SUMMARY

	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

CONTENTS

This website contains 2*4 study units in 'mathematics from below, the LAB-approach', organised as lab-activities where the learner learns 'CATS', i.e. learns to count and add in time and space. The study units CATS1 are for primary school and the study units CATS2 are for secondary school. The units were developed for a web-based teacher-training course in mathematics at a Danish teacher college.

Counting C1. We look at ways to count Many. In space, Many is representing temporal repetition through strokes. Many strokes can be rearranged in icons so that there are four strokes in the icon 4 etc. Then a given total T can be counted in e.g. 4s by repeating the process 'form T take away 4', which can be iconised as 'T-4' where the repeated process 'form T take away 4s' can be iconised as 'T/4' making it possible to predict the counting result through a calculation using the 'recount-equation' T = (T/b)*b, where the number T/b is called a per-number describing the total and the bundle-size. Thus counting a total of 8 in 2s there are T/b = 8/2 = 4 = 4 per 1.

Changing units is another example of a recounting first telling how a given total can be counted into different units e.g. T = 4$ = 5kg producing a per-number 4$/5kg.Thus to answer the question ' 7kg=?$' we just have to recount the 7 in 5s: T = 7kg = (7/5)*5kg = (7/5)*4$ = 5 3/5$.

In most cultures ten is chosen as a standard bundle-size thus bundling and stacking in bundles of tens. In this way a total T becomes a many-stack, a polynomial, consisting of a number of unbundled, a number of bundled, a number of bundles of bundled, etc.

Counting C2. We look at ways to count numbers that change unpredictably as e.g. in surveys. Through counting we can set up a table accounting for the frequency of the different numbers. From this we can calculate the average level and the average change. The average level can then be used as the winning probability p in a game that is repeated n times. By counting the different possibilities it turns out that there is a 95% probability that future numbers lie within an interval determined by the average level and change.

Addition A1. We look at how stacks can be added by removing the overload that often appears when one stack is placed on top of another stack. The stack can be added concretely through a principle of internal trade where a full stack of 10 1s is traded with one 10-bundle. And the result can be predicted by a calculation on paper using either a vertical way of writing the stacks using carrying to symbolise the internal trade; or using a horizontal way of writing the stacks using the FOIL principle. In both cases the overload can be recounted by the recount-equation T=(T/b)*b or transferred by the restack-equation T=(T-b)+b.

Addition A2. We look at how we add per-numbers by transforming them to totals. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b. 2days @ 6$/day + 3days @ 8$/day = 5days @ 7.2$/day. And 1/2 of 2 cans + 2/3 of 3 cans = 3 of 5 cans = 3/5 of 5 cans. Repeated and reversed addition of per-numbers leads to integration and differentiation:
T2 = T1+a*b; T2-T1 = +a*b; ΔT = ∑a*b = ∫y*dx   and   T2 = T1+a*b; a = (T2-T1)/b = ΔT/Δb = dy/dx

Time T1. We look at how addition can be reversed by moving numbers to the other side reversing their signs: x*3+2=14 is reversed to x = (14-2)/3. This enables us to do both forward and backward calculations. Thus we can consider the classical quantitative literature consisting of word-problems from especially economy and physics.

Time T2. We look at how a stack can change in time by adding a constant number or a constant percentage. Or by adding a variable predictable number.

Space S1. We look at how to describe plane properties of stacks as area and diagonals by the 3 Greek Pythagoras', mini, midi & maxi; and by the 3 Arabic recount-equations: sinA=a/c, cosA=b/c and tanA=a/b. Then we look at how to describe spatial properties of solids such as surface and volume by formulas and by a 2-dimensional representation of 3-dimensional shapes.

Space S2. We look at how to calculate the position of points and lines by using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = (y-4)/(x-3) = 2, then P1(8,y) = (8,2*(8-3)+4) = (8,14). Then we look at how to use the new calculation technology such as computers to calculate a set of numbers, vectors, and a set of vectors, matrices.

Where do concepts come from? The present form of presentation has been chosen in order to allow learning to take place in all four learning rooms coming from the four different answers to the question: 'Where do concepts come from? From above or from below? Form the outside or from the inside?' The two traditional learning rooms, the transmitter room and the constructivist room, say 'above&outside' and 'above&inside'. The two hidden alternatives, the fairy-tale room and the apprentice room, say 'below&outside' and 'below&inside'. The traditional rooms take mathematics for granted and see the world as applying mathematics. The hidden rooms have the opposite view seeing the Manyness of the world for granted and as a creator of mathematics through the principle 'coming to grips through gripping' or 'grasping by grasping'. The transmission/fairy-tale room arranges sentence-loaded educational meetings with sentences having abstract/concrete subjects. The constructivist/apprentice room arranges sentence-free educational meetings with abstract/concrete subjects.

Historically mathematics arose from below as abstractions from examples. Today mathematics is turned upside down by being presented as examples of abstractions. Today's mathematics thus ought to be called 'meta-matics' to be distinguished from the historical 'mathe-matics', which could be called 'mathematics from below - the natural way'.

Where did the LAB-approach come from? Mathematics from below came out of a postmodern study searching for a solution to the global relevance-problem in mathematics by asking the 'Cinderella-question': 'Are there other hidden alternatives? Is there a postmodern mathematics?' The study was based upon the hypothesis that today's postmodern students like mathematics but reject meta-matics. To test this hypothesis an alternative postmodern mathematics from below was developed inspired by the historical development of mathematics. When tested in the laboratory, the difference turned out to be a genuine 'Cinderella-difference' making a difference in the mathematics classrooms in schools and teacher colleges. These study units are built upon this version of postmodern mathematics.

Word-language and Number-language. A ruler and a dictionary help us to assign numbers and words to things using our number-language and our word-language. Thus we have word-sentences containing a subject, a verb and an object; and we have number-sentences, equations, containing a quantity, an equation sign and numbers and calculations. Both sentences are describing the world and are being described by a meta-language. The meta-language of the word-language is called grammar. The meta-language of the number-language is called mathematics.

Our two languages and their meta-languages constitute a language-house with two floors. In the lower floor the language is used to describe the world, and in the upper floor the meta-language is used to describe the language. Syntax errors occur if the meta-language is used to describe the world: 'the verb got drunk'. So mathematics does not describe the world, mathematics describes the number-language, and the number-language describes the world.

		THE LANGUAGE HOUSE		
META-LANGUAGE	GRAMMAR	Subject	Constants and variables	MATHEMATICS
LANGUAGE	WORD-LANGUAGE	The pencil is red	Area = length*height	NUMBER-LANGUAGE
WORLD		THINGS IN TIME AND SPACE		

A Many-Based Mathematics: The Count&Add Laboratory

1. Repetition in time exists and can be experienced by putting a finger to the throat.

2. Repetition in time has a 1-1 correspondence with Many in space (1 beat <->1 stroke).

3. Many in space can be bundled in icons with 4 stokes in the icon 4 etc.: IIII -> 4.

4. Many can be counted in icons producing a stack of e.g. T = 3 4s = 3*4. The process 'from T take away 4' can by iconised as 'T-4'. The repeated process 'from T take away 4s' can by iconised as 'T/4, a 'per-number'. So the 'recount-equation' T = (T/4)*4 is a prediction of the result when counting T in 4s to be tested by performing the counting and stacking: T = 8 = (8/4)*4 = 2*4, T = 8 = (8/5)*5 = 1 3/5 * 5.

5. A calculation T=3*4= 12 is a prediction of the result when recounting 3 4s in tens and ones.

6. Many can be re-counted: If 2 kg = 5 $ = 6 litres = 100 % then what is 7 kg? The result can be predicted through a calculation recounting 7 in 2s:

T = 7 kg = (7/2)*2kg = (7/2)*6 litres = 21 litres	T = 7 kg = (7/2)*2kg = (7/2)*100 % = 350 %	T = 7 kg = (7/2)*2kg = (7/2)*5 $ = 17.50 $

7. A stack is divided into triangles by its diagonal. The diagonal length is predicted by the Pythagorean theorem a^2+b^2=c^2, and its angles are predicted by recounting the sides in diagonals: a = a/c*c = sinA*c, and b = b/c*c = cosA*c.

8. Diameters divide a circle in triangles with bases adding up to the circle circumference:

C = diameter * n * sin(180/n) = diameter * π.

9. Stacks can be added by removing overloads (predicted by the 'restack-equation' T = (T-b)+b):

T = 38 + 29 = 3ten8 + 2ten9 = 5ten17 = 5ten1ten7 = (5+1)ten7 = 6ten7 = 67

10. Per-numbers can be added after being transformed to stacks. Thus the $/day-number b is multiplied with the day-number n before being added to the total $-number T: T2 = T1 + n*b.

2days at 6$/day + 3days at 8$/day = 5days at (2*6+3*8)/(2+3)$/day = 5days at 7.2$/day

1/2 of 2 cans + 2/3 of 3 cans = (1/2*2+2/3*3)/(2+3) of 5 cans = 3/5 of 5 cans

Repeated addition of per-numbers -> integration		Reversed addition of per-numbers -> differentiation	
T2	= T1 + n*b	T2	= T1 + n*b
T2 - T1	= + n*b	(T2-T1)/n	= b
ΔT	= ∑ n*b	ΔT/Δn	= b
ΔT	= ∫ b*dn	dT/dn	= b

Only in case of adding constant per-numbers as a constant interest of e.g. 5% the per-numbers can be added directly by repeated multiplication of the interest multipliers: 4 years at 5 % /year = 21.6%, since 105%*105%*105%*105% = 105%^4 = 121.6%.

Conclusion. A Kronecker-Russell Many-based mathematics can be summarised as a 'count&add-laboratory' adding to predict the result of counting totals and per-numbers, in accordance with the original meaning of the Arabic word 'algebra', reuniting.

ADDING	Constant	Variable
Stacks m, s, kg, $	T = n*b *T/n = b*	T2 = T1 + n*b *T2-T1 = n*b*
Per-numbers m/s, $/kg, $/100$ = %	T = b^n $\sqrt[n]{T} = b$ $\log_b T = n$	T2 = T1 + ∫b*dn *dT/dn = b*

The Count&Add Laboratory