COUNT&ADD IN TIME

T2

Question	Answer
How to predict the terminal number when the change is constant?	By solving constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7 = Ko*(1+r)^n = 30*1.02^7 = 34.46
How to predict the terminal number when the change is variable, but predictable?	By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx

1 COUNTING CHANGE

Question. How can we count change?
Answer. By three change-numbers restacking or recounting the terminal number:
Change, increment = ΔT = TerminalNumber-InitialNumber: T2 = (T2-T1)+T1 = ΔT + T1
Change-multiplier or index = I = TerminalNumber/InitialNumber T2 = (T2/T1)*T1 = I*T1
Change-percent, interest = r = Change-number/InitialNumber r = ΔT/T1 = (T2 - T1)/T1 = T2/T1-1 = I-1
From the level-numbers T the change-numbers are calculated directly. From the change-numbers the level-numbers are predicted by solving the change-equation telling how the change can be calculated. The change might be constant or variable.
Example.

Level		Single change				Total change		
T		**ΔT**	**I**	**r**		**ΔT**	**I**	**R**
200								
230		+ 30	*115 (%)	+ 15%		+ 30	*115 (%)	+ 15%
210		- 20	*91,3 (%)	- 8,70%		+ 10	*105 (%)	+ 5%
450		+240	*214,3 (%)	+ 114,30%		+ 250	*225 (%)	+ 125%

Exercise. Find the single and total change-numbers for T-levels 300, 360, 324, 420, etc.

2 CONSTANT CHANGE

Question. How can we predict the terminal number in constant change?
Answer. By solving the constant change-equations.
Example1. Constant change: Money in a box can change by adding a constant number (linear change by adding), by adding a constant percentage (exponential change by multiplying) or both (+&*change, saving).

+CHANGE (ΔK = a$)	*+&*CHANGE* **= a*R/r**	**CHANGE (ΔK/K = r%)*
Ko + A = Ko + a*n =	**K**	**= Ko*(1+r)^n = Ko*(1+R)**
	+ a $ + r %	
+ A = + a*n	**+ … + …**	+ R %
	+ a $ + r %	*1+R = (1+r)^n*
	+ a $ + r %	
Linear change by adding +5$+5$+5$ change, +change K*term* = K*init* +a+a+a n times: K = Ko + a*n = Ko + A, where A = a*n is the total change	**Ko**	**Exponential change by multiplying** +5%+5%+5% change, *change K*term* = K*init* +r%+r%+r% n times K*term* = K*init* +r%+r%+r% n times K = Ko*(1+r)*(1+r)*(1+r) = Ko*(1+r)^n = Ko*(1+R) where R is the total %change or compound interest

Example2. Compound interest: We cannot add percentages to dollars so Ko is recounted as 100% making K1 100% + 5% = 105% of Ko: K1 = Ko*105% = Ko*1,05 = Ko*(1+r)
R is the total compound interest, i.e. the sum of the simple interest SR = n*r and compounded interest RR:
R = SR+RR = n*r + RR, where (1+r)^n = 1+R
7 times @ 5$ = 7*5 $ = 35 $
7 times @ 5% = 7*5% + RR = 35% + 5% = SR + RR = R = 40% (since 1+R=107%^5 = 140%)
Example3. Double-taxation:
CarPrice + 200% tax + 25% vat. = CarPrice*(1+2)*(1+0,25) = CarPrice*3,75 = CarPrice*(1+2,75) = CarPrice + 275% tax. And 275% tax = 200% CarTax + 75% Tax-of-Tax
Example4. Doubling:
R=100% gives the change-multiplier 2 and the doubling-time predicted by:
(1+r)^N = 2, N = log2/log(1+r) ≈ 70/r.

Example5. Saving, or +&*change, is change by adding&multiplying both a constant deposit a$ and a constant interest r% e.g. +(5%&7$)+(5%&7$). Thus the compound interest Ko*R is a saving generated by the deposit a = Ko*r$ and the interest r%. So the saving is K = Ko*R = a/r*R = a/r*((1+r)^n-1). Only the interest is taxed since the deposits have already been taxed.
Account II hold a debt G having grown exponentially to K from its interests: K = G*(1+R). In an instalment plan the goal is to balance the debt on II by the saving so G*(1+R)=a/r*R: G=a/r*R/(1+R)=a/r*(1-(1+r)^-n).
Example6. Seen as +change & *change, linear and exponential change is grouped with +&change or saving. Seen as ++change and +*change, linear and exponential change is grouped with **change or power-change.

Linear change (++ change) x: +1, y: +a, e.g. trade	*Exponential change (+* change)* x: +1, y: +a %, e.g. interest	*Power change (** change)* x: +1 %, y: +a %, e.g. dimensions
b + x times @ a $/time total T b + x*a = T	b + x times @ á r %/time total T b * (1+r%)^x = T	Dimension T = a*Dimension x b * x^a = T #
Δy/Δx = a, y = b+a*x Δy/Δx: slope ≈ dy/dx = y’	Δy/Δx = a*y, y = b*a^x (Δy/Δx)/y: interest	Δy/y = c*Δx/x, y = b*x^a (Δy/y)/(Δx/x): elasticity

Example7. Calculation tables: # b is a conversion number between units

	Linear change		*Exponential change*		*Power change*
K = ?	K = Ko+a*n	K = ?	K = Ko*a^n	K = ?	K = Ko*n^a
Ko = 40 a = 3 n = 5	K = 40+3*5 **K = 55**	Ko= 40 a= 1.03 n = 5	K = 40*1.03^5 **K = 46.371**	Ko= 40 a= 3 n = 5	K = 40*5^3 **K = 5000**

Ko = ?	K = Ko+a*n	Ko = ?	K = Ko*a^n	Ko = ?	K = Ko*n^a
K = 70 a = 3 n = 5	K = Ko+(a*n) K - (a*n) = Ko 70 – (3*5) = Ko **55 = Ko**	K = 70 a = 1.03 n = 5	K = Ko*(a^n) K/(a^n) = Ko 70/(1.03^5) = Ko **60.383 = Ko**	K = 70 a = 3 n = 5	K = Ko*(n^a) K/(n^a) = Ko 70/(5^3) = Ko **0.56 = Ko**
Check:	*70 =? 55+3*5* *70 =! 70*	*Check:*	*70 =? 60.383*1.03^5* *70 =! 70*	*Check:*	*70 =? 0.56*5^3* *70 =! 70*

a = ?	K = Ko+a*n	a = ?	K = Ko*a^n	a = ?	K = Ko*n^a
Ko = 40 K = 70 n = 5	K = Ko+(a*n) K-Ko = a*n (K-Ko)/n = a (70-40)/5 = a **6 = a**	Ko = 40 K = 70 n = 5	K = Ko*(a^n) K/Ko = a^n n√(K/Ko) = a 5√(70/40) = a **1.118 = a**	Ko = 40 K = 70 n = 5	K = Ko*(n^a) K/Ko = n^a log(K/Ko)/log n = a log(70/40)/log 5 = a **0.348 = a**
Check:	*70 =? 40+6*5* *70 =! 70*	*Check:*	*70 =? 40*1.118^5* *70 =! 69.866*	*Check:*	*70 =? 40*5^0.348* *70 =! 70.032*

n = ?	K = Ko+a*n	n = ?	K = Ko*a^n	n = ?	K = Ko*n^a
Ko = 40 a = 3 K = 70	K = Ko+(a*n) K-Ko = a*n (K-Ko)/a = n (70-40)/3 = n **10 = n**	Ko = 40 a = 1.03 K = 70	K = Ko*(a^n) K/Ko = a^n log(K/Ko)/log a = n log(70/40)/log 1.03=n **18.9 = n**	Ko = 40 a = 3 K = 70	K = Ko*(n^a) K/Ko = n^a a√(K/Ko) = n 3√(70/40) = n **1.205 = a**
Check:	*70 =? 40+3*10* *70 =! 70*	*Check:*	*70 =? 40*1.03^18.9* *70 =! 69.933*	*Check:*	*70 =? 40*1.205^3* *70 =! 69.988*

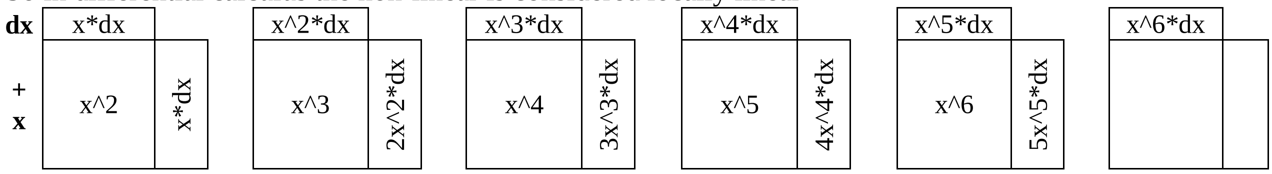
In savings there are 3 calculation tables since the interest r cannot be isolated appearing in both r and R:

K = ?	K = a/r*R	a = ?	K = a/r*R	n = ?	K = a/r*R
a = 100 n = 12 r = 0.05 1+R=1.05^12 R = 0.796	K=100/0.05*0.796 **K = 1592**	K = 1000 n = 12 r = 0.05 1+R=1.05^12 R = 0.796	K/R*r = a 1000/0.796*0.05=a **62.81 = a**	K=1000 a = 100 r = 0.05	K*r/a = R =(1+r)^n-1 1 + K*r/a = (1+r)^n log(1+K*r/a)/log(1+r)=n **8.3 = n** R = 1.05^8.3-1 = 0.499
		Check:	1000= 62.814/0.05*0.796 1000 =! 999.999	Check:	1000 =? 100/0.05*0.499 1000 =! 998

Example7. Graph papers:
On an equidistant +scale the same number is added for each step: 0,10,20,30,40
On a logarithmic *scale the same number is multiplied for each step: 1,2,4,8,16,32
Linear change gives a straight line on ++ paper (graph paper).
Exponential change gives a straight line on +* paper (logarithmic paper).
Power change gives a straight line on ** paper (double logarithmic paper).
Exercise. Make the calculation tables with different numbers. Make the graphs on graph paper too.

3 PREDICTABLE CHANGE

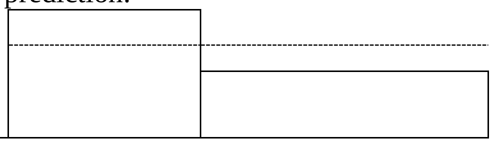
Question. How can we predict the terminal number in predictable change?
Answer. By solving the predictable change-equations.
Example. Totalling per-numbers:
5 seconds @ 6 m/s total 5*6 = 30 m.
5 seconds @ 6 m/s increasing to 8 m/s total ? m.
5 seconds @ 6 %/s total 5*6% = 30% + 3.8% = linear interest + compounded interest (since 1.06^5 = 1.338)
So compounded interest is what keeps exponential change from being linear.
For small interests dx the compounded interest can be neglected: 1.006^5 = 1.03036 ≈ 1.03 so 5*0.6% ≈ 3%
Neglecting compounded interest (or the upper right corner of the change stack) is called differential calculus.
So in differential calculus the non-linear is considered locally linear



x + dx **x^2 + 2x*dx** **x^3 + 3x^2*dx** **x^4 + 4x^3*dx** **x^5 + 5x^4*dx** **x^6 + 6x^5*dx**

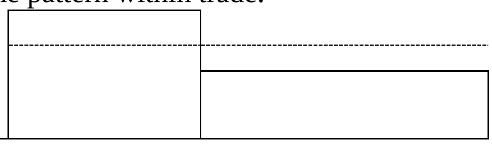
In differential calculus (x+dx)^2=x^2+2*x*dx; (x+dx)^3=x^3+3*x^2*dx etc.: (x+dx)^n = x^n+n*x^(n-1)*dx
If y = x^n, then a change in x, dx, produces a change in y, dy, and
y + dy = (x+dx)^n = x^n + n*x^(n-1)*dx = y + n*x^(n-1)*dx
So dy = + n*x^(n-1)*dx, or dy/dx = n*x^(n-1)
The elementary school introduces the practise of counting Many by bundling and stacking to be predicted by per-numbers, thus making the students acquainted with the geometrical representation of per-numbers as the height of a stack. Also simple additions as T = 3 4s + 2 5s = ? 9s are carried out both by recounting and by prediction thus realising that mathematics is our language of prediction:

T = 3 4s+2 5s = 3*4+2*5 = (3*4+2*5)/9*9 = 2 4/9 *9



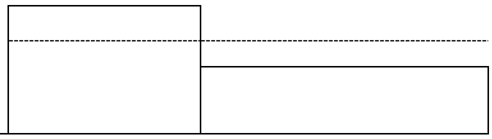
In secondary school addition of per-numbers follows the same pattern within trade:

T = 4kg @ 3$/kg + 5kg @ 2$/kg = 9kg @ 2 4/9$/kg



And so does addition of per-numbers within physics:

T = 4s @ 3m/s + 5s @ 2m/s = 9s @ 2 4/9m/s



This also applies if the m/s-number is locally constant and not just piecewise constant, i.e. if ε and δ changes places in the formal definition of constancy:

A variable y is globally constant c	∀ε>0:	|y-c|	< ε all over
A variable y is piecewise constant c	∃δ>0 ∀ε>0:	|y-c|	< ε in the interval δ
A variable y is locally constant c (continuous)	∀ε>0 ∃δ>0:	|y-c|	< ε in the interval δ

Thus per-numbers are added by the area under their curve. Since any smooth curve is locally constant, its area can be approximated by stacks that are summed (integrated): ∫ y*dx ≈ ∑ y*Δx. However if y*Δx can be written as the change of another variable z (y*Δx = Δz, or y = Δz/Δx) then the sum can be predicted since the sum of single changes = the total change = TerminalNumber - InitialNumber:

∑ y*Δx = ∑ Δz = Δz = z2 - z1.

This relation does not depend on the size of the change, so also ∫ y*dx = ∫ dz = z2 – z1.
Now we are able to predict the result of adding variable per-numbers through the calculation integration:

5 sec. @ 3m/sec increasing to 4 m/sec total $\int_0^5 (3+\frac{4-3}{5}x)\,dx = \int_0^5 (3+0.2x)\,dx$ = ? m

Since d/dx (3x+0.1x^2) = 3+0.2x we get that d (3x+0.1x^2) = (3+0.2x) dx, so

$$\int_0^5 (3+0.2x)\,dx = \int_0^5 d(3x+0.1x^2) = \Delta_0^5\ (3x+0.1x^2) = (3\cdot5+0.1\cdot5^2) - 0 = 17.5 \text{ m}$$

Exercise. Do other examples of integration from p. 6 and 7.

4 DRAWING GRAPHS

Question. How can we draw a non-linear graph?
Answer. By identifying its turning points.
A variable number is called x. A formula with a variable number is called a function f(x) e.g. f(x) = 3*x^2 – 4*x + 5. A function can be drawn as a f-curve y = f(x) in a coordinate system.

y = f(x)	predicts the level of the curve
y’ = f’(x)	predicts the slope of the curve and of the tangent extending the local linearity of the curve
y” = f”(x)	predicts the curvature of the curve
∫y dx	predicts the area under the curve, and the average level of the curve

Example 1. An f-table has four rows with x, y, y’ og y”, telling that for e.g. x = 3 the level is 20, the slope is 14 and the curvature is 3. With a positive slope the curve is going upward. With a positive curvature the curve is bending upward.

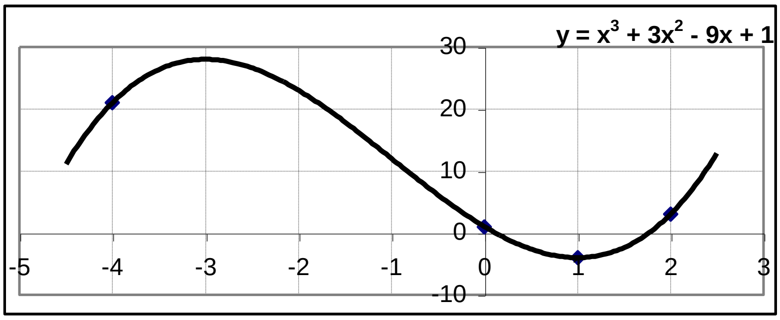
Position	x	3	5
Level	y = 3*x^2–4*x+5	y = 3*3^2–4*3+5 = 20	y =
Slope	y’ = 6*x–4	y’ = 6*3 – 4 = 14	y’ =
Curvature	y” = 3	y” = 3	y” =

Example 2. At a curve tops and bottoms can be located from the drawing. Or from the curve’s f-table:

x		-3		-1		1	
y =x^3+3*x^2–9*x+1	↗			↘			↗
y’ =3*x^2+6*x–9 *y’=0 for x=-3 og x=1*	+	y’ =0 top		-		y’= 0 bottom	+
y” = 6*x+6 *y”=0 for x=-1*		-		y” = 0 turning tangent		+	

The f-table shows that for x = -3 and x = 1 the curve has a horizontal tangent with slope y’= 0.
For x = -3 the curve is bending downward (y” is negative) giving a top. Also the signs of y’ indicate a top being positive before and negative after x = -3.
For x = 1 the curve is bending upward (y” is positive) giving a bottom. Also the signs of y’ indicate a top being negative before and positive after x = -3.
For x = -1 we have a turning tangent since the sign of the curvature changes.

x	y
-4	21
0	1
1	-4
2	3



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Exercise. Do other examples of graph drawing from p. 6 and 7.

5 TWO VARIABLES

Question. How can we predict the behaviour of a two-variable graph?
Answer. By identifying its turning points.
A total T containing two variable numbers x and y will form a surface when drawn in a three dimensional coordinate system. Restricting one variable creates partial curves with slopes found by partial differentiation.

Example. T = x^2 + y^2 – 2*x + 3
Slope in the i x-direction Tx' = ∂T/∂x = ∂/∂x (x^2 + y^2 – 2*x + 3) = 2*x – 2 (y is constant)
Slope in the i y-direction Ty' = ∂T/∂y = ∂/∂y (x^2 + y^2 – 2*x + 3) = 2*y (x is constant)
The surface is locally horizontal where Tx' = 0 og Ty' = 0, i.e. in (x,y) = (1,0)
A surface has level-curves with a constant T-number e.g. T = 6:

Level-curve = ?	x^2 + y^2 – 2*x + 3 = T
T = 6	x^2– 2*x + 1 + y^2 = 6 – 3 + 1
	(x-1)^2 + (y-0)^2 = 4 = 2^2

So the level-curve T = 6 is a circle with centre (x,y) = (1,0) and radius r = 2.

T = a*x^2 + b*x + c*y^2 + d*y + e

a	b	c	d	e
1	-2	1	0	3

	y						
T	-3	-2	-1	0	1	2	3
x -3	27	22	19	18	19	22	27
-2	20	15	12	11	12	15	20
-1	15	10	7	6	7	10	15
0	12	7	4	3	4	7	12
1	11	6	3	2	3	6	11
2	12	7	4	3	4	7	12
3	15	10	7	6	7	10	15

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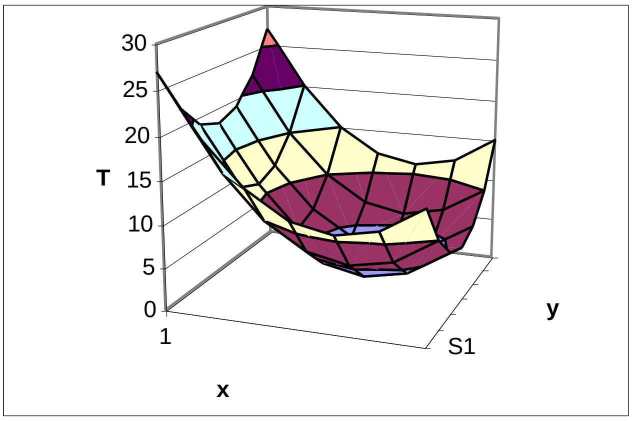


Exercise. Make other examples of graph drawing.

6 CHANGE EQUATIONS

Question. How can we solve the general change equation?
Answer. By adding the change ΔT to the initial number To to produce the terminal number T.
A change equation tells how the change can be calculated. A change equation is solved by calculating the terminal number from the initial number and the change. It can be solved by calculating techniques (e.g. integration) or by manual addition (numerical integration). Examples of growth equations:

Type of change	*Initial number*	*Change equation*	*Terminal number*
Constant change	To = b	ΔT = a	T = b + a*x
Constant change-percentage	To = b	ΔT = r%*T	T = b * a^x, a = 1+r
Constant change and change-percentage	To = 0	ΔT = r%*T + a	T/a = R/r, 1+R = (1+r)^n
Variable predictable change	To = b	dT = f*dx	T = b + ∫ f*dx
Variable unpredictable change	-	ΔT = ?	T = Tave ± 2*ΔTave

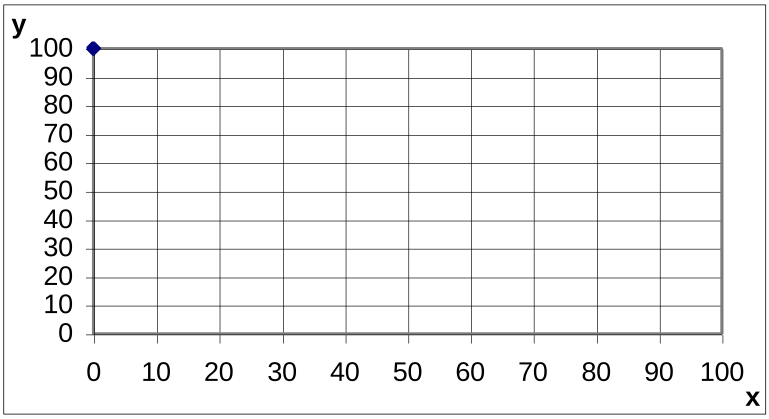
Example1. Numerical integration:

Stop/Start: 0/1 0

	x	y
Initial number	0	100
Change	1	-5
Terminal number	**0**	**100**

Double-click and edit

Guide
1. Enter 0 in B1, press F9
2. Enter initial number in C3
3. Enter change equation in C4
4. Enter 1 in B1, press F9
5. Repeat pressing F9



Exercise. Do other examples of solving change equations.

Exemple2. Predicting planetary orbits in EXCEL

ChangeEquations

d vx =	ax*dt
d vy =	ay*dt
d x =	vx*dt
d y =	vy*dt

LevelEquations

ax =	-x/r^3
ay =	-y/r^4
vx =	vx + d vx
vy =	vy + d vy
x =	x + d x
y =	y + d y
r =	V(x^2+y^2)
a =	1/r^2

Guide
1. Choose the field DEL and delete by pressing Delete
2. Choose the field Table
3. Pull down in the black spot in the lower right corner, one or two steps at a time

SUN	0	0
dt:	0,1	

ax	ay	dvx	vx	dvy	vy	dx	x	dy	y	r
			0		1,10		1		0	1,00
-1,00	0,00	-0,10	-0,10	0,00	1,10	-0,01	0,99	0,11	0,11	1,00
-1,00	-0,11	-0,10	-0,20	-0,01	1,09	-0,02	0,97	0,11	0,22	0,99
										0,99
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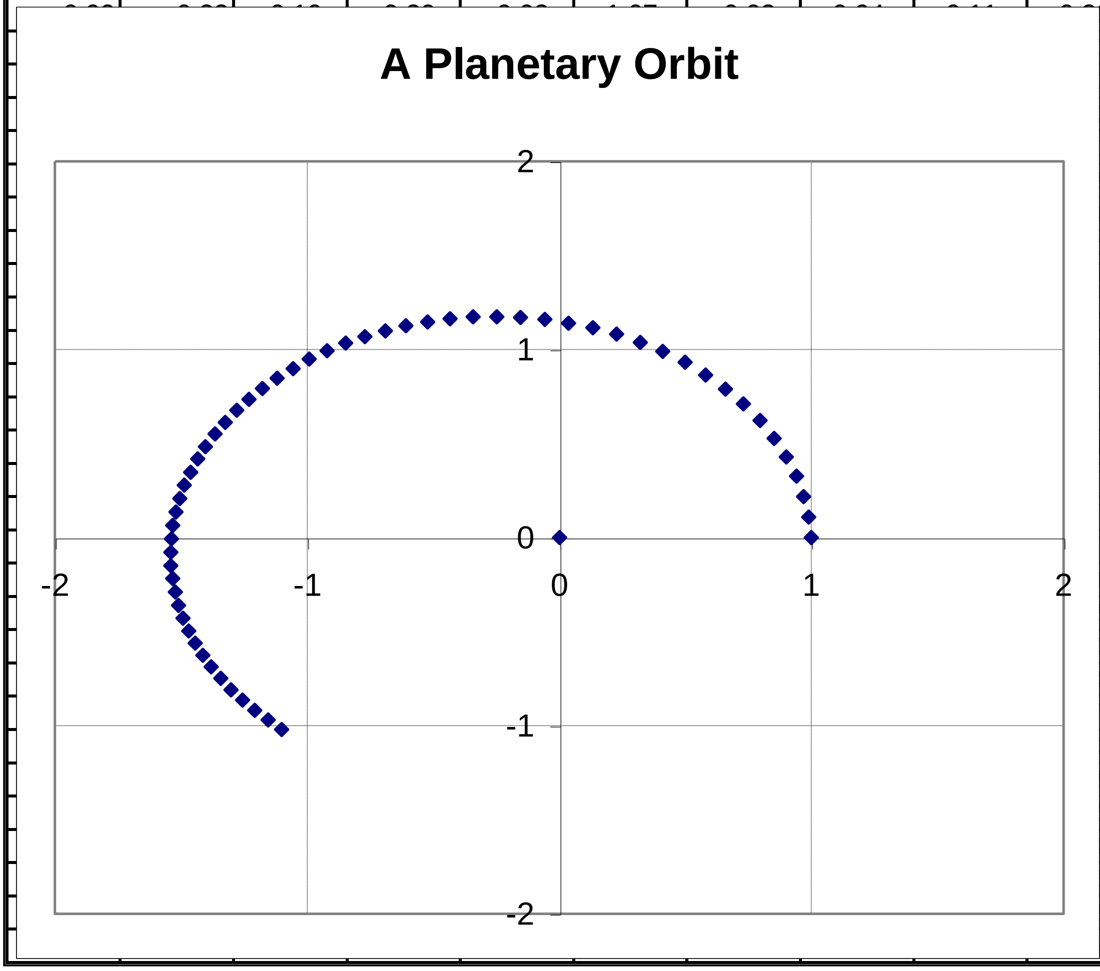
										1,43



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Exercise1 *Answers*

		x = 3 y = ?	y = 0 x = ?		y '	y ' = 0	y ''	∫ y dx	Turning points		Tangent i x=2	∫ y dx from 1 to 2	Signs	Factorise
1	y = x^2 -6x+5	-4	1	5	2x-6	3	2	0,33x^3-3x^2+5x+k	3	-4	y = -2x+1	1,67	+ - +	(x-1)(x-5)
2	y = x^2 -3x+2	2	1	2	2x-3	1,5	2	0,33x^3-1,5x^2+2x+k	1,5	-0,25	y = +1x-2	0,17	+ - +	(x-2)(x-1)
3	y = 2x^2 -10x+12	0	2	3	4x-10	2,5	4	0,67x^3-5x^2+12x+k	2,5	-0,5	y = -2x+4	-1,67	+ - +	2(x-3)(x-2)
4	y = 2x^2 -6x-8	-8	-1	4	4x-6	1,5	4	0,67x^3-3x^2-8x+k	1,5	-12,5	y = +2x-16	12,33	+ - +	2(x-4)(x+1)
5	y = 3x^2 -18x+15	-12	1	5	6x-18	3	6	1,00x^3-9x^2+15x+k	3	-12	y = -6x+3	5,00	+ - +	3(x-5)(x-1)
6	y = 3x^2 -24x+36	-9	2	6	6x-24	4	6	1,00x^3-12x^2+36x+k	4	-12	y = -12x+24	-7,00	+ - +	3(x-6)(x-2)
7	y = 4x^2 -40x+84	0	3	7	8x-40	5	8	1,33x^3-20x^2+84x+k	5	-16	y = -24x+68	-33,33	+ - +	4(x-7)(x-3)
8	y = 4x^2 -40x+64	-20	2	8	8x-40	5	8	1,33x^3-20x^2+64x+k	5	-36	y = -24x+48	-13,33	+ - +	4(x-8)(x-2)
9	y = -4x^2 -12x-8	-80	-1	-2	-8x-12	-1,5	-8	-1,33x^3-6x^2-8x+k	-1,5	1	y = -28x+8	35,33	- + -	-4(x+1)(x+2)
10	y = -4x^2 -4x+8	-40	1	-2	-8x-4	-0,5	-8	-1,33x^3-2x^2+8x+k	-0,5	9	y = -20x+24	7,33	- + -	-4(x+2)(x-1)
11	y = -3x^2 -6x+9	-36	1	-3	-6x-6	-1	-6	-1,00x^3-3x^2+9x+k	-1	12	y = -18x+21	7,00	- + -	-3(x+3)(x-1)
12	y = -3x^2 -6x+24	-21	2	-4	-6x-6	-1	-6	-1,00x^3-3x^2+24x+k	-1	27	y = -18x+36	-8,00	- + -	-3(x+4)(x-2)
13	y = -2x^2 -4x+30	0	3	-5	-4x-4	-1	-4	-0,67x^3-2x^2+30x+k	-1	32	y = -12x+38	-19,33	- + -	-2(x+5)(x-3)
14	y = 2x^2 +8x-24	18	-6	2	4x+8	-2	4	0,67x^34x^2-24x+k	-2	-32	y = +16x-32	7,33	+ - +	2(x+6)(x-2)
15	y = 3x^2 +18x-21	60	-7	1	6x+18	-3	6	1,00x^39x^2-21x+k	-3	-48	y = +30x-33	-13,00	+ - +	3(x+7)(x-1)
16	y = x^2 +6x-16	11	-8	2	2x+6	-3	2	0,33x^33x^2-16x+k	-3	-25	y = +10x-20	4,67	+ - +	(x+8)(x-2)

Exercise 2 *Answers*

		x=4 y=?	Zeros			Signs	Turning points		Factorising	Tangent in x=2	Differentiated	Integrated
1	y = x^3 -2x^2-5x+6	18	1	-2	3	- + - +	-0,79 8,21	2,12 -4,06	(x-1)(x+2)(x-3)	y=-4-1(x-2)	3x^2-4x-5	0,25x^4-0,67x^3-2,50x^2+6x+k
2	y = x^3 -6x^2+11x-6	6	1	2	3	- + - +	1,42 0,38	2,58 -0,38	(x-1)(x-2)(x-3)	y=-1(x-2)	3x^2-12x+11	0,25x^4-2,00x^3+5,50x^2-6x+k
3	y = 2x^3 -8x^2-22x+60	-28	2	-3	5	- + - +	-1,00 72,00	3,67 -29,63	2(x-2)(x+3)(x-5)	y=-30(x-2)	6x^2-16x-22	0,5x^4-2,67x^3-11,00x^2+60x+k
4	y = 2x^3 -20x^2+62x-60	-4	2	3	5	- + - +	2,45 1,26	4,22 -4,23	2(x-2)(x-3)(x-5)	y=6(x-2)	6x^2-40x+62	0,5x^4-6,67x^3+31,00x^2-60x+k
5	y = 3x^3 -18x^2-57x+252	-72	3	-4	7	- + - +	-1,21 289,30	5,21 -109,30	3(x-3)(x+4)(x-7)	y=90-93(x-2)	9x^2-36x-57	0,75x^4-6,00x^3-28,50x^2+252x+k
6	y = 3x^3 -42x^2+183x-252	0	3	4	7	- + - +	3,46 2,64	5,87 -18,19	3(x-3)(x-4)(x-7)	y=-30+51(x-2)	9x^2-84x+183	0,75x^4-14,00x^3+91,50x^2-252x+k
7	y = 4x^3 -32x^2-116x+720	0	4	-5	9	- + - +	-1,43 808,75	6,76 -290,82	4(x-4)(x+5)(x-9)	y=392-196(x-2)	12x^2-64x-116	1x^4-10,67x^3-58,00x^2+720x+k
8	y = 4x^3 -72x^2+404x-720	0	4	5	9	- + - +	4,47 4,51	7,53 -52,51	4(x-4)(x-5)(x-9)	y=-168+164(x-2)	12x^2-144x+404	1x^4-24,00x^3+202,00x^2-720x+k
9	y = -4x^3 +24x^2+76x-336	96	-4	3	7	+ - + -	5,21 145,74	-1,21 -385,74	-4(x+4)(x-3)(x-7)	y=-120+124(x-2)	-12x^2+48x+76	-1x^4+8,00x^3+38,00x^2-336x+k
10	y = -4x^3 +148x+336	672	-4	-3	7	+ - + -	3,51 682,51	-3,51 -10,51	-4(x+4)(x+3)(x-7)	y=600+100(x-2)	-12x^20x+148	-1x^40,00x^3+74,00x^2+336x+k
11	y = -3x^3 +12x^2+33x-90	42	-3	2	5	+ - + -	3,67 44,44	-1,00 -108,00	-3(x+3)(x-2)(x-5)	y=45(x-2)	-9x^2+24x+33	-0,75x^4+4,00x^3+16,50x^2-90x+k
12	y = -3x^3 +57x+90	126	-3	-2	5	+ - + -	2,52 185,63	-2,52 -5,63	-3(x+3)(x+2)(x-5)	y=180+21(x-2)	-9x^20x+57	-0,75x^40,00x^3+28,50x^2+90x+k
13	y = -2x^3 +4x^2+10x-12	-36	-2	1	3	+ - + -	2,12 8,12	-0,79 -16,42	-2(x+2)(x-1)(x-3)	y=8+2(x-2)	-6x^2+8x+10	-0,5x^4+1,33x^3+5,00x^2-12x+k
14	y = -2x^3 +14x+12	-60	-2	-1	3	+ - + -	1,53 26,26	-1,53 -2,26	-2(x+2)(x+1)(x-3)	y=24-10(x-2)	-6x^20x+14	-0,5x^40,00x^3+7,00x^2+12x+k
15	y = -x^3 +3x+2	-50	-1	-1	2	+ - + -	1,00 4,00	-1,00 0,00	-(x+1)(x+1)(x-2)	y=-9(x-2)	-3x^20x+3	-0,25x^40,00x^3+1,50x^2+2x+k
16	y = -x^3 +3x^2+1x-4	-16	-1	2	2	+ - + -	2,15	-0,15	-(x+1)(x-2)(x-2)	y=2+1(x-2)	-3x^2+6x+1	-0,25x^4+1,00x^3+0,50x^2-4x+k