Count in Icons before Tens, then Add NextTo before OnTop

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Abstract

Preschool allows rethinking mathematics outside the tradition of ordinary school. Seeing schooling as adapting the child to the outside world containing many examples of Many, we can ask: How will mathematics look like if built as a natural science about physical fact Many? To deal with Many we count and add. The school counts in tens, but preschool allows counting in icons also. Once counted, totals can be added. To add on-top the units are made the same through recounting, also called proportionality. Adding next-to means adding areas, also called integration. So icon-counting and next-to addition offer golden learning opportunities in preschool that are lost once primary school insists that totals can only be counted in tens and added on-top. Likewise, icon-counting and next-to addition allow special need learners a fresh start and a return to the classroom as experts seeing digits as icons containing as many stokes as they represent, seeing natural numbers as decimal numbers with a unit using the decimal point to separate bundles from unbundled and with cup-writing allowing overloads to be created or removed when recounting in the same unit, seeing a calculator as predicting recounting results, seeing ten-counting as a special case of icon-counting, seeing tables as recounting icon-numbers in tens, seeing equations as recounting tens in icons, seeing proportionality as another word for changing units using per-numbers in the case of physical numbers, and seeing integration as another name for adding next-to.

BACKGROUND

Increased mathematics education research seems to create a decrease in Nordic PISA results as witnessed by the latest PISA study and the OECD 2015 report ‘Improving Schools in Sweden’. Research showing the importance of an early start in mathematics indirectly indicates that a bad start will create learning problems later. Consequently, preschool mathematics should be examined for hidden alternatives as shown when a 3-year old child insists that four fingers held together two by two is not four but two twos. Based upon existentialist philosophy this paper asks: What is existentialist preschool mathematics and what difference will it make?

EXISTENTIALISM

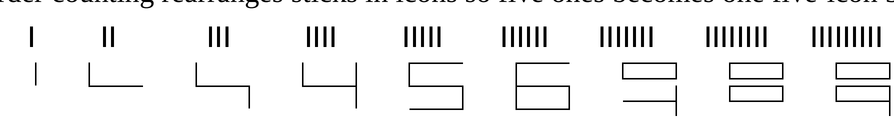
Building on the work of Kierkegaard and Nietzsche, Sartre defines existentialism as holding that ‘existence precedes essence, or (..) that subjectivity must be the starting point’. Focusing on the classical virtues truth and beauty and goodness, Kierkegaard placed goodness before beauty. Nietzsche saw people as imprisoned in moral serfdom until the coming of a personality having an ‘absorption, immersion, penetration into reality, so that (..) he may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.’ (Marino 2004: 344, 186-187).

The existentialist distinction allows a distinction between good and bad essence with and without existence behind it, and a hypothesis to be formulated: Learning difficulties disappear when replacing bad with good essence. The hypothesis may be tested by exposing special need learners to good essence.

MATHEMATICS AS A NATURAL SCIENCE ABOUT MANY

According to Freudenthal, the Greek Pythagoreans used mathematics as a common label for their four knowledge areas astronomy, music, geometry and arithmetic. Today, the label covers only geometry and arithmetic, replaced by algebra meaning to reunite in Arabic illustrated by writing out fully 345 = 3*B^2+4*B+5*1 showing that we deal with Many, first by iconizing, then by bundling and stacking. So we can distinguish between first, second and third order counting creating icons, counting in icons and counting in tens.

First order counting rearranges sticks in icons so five ones becomes one five-icon 5, etc.



1 2 3 4 5 6 7 8 9

Where third order counting counts in tens, second order counting counts in icon-bundles so a total T of 7 is bundled in 3s as T = 2 3s and 1 shown with two sticks in a in a left bundle-cup and one stick in a right single-cup, both described by icons, first using 'cup-writing' 2)1), then using 'decimal-writing' with a decimal point to separate the bundles from the unbundled, and including the unit 3s, T = 2.1 3s.

IIIIIII → III III I → II) I) → 2)1) → 2.1 3s

A calculator can predict the counting result. A stack of 2 3s is iconized as 2x3 showing a jack used 2 times to lift the 3s. Taking away is iconized with '/3' or '–3' showing the broom or the trace when wiping away 3 several times or just once, called division and subtraction. Entering '7/3', we ask the calculator 'from 7 take away 3s' and get the answer '2.some'. Entering '7 – 2x3' we ask 'from 7 take away 2 3s' and get the answer 1 leftover. Thus the calculator predicts that 7 = 2.1 3s.

ReCounting in the Same or in a Different Unit

Once counted, totals can be recounted in the same or in a different unit. Recounting in the same unit, changing a bundle to singles creates overloads allowing recounting a total of 4 2s as 3.2 2s or 2.4 2s. And 4 2s can be recounted as 5 2s less 2, or as 6 2s less 4 thus leading to negative numbers.

T = 4 2s = 3.2 2s = 2.4 2s = 1.6 2s = 0.8 2s, or T = 4 2s = 5.-2 2s = 6.-4 2s = 7.-6 2s

To recount in a different unit means changing unit, called proportionality. Asking '3 4s is how many 5s?' sticks give the result 2.2 5s as predicted by a calculator.

IIII IIII IIII → IIIII IIIII I I → 2) 2) 5s → 2.2 5s

Likewise, recounting in a different physical unit creates 'per-numbers' as 4$/5kg allowing 16$ to be recounted in 4s to bridge to the kg-numbers: 16$ = (16/4)*4$= (16/4)*5kg = 20kg.

Two totals 2 3s and 4 5s can be added next-to as 3.2 8s, or on-top after the units have been made the same through recounting as 1.1 5s and 4 5s = 5.1 5s.

TESTED WITH SPECIAL NEED LEARNERS

Based upon these findings material was tested with learners removed from their ordinary grade 6 class to receive special education. The test used the word 'many-matics' instead of mathematics.

Activity	Examples of statements
Icon-creation with a folding rule	Oh that's where the digits come from.
Icon-counting	So that means that 3x5 is 3 5s and not a tables-question?
Recounting in the same unit	That is the same as changing coins or getting back change.
Recounting in a different unit	Wow, a calculator can predict the result before I carry it out. Can I please keep this calculator?
Recounting icon-numbers in tens	Hey, you just have to enter 3x4 to recount in tens.
Recounting tens in icon-numbers	So recounting in icons is just another word for solving equations?
Allowing overloads in cup-writing 35 + 47 = 7) 12) = 8) 2) = 82 3 x 58 = 15) 24) = 17) 4) = 174	Hey, its fun to trade bundles for singles and vice versa.

Finally learning about recounting with per-numbers allowed the learners to return to ordinary class as experts helping the other learners having problems with the traditional approach to proportionality. And both learners and the teacher were amazed when hearing about integration as next-to addition.

CONCLUSION

Existentialist preschool mathematics presenting digits as icons, and performing icon-counting before ten-counting, and performing NextTo addition before OnTop addition will allow special need learners to return to class as experts in proportionality and solving equations thus replacing learning difficulties with mastery.

REFERENCE

Marino, G. (2004). *Basic Writings of Existentialism.* New York: Modern Library.