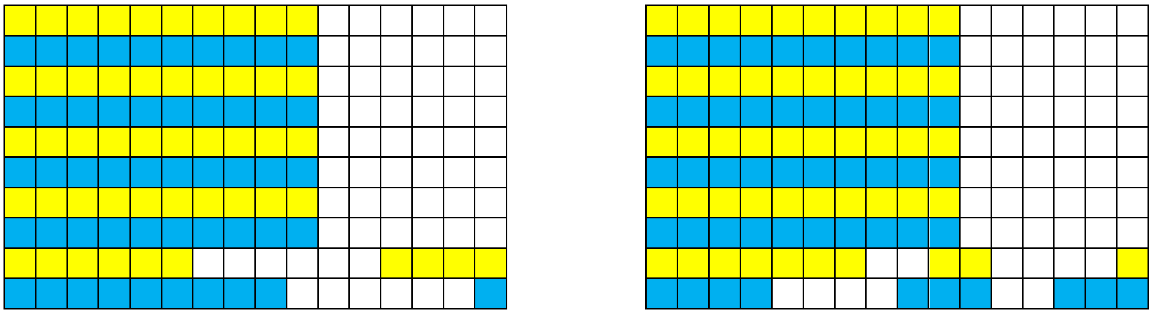


Figure 704. Solving the equation 2.3 5s + ? 5s total 4.1 5s on a western abacus in algebra mode

When using a calculator to predict the result, we include the two totals in a bracket before counting in 3s: Asking '(4x5+1 – 2x5 – 1)/5' gives the answer 1.some. Then asking '(4x5+1 – 2x5 – 1) – 1x5 gives the answer 1. So the answer is 1.3 5s.

(4 x 5 + 1 – 2 x 5 – 1)/ 5	1.some
(4 x 5 + 1 – 2 x 5 – 1) – 1 x 5	3

Figure 705. Using a calculator to solve the equation 2.3 5s + ? 5s total 4.1 5s

<u>Reversing task 2</u>: '4 5s and ? 3s total 6 5s?'

On an abacus in geometry mode, leaving the bottom line empty, a stack of 6 5s is moved to the right. To get back to the original 4 5s, 2 5s are moved to the middle and moved as 1s to the single line to be bundled if possible, thus giving the answer 3.1 3s. To test this result by performing the forward addition '4 5s and 3.1 3s total ? 5s?'

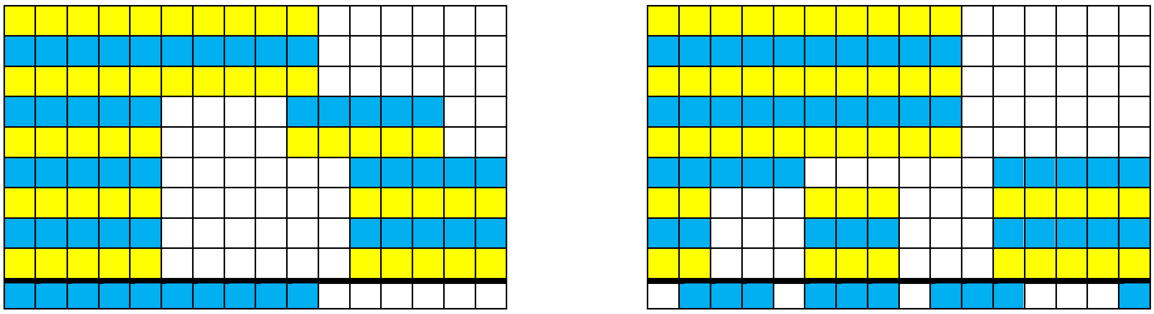


Figure 706. Solving the equation 4 5s + ? 3s total 6 5s on a western abacus in geometry mode

On an abacus in algebra mode and split in the middle by a horizontal rubber band, the 6 5s are moved to the right on the bundle line below the band. To get back to the original 4 5s, 2 5s are moved to the middle and moved as 1s to the single line above the band to be bundled if possible, thus giving the answer 3.1 3s. To test this result by performing the forward addition '4 5s and 3.1 3s total ? 5s?'

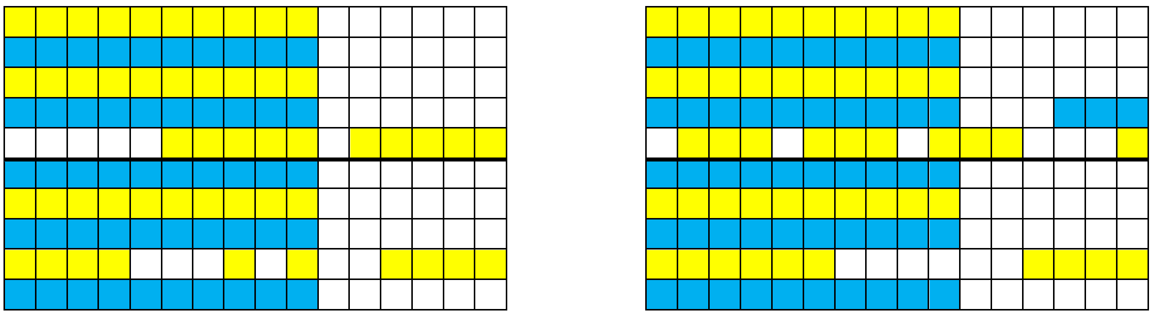


Figure 707. Solving the equation 4 5s + ? 3s total 6 5s on a western abacus in algebra mode

When using a calculator to predict the result, we include the two totals in a bracket before counting in 3s: Asking '(6x5 – 4x5)/3' gives the answer 3.some. Then asking '(6x5 – 4x5) – 3x3 gives the answer 1. So the answer is 3.1 3s. A display showing that (4x5 + 2x3) – 5x5 = 1 indirectly predicts that 2 3s can be added to 4 5s as 5.1 5s.

(6 x 5 – 4 x 5)/ 3	3.some
(6 x 5 – 4 x 5) – 3 x 3	1

Figure 708. Using a calculator to solve the equation 4 5s + ? 3s total 6 5s

Rolling dices can inspire additional adding tasks, e.g. by adding 1 or 2 to the numbers, so that rolling 4 dices creates the reversed addition tasks ‘2 4s and ? 3s total 6 4s?’, ‘3 5s and ? 4s total 7 5s?’, etc.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to solve equations using geometry or algebra or a calculator for prediction allow for additional observations as to preferences and performances. Furthermore, using icon-numbers in equations in preschool is a new idea that has never been researched before.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.

Micro-curriculum 8. Reversing Adding Icon-numbers NextTo

A. Deconstructing the Tradition

Traditionally, reversed addition next-to as '2 3s + ? 5s = 4 8s' is reformulated to '2 x 3 + 5 x u = 4 x 8', called a linear equation and defined as an example of an equivalence relation between the two 'number-names' 2 x 3 + 5 x u and 4 x 8. Performing identical operation to both number-names will keep the equivalence intact. By applying the laws of associativity and community as well as the definitions of a neutral and an inverse element, it is possible to change one of the number-names to a single u called the solution of the equation. Testing the solution is not necessary because of the equivalence relation. Because of the degree of abstractness, equations are traditionally postponed to secondary school. Alternatively, the left hand side is seen as an example of a linear function, likewise postponed to secondary school.

2 x 3 + u x 5 = 4 x 8	Simplifying the number-names where possible
6 + 5 x u = 32	Addition has 0 as its neutral element, and 6 has -6 as its inverse element
(6 + 5 x u) + (-6) = 32 + (-6)	Adding 6's inverse element to both number-names
(5 x u + 6) + (-6) = 26	Applying the commutative law to 6 + 5 x u
5 x u + (6 + (-6)) = 26	Applying the associative law
5 x u + 0 = 26	Applying the definition of an inverse element
5 x u = 26	Applying the definition of a neutral element Multiplication has 1 as neutral element, and 5 has 1/5 as inverse element
(5 x u) x (1/5) = 26 x (1/5)	Multiplying 5's inverse element to both number-names
(u x 5) x (1/5) = 5.2	Applying the commutative law to 5 x u
u x (5 x (1/5)) = 5.2	Applying the associative law
u x 1 = 5.2	Applying the definition of an inverse element
u = 5.2	Applying the definition of a neutral element

Figure 801. Solving the equation 2 x 3 + u x 5 = 4 x 8 the traditional algebraic way

Skepticism could point out that instead of seeing reversed equation as an example of an equation, the solution of which is seen as an example of applying abstract algebra, reversed addition could be treated as what it is, reversed addition, asking '2 3s + ? 5s = 4 8s' and not asking '6 + 5 x u = 32' . Finally, a solution to a reversed addition should always be tested by the corresponding forward addition.

Because of the close relationship between forward addition nex-to, also called integration, and reversed addition nex-to, also called differentiation, differentiation should be introduced in preschool along with integration.

This deconstruction raises the question:

What learning possibilities occur if allowing preschool children to reverse integration?

B. Designing a Micro-curriculum

<u>Reversing task 1</u>: '2 3s and ? 5s total 4 8s?'

Using algebra-counting, 2 3s is recounted as 0.6 8s, that removed from the 4 8s leaves what must be counted in 5s. Since 4 8s can be recounted to 3.8 8s, removing 0.6 8s leaves 3.2 8s that can be recounted as 5.1 5s. So reversing integration to differentiation means first subtracting, then dividing, later called a difference quotient (T2 – T1)/n.

On an abacus in geometry mode, leaving the bottom line empty, a stack of 4 8s is moved to the right. First we remove what was added by moving 2 3s to the left. The surplus 2 3s are moved to the single line and recounted in 5s to give the result 5.1. 5s

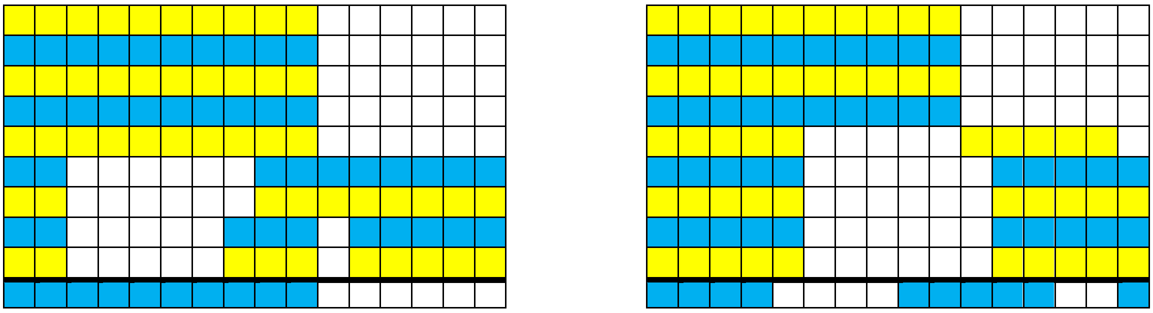


Figure 802. Solving the equation 2 3s + ? 5s total 4 8s on a western abacus in geometry mode

When using a calculator to predict the result, we include the two totals in a bracket before counting in 3s: Asking '(4x8 – 2x3)/5' gives the answer 5.some. Then asking '(4x8 – 2x3) – 5x5 gives the answer 1. So the answer is 5.1 5s. A display showing that (4x8 + 2x3) – 5x5 = 1 indirectly predicts that 2 3s can be added to 5.1 5s as 4 8s.

(4 x 8 – 2 x 3)/ 5	5.some
(4 x 8 – 2 x 3) – 5 x 5	1

Figure 803. Using a calculator to solve the equation 2 3s + ? 5s total 4 8s

Rolling dices can inspire additional adding tasks, e.g. by adding 1 or 2 to the numbers, so that rolling 4 dices creates the reversed addition tasks '4 3s and ? 5s total 2 8s?', '5 4s and ? 5s total 3 9s?', etc.

C. Researching the Micro-curriculum

The researcher(s) make observations of what the children do and say, if possible some sessions will be videotaped. Based upon the observations the researcher(s) will look for patterns in statements, in actions and in personal behavior allowing the construction of proto-categories. Bringing these back to the preschool, the researcher(s) will use further observation to refine the categories, typically by dividing them into sub-categories, and if possible look for relations between these.

In this case, the three ways to solve equations using geometry or algebra or a calculator for prediction allow for additional observations as to preferences and performances. Furthermore, using icon-numbers to reverse integration in preschool is a new idea that has never been researched before.

D. Reporting the Findings

The findings are reported in the traditional way in journals and more interactively on a Facebook profile supported by YouTube videos.