Invitation to a Dialogue on Mathematics Education and its Research

Inspired by the Chomsky-Foucault debate on Human Nature, www.youtube.com/watch?v=3wfNl2L0Gf8

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Bo: Welcome to the MATHeCADEMY.net channel. My name is Bo. Today we discuss Mathematics education and its research. Humans communicate in languages, a word-language and a number-language. In the family, we learn to speak the word language, and we are taught to read and write in institutionalized education, also taking care of the number-language under the name Mathematics, thus emphasizing the three r's: Reading, Writing and Arithmetic. Today governments control education, guided by a growing research community. Still international tests show that the learning of the number language is deteriorating in many countries. This raises the question: If research cannot improve Mathematics education, then what can? I hope our two guests will provide some answers. I hope you will give both a statement and a comment to the other's statement.

Welcome to John. John has …

John: Thank you Bo

Bo: And welcome to Allan. Allan has been working as an ethnographer in different parts of education from secondary school to teacher education. Allan has created the web-based MATHeCADEMY.net teaching teachers to teach Mathematics as a natural science about Many. In addition, Allan has written a series of papers for the ICME congresses collected in an ICME-trilogy.

Allan: Thank you Bo

1. Mathematics Itself

Bo: We begin with Mathematics. The ancient Greeks Pythagoreans used this word as a common label for what we know, which at that time was Arithmetic, Geometry, Astronomy and Music. Later Astronomy and Music left, and Algebra and Statistics came in. So today, Mathematics is a common label for Arithmetic, Algebra, Geometry and Statistics, or is it? And what about the so-called 'New Math' appearing in the 1960s, is it still around, or has it been replaced by a post New-Math, that might be the same as pre New-Math? In other words, has pre-modern Math replaced modern Math as post-modern Math? So, I would like to ask: 'What is Mathematics, and how is it connected to our number-language?'

John: Sentence. Sentence. Sentence. …

Allan: To me, it is the need to communicate about the natural fact 'Many' that created the number-language. In space, we constantly see many examples of Many; and in time Many is present as repetition. So, if Mathematics means what we know, we might want to add about Many, and use the word 'Manyology' as a parallel word for Mathematics.

To deal with Many we perform two actions, we count and we add to answer the basic question 'how many'. This resonates with the action-words algebra and geometry meaning to reunite numbers in Arabic and to measure land in Greek. We count a given total in singles, bundles, bundles of bundles, etc. as shown by a number as five hundred and forty three, consisting of 3 singles, 4 ten-bundles and 5 ten-bundles of ten-bundles. We see that all numbers carry units as ones, tens, ten-tens etc. Having the same unit, the 4 ten-bundles are added on-top of each other; and having different units, the 5 tens-tens and the 4 tens are added next-to each other as areas, also called integration, where shifting unit is called linearity. So, a three digit number shows the core of Mathematics, which is linearity and integration. The number also shows the four different ways to unite numbers: by multiplication as in 4 tens, by power as in ten-tens, by vertical on-top addition as in 3 ones, and by horizontal next-to addition as in the juxtaposition of the three blocks with different units. Showing its bundle-size ten when written as 54.3 tens, the total also shows that singles can be written as decimals or as fractions where the 3 singles become 0.3 tens or 3 counted in tens, 3/10. With unspecified bundle-number, a three-digit number becomes a formula, where the bundle-number can be found by reversing addition, also called solving equations.

So, Mathematics is very easy; and also very easy to make hard. You just replace Mathematics with 'Metamatism', a mixture of 'Meta-matics' and 'Mathema-tism'.

Mathematism is true in a library but not in a laboratory. Thus statements as '2 + 3 is 5' are found in any textbook even if it is falsified by countless outside examples, as e.g. 2 weeks and 3 days total 17 days.

Metamatics defines its concepts as examples of abstractions instead of as abstractions from examples, i.e. top-down and from above instead of bottom-up and from below. Thus, Metamatics defines a formula as an example of a set-product where first-component identity implies second-component identity, instead of, as Euler did, as a name for a calculation containing both numbers and letters. Defining concepts as examples of the ultimate abstraction, a set, makes Metamatics self-referring, and thus meaningless according to Russell's set-paradox saying that the set of sets not belonging to itself will belong to itself if it does not belong, and vice versa. To avoid this paradox, Russell proposed a type-theory to distinguish between examples and abstractions, meaning e.g. that a fraction is not a number. Unwilling to accept this, modern set theory removes the difference between an element and a set, i.e. between an example and an abstraction, which still makes Metamatics meaningless since you can survive on examples of food but not on the label food; they enter different holes in the head.

Summing up, Mathematics can be a grounded natural science about the natural fact Many, thus becoming a number-language showing how numbers are built by using four different ways to unite:

multiplication, power, on-top and next-to addition, that can all be reversed. However, Mathematics can also be an ungrounded self-referring Metamatism with set-derived definition and with statements that are claimed to be true even when confronted by counter-examples. In other words, Mathematics can be easy and accessible to all, or it can be made hard and accessible to an elite only.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

2. Education in General

Bo: Thank you, John and Allan. Now let us talk about education in general. On our planet, life takes the form of single black cells, or green or grey cells combined as plants or animals. To survive, plants need minerals, pumped in water from the ground through their leaves by the sun. Animals instead use their heart to pump the blood around, and use the holes in the head to supply the stomach with food and the brain with information. Adapted through genes, reptiles reproduce in high numbers to survive. Feeding their offspring while it adapts to the environment through experiencing, mammals reproduce with a few children per year. Humans only need a few children in their lifetime, since transforming the forelegs to hands and fingers allows humans to grasp the food, and to share information through communication and education by developing a language when associating sounds to what they grasp. Where food must be split in portions, information can be shared. Education takes place in the family and in the workplace; and in institutions with primary, secondary and tertiary education for children, for teenagers and for the workplace. Continental Europe uses words for education that do not exist in the English language such as Bildung, unterricht, erziehung, didactics, etc. Likewise, Europe still holds on to the line-organized office preparing education that was created by the German autocracy shortly after 1800 to mobilize the population against the French democracy, whereas the North American republics have block-organized talent developing education from secondary school. As to testing, some countries use centralized test where others use local testing. And some use written tests and others oral tests. So, my next question is 'what is education?'

John: Sentence. Sentence. Sentence.

Allan: We adapt to the outside world through experience and advice, i.e. we are educated by the outside world and by other human beings. Children like to feel the outside world; teenagers like to gossip about it and about themselves; and adults must exchange actions with money to support a family. Thus, it makes sense to institute both primary, secondary and tertiary education to serve the needs of children, teenagers and adults. As an institution, education contains an element of force. Our language came from naming what we can grasp or point to, i.e. through a from-the-hand-to-the-head principle, called greifen-begreifen in German. So guiding children with concrete material to grasp, and teenagers with gossip to listen to makes education successful as described in Psychology by Piaget and Ausubel. On the other hand, forcing abstractions upon children and teenagers before introducing concrete materials or gossip excludes many children and teenagers from learning, thus creating a monopoly of knowledge as described in Sociology by e.g. Foucault and Bourdieu.

As to the space-and-time structure of education, primary education for children should be line-organized with yearly age-group-nannies as guides bringing the outside world to the classroom to develop concepts about nature described by a number-language, and concepts about society described by a word-language. In late primary school, this double-nanny becomes two different nannies. Daily, the children also express themselves through music, art, or motion. The priority of to-do-subjects over to-be-subjects changes from primary to secondary school.

Transformed from children to teenagers able to have children of their own, the curiosity changes from the outside to the inside world, from things to persons. Being biologically programmed to remember gossip is useful if information about nature and society takes the form of gossip, i.e. statements with known subjects. Experimenting now is with what is inside oneself, e.g. as to talents. Consequently, secondary school should offer daily lessons in self-chosen half-year blocks to allow the individual teenager to test personal talents. If successful, the school says 'good job, you need more of this'. If not, the school says 'good try, you need to try something else' to express admiration for the courage it takes to try out something new. This is how the North American republics organize a bottom-up secondary and tertiary education.

Being highly institutionalized, Europe hangs on to its line-organized school system preparing for public, created by Humboldt in Berlin shortly after 1800. Furthermore, the word 'education' is replaced by words as ‘unterricht’ and ‘erziehung’ and ‘Bildung’. Unterricht means handing down to those below you, and erziehung means dragging them up. These top-down words come from the Platonic patronizing view that the goal of education is to transmit and exemplify abstract knowledge.

The success of the French Enlightenment republic came from enlightening its population. To protect autocracy, the Prussian king asked Humboldt to create a school that could replace the blood-nobility unable to stop the French with a knowledge-nobility to occupy a strong public administration and to receive Bildung so it could go to court. This Bildung school should have two more goals: to prevent democracy, the population must not be enlightened; instead, the population must be transformed into a people proud of its history and willing to protect it against other people, especially the people from the French republic. To hide its anti-enlightening agenda, teacher education is based upon a special subject called didactics, confusing the teachers by claiming to determine the content of Bildung.

So to sum up, education can be bottom-up enlightenment allowing children to experiment with the outside world brought to the classroom, and allowing teenager to experiment with their inside talents through daily lessons in self-chosen half-year blocks that inform about the outside world in the form of gossip. Or, education can be top-down Bildung trying to make the students accept patronization by abstract knowledge created at a distant university, where the best of them might be accepted later.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

3. Mathematics Education

Bo: Thank you, John and Allan. Now let us talk about education in Mathematics, seen as one of the core subjects in schools together with reading and writing. However, there seems to be a difference here. If we deal with the outside world by proper actions, it has meaning to learn how to read and how to write since these are action-words. However, you cannot Math, you can reckon. At the European continent reckoning, called 'Rechnung' in German, was an independent subject until the arrival of the so-called new Mathematics around 1960. When opened up, Mathematics still contains subjects as fraction-reckoning, triangle-reckoning, differential-reckoning, probability-reckoning, etc. Today, Europe only offers classes in Mathematics, whereas the North American republics offer classes in algebra and geometry, both being action words meaning to reunite numbers and to measure earth in Arabic and Greek. Therefore, I ask, 'what is Mathematics education?'

John: Sentence. Sentence. Sentence. …

Allan: The outside world contains many examples of Many: many persons, many houses, many days, etc. So, to adapt to the outside world, humans need to be deal with the natural fact Many, and this should be the goal of Mathematics education since the main contents of Mathematics was created as precisely that: statistics to count Many, algebra to reunite Many and geometry to count spatial forms. To deal with Many, we count and add. Counting takes place in the family and therefore integrates into preschool in a natural way. Since primary school only allows counting in tens, preschool can profit from the golden learning opportunities coming from icon-counting in numbers less than ten. Here first-order counting allows five ones to be bundled as one fives, transformed into one five-icon containing five strokes if written in a less sloppy way. Now second-order counting can count in icons so that seven sticks can be recounted in 1 five-bundle and two unbundled singles, written as 1 and 2 5s, or as 1.2 5s using the decimal point to separate bundles and unbundled. Which again can be recounted as 2.1 3s where changing units later is called proportionality and linearity. Once counted, totals can be added. To add on-top the units must be the same, so one of the totals must be recounted in the other's unit. Added next-to each other, the totals are added as areas which is called integration. And reversing addition means creating opposite operations to predict the result. Here the operations occur in their natural order, which is the opposite of what the school presents: to count in 5s we take away 5s many times, which is division. Then the bundles are stacked, which is multiplication. We might want to recount a stack by taking away one bundle to change it into singles, which is subtraction. Finally stacks can be added on-top or next-to. By meeting concrete examples of Many, children learn to count and recount by bundling and stacking; and to add on-top and next-to. Later physical units introduce children to per-numbers when double-counting in two different units as e.g. 5 $ per 3 kg, or 5/3 $/kg.

Telling Mathematics as gossip makes learning easy for teenagers, biologically programmed to remember statements about known subjects. The formula for a number as 543, i.e. 5 tens-tens and 4 tens and 3 ones show the four ways to unite numbers: Multiplication, power, on-top addition and next-to addition, also called integration. With an unknown bundle-number, the number-formula becomes a polynomial containing basic relations between variable numbers as proportional, linear, exponential, power and quadratic formulas that tabled and graphed show the different forms of constant changing unit-numbers in pre-calculus. As to calculus, per-numbers can be constant in three different ways: globally, piecewise and locally also called continuous; all added to totals by the area under the per-number graph i.e. by combining multiplication and addition. Reversed, the combination of subtraction and division, called differentiation, allows the per-number to be determined from the area. Many teenagers enjoy the beauty of uniting geometry and algebra in coordinate-geometry allowing a geometrical prediction of algebraic solutions and vice versa; as well as the fascinating post-diction by statistics of unpredictable numbers in probability.

To sum up: Mathematics education can be easy if grounded in the roots of Mathematics, the natural fact Many, to be dealt with by counting and adding making a natural number a decimal number with a unit. Counting and recounting in icons before counting in tens brings the core of Mathematics, linearity and integration, to preschool; and allows solving equations and fractions to be introduced in the beginning of primary school as reversed addition and double-counting in different physical units. Or Mathematics can be hard by allowing only counting in tens, by presenting a natural number without a decimal point and a unit, and by transforming Mathematics to Mathematism by adding numbers without units, claiming e.g. that 2 plus 3 is 5 in spite of many counterexamples; and by postponing proportionality and integration to the beginning and end of secondary school.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

4. The Learner

Bo: Thank you, John and Allan. Now let us talk about at the humans involved in Mathematics education: Governments choose curricula, build schools, buy textbooks and hire teachers to help learners learn. We begin with the learners. The tradition sees learning taking place when learners follow external instructions from the teacher in class and from the textbook at home. Then constructivism came along suggesting that instead learning takes place through internal construction. Therefore, I ask 'what is a learner?'

John: Sentence. Sentence. Sentence. …

Allan: Again, let us assume that we adapt to the outside world through actions, physical and verbal. So learning means acquiring proper actions, some verbalized and some tacit. Repetition makes learning effective. Repetition takes place in the family and in the workplace, and can take place in school through daily lessons both for children and for teenagers. Also, allowing learners to carry out most of the homework at school will minimize the effect of the learners' different social backgrounds.

Again we must distinguish between a child, a teenager and an adult. Its biology programs a child to learn by grasping as described by Piaget, and a teenager to learn by gossip as described by Ausubel stressing the importance of connecting new knowledge to what the learner already knows. An adult is motivated to learn something from its use in the workplace.

Piaget describes individual learning as creating schemata that can assimilate new examples, or be accommodated to assimilate divergent examples. In contrast, Vygotsky describes learning as being able to connect the learner's individual knowledge zone with the abstract concepts of the actual knowledge regime.

The four answers to the question: "Where do concepts come from? From above or from below? Form the outside or from the inside?" create four learning rooms. The two traditional rooms, the transmitter room and the constructivist room, say "above and outside" and "above and inside". The two hidden alternatives, the "fairy-tale room" and the apprentice room, say "below and outside" and "below and inside". The traditional rooms take Mathematics for granted and see the world as applying Mathematics. The hidden rooms have the opposite view seeing Many as granted and as a creator of Mathematics through the principle 'grasping by grasping'. The transmission room and the fairy-tale room facilitate learning through sentences with abstract and concrete subjects. The constructivist room and apprentice room facilitate learning through sentence-free meetings with abstract or concrete subjects.

A block-organized education allows the learners to change classes twice a year with a "good job" greeting if successful and a "good try" greeting if less successful aiming at keeping alive the curiosity of the teenager as to which talent is hidden inside. In Europe, its line-organized education forces the learner to stay in the class even if being less successful, or to be removed from class to special education, of to be to leave education and find a job as an unskilled worker.

To summarize: As to children, learning can be concept-building through daily contact with concrete materials. Or, learning can prevent concept-building by excluding concrete materials and by sporadic lessons. As to teenagers, learning can be expanding their personal narrative with authorized gossip enforced by daily lessons in self-chosen half-year blocks. Or learning can be preventing their narratives from growing by teaching unknown fact about unknown subjects, again enforced by sporadic lessons. Finally, to adults learning can be grounded in workplace examples, or learning can be ungrounded encapsulated knowledge claimed to become maybe useful later.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

5. The Teacher

Bo: Thank you, John and Allan. Now let us talk about the teacher. It seems straightforward to say that the job of a teacher is to teach learners so that learning takes place, checked by written tests. However, continental Europe calls a teacher a 'Lehrer' thus using the same word as for learning. In addition, a Lehrer is supposed to facilitate 'unterrichtung and erziehung and to develop qualifications and competences. In teacher education, the subject didactics, meant to determine the content of Bildung, is unknown outside the continent. And until lately, educating lehrers took place outside the university in special lehrer-schools. Thus, being a teacher does not seem to be that well-defined. Therefore, my next question is 'what is a teacher?'

John: Sentence. Sentence. Sentence. …

Allan: As with learning, we must differentiate between teaching children in primary school, teaching teenagers in secondary school and teaching adults in tertiary schools.

A parent is an adult helping the child to supply its stomach with food and its brain with information, based upon a relationship of trust. Removed from the home in an institution, a child will look for a substitute parent, a nanny. To prevent them from becoming competing parents, a nanny only teaches one year-group and has only one class. The first year of primary school the nanny slowly splits up the outside world in things that we count and humans that we communicate with or about, thus laying the foundation to the two basic knowledge areas: nature with a number-language and society with a word language. At the end of primary school a class has two nannies specialized in each of the two basic knowledge areas.

In secondary school, the teacher role changes from a nanny to an expert with special training in one or two subjects. Now teachers have their own classroom where they teach the different daily half-year groups in their subject in the form of gossip. Half-year classes allow the teachers and the learners to maintain a good relationship, since at the end of the half year all learners leave the class thanked with a "good job" if successful and a "good try" if less successful.

In tertiary education, the degree of specialization is higher demanding a master degree in a theoretical subject or a license in a trade or in a craft.

At a block-organized university taking additional blocks allows a teacher to change career from primary to secondary or tertiary education, or to business, engineering or other crafts, and vice versa. And the final choice between teaching preschool or primary or secondary school can be postponed to later in teacher education. In contrast, Europe's line-organized education forces a choice between the different level to be made before tertiary school, and forces teachers to stay in their public office for the rest of their working life.

To summarize, a teacher have different roles at block- and line-organized schools. At the former, a teacher for children is a nanny splitting up the world in two subject areas: nature with a number-language and society with a word-language. And for teenagers teachers are experts telling about their specific knowledge area in the form of gossip. Both are educated at a university and able to change career by taking additional blocks. In line-organized education, a teacher specializes in several subjects, have several classes each day, and follows a class for several years. And once a teacher, always a teacher, since line-organized universities typically force students to start all over if wanting to change form one line to another.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

6. The Political System

Bo: Thank you, John and Allan. Now let us talk about governments. Humans live together in societies with different degrees of patronization. In the debate on patronization, the ancient Greek sophists argued that humans must be enlightened about the difference between nature and choice to prevent patronization by choices presented as nature. In contrast, the philosophers saw choice as an illusion since physical phenomena are but examples of metaphysical forms only visible to philosophers educated at Plato's Academy who consequently should be accepted as patronizors. Still today, democracies come in two forms with a low and high degree of institutionalized patronization using block-organized education for individual talent developing or using line-organized education for office preparation. As to exams, some governments prefer them centralized and some prefer them decentralized. As to curricula, the arrival of new Mathematics in the 1960s integrated its subfields under the common label Mathematics. Likewise, constructivism meant a change from lists of concepts to lists of competences. However, these changes came from Mathematics and education itself. So my question is: 'Should governments interfere in Mathematics education?'

John: Sentence. Sentence. Sentence. …

Allan: A government must create an educational institution forcing children and teenagers to spend so much of their life in it that some Greenland teenagers even talk about being condemned to school. Thus, a government must decide how much force it will allow the educational institution to exercise. Likewise, a government should know the root and agenda of their present educational institution as well as alternatives practiced elsewhere in the world.

As to curricula, a government must decide if schools present concepts as exemplified from above or abstracted from below. As to structure, a government must choose between the block-organized enlightenment education of the North American Democracies aiming at developing individual talents; and the line-organized Bildung education in Europe created in Berlin around 1800 to prevent democracy from spreading from France and aiming at preparing for public offices.

Besides politicians, a government also includes public servants, called mandarins in the ancient Chinese empire. In Europe the French sociologist Bourdieu has pointed out that the mandarin class forms a new knowledge-nobility using the educational system to exercise symbolic violence so that their children inherit the parents' lucrative public offices; and that Mathematics is especially well suited for this purpose. Some countries, as e.g. Denmark, even hold on to oral exams, thus giving additional advantages to mandarin children.

In Europe, spreading out economical capital by creating a welfare state made socialist parties strong. However, they seem to neglect to spread out knowledge capital as well. After all, where economical capital is split up in a 'what I win, you lose' game, knowledge capital can be enjoyed by all in an all-win game. To me this paradox shows the strength of the mandarin class in Europe.

So to sum up. Yes, governments must create educational institutions, but should minimize its force as much as possible. Consequently, education should be block-organized from secondary school, and school subjects should be teaching grounded categories and knowledge. That is, Mathematics education must meet the human need to deal with the natural fact Many by counting and adding, i.e. by recounting in different units to root proportionality, by adding also next-to to root integration, and to reverse addition to root solving equations. And no, Europe should not hold on the its Humboldt line-organized Bildung preparing the mandarin children to inherit their parents' public offices, created 200 years ago by the German nobility to induce nationalism into the population to keep democracy from spreading from France.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

7. Research

Bo: Thank you, John and Allan. Now let us talk about research. Tradition often sees research as a search for laws built upon reliable data and validated by unfalsified predictions. The ancient Greek Pythagoreans found three metaphysical laws obeyed by physical examples. In a triangle, two angles and two sides can vary freely, but the third ones must obey a law. In addition, shortening a string must obey a simple ratio-law to create musical harmony. Their findings inspired Plato to create an academy where knowledge meant explaining physical phenomena as examples of metaphysical forms only visible to philosophers educated at his academy by scholasticism as ‘late opponents’ defending their comments on an already defended comment against three opponents. However, this method discovered no new metaphysical laws before Newton by discovering the gravitational law brought the priority back to the physical level, thus reinventing natural science using a laboratory to create reliable data and test library predictions. This natural science inspired the 18th century Enlightenment period, which again created counter-enlightenment, so today research outside the natural sciences still uses Plato scholastics. Except for the two Enlightenment republics where American Pragmatism used natural science as an inspiration for its Grounded Theory, and where French post-structuralism has revived the ancient Greek sophist skepticism towards hidden patronization in categories, correctness and institutions that are ungrounded. Using classrooms to gather data and test predictions, Mathematics education research could be a natural science, but it seems to prefer scholastics by researching, not Math education, but the research on Math education instead. To discuss this paradox I therefore ask, ‘what is research in general, and within Mathematics education specifically?’

John: Sentence. Sentence. Sentence. …

Allan: A ‘pencil-paradox’ illustrates the trust-problem in research. Placed between a ruler and a dictionary, a pencil can itself falsify a number by pointing to a different number, but it cannot falsify a word by pointing to a different word, so where number-statements may express natural correctness, word-statements express a political correctness valid inside a ruling truth regime. In other words, using numbers, natural science produces universal truth, and using words, human and social sciences produce local and temporary truths always threatened by competing truth regimes or paradigms as Kuhn called them. Psychology has a paradigm war between behaviorists and constructivists, and within constructivism between Vygotsky and Piaget disagreeing as to whether the learner shall adapt to the ruling paradigm or the other way around. Sociology has a paradigm war called the actor-structure controversy, where the North American republics see social life as created by the symbolic interaction between independent actors, while the institutionalized Europe traditionally sees social life as determined by structures similar to the gravitational laws of natural science. But accepting word-statements as being not nature but choice has created a research genre studying the social construction of different word-paradigms.

The two Enlightenment republics have found ways around the pencil-paradox. North American reaction against traditional philosophy has created American Pragmatism and its symbolic interactionism insisting that categories and theory be grounded in observations. Thus, you must not enter a field with preconceived categories, and generated categories must accommodate to field resistance, thus paralleling the generation of collective and individual knowledge as described by Piaget both accepting the priority of observations as in natural science. Here counter-examples do not reject a category but splits it into sub-categories. In other words, both the courtroom and Grounded Theory base their categories upon action-statements and reject is-statements as prejudice, reserved for the judge and the researcher.

In the second Enlightenment republic, the French, patronization hidden in ungrounded words, sentences and institutions has developed the post-structural thinking of Derrida, Lyotard and Foucault. Derrida recommends deconstructing patronizing categories. Lyotard recommends challenging political correctness by inventing paralogy as dissension to the ruling consensus. Foucault recommends using concept archeology to uncover the pastoral power of the so-called human sciences, instead being disciplines disciplining themselves and their subject, thus silencing competing disciplines and forcing ungrounded identities upon humans as diagnoses to be cured by normalizing institutions applying these human sciences.

Inspired by this French skeptical thinking, postmodern contingency research has found another solution to the pencil paradox. Often postmodern thinking is seen as meaningless since its skepticism also must apply to itself. However, postmodern skepticism is a meta-statement about statements about the world and therefore not one of the statements about the world, against which it directs its skepticism. Of course, the liar paradox saying 'this sentence is false' and being false if true and vice versa makes self-reference problematic, but postmodern thinking avoids self-reference by its meta-statement 'Everything can be different, except the fact that everything can be different'. Thus the ancient sophist warning against mixing up nature and choice makes it possible for postmodern contingency research to discover false nature by finding hidden alternatives to choices presented as nature. Within Mathematics education research, contingency research has successfully pointed out hidden alternatives to unquestioned traditions within numbers, operations, equations, teacher education, etc. as seen on the MATHeCADEMY.net website.

To sum up, research can be a bottom-up activity using outside world observations to generate categories and theories to test predictions, especially successful with the number-statements of natural sciences. Or research can be a top-down activity forcing the outside world to assimilate to operationalized categories from the ruling paradigm, and using scholasticism to produce new researchers as late opponents defending comments on already defended comments against three opponents.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

8. Conflicting Theories

Bo: Thank you, John and Allan. Of course, Mathematics education research builds upon and finds inspiration in external theories. However, some theories are conflicting. Within Psychology, constructivism has a controversy between Vygotsky and Piaget. Vygotsky sees education as building ladders from the present theory regime to the learners' learning zones. Piaget replaces this top-down view with a bottom-up view inspired by American Grounded Theory allowing categories to grow out of concrete experiences and observations. Within Sociology, disagreement about the nature of knowledge began in ancient Greece where the sophists wanted it spread out as enlightenment to enable humans to practice democracy instead of allowing patronizing philosophers to monopolize it. Medieval times saw a controversy between the realists and the nominalists as to whether a name is naming something or a mere sound. In the late Renaissance, a controversy occurred between Hobbes arguing that their destructive nature forces humans to accept patronization, and Locke arguing, like the sophists, that enlightenment enables humans to practice democracy without any physical or metaphysical patronization. As counter-enlightenment, Hegel reinstalled a patronizing Spirit expressing itself through art and through the history of different people. This created the foundation of Europe's line-organized office preparing Bildung schools; and for Marxism and socialism, and for the critical thinking of the Frankfurter School, reviving the ancient sophist-philosopher debate by fiercely debating across the Rhine with the post-structuralism of the French Enlightenment republic. Likewise, the two extreme examples of forced institutionalization in 20th century Europe, both terminated by the low institutionalized American Enlightenment republics, made thinkers as Baumann and Arendt point out that what made termination camps work was the authorized routines of modernity and the banality of evil. Reluctant to follow an order, you can find another job in the private sector, but not in an institution. Here the necessity of keeping a job forces you to carry out both good and evil orders. As an example of a forced institution, this also becomes an issue in Mathematics Education. So I ask: What role do conflicting theories play in Mathematics education and its research?

John: Sentence. Sentence. Sentence. …

Allan: To me, Sociology is the basic theory when discussing Mathematics education and its research. Sociology asks the basic question: in the social space, do we need patronization or can we find mutual solutions using the threefold information-debate-choice method of a democracy? As pointed out, the debate on patronization began in ancient Greece between the philosophers and the sophists; and the debate is still with us today between socialist top-down critical theorists and skeptical bottom-up postmodern theorists. As a social institution, education contains an element of force, that can be patronizing or emancipating providing what is called 'Mündigkeit' in German. Europe maximizes the force-component by using line-organized office preparing education to force humans to stay in the line as long as possible, and to accept that their difficulties are caused by their inferiority to the children of the public office holders helping their children inherit their offices created to patronize the population. Whereas North America from secondary school minimizes the force-component by using daily lessons in self-chosen half-year blocks to uncover and develop the individual talent of the learner.

Likewise, Mathematics can serve both purposes. Presented from above as top-down falsified Metamatism, it becomes so hard to learn that it forces many learners to stop learning it. This is a minor problem with half-year blocks since leaving Mathematics does not force you to leave school, but it is a big problem at line-organized schools where leaving the line means leaving school for good. Presented bottom-up from below grounded in the natural fact Many, Mathematics becomes easy to learn; and the learner can keep on choosing more blocks until the interest may disappear, or in Europe the ordinary learner can stay longer on the line to the dislike of the public office holders, the mandarins.

Likewise, the controversy within Psychology between Vygotsky and Piaget as to how learning takes place also serves both sociological purposes. Presented top-down from above, concepts become hard to learn and force many learners to stop learning the concepts and to accept patronization by those who succeed learning them. In contrast, bottom-up concepts grounded from below in the outside world are easy to learn for children through the concrete material that roots the concepts; and for teenagers since knowing the subject of the sentence gives a Grounded Theory the form of gossip.

The need to keep their job forces teacher to follow the orders of their specific institution. When trained, teachers should as potential change agents be informed about the many choices of an educational institution and within Mathematics, so the individual teacher knows the difference between choice and nature, i.e. what can be changed and what cannot, in order to prevent being a victim of the banality of evil.

To sum up, a civilized teacher education should inform about the many examples of conflicting theories in Mathematics, in education and in research and should put more emphasis on the sociological consequences of unnecessary force in these three institutions.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

9. Me and Mathematics Education and Research

Bo: Thank you, John and Allan. Now let us talk about your own experiences with Mathematics education and its research. In addition, I would like to ask you who are the most important theorists in Math education research in your opinion?

John: Sentence. Sentence. Sentence. …

Allan: I met Mathematics before the arrival of the so-called new Math. In elementary school we had reckoning, and in middle school we had written and oral reckoning together with arithmetic and geometry, and finally about 5% of us went on to the European high school called a 'gymnasium' where we met the word Mathematics for the first time; finally, at the university, Mathematics was to new Math from day one. Repetition and its roots to the outside world made reckoning easy to learn, likewise with geometry where we learned to construct different figures and met formal definitions and proofs. Introduced as letter-reckoning made arithmetic strange and difficult, especially when reducing letter fractions came along. At the gymnasium, the epsilon-delta definition of real numbers from day one killed the interest of most students; and likewise during the first year at the university when geometry was replaced by n-dimensional linear algebra. Here Mathematics changed to Metamatics with top-down set-derived definitions and general proofs without examples to sort out the elite for graduate studies. Most students dropped out or failed the exam. I passed, but to get a meaningful job I decided to shift to architecture. However, at a Belgian library I met American textbooks presenting algebraic topology bottom-up as abstractions from examples instead of the other way around and I decided to become a Math teacher teaching bottom-up meaningful Mathematics instead of the top-down meaningless Metamatics, that made the textbooks so hard to access for the students in the gymnasium.

As a teacher I learned, that using words derived from its roots made concepts much more understandable. Thus, most students had problems with the traditional textbook definitions and theorems of exponential functions introduced after the set-derived definition of a function. In contrast, telling that when growing by a constant multiplier, the end value y is the initial value b multiplied with the multiplier c x times, written as y equal b multiplied with c to the power of x made one student remark: 'Hey mr. Teacher, this we already know, when do you teach us something we don't know?' So I began to look for root-based names for the Mathematical concepts and was surprised to find the root of calculus as adding variable per-numbers, and to find that when epsilon and delta changes places we define a piecewise instead of a locally constant formula. Likewise, introducing integral calculus before differential calculus took the hardness out of calculus.

The discovery that hidden alternatives can change Mathematics from hard to easy brought me to Mathematics education research. Here the beauty and simplicity of the ancient Greek sophist warning against false nature by saying that unenlightened about the difference between nature and choice we risk being patronized by choices presented as nature made me develop contingency research aiming at discovering hidden alternatives to choices presented as nature. Likewise, I admired the beauty and simplicity of American Sociology where Berne talks about the three states of communication, parent, child and adult. These three states create two effective ways of communicating, child-parent where both accept the presence of authority, and adult-adult where both accept its absence; and several ineffective ways not agreeing upon the role of authority. In addition, I was fascinated about the resemblance between Piaget in Psychology and American Grounded Theory both inspired by natural science and describing how individual and collective learning means adapting knowledge to the outside world by assimilation and accommodation. And finally I was caught by postmodern or post-structural skeptical thinking developed in the threatened French Enlightenment republic warning against patronization in our most basic institutions: our words, beliefs, cures and schools. Here I saw the patronizing techniques of the school: hiding understandable alternatives forces children and teenagers to accept the ruling choices as nature.

Searching for contingency, I found hidden words as icon-counting, next-to addition, reversed addition, and per-numbers. In addition, I found that Mathematics was created as a natural science about the natural fact Many. By teaching in the US I found that teenagers can be allowed to develop their personal talent if Europe's line-organized office preparing education with forced classes are replaced with North American block-organized talent developing education with daily lessons in self-chosen half-year blocks. Furthermore, I found that Bourdieu might be right when warning against a knowledge nobility that use their public offices to protect the line-organized education to ensure that their children inherit their offices. And finally, Baumann's and Arendt's work on the extreme institutionalization in 20th century Europe made me realize that the problems in Mathematics education and its research might be caused by an exaggerated institutionalization that by forcing teachers to follow authorized routines makes them subjects to the banality of evil without knowing it and without wanting to be so.

10. How to Improve Mathematics Education

Bo: Thank you, John and Allan. Let us finish by looking at what this is all about, Mathematics education. The first International Congress on Mathematics Education, ICME 1, took place in 1968, so we can say that Mathematics education research has about the same age as the new Mathematics emerging in the 1960s. With half a century of research, we should expect the problems in Mathematics education to have disappeared or at least decreased considerably. However, the decreasing results of international tests indicates that the opposite is the case. The paradox that researching Mathematics education seems to create more problems than solutions motivates my last question 'how can Mathematics education be improved?'

John: Sentence. Sentence. Sentence. …

Allan: Indeed, we have a paradox when the problems in Math education increase with its research. To solve it we can ask how well defined Mathematics and education and research is? Or, as in the fairy tale Cinderella we can look for hidden alternatives that might please the Prince and make the paradox disappear? The ruling tradition presents Mathematics as ungrounded Metamatism with meaningless self-referring concepts, and with statements falsified by the outside world. The hidden alternative presents Mathematics as grounded science about the natural fact Many. These two alternatives entail two different forms of teaching. One presents concepts as created from above as examples from abstractions as shown in the textbooks; the other show how concepts are created from below as abstractions from examples, facilitated by concrete material for children and relevant gossip for teenagers.

Theorists also come in two forms. One uses the Platonic tradition to present physical phenomena as examples of metaphysical forms discovered by and investigated by philosophers. The other sees theory as grounded in and adapting to its underlying reality that generates the theory's concepts and validates its statements.

Research also comes in two forms. One is self-referring scholasticism commenting on comments already defended against three opponents. The other is Grounded Theory seeing individual and collective knowledge creation as parallel processes, creating schemata that adapt to the outside world. Finally, education also comes in two forms, as line-organized office-preparation or as block-organized talent-developing.

So to me, the choice within four factors determines the success of Mathematics education. Problems occur if Mathematics presents itself as Metamatism, if only top-down theorists are used, if research is scholastic, or if education uses force by choosing line-organized office preparation. When chosen simultaneously as in Europe, Mathematics education is in deep trouble, which of course suits the knowledge nobility well. To be successful, Mathematics must grows from its roots in the natural fact Many, only grounded bottom-up theorists must be used, research must be a natural science using the classrooms to generate categories and test predictions; and education must minimize its force by choosing block-organized talent development from secondary school. Having implemented the three latter, the North American republics only need to change Metamatism to grounded Mathematics to make their Mathematics education successful.

John: I would like to comment on what Allan said. Sentence. Sentence. Sentence.

Allan: I would like to comment on what John said. Sentence. Sentence. Sentence.

Bo: Thank you, John and Allan. I began by expressing the hope that you could provide some answers to the question 'If research cannot improve Mathematics education then what can?' I now see that this debate has resulted in a several suggestions that I am sure practitioners and politicians will be eager to work with and be inspired by. Thank you, John and Allan, for your time and for sharing your views with us.

John: You are welcome, Bo. I enjoyed very much to take part in this debate. Allan: So did I, Bo.