COUNTING MANY

C1

Questions	Answers
How to count Many?	By bundling and stacking the total T predicted by T = (T/b)*b.
How to recount 8 in 3s, T = 8 = ? 3s	T = 8 = ?*3 = ?3s, T = 8 = (8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3
How to recount 6 kg in $: T= 6 kg= ?$	If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$
How to count in standard bundles?	Bundling bundles gives a multiple stack, a stock or polynomial: T= 423 = 4BundleBundle2Bundle3 = 4tenten2ten3 = 4*B^2+2*B+3

1 REPETITION BECOMES MANY

Question. How can repetition in time be represented in space?
Answer. By iconisation: put a finger to the throat and add a match or a stroke for each beat of the heart.

Example: . . . . -> I I I I I I I I

Exercise. Find other examples of spatial representation of temporal repetition

2 MANY BECOMES BUNDLES

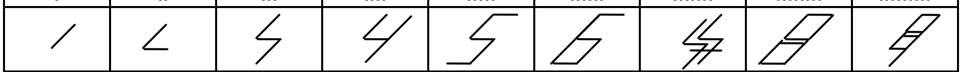
Question. How can we organise Many?
Answer. By bundling: line up the total and divide it into bundles.

Examples: I I I I I I I I I -> II II II II I or I I I I I I I I I -> IIIII IIII or I I I I I I I I I -> III III III or …

Exercise. Take a lot of matches and bundle them in 2s, then in 3s, then in 4s, etc.

3 BUNDLES BECOME ICONS

Question. How can we represent the different degrees of Many?
Answer. By iconisation: the strokes of the different degrees of Many are rearranged as icons, realising that there would be four strokes in the number-icon 4, etc., if written in a less sloppy way.
Example:

I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
								
1	2	3	4	5	6	7	8	9

Exercise. Find other ways to build icons for the numbers above. Invent icons for ten, eleven and twelve.

4 MANY IS COUNTED AS A STACK OR AS A STOCK

Question. How can we arrange the different degrees of Many?
Answer. By counting, by bundling and by stacking: First the total is lined up, then it is bundled and equal bundles are stacked and finally the height is counted as e.g. T = 3 4s = 3*4.

Examples:

IIIIIIIIIIII -> IIIIIIII [IIII] -> IIII [IIII] [IIII] -> [IIII] [IIII] [IIII] -> [IIII] / [IIII] / [IIII] (stacked) — T = 3 4s = 3*4 (a stack)

Leftovers are arranged in a separate stack creating a stock:

IIIIIIIIIIIIIII -> IIIIIIIIIII [IIII] -> IIIIIII [IIII] [IIII] -> III [IIII] [IIII] [IIII] -> [IIII] I / [IIII] I / [IIII] I (stacked) — T = 3*4 + 3*1 (a stock)

Or the 3 leftovers are counted in 4s: 3 = ¾*4:

IIIIIIIIIIIIIII -> IIIIIIIIIII [IIII] -> IIIIIII [IIII] [IIII] -> III [IIII] [IIII] [IIII] -> [III] / [IIII] / [IIII] / [IIII] (stacked) — T= 3*4 + ¾ *4 = 3 ¾ *4

We count in 4s by taking away 4s.
The process 'from T take away 4' may be iconized as 'T-4' and worded as 'T minus 4'.
The 4 taken away does not disappear, they are just put aside so the original total T is divided into two totals, one containing T-4 and the other containing 4:

[IIIIIIIII] -> [IIIII] [IIII] -> [IIIII] [IIII]
9 = (9-4) + 4 = 5 + 4 as predicted by the 'restack-equation' or 'readd-equation' T = (T-b)+b.

The repeated process 'from T take away 4s' may be iconized as 'T/4' and worded as 'T counted in 4s'. So the 'recount-equation' or 'rebundle-equation' T = (T/4)*4 predicts the result of recounting the total T in 4-bundles: T =(T/4)*4 =3*4+3*1 =3 ¾*4. T/4 is called a per-number, T a stock-number or a total, and 4 a base.
Exercise. Take a lot of matches and count and stack them in 2s, then in 3s, then in 4s, etc.

5 STOCKS ARE RECOUNTED

Question. How can we change the bundle-size in a stack (T = 3 4s = ? 5s).
Answer. By de-stacking, de-bundling, re-bundling and re-stacking: First the stack is de-stacked into separate bundles, then the bundles are de-bundled into a total, then the total is bundled and equal bundles are stacked and finally the heights are counted. Recounted in 2s even numbers give a stack and odd numbers give a stock.
Example:

IIII
IIII
IIII -> IIII IIII IIII -> IIIIIIIIIIII -> IIIIIII IIIII -> II IIIII IIIII -> IIIII I / IIIII I

3*4 = 4+4+4 = T = (T-5)+5 = (T/5)*5 = 2*5 + 2*1

Again the counting result can be predicted by the recount-equation T = (T/5)*5 = (3*4/5)*5 = 2*5 + 2*1 and displayed on a calculator able to do integer division, e.g. the Texas Instruments' Math Explorer.
Exercise1. Recount a 2-stack in 3s, in 4s, in 5s, etc. Recount a 3-stack in 2s, in 4s, in 5s, etc.
Exercise2. Recount a 2-stack in ½ s, in 1/3 s, in ¼ s, etc. Show that T = (T*n)*1/n.

6 STOCKS ARE CODED

Question. How can we code a stock?
Answer. By using codes as C and S to code different bundle sizes, e.g. C = 4 and S = 5.
Then the recount-question 'T = 3*4 = ?*5' is reformulated to 'T = 3*C = ?*S '.
Example:

C
C
C -> C C C -> IIIIIIIIIIII -> IIIIIII S -> II S S -> S I / S I

3*C = C+C+C = T = (T-S) +S = (T/S)*S = 2*S + 2*1

Also the recounting can be done as in trade by skipping the 1s: Thus if 1*C = 2*S we have

CCC -> SS SS SS -> SSSSSS
T = 3*C = 3*2*S = 6*S predicted by 3*C = 3*2*S = 6*S

SSSSSS -> SS SS SS -> CCC
T = 6*S = 3*2*S = 3*C predicted by 6*S = ((6/2)*2)*S = 3*2*S = 3*C

SSSSSSS -> SS SS SS S -> CCC S
T = 7*S = 3*2*S + S = 3*C + 1*S predicted by 7*S=((7/2)*2)*S=3*2*S+1*S=3*C+1*S

With codes a stack can by counted in Cs to practise tables. Thus C C C C can be counted as '2 4 6 8' or ' 3 6 9 twelve'. Here 'twelve' regains its original meaning 'two left' making it possible to count from ten to 'two-ten' as: ten, one left, two left, three left, …, nine left, two ten, etc.
Exercise. Using symbols, recount a 2-stack in 3s, in 4s, in 5s, etc. Recount a 3-stack in 2s, in 4s, in 5s, etc.

7 STOCKS ARE CODED

Question. How can we code a stock?
Answer. By using a container-symbol ')' to leave out the symbols for bundle and 1: T= 2*C+3*1 -> 2)3).
Thus we use the container-symbol '))' or '0' for 'none' to distinguish T = 2*C = 2)) = 20 from T = 2*1 = 2.
Later we leave out the containers and get many-digit numbers: T = 2)3) = 23, and T = 2)) = 2)0) = 20
Example1.

If C = 3, then T = 18 = 25 = 32
C IIIIIIII -> C IIIII III -> CC IIIII -> CC III II -> CCC II
T = 1 8 = 2 5 = 3 2

C = 4, then T = 30 = 24 = 18
CCC -> CC IIII -> C IIII IIII
T = 30 = 2 4 = 1 8

Or with containers

T = 18 = 1)8)
T = 1)8) = 1+1)-3+8) = 2)5) = 25
T = 2)5) = 2+1)-3+5) = 3)2) = 32

I	IIIII III
II	II III
III	II

T = 30 = 3)) = 3)0)
T = 3)0) = 3-1)+4+0) = 2)4) = 24
T = 2)4) = 2-1)+4+4) = 1)8) = 18

III	
II	IIII
I	IIII IIII

Example2. With many-digit numbers we can still practise recounting stocks by reformulating the recount-question 'T = 3*C + 1 = ?*S' to 'T = 31 = ?*S' , e.g. with C = 4 and S = 5:

C
C
C I -> C C C I -> IIIIIIIIIIIII -> IIIIIIII S -> III S S -> I / S I / S I

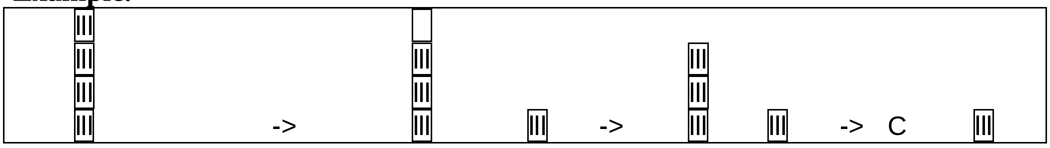
T = 3*C + 1 = 31
T = 2*S + 3*1 = 23

3 1 = T = (T/S)*S = 2 3

Example3. When the icons stop we use symbols. Thus the Romans used the symbol X for the number ten. The recount-question 'T = 3*8 = ?*X' recounts 3 8s as 2 tens and 4 1s: T = 3*8 = 2*X+4*1 = 24. This leads to traditional multiplication. However 3*8 is 3 8s; only if recounted in tens, it is 24.
The recount-question 'T = 3*X = ?*8' recounts 3 tens as 3 8s and 6 1s: T= 3*X= 30/8*8= 3*8+6*1. This connects traditional division with recounting, not with sharing, which is applying and not creating division.
Exercise1. If C=2 rewrite T=13 and T=30, etc. If C=3 rewrite T=15 and T = 41. If C=4, etc.
Exercise2. If C=2 and S=3 recount 31 in Ss, etc.
Exercise3. Recount a 3-stack in tens. Recount a ten-stack in 2s, in 3s, in 4s, etc.
Exercise4. Recount numbers on an abacus with 3 containers for 1s, 5s and tens, e.g. 2)4)8) = 2)5)3)= 4)1)3).

8 STACKS ARE OVERLOADED

Question. What do we do about an overload, i.e. if a stack is higher than its unit?
Answer. The overload then can be restacked to a new stack leaving a full stack C.
Example.



T = 4*3 = overload = (4*3 – 1*3) + 1*3 = 3*3 + 1*3 = 1*C + 1*3

So also bundles can be bundled and stacked in bundles-of-bundles, bundles & unbundled:
T = 234 = 2 bundles-of-bundles + 3 bundles + 4 unbundled (e.g. T = 2tenten3ten4)
In short a given degree of Many can always be rearranged as a multiple stack, a stock or a polynomial:
T = 2345 = 2)3)4)5 = 2)))+3))+4)+5 = 2*B^3 + 3*B^2 + 4*B + 5*1
Exercise. Recount the overloads: T= 7 5s, T= 18 8s, T= 34 tens, T= 562 hundreds, T= 562 tens, T= 562 1s

9 STOCKS ARE COUNTED IN TENS

Question. How can we code a stock counted in tens?
Answer. By many-digit numbers leaving out the symbols for bundle and 1: T = 2*X+3*1 -> 2)3) -> 23
Example. Multiplication shows the result of a standard recounting in tens and 1s:
T = 3 6s = 3*6 = 18 = 1*10 + 8*1.
Recounting in tens leads to decimals and percentages:
T = 3 6s = 3*6 = 18 = (18/10)*10 = (1 8/10)*10 = 1.8*10
T = 3 6s = 3*6 = 18 = (18/100)*100 = 18%*100
T = 3.6 = (3.6/10)*10 = 0.36*10
T = 3.6 = (3.6/(1/10))*1/10 = 36*1/10 = 36 tenths = (3.6*10)*1/10, so T/(1/b) = T*b
Remark. T = 3*6 = 3 6s and T = 3*6 = 18 only if recounted in tens. Thus multiplication is a special division.
Exercise. Recount a 3-stack in tens, in tenths, in hundreds, in hundredths, in thousands, in thousandths.

10 STOCKS ARE COUNTED IN UNITS

Question. How can we recount a stock in a different unit?
Answer1: Recount the number using the key-numbers.
Answer2: Recount the unit using the key-numbers
Example: Sugar can be bundled in kilos, litres, dollars and %.
Key-numbers: 2 kg = 5 $ = 6 litres = 100 %, T = 7 kg = ?

Recount the number	Recount the unit
T = 7 kg = (7/2)*2kg = (7/2)*5 $ = 17.50 $	$ = ($/kg)*kg = (5/2)*7 = 17.5
T = 7 kg = (7/2)*2kg = (7/2)*6 litres = 21 litres	litres = (litres/kg)*kg = (6/2)*7 = 21
T = 7 kg = (7/2)*2kg = (7/2)*100 % = 350 %	% = (%/kg)*kg = (100/2)*7 = 350
P = 5% = (5/100)*100% = (5/100)*2 kg = 0.1 kg	kg = (kg/%)*% = (2/100)*5 = 0.1

Exercise. 4 kg = 6 $ = 7 litres = 100 %, T = 8 kg = ?

11 STOCKS ARE COUNTED IN DIFFERENT BASES

Question. How can we recount a stock in a different base?
Answer1. Recount the number using the key-numbers.
Example1. T = 2 7s. The 'ten-counter' counts 'bundle + 4', i.e. 14, and the 'twelve-counter' counts 'bundle + 2', i.e. 12. Hence T = 14($\underline{10}$) = 12($\underline{12}$).
Example2. T = 31($\underline{4}$) = ? ($\underline{5}$)

Key-numbers: 4 5s = 5 4s
Recount the number
31(4) = 3 4s+1 = (3/5)*5 4s + 1 = (3/5)*4 5s + 1 = 12/5 5s + 1 = 2 5s + 2+1 = 2 5s + 3 = 23(5)
Check by recounting in tens: 31(4) = 3*4+1 = 13 and 23(5) = 2*5+3 = 13
Exercise1. Count your ten fingers as a 5-counter, a 4-counter, a 3-counter and a 2-counter.
Exercise2. 23(4) = ?(5) = ?(6) , 23(6) = ?(5) = ?(4) , 23(10) = ?(9) = ?(8) = ? , etc.

12 STOCKS ARE DECODED

Question. How can we decode a coded stock?
Answer. By recounting and restacking.
Examples.

C III = IIIIIIII = IIIII III
C + 3 = 8 = (8-3)+3 So C = 5 as predicted by C+3 = 8 = (8-3)+3 = 5+3

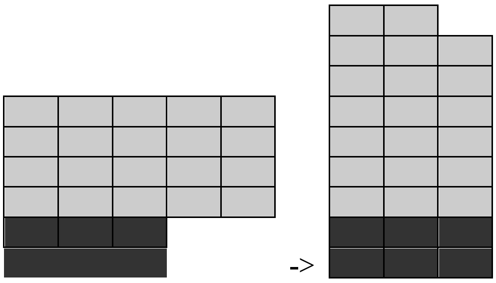
II
C II III
C = IIIIII -> II II II II -> II III
2*C = 6 = (6/2)*2 = 3*2 = 2*3 So C = 3 as predicted by 2*C = 6 = (6/2)*2 = 3*2

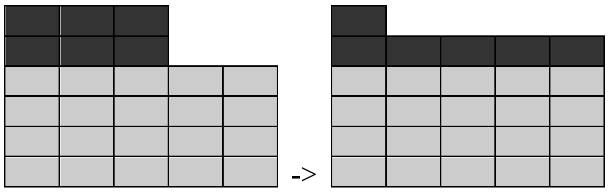
CC I = IIIIIIIII = IIIIIIII I = II II II II I = IIII IIII I
2*C + 1 = 9 = (9-1)+1 = 8+1 = (8/2)*2 +1 = 4*2 + 1 = 2*4 + 1

So C = 4 as predicted by 2*C + 1 = 9 = (9-1)+1 = 8+1 = (8/2)*2+1 = 4*2 + 1.
Exercise1. Decode C+1=3, C+1=4, etc. Decode C+2=4, C+3=5, etc.
Exercise2. Decode 2*C=4, C+1=4, etc. Decode C+2=4, C+3=5, etc.
Exercise3. Decode C+1=3, C+1=4, etc. Decode C+2=4, C+3=5, etc.

13 STACKS ARE ADDED UPWARD

Question. How can we add stacks upward?
Answer. Stacks can be added upward in space by recounting and restacking.
Examples. T = 2 3s + 4 5s = ? 3s, T = 2 3s + 4 5s = ? 5s. The result can be predicted by the recount-equation.
The 4 5s are recounted in 3s: T = 2 3s + 4 5s = 2 3s + (4*5/3)*3 = 2 3s + 6*3 +2 = 2 3s + 6 3s + 2 = 8 3s + 2
The 2 3s are recounted in 5s: T = 2 3s + 4 5s = (2*3)/5*5 + 4 5s = 1*5 + 1 + 4 5s = 1 5s + 1 + 4 5s = 5 5s + 1
To check both stocks can be recounted in tens: T = 8 3s + 2 = 8*3+2 = 26 and T = 5 5s + 1 = 5*5+1 = 26



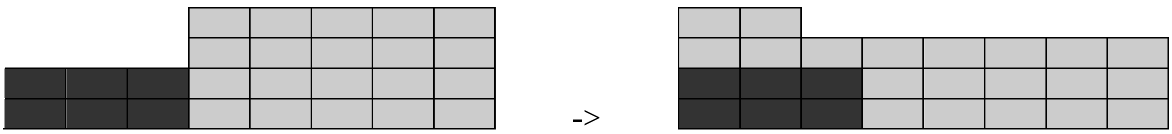


T = 2 3s + 4 5s -> = 8 3s + 2 T = 2 3s + 4 5s -> = 5 5s + 1

Exercise. Add 1 2s + 3 4s upward and predict the result. Add 2 3s + 2 4s upward and predict the result, etc.

14 STACKS ARE ADDED SIDEWARD

Question. How can we add stacks sideward?
Answer. Adding stacks sideward in time is called integration. It can be done by recounting and restacking.
Example. T = 2 3s + 4 5s = ? 8s. The result can be predicted by the recount-equation.
The 3s and 5s are added as 3+5 = 8s: T = 2 3s + 4 5s = (2*3 + 4*5)/8*8 = 3*8 +2 = 3 8s + 2
To check both stocks can be recounted in tens: T = 2 3s + 4 5s = 2*3 + 4*5 = 26 and T = 3*8 +2 = 26



T = 2 3s + 4*5 -> = 3 8s + 2

Thus integration adds the per-numbers 2 and 4 as heights in stacks: 2 + 4 = 3 2/8.
Thus 2 + 4 can give many different results, unless the units are the same:
T= 2*3 + 4*3 = 6*3 if added in time; and T= 2*3 + 4*3 = (2*3 + 4*3)/6*6 = 3*6 if added in space.
Exercise1. Add 1 2s + 3 4s sideward and predict the result. Add 2 3s + 2 4s sideward and predict the result, etc.
Exercise2. Reverse integration to get differentiation: T = 2 3s + ? 5s = 3 2/8 8s, etc.