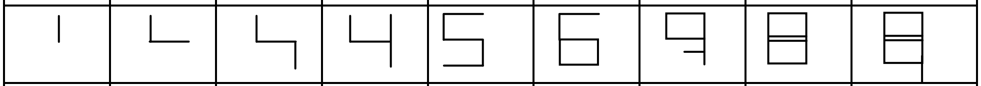
Reinventing Mathematics as a Natural Science about MANY

1. Let us try to reinvent mathematics as a natural science dealing with the natural fact many. What do we do when we meet many? Two things, first we Count, then we Add, and we do that where we live, in Time and Space. So this approach can be called the CATS-approach to mathematics: Count&Add in Time&Space.

With a pile of sticks there are 3 ways of counting: 1.order-counting, 2.order-counting and 3.order-counting.

A 1.order-counting means rearranging the sticks in icons, so that there are five sticks in the five-icon 5 etc. So an icon contains the degree of many it describes. 1.order-counting stops after nine, thus ten has no icon.

I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9



Rearranging sticks into icons transforms 5 1s into 1 5s, etc.

A 2.order-counting counts by bundling and stacking in icon-bundles, i.e. counting in e.g. 5s, but not in tens.

A 3.order-counting counts in tens, a very special number: the only number with a name but without an icon.

2. As an example of 2.order-counting let us count 7 1s in 3s, 5s and 2s.

I I I I I I I -> ~~III~~ ~~III~~) I) -> II) I) -> 2) 1) -> 2.1 3s

I I I I I I I -> ~~IIIII~~) II) -> I) II) -> 1) 2) -> 1.2 5s

I I I I I I I -> ~~II~~ ~~II~~ ~~II~~) I) -> III) I) -> ~~II~~ I) I) -> I) I) I) -> 1) 1) 1) -> 11.1 2s

Counting 7 1s in 3s, we take away a 3-bundle 2 times leaving 1 stick unbundled. The unbundled is placed in a right single-cup, and the 3-bundles are placed in a left bundle-cup, either as actual bundles, or as sticks counting bundles by being placed in the left bundle-cup. Thus the counting result is 2.1 3s, using a decimal point to separate the bundles to the left from then unbundled to the right; and including the unit 3s.

Likewise counting 7 1s in 5s gives 1.2 5s.

Counting 7 1s in 2s gives 3.1 2s. However, in the bundle-cup we also have a bundle of bundles that can be moved to a new cup to the left, counting the bundles of bundles. Thus counting 7 1s in 2s gives 11.1 2s.

Counting 3 8s in tens gives 2.4 tens, only this time we have no icon for ten: 3 8s = 2.4 tens.

In all cases, counting means bundling in a chosen bundle-size, and counting always produces decimal numbers carrying units. So natural numbers are decimal numbers carrying units.

3. Is this what the Book says? No. The book says: we only count in tens, and we do not write 2.4 tens. First we throw away the unit tens; then we misplace the decimal point one to the right. So instead of 2.4 tens we just write 24, which we call a natural number. Thus what the Book calls natural numbers are instead patronizing numbers hiding its natural alternative and creating problems to learners.

Counting in different bundle-sizes might also be called counting in different bases. However, base is a patronizing term hiding its alternative ‘counting in different bundle-sizes’. The term ‘bundle’ is grounded in experience, a bundle can be grasped. The term ‘base’ is not, it comes from the Book and it can’t be grasped.

4. Since patronizing numbers create learning problems by being unnatural, we ask: If ten is a cognitive bomb by having no icon but needing 2digits, how much mathematics can be learned from 1digit numbers alone?

Surprisingly the answer is that the core of mathematics can be learned as 1digit mathematics.

An example: My sister has 3.2 4s, and I have 2.3 5s. Now we would like to add them. However, to add, the units must be the same, so I must recount my 5s the 4s, or my sister must recount her 4s in 5s. Or we can add them as 9s by uniting the bundle-sizes.

Double-counting a given quantity in two different units, e.g. 4s and 5s, or kgs and £ is called proportionality, normally learned in middle school; and adding in the combined bundle-size is called integration, normally learned late in high school if ever. But using 1digit mathematics, both core concepts are learned in grade 1.

5. Furthermore, recounting 3.2 4s in 5s can be predicted by a calculator. We enter '(3*4+2*1)/5' since counting in 5s means taking away 5s many times, which is iconised as division. The answer is 2.8 5s. To see if we can trust the .8 we take away the 2 5s by subtracting 2*5. Entering '(3*4+2*1)-2*5' gives 4, so the recounting result can be predicted to be 2.4 5s. To test this prediction we perform the actual recounting by de-bundling the 3.2 4s in 1s and the re-bundling the 1s in 5s:

3.2 4s -> 3)2) -> IIII IIII IIII II -> IIII IIII IIII II -> IIIII IIIII IIII -> 2)4) -> 2.4 5s

So the prediction holds. So from now on we don't have to perform the actual recounting by de-bundling and re-bundling since we can predict the result on a calculator thus becoming a number-predictor.

6. When the units are the same we can add the two 'stocks' using cup-writing:

3.2 4s + 2.3 5s = 2.4 5s + 2.3 5s = 5.7 5s = 5)7) = 5+1) 7-5) = 6)2) = 1) 6-5) 2) = 1)1)2) = 11.2 5s

Here the 7 1s can be recounted in 1.2 5s transferring 5 1s as 1 5s form the single-cup to the bundle-cup. Here the 6 5s can be recounted to 1.1 5*5s transferring the 5 5s as 1 5*5 from the bundle-cup to the bundles of bundles-cup, thus giving the total of 1 bundle of 5 5s and 1 bundle of 5s and 2 unbundled 1s.

7. With 2.3 5s, what happens if I add an extra cup to the right?

2.3 5s = 2)3) <adding a cup to the right> 2)3)) = 23.0 5s.

Apparently adding an extra cup to the right means that the 3 1s becomes 3 5s, and that the 2 5s becomes 2 5*5s, i.e. means multiplying with the bundle-number and moving the decimal point 1 place to the right.

Likewise, removing 1 cup from the right means dividing with the bundle-number and moving the decimal point 1 place to the left:

23.0 5s = 2)3)) <removing a cup from the right> 2)3) = 2.3 5s.

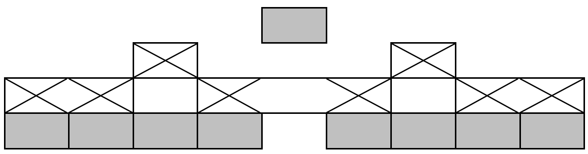
8. Thus we see that 1digit mathematics respects the Piaget 'through the hands to the head'-principle of natural learning: to grasp with the head, first grasp with the hand.

9. The CATS approach using 1digit mathematics conflicts with the traditional patronizing approach that introduces 2digit numbers in grade 1 by claiming that 10 is the follower of 9. Now ten is the follower of nine by nature, but to say that 10 IS the follower of 9 is a patronizing choice hiding the alternatives. With 8 as the bundle-number, 10 is the follower of 7, and the follower of nine is 12. Thus the CATS approach treating mathematics as a natural science cannot be learned at traditional academies. Hence a web-based academy www.MATH**e**CADEMY.net has been set up to teach the CATS approach to mathematics as an anti-patronizing natural science respecting the huge learning potential of 1digit mathematics.

10. The MATH**e**CADEMY.net offers free in-service teacher education to teachers wanting to learn mathematics as a natural science investigating the natural fact many.

The learners are organized in groups of 8 using PYRAMID**e**DUCATION: the 8 learners are organized in 2 teams of 4 learners choosing 3 pairs and 2 instructors by turn. The teacher coaches the instructors instructing the rest of their team. Each pair works together to solve count&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation. The coach assists the instructors in correcting the count&add assignments. In each pair each learner corrects the other learner's routine-assignment. Each pair is the opponent on the essay of another pair. Each learner pays for the education by coaching a new group of 8 learners. It is not difficult to be a coach since the learners are educated, not by books but by counting and adding in time and space.

1 coach
2 instructors
3 pairs
2 teams



8 learners and 1 coach

The activities are divided into 2x4 parts, Count&Add in Time&Space 1 for primary school, C1, A1, T1 and S1; and Count&Add in Time&Space 2 for secondary school, C2, A2, T2 and S2. The study units are activity-based and very short. They is accessible at the MATH**e**CADEMY.net website. The content is given in the summary below.