CATS: Learning Mathematics Automatically through Counting & Adding MANY

CATS shows that mathematics is simple if respecting its nature as a natural science about MANY.

To deal with MANY we simply count and add, as demonstrated when writing a Total as we say it:

T = 456 = 4*10^2 + 5*10 + 6*1.

This shows why algebra means re-unite in Arabic: We count MANY by uniting the ones in bundles, in this case ten-bundles, ten-bundles of ten-bundles etc. Also we see that

All numbers carry units: ones, tens, ten-tens etc.

There are 4 ways to add: +, *, ^, integration, where

+ adds unlike numbers,

* adds like numbers,

^ adds like factors, and

integration adds stacks with different units next-to each other as areas.

Thus, adding next-to roots integration, adding on-top roots linearity forcing totals to be recounted in the same unit, and reversing addition roots equations.

Mathematics is as simple as that, when introduced as a natural science about MANY by CATS.

The unhappy-student problem

Today mathematics education creates many unhappy students. Why? And could it be otherwise?

One understanding of the unhappy-student problem comes from the fairy tales. The unhappy students are sleeping beauties that may be wakened, to live happily ever after. They are sleeping because they have touched a thorn or taken a bite of something that seemed healthy but carried poison inside. To awaken the thorns must be changed to roses or the poison must be taken away. But what are the thorns of mathematics or the poison inside mathematics? And is there a different mathematics inside the living room of mathematics that makes the Prince happy? Or will we have to look for a hidden Cinderella mathematics outside?

Words may be poisonous

The thorns and the poison might be the words used in mathematics education. This suggestion is inspired by French post-structuralist thinking opposing structuralist thinking saying that the words and sentences representing outside phenomena and relations can be used as an enlightened rational basis for institutions created to cure humans for e.g. ‘uneducatedness’. Contrary to this Derrida warns us against words, they are not representing but installing what they describe. Lyotard warns us against sentences, they are not representing but installing relations. And Foucault warns us against institutions, they are not curing but installing the patients.

This French scepticism towards words is easily verified by the ‘number&word’ pencil-paradox: placed between a ruler and a dictionary a pencil can point to numbers but not to words, thus numbers are reliable and words are unreliable; numbers are installed by the described based on the physical property ‘extension in space’, and words are installed by the describer based on an agenda.

In this way of thinking the poison inside mathematics are the words and sentences installed by ‘meta-matics’ and ‘mathe-matism’.

Meta-matics or set-ism

Meta-matics or set-ism is introducing words that are not derived from the outside world. It defines the words of mathematics as, not abstractions from examples, but as examples of abstractions; e.g. the word ‘function’ is defined as an example of a set-relation and not, as it was done historically, as a name for a calculation with a variable quantity. ‘Function’ is from around 1750 and ‘set’ is from around 1900; basing the definition of a 1750 word on a 1900 word is turning mathematics upside down transforming mathe-matics to meta-matics that is deduced from above instead of induced from below. Today’s mathematics is basing its definition on the word set. But the history of mathematics shows that the word set is highly controversial and can only be rationally introduced as an undefined term, i.e. as something you believe in, thus making set and set-based mathematics an ‘ism’, set-ism.

Mathe-matism or add-ism

Mathematism or add-ism is introducing sentences that are not derived from the outside world. It claims that numbers can be added and multiplied without regard to the units thus portraying 2+3 = 5 and 2*3 = 6 as universal truths.

That 2*3 = 6 can easily be validated in the laboratory since 2*3 is physically present as a stack of 2 3s that can be recounted as 6 1s (2*3 = 2 3s = 6 1s = 6*1): *** *** -> * * * * * *. Thus in multiplication the unit is automatically present as the last number.

That 2+3 = 5 depends on the unit is easily shown by examples: 2*week+3*week = 5*week, but 2*week+3*day = 17*day = 2 3/7*week. Thus if units are not included before adding, addition becomes an ‘ism’, mathematism or add-ism, i.e. something that you have to believe in since it seldom takes place in real life. Thus in the case of adding fractions 1/2 of 2 bottles + 2/3 of 3 bottles is 3/5 of 5 bottles making 1/2 + 2/3 = 3/5 and not 7/6 as add-ism teaches in spite of the fact that 7/6 of 6 bottles is a physical absurdity. So also when adding fractions the units must be included before adding.

Non-poisonous words

Words could be otherwise and should be debated before chosen; all democratic thinking recognizes this. The ancient Greek sophists taught about the democratic difference between information and debate by distinguishing between necessity and choice. Later the Enlightenment revived democratic thinking and installed two democracies in the late 1700s, the American and the French. Where the American still has its first republic France now has its 5^{th} republic always being threatened by ‘pastoral power’ as Foucault calls it. So where the French scepticism towards words is desperate the American scepticism towards words is pragmatic developing grounded theory as a research method, only allowing the words that comes from the data and only allowing the sentences coming from the tales that has been validated by their ability to survive for countless generations, the fairy tales, where rational agents pursuing a goal are being helped and hindered by helpers and opponents.

So where French scepticism is looking for hidden irrationality behind apparently rational institutions, American scepticism is looking for hidden rationality behind apparently irrational agents.

Combining American and French scepticism towards words

The MATHeCADEMY.net is designed by a postmodern ‘sceptical Cinderella research’ combining American and French scepticism towards words. It uses French scepticism to identify the poison inside mathematics in the form of set-ism and add-ism. And it uses American scepticism to look for new words that are Cinderella-differences, i.e. differences that make a difference to the unhappy-student problem and that cannot be accepted in the official room but has be found outside.

These new words have to grow out of the root of mathematics, i.e. the human wish to be able to deal with many-ness by counting and adding Many in time and space. In this way an ism-free mathematics can be rebuild from below in a Many laboratory where mathematics is learned automatically through performing the activities of counting and adding.

The Count&Add Laboratory

A Many-Based Count&Add Laboratory is one example of building mathematics from below only allowing for the words that come out of the two fundamental competences needed to deal with Many, counting and adding. This laboratory could be named after the two mathematicians that were sceptical towards sets, Kronecker and Russell.

In the Count&Add laboratory two different numbers exist, stack-numbers and per-numbers. This gives a whole new approach to mathematics. Primary school is about learning to deal with stack-numbers. Constructivist mathematics says that developing number sense is more important than learning calculation algorithms.

A Kronecker-Russell Many-Based Mathematics: The Count&Add Laboratory

1. Repetition in time exists and can be experienced by putting a finger to the throat.

2. Repetition in time has a 1-1 correspondence with Many in space (1 beat <->1 stroke).

3. Many in space can be bundled in icons with 4 stokes in the icon 4 etc.: IIII -> 4.

4. Many can be counted in icons producing a stack of e.g. T = 3 4s = 3*4. The process 'from T take away 4' can by iconised as 'T-4'. The repeated process 'from T take away 4s' can by iconised as 'T/4, a 'per-number'. So the 'recount-equation' **T = (T/b)*b** is a prediction of the result when counting T in bs to be tested by performing the counting and stacking: T = 8 = (8/4)*4 = 2*4, T = 8 = (8/5)*5 = 1 3/5 * 5.

5. A calculation T = 3*4 = 12 is a prediction of the result when recounting 3 4s in tens and ones.

6. Many can be re-counted: If 2 kg = 5 $ = 6 litres = 100 % then what is 7 kg? The result can be predicted through a calculation recounting 7 in 2s:

T = 7 kg = (7/2)*2kg = (7/2)*6 litres = 21 litres	T = 7 kg = (7/2)*2kg = (7/2)*100 % = 350 %	T = 7 kg = (7/2)*2kg = (7/2)*5 $ = 17.50 $

7. A stack is divided into triangles by its diagonal. The diagonal length is predicted by the Pythagorean theorem a^2+b^2=c^2, and its angles are predicted by recounting the sides in diagonals: a = a/c*c = sinA*c, and b = b/c*c = cosA*c.

8. Diameters divide a circle in triangles with bases adding up to the circle circumference:

C = diameter * n * sin(180/n) = diameter * π.

9. Stacks can be added by removing overloads (predicted by the 'restack-equation' **T = (T – b) + b**):

T = 38+29 = 3ten8+2ten9 = 5ten17 = 5ten1ten7 = (5+1)ten7 = 6ten7 = 67 (5ten17 = 5ten(17-10+10) = 6ten7)

Or with *cup-writing* and *internal trade*: T = 38+29 = 3)8) + 2)9) = 5)17) = 5+1)-10+17) = 6)7) = 67

10. Per-numbers can be added after being transformed to stacks. Thus the $/day-number b is multiplied with the day-number n before being added to the total $-number T: T2 = T1 + n*b.

2days @ 6$/day + 3days @ 8$/day = 5days @ (2*6+3*8)/(2+3)$/day = 5days @ 7.2$/day

1/2 of 2 cans + 2/3 of 3 cans = (1/2*2+2/3*3)/(2+3) of 5 cans = 3/5 of 5 cans

Repeated addition of per-numbers -> integration	Reversed addition of per-numbers -> differentiation
T2 = T1 + n*b T2 - T1 = + n*b ΔT = ∑ n*b ΔT = ∫ b*dn	T2 = T1 + n*b (T2-T1)/n = b ΔT/Δn = b dT/dn = b

Only in case of adding constant per-numbers as a constant interest of e.g. 5% the per-numbers can be added directly by repeated multiplication of the interest multipliers: 4 years @ 5 % /year = 21.6%, since 105%*105%*105%*105% = 105%^4 = 121.6%.

Conclusion. A Kronecker-Russell Many-based mathematics can be summarised as a 'count&add-laboratory' adding to predict the result of counting totals and per-numbers, in accordance with the original meaning of the Arabic word 'algebra', reuniting. (Respecting a Kronecker-principle of building on natural number; and a Russell-principle of not talking about sets of sets e.g. fractions)

ADDING/ DeAdding	**Constant**	**Variable**
Unit-numbers m, s, kg, $	**T = n*b** *T/n = b*	**T2 = T1 + n*b** *T2-T1 = n*b*
Per-numbers m/s, $/kg, $/100$ = %	**T = b^n** *$\sqrt[n]{T} = b$ $\log_b T = n$*	**T2 = T1 + ∫b*dn** *dT/dn = b*

The Count&Add Laboratory