Truth, Beauty and Goodness in Mathematics Education

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Abstract
In math education we can ask how education can lead to mathematics. But we could also ask how mathematics could lead to general educational goals as the three classical virtues: Truth, Beauty and Goodness. To do so, math must change from a self-referring MetaMatism true inside but not outside the classroom to a grounded ManyMatics, a natural science about Many, with numbers as blocks and with algebra as the art of reuniting numbers.

Goals and Means in Mathematics Education

Mathematics education is a core part of a school and is described by goals and means. Typically, mathematics is the goal with assessment focusing on the degree to which it has been learned. As means, different kinds of education are considered: Should the main emphasis be on teaching with high quality in teacher training and textbooks? Or should the main emphasis be on learning with focus on constructivism be it social or radical?

Once a means has been chosen education can begin, hopefully resulting in leading to the goals. However, PISA studies show that student performances are decreasing e.g. in the former model country Sweden seeing its mathematics levels decrease from 509 in 2003 to 478 in 2012 far below the OECD average at 494. This made OECD write a report describing the Swedish school system as being in need of urgent change (OECD, 2015).

Increased funding of mathematics education research in the period seems to have made the situation even worse. So to change the situation, unorthodox methods must be used by e.g. turning the goal and means discussion around and ask: How can mathematics contribute to general educational goals?

As to general educational goals Howard Gardner, known for his theory on MI, multiple intelligences, writes

> In my book *The Disciplined Mind*, published in 1999, I put forth a simple educational agenda: To help students understand, and act, on the basis of what is true, what is beautiful and what is good. I believed – and still believe – in that agenda. (Gardner 2001, xiv)

From this we can ask: how can mathematics be a means leading to the goal of implementing the three classical virtues Truth, Beauty and Goodness?

Truth in Mathematics

As to mathematics, its strength comes from including only well-defined concepts and well-proven statement, and from being highly applicable to the outside world. However, the declining PISA performance in many countries leads to ask: Is it mathe-matics that is taught or 'meta-matism', a mixture of 'meta-matics' and 'mathe-matism'?

MetaMatics is mathematics that uses self-reference to define its concepts top-down as examples of abstractions instead of using its historically roots to define its concepts bottom-up as abstractions from examples. Originally Euler defined a function as a common name for calculations containing numbers and letters. The invention of the abstraction Set turned this upside down so that today a function is defined as an example of a many-to-one set relation.

MatheMatism is mathematics that is true inside but not necessarily outside the classroom. Thus the statement '2+3 = 5' is not true with different units, e.g. 2 weeks + 3 days = 17 days. The statement '2x3 = 6' is always true since here 3 is the unit. Likewise with fractions where 1 empty bottle of 2 added to 2 empty bottles of 3 totals 3 empty bottles of 5 and not 7 empty bottles of 6. So to teach mathematics instead of mathematism we must always include the units as shown when writing out numbers fully: 345 = 3x100 + 4x10 + 5x1.

In the questionnaire below, teacher-answers marked with xs differ from the correct answers marked with dots. This and textbook definitions of functions show that what schools teach is indeed metamatism, not mathematics.

This is true	Always	Never	Sometimes
2+3 = 5	x		●
2x3 = 6	x ●		
1/2+2/3 = 3/5		x	●
1/2+2/3 = 7/6	x		●

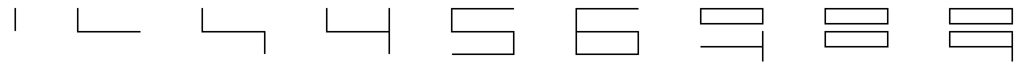
To bring back truth to mathematics it must be rebuilt from its original roots.

Building a Natural Science about Many

The core of mathematics is geometry and algebra, meaning to measure earth in Greek and to reunite numbers in Arabic. This shows that the root of mathematics is the physical fact 'Many' as it occurs in space and time.

To deal with Many, first we iconize, then we count by bundling. With 'first order counting' we rearrange sticks in icons. Thus five ones becomes one five-icon 5 with five sticks if written in a less sloppy way. We create icons until ten since we do not need an icon for the bundle-number as show when counting in fives: one, two, three, four, bundle, one bundle one, one bundle two etc..

I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII



1	2	3	4	5	6	7	8	9

With 'second order counting' we bundle a total in icon-bundles. Here a total T of 7 1s can be bundled in 3s as T = 2 3s and 1. So we place 2 sticks in a left bundle-cup and the unbundled we place in a right single-cup.

Writing the total in 'algebra-form', the cup-content is described by an icon, first using 'cup-writing' 2)1), then using 'decimal-writing' with a decimal point to separate the bundles from the unbundled, and including the unit 3s, T= 2.1 3s.

Alternatively, we can use plastic letters as B, C or D for the bundles.

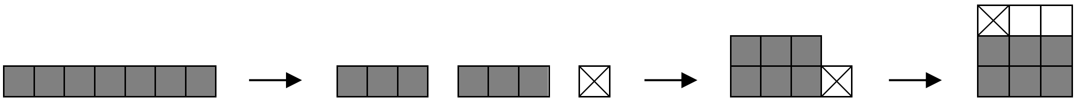
IIIIIII → III III I → II) I) → 2)1) → 2.1 3s or BBI → 2BI

A calculator can predict the counting result. To count in 3s we take away 3s, iconized as '/3' showing the broom wiping away the 3s several times. Building a stack of 2 3s we iconize as 2x3 showing a jack used to lift the 3s. And the trace coming from taking away the stack of 2 3s to look for unbundled is iconized as '-2x3'. These three operations are called division, multiplication and subtraction respectively.

Entering '7/3' the answer is '2.some'. To find the unbundled we take away the 2 3s by asking '7 – 2x3'. From the answer '1' we conclude that 7 = 2.1 3s.

7 / 3	2.some
7 – 2 x 3	1

Writing the total in 'geometry-form we use squares or LEGO blocks or an abacus to stack the 2 3-bundles on-top of each other with an additional stack of unbundled 1s next-to or on-top, thus describing the total as a decimal number 2.1 3s, or as a fraction number 2 1/3 3s counting the unbundled in 3s.



T = 7 = 2 3s & 1 = 2.1 3s = 2 1/3 3s

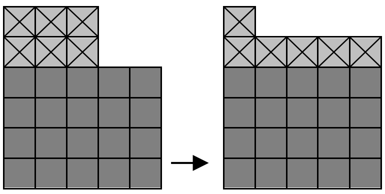
DoubleCounting creates PerNumbers Connecting Units

A physical quantity can be counted in different units. With 4kg = 5$ we have the 'per-number' 4kg/5$ = 4/5 kg/$. To shift from one physical unit to another we simply use the per-number to recount in a new number unit. To change unit is called proportionality, which is one of the core concepts in mathematics.

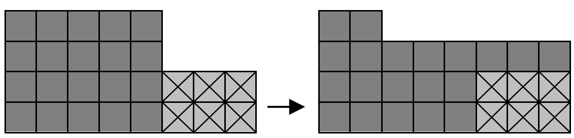
7 kg = ? $	8 $ = ? kg
7 kg = (7/4) x 4 kg = (7/4) x 5 $ = 8.75 $	8 $ = (8/5) x 5 $ = (8/5) x 4 kg = 6.4 kg

Adding Totals

Once Counted, totals can be added on-top or next-to. To add on-top, the units must be changed to be the same, typically by recounting one total in the other's unit. Adding next-to is called integrating areas.



OnTop: 4 5s + 2 3s = 5.1 5s



Next-to: 4 5s + 2 3s = 3.2 8s

NextTo addition is used when adding piecewise constant per-numbers:

4 kg at 5 $/kg + 2 kg at 3 $/kg = (4x5 + 2x3) $ = Σ (per-number x quantity)

Or when adding locally constant (continuous) per-numbers:

6 kg at 5 $/kg decreasing to 3 $/kg = $\int_0^6 (5 + \frac{3-5}{6} u)\, du$

Reversing Addition, or Solving Equations

Reversing addition we ask e.g. ‘2+? = 8’. With the restack-formula **T= (T-b)+b** we can restack 8 as (8-2)+2 to get the answer 8-2. Reversing multiplication we ask e.g. ‘2x? = 8’. With the re-count formula **T= (T/b)xb** we can recount 8 as (8/2)x2 to get the answer 8/2. We see that solving equations means moving numbers to the opposite side with opposite sign.

OnTop		NextTo
2 + ? = 8 = (8-2) + 2	**2 x ? = 8** = (8/2) x 2	**2 3s + ? 5s = 3 8s**
? = 8-2	? = 8/2	? = (3 8s – 2 3s)/5

Reversing adding next-to we ask e.g.‘2 3s + ? 5s = 3 8s’. To find what was added we take away the 2 3s and count the rest in 5s. Combining subtraction and division in this way is called reversed integration or differentiation.

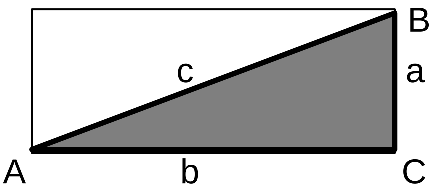
The Algebra Project: the Four Ways to Add

Meaning ‘to re-unite’ in Arabic, the ‘Algebra-square’ shows that with variable and constant unit-numbers and per-numbers there are four ways to unite numbers into a total, all present when writing 345 as 3x10^2 + 4x10 + 5x1; and that there are five ways to split up a united total.

Uniting/*splitting*	Variable	Constant
Unit-numbers	**T = a + n**, *T – a = n*	**T = a x n**, *T/n = a*
Per-numbers	**T = ∫ a dn**, *dT/dn = a*	**T = a^n**, *$\log_a(T) = n$, $\sqrt[n]{T} = a$*

Geometry: Measuring Earth divided into HalfBlocks

Geometry means earth-measuring in Greek. The earth can be divided in triangles that can be divided in right triangles that can be seen as blocks halved by their diagonals thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras theorem a^2 + b^2 = c^2; and connected with the angles by formulas recounting the sides in diagonals:

a = (a/c) x c = sin A x c b = (b/c) x c = cos A x b *a = (a/b) x b = tan A x b*	

Different answers to the same Questions

Asking the same questions Q, ‘ManyMatics’ and ‘MetaMatics’ gives different answers A1 and A2

Q: Digits? A1: ‘Icons, different from letters’. A2: ‘Symbols like letters’.

Q: Count? A1: ‘Count in icons before in tens’. A2: ‘Only count in tens’.

Q: Natural numbers? A1: ‘2.3 tens’. A2: ‘23’.

Q: Fractions? A1: ’Per-numbers needing a number to produce a number’. A2: ‘Rational numbers’.

Q: Per-numbers? A1: ‘Double-counting’. A2: ‘Not accepted’.

Q: Operations? A1: ‘Icons for the counting processes’. A2: ‘Mappings from a set-product to a set’.

Q: Order of operations? A1: ‘/, x, - , +’. A2: ‘+, - , x, /’.

Q: Addition? A1: ‘On-top and next-to’. A2: ‘On-top only’.

Q: Integration? A1: ‘Preschool: Next-to addition; Middle school: Adding piece-wise constant per-numbers. High school: Adding locally constant per-numbers’. A2: ‘Last year in high school, only for the few’.

Q: A formula? A1: ‘A stand-by calculation with numbers and letters’. A2: ‘An example of a function that is an example of a relation in a set-product where first component identity implies second component identity’.

Q: Algebra? A1: ‘Re-unite constant and variable unit-numbers and per-numbers’. A2: ‘A search for patterns’.

Q: The root of Mathematics? A1: ‘The physical fact Many’. A2: ‘The metaphysical invention Set’.

Q: Concepts? A1: ‘Abstraction from examples’. A2: ‘’Example of abstractions’.

Q: An equation? A1: ‘A reversed operation’. A2: ‘An example of an equivalence relation between two number-names’.

Can Education be Different?

From secondary school, continental Europe uses line-organized education with forced classes and forced schedules making teenagers stay together in age-groups even if girls are two years ahead in mental development.

The classroom belongs to the class. This forces teachers to change room and to teach several subjects outside their training in lower secondary school.

Tertiary education is also line-organized preparing for offices in the public or private sector. This makes it difficult to change line in the case of unemployment, and it forces the youth to stay in education until close to 30 making reproduction fall to 1.5 child per family so the European population will be reduced to 10% in 200 years.

Alternatively, North America uses block-organized education saying to teenagers: “Welcome, you carry a talent! Together we will uncover and develop your personal talent through daily lessons in self-chosen half-year blocks.” If successful the school will say ‘good job, you have a talent, you need more’. If not, the school will say ‘good try, you have courage, now try something else’. The classroom belongs to the teacher teaching only one subject and helped by daily lessons to adapt quickly to learner differences.

Likewise, college is block-organized to be tested already in high school and easy to supplement with additional blocks in the case of unemployment.

At the age of 25, most students have an education, a job and a family with three children to secure reproduction: one for mother, one for father and one for the state.

So different education forms might not all lead to Truth, Beauty and Goodness.

Conclusion: Blocks in Mathematics Education, Please

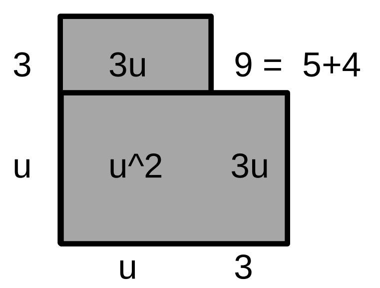
We asked: How can mathematics be a means leading to the goal of implementing the three classical virtues Truth, Beauty and Goodness? The answer is very simple: Blocks in mathematics and in education, please.

Blocks will bring Truth and Goodness back if mathematics will

- respect the nature of numbers as integrated blocks
- replace self-referring meta-matics and falsified mathe-matism with grounded many-matics presenting mathematics as a natural science about the physical fact Many
- make geometry grounded in blocks halved by their diagonals
- bring back algebra to its original Arabic meaning: to reunite constant and variable unit-numbers and per-numbers.

Blocks will bring Truth and Goodness to education that uncovers and develops a teenager's individual talent through daily lessons in self-chosen half-years blocks made possible when replacing line- with block-organization.

Blocks will bring Beauty to the streets with Block-Art posters showing how algebra and geometry work nicely together:



2 cards solve quadratic equations

u^2 + 6u + 5 = 0

(u + 3)^2 = u^2 + 6u + 5 + 4 = 0 + 4 = 4

u + 3 = ± 2

u = -3 ± 2

u = -1 and u = -5

Recommendation: Mathematics, Unmask Yourself, Please

Mathematics, in Greek you mean 'knowledge' and you were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic (Freudenthal 1973: 7). Today only 2 activities remain: Geometry and Algebra. Then Set transformed you from a natural science about the physical fact Many to a self-referring metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism (MrAlTarp YouTube videos 2013).

So please, unmask your true identity, and tell us how you would like to be presented in education: MetaMatism for the few - or ManyMatics for the many.

References

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