Count in Icons before Tens, then Add NextTo before OnTop

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Abstract
Preschool allows rethinking mathematics outside the tradition of ordinary school. Seeing schooling as adapting the child to the outside world containing many examples of Many, we can ask: How will mathematics look like if built as a natural science about physical fact Many? To deal with Many we count and add. The school counts in tens, but preschool allows counting in icons also. Once counted, totals can be added. To add on-top the units are made the same through recounting, also called proportionality. Adding next-to means adding areas, also called integration. So icon-counting and next-to addition offers golden learning opportunities in preschool that is lost in ordinary school allowing only ten-counting to take place.

Math in Preschool – a Great Idea

Mathematics is considered one of the school's most important subjects. So it seems a good idea to introduce mathematics in preschool - provided we can agree upon what we mean by mathematics.

As to its etymology Wikipedia writes that the word mathematics comes from the Greek máthēma, which, in the ancient Greek language, means "that which is learnt". Later Wikipedia writes:

> In Latin, and in English until around 1700, the term mathematics more commonly meant "astrology" (or sometimes "astronomy") rather than "mathematics"; the meaning gradually changed to its present one from about 1500 to 1800. (http://en.wikipedia.org/wiki/ Mathematics)

This meaning resonates with Freudenthal writing:

> Among Pythagoras' adepts there was a group that called themselves mathematicians, since they cultivated the four "mathemata", that is geometry, arithmetic, musical theory and astronomy. (Freudenthal 1973: 7)

Thus originally mathematics was a common word for knowledge present as separate disciplines as astronomy, music, geometry and arithmetic.

This again resonates with the educational system in the North American republics offering courses, not in mathematics, but in its separate disciplines algebra, geometry, etc.

In contrast to this, in Europe with its autocratic past the separate disciplines called Rechnung, Arithmetik und Geomtrie in German were integrated to mathematics from grade one with the arrival of the 'new math' wanting to revive the rigor of Greek geometry by defining mathematics as a collection of well-proven statements about well-defined concepts all being examples of the mother concept set.

Kline sees two golden periods, the Renaissance and the Enlightenment that both created and applied mathematics by disregarding Greek geometry:

> Classical Greek geometry had not only imposed restrictions on the domain of mathematics but had impressed a level of rigor for acceptable mathematics that hampered creativity. Progress in mathematics almost demands a complete disregard of logical scruples; and, fortunately, the mathematicians now dared to place their confidence in intuitions and physical insights. (Kline 1972: 399)

Furthermore, Gödel has proven that the concept of being well-proven is but a dream. And Russell's set-paradox questions the set-based definitions of modern mathematics by showing that talking about sets of sets will lead to self-reference and contradiction as in the classical liar-paradox 'this sentence is false' being false if true and true if false:

If $\quad M = \{A \mid A \notin A)\}$ $\quad$ then $\quad M \in M \Leftrightarrow M \notin M$.

Without an agreement as to what mathematics is and with the negative effects of imposing rigor, preschool math should disintegrate into its main ingredients, algebra meaning reuniting numbers in Arabic, and geometry meaning measuring earth in Greek; and both should be grounded in their common root, the physical fact Many. To see how, we turn to skeptical research.

Postmodern Contingency Research

Ancient Greece saw a knowledge controversy between the sophists and the philosophers. The sophists warned that in a republic people must be enlightened about choice and nature to prevent being patronized by choices presented as nature. In contrast to this skepticism philosophers saw the physical as examples of meta-physical forms only visible to the philosophers educated at Plato's academy, who then should be allowed to patronize.

Enlightenment later had its own century, the 18th, that created two republics, an American and a French. Today the sophist warning against hidden patronization is kept alive in the French republic in the postmodern skeptical thinking of Derrida, Lyotard, Foucault and Bourdieu warning against patronizing categories, discourses, institutions and education presenting their choices as nature (Tarp 2004).

Thus postmodern skeptical research discovers contingency, i.e. hidden alternatives to choices presented as nature. To make categories, discourses and institutions non patronizing they are grounded in nature using Grounded Theory (Glaser et al 1967), the method of natural research developed in the other Enlightenment republic, the American; and resonating with Piaget's principles of natural learning (Piaget 1970) and with the Enlightenment principles for research: observe, abstract and test predictions.

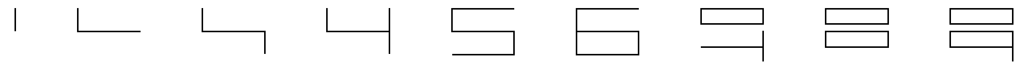
To look for patronization hidden in the words, truths and discourses of math education we ask: How will mathematics look like if built, not as a self-referring science about its own invention Set, but as a natural science about the physical fact Many; and how can this affect early childhood education?

The answers will be presented in papers and in YouTube videos (2013).

Building a Natural Science about Many

To deal with the physical fact Many, first we iconize, then we count by bundling. With ‘first order counting’ we rearrange sticks in icons. Thus five ones becomes one five-icon 5 with five sticks if written in a less sloppy way. In this way we create icons for numbers until ten since we do not need an icon for the bundle-number as show when counting in e.g. fives: one, two, three, four, bundle, one bundle and one, one bundle and two etc..

I II III IIII IIIII IIIIII IIIIIII IIIIIIII IIIIIIIII

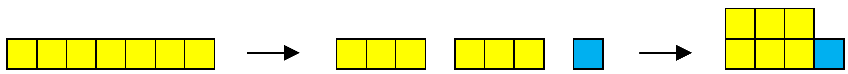


1 2 3 4 5 6 7 8 9

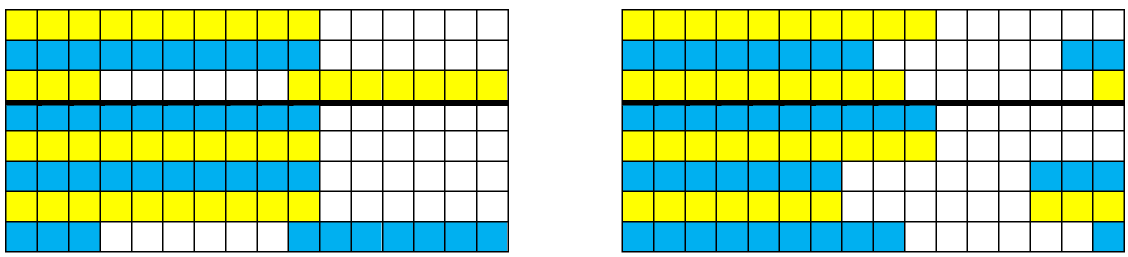
With ‘second order counting’ we bundle a total in icon-bundles. Here a total T of 7 1s can be bundled in 3s as T = 2 3s and 1. The unbundled can be placed in a right single-cup, and in a left bundle-cup we trade the bundles, first with a thick stick representing a bundle glued together, then with a normal stick representing the bundle. The cup-contents is described by icons, first using ‘cup-writing’ 2)1), then using ‘decimal-writing’ with a decimal point to separate the bundles from the unbundled, and including the unit 3s, T = 2.1 3s. Alternatively, we can also use plastic letters as B, C or D for the bundles.

IIIIIII → III III I → ▌▌) I) → II) I) → 2)1) → 2.1 3s or BBI → 2BI

Using squares or LEGO blocks or an abacus, we can stack the 3-bundles on-top of each other with an additional stack of unbundled 1s next-to, thus showing the total as a double stack described by a decimal number.



We live in space and in time. To include both when counting, we can introduce two different ways of counting: counting in space, geometry-counting, and counting in time, algebra-counting. Counting in space, we count blocks and report the result on a ten-by-ten abacus in geometry-mode, or with squares. Counting in time, we count sticks and report the result on a ten-by-ten abacus in algebra-mode, or with strokes.



To predict the counting result we can use a calculator. Building a stack of 2 3s is iconized as 2x3 showing a jack used 2 times to lift the 3s. As to the two icons for taking away, division shows the broom wiping away several times, and subtraction shows the trace left when taking away just once. Thus by entering ‘7/3’ we ask the calculator ‘from 7 we can take away 3s how many times?’ The answer is ‘2.some’. To find the leftovers we take away the 2 3s by asking ‘7 – 2x3’. From the answer ‘1’ we conclude that 7 = 2.1 3s. Showing ‘7 – 2x3 = 1’, a display indirectly predicts that 7 can be recounted as 2 3s and 1.

7 / 3	2.some
7 – 2 x 3	1

Re-counting in the Same Unit and in a Different Unit

Once counted, totals can be re-counted in the same unit, or in a different unit. Recounting in the same unit, changing a bundle to singles allows recounting a total of 4 2s as 3.2 2s, 2.4 2s. Likewise 4.2s can be recounted as 5 2s less or short of 2; or as 6 2s less 4 thus leading to negative numbers:

Letters	Sticks	Calculator		T =
B B B B	# # # #			4.0 2s
B B B I I	# # # I I	4x2 – 3x2	2	3.2 2s
B B I I I I	# # I I I I	4x2 – 2x2	4	2.4 2s
B B B B B	# # # # #	4x2 – 5x2	-2	5.2 2s
B B B B B B	# # # # # #	4x2 – 6x2	-4	6.4 2s

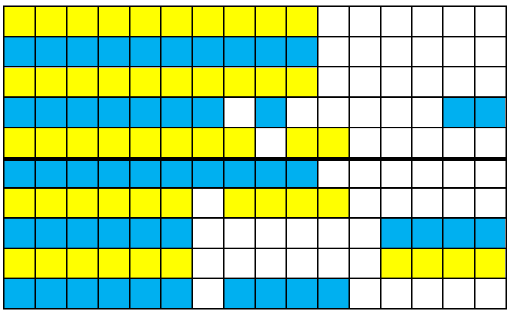
To recount in a different unit means changing unit, called proportionality or linearity also. Asking ‘3 4s is how many 5s?’ we can use sticks or letters to see that 3 4s becomes 2.2 5s.

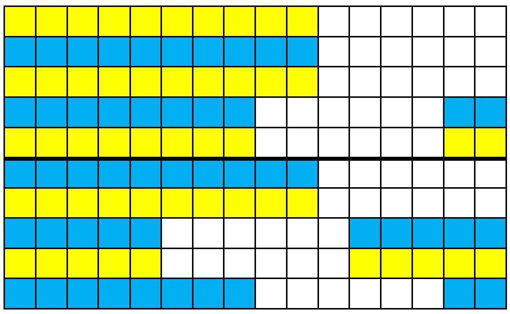
IIII IIII IIII → IIIII IIIII I I → 2) 2) 5s → 2.2 5s

With letters, C = BI so BBB → BBIIII → CCII

Using geometry-counting on an abacus, reserving the bottom line for the single 1s, a stack of 3 4s is moved from left to right on an abacus. The top bundle is changed to 1s in the single line and twice a stick is removed to enlarge the two 4-bundles to 5-bundles. This shows that ‘3 4s can be recounted as 2.2 5s.’

Using algebra-counting, 3 bundles are moved to the right. Then one 4-bundle is changed to 4 1s on the single-line. Moving 2 beads to the left on the single-line allows enlarging the 4s to 5s thus showing that 3 4s = 2.2 5s.





Using a calculator to predict the result we enter ‘3x4/5’ to ask ‘from 3 4s we take away 5s how many times?’ The calculator gives the answer ‘2.some’. To find the leftovers we take away the 2 5s and ask ‘3x4 – 2x5’. Receiving the answer ‘2’ we conclude that T = 3 4s = 2.2 5s.

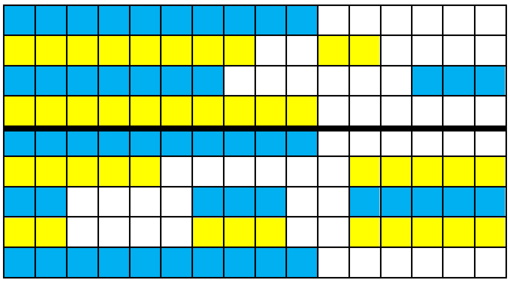
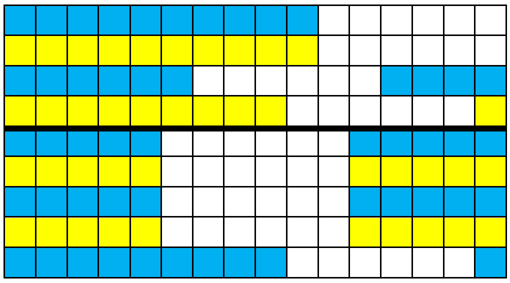
3 x 4 / 5	2.some
3 x 4 – 2 x 5	2

Adding On-top and Next-to

Once counted, totals can be added on-top or next-to. Asking '3 5s and 2 3s total how many 5s?' we see that to be added on-top, the units must be the same, so the 2 3s must be recounted in 5s giving 1.1 s that added to the 3 5s gives a grand total of 4.1 5s. With letters: 3B + 2C = 3B III III = 4BI. With sticks:

IIIII IIIII IIIII III III → IIIII IIIII IIIII IIIII I → 4) 1) 5s → 4.1 5s,

On an abacus in geometry mode a stack of 3 5s is moved to the right and a stack of 2 3s is moved to the middle. Now, the 2 3s is changed to 6 1s on the bottom line allowing one additional 5s to be moved to the top of the stack of 5s to show the grand total is 4.1 5s. Using algebra mode, the 3 5s become 3 beads on the bundle line and the 2 3s become 2 beads on the line above. Again the 2 3s is changed to 6 1s on the bottom line allowing one additional bead to be added to the bundle-line to give the result 4.1 5s

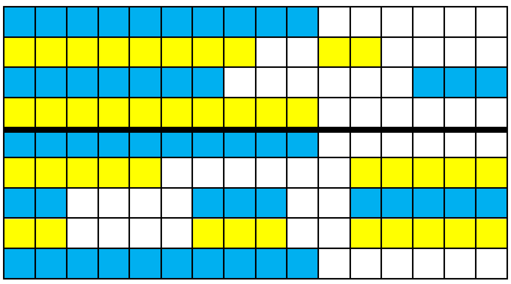
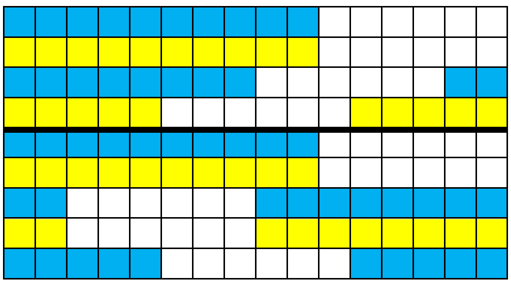
Using a calculator to predict the result we use a bracket before counting in 5s: Asking '(3x5 + 2x3)/5', the answer is 4.some. Taking away 4 5s leaves 1.

(3 x 5 + 2 x 3)/ 5	4.some
(3 x 5 + 2 x 3) – 4 x 5	1

To add next-to means adding areas called integration. Asking '3 5s and 2 3s total how many 8s?' we use sticks or letters to see that the answer is 2.5 8s.

IIIII IIIII IIIII III III → IIIII III IIIII III IIIII → 2) 5) 8s → 2.5 8s

On an abacus in geometry mode a stack of 3 5s is moved to the right and a stack of 2 3s is moved to the middle. Now a 5-bundle is moved to the single line allowing the two stacks to be integrated as 8s, showing that the grand total is 2.5 8s. Likewise when using algebra mode.

Using a calculator to predict the result we include the two totals in a bracket before counting in 8s: Asking '(3x5 + 2x3)/8', the answer is 2.some. Taking away the 2 8s leaves 5. Thus we get 2.5 8s.

(3 x 5 + 2 x 3)/ 8	2.some
(4 x 5 + 2 x 3) – 2 x 8	5

Reversing Adding On-top and Next-to

To reverse addition is called backward calculation or solving equations also. To reverse next-to addition is called reversed integration or differentiation. Asking '3 5s and how many 3s total 2.5 8s?' sticks will get the answer 2 3s:

IIIII IIIII IIIII III III ← IIIII III IIIII III IIIII ← 2) 5) 8s ← 2.5 8s

On an abacus in geometry mode with 2 8s and 5 moved to the right, first 3 5s is moved to the left, then the remaining is recounted in 3s as 2 3s. Using a calculator to predict the result the remaining is bracketed before counted in 3s.

(2 x 8 + 5 – 3 x 5)/ 3	2
(2 x 8 + 5 – 3 x 5) – 2 x 3	0

Adding two stacks 2 3s and 3 5s next-to each other as integration means multiplying before adding. And reversing integration as differentiation means subtracting before dividing, as in the gradient formula y' = dy/t = (y2 – y1)/t.

Conclusion

To find how mathematics looks like if built as a natural science about Many, and how this could affect early childhood education, postmodern contingency research has uncovered a 'ManyMatics' as a hidden alternative to the ruling tradition in mathematics. Dealing with Many means bundling and counting in icons, and recounting when adding on-top or next-to thus introducing proportionality and calculus. Likewise reversing on-top or next-to addition leads to solving equations and differentiation. That totals must be counted before being added means introducing the operations division, multiplication, subtraction before addition. These golden learning opportunities must be realized in preschool since they are lost from grade one, where the monopoly of ten-counting and the opposite order of operations prevent both from happening. Furthermore, here grounded ManyMatics is replaced by 'MetaMatism', a mixture of 'MetaMatics' turning mathematics upside down by presenting concepts as examples of abstractions instead of as abstractions from examples, and 'MatheMatism' true inside a classroom but not outside where claims as '1+2 IS 3' meet counter-examples as e.g. 1 week + 2 days is 9 days.

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YouTube videos (2013) by *MrAlTarp*.