

Improving Schools in  
Sweden:  
An OECD Perspective



*UnDiagnosed  
Cure?*



# To improve Schools, chose **Good, Bad or Evil** **MatheMatics** **Education**

Allan.Tarp

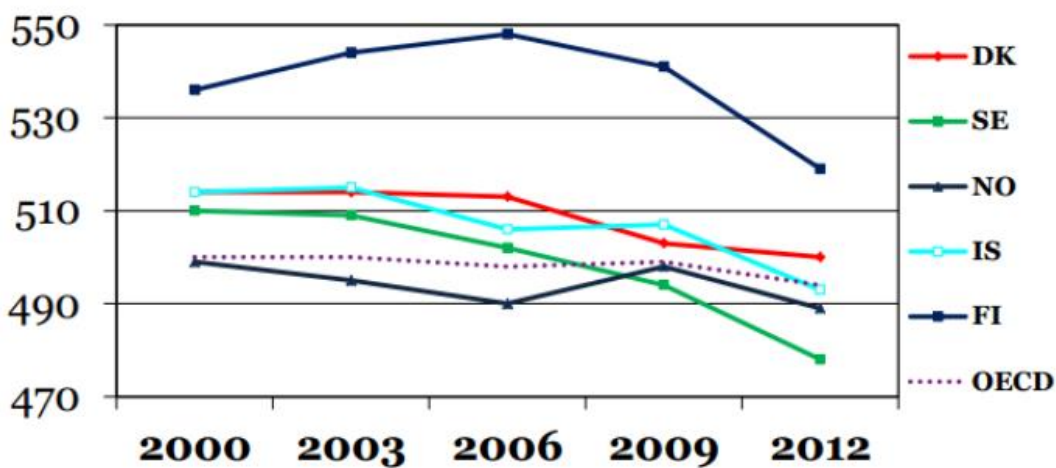
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Teaches Teachers to Teach MatheMatics as ManyMatics,  
a Natural Science about the physical fact Many  
January 2016

# The MeltDown of Swedish PISA results in spite of Increased Funding – caused by Bad & Evil Math?

www.uvm.dk/~media/UVM/Filer/Udd/Folke/PDF13/Dec/131203%20PISA%20Resultatnotat.pdf

Figur 2. Udvikling i matematikresultaterne i nordiske lande (2000-2012).



Ser man bort fra Finland (519 point), er Danmark det eneste af de nordiske lande, som er placeret i gruppen, der ligger signifikant over det internationale gennemsnit. Eleverne i Island (493 point) og i Norge (489 point) præsterer omkring gennemsnittet, mens den svenske score (478 point) er signifikant lavere end gennemsnittet. I tabel 1 nedenfor vises tallene bag figur 2.

Tabel 1. Gennemsnit for nordiske lande 2003-2012

	2003	2006	2009	2012	2012-2009	2012-2003
Finland	544	548	541	519	-22	-25
Danmark	514	513	503	500	-3	-14
Island	515	506	507	493	-14	-22
Norge	495	490	498	489	-9	-6
Sverige	509	502	494	478	-16	-31
OECD	500	498	499	494	-5	-6

All melt down, but as to the OECD average, Finland & Denmark are significantly above, Iceland & Norway are on level, only Sweden is significantly below

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Aktuellt

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Forskningsseminarium i Matematikdidaktik (19/1)

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"Bättre kunskaper med externa lärare" (12/1)

Läs

Nämnen

NOMAD

2

Tänka, resonera

# Schools Exclude 1 of 4 Socially

“PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life.” (page 3)

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# A Need for Urgent Reforms and Change

*Part I: A school system in need of urgent change. (p. 11)*

“Sweden has the highest percentage of students arriving late for school among all OECD countries, especially among socio-economically disadvantaged and immigrant students, and the lack of punctuality has increased between 2003 and 2012. There is also a higher-than-average percentage of students in Sweden who skip classes, in particular among disadvantaged and immigrant students. Arriving late for school and skipping classes are associated highly negatively with mathematics performance in PISA and can have serious adverse effects on the lives of young people, as they can cut into school learning and also distract other students.” (p. 69)  
(Note: Male immigrants make Sweden beat China with 123 boys/100 girls of the 16-17 years old)

# Serious Situation and Serious Deterioration

“If serious shortcomings are identified in a school, the Schools Inspectorate can determine that the deficient school should be closed for up to six months until the deficiencies are corrected. However, this is very much a last resort and has rarely been applied.” (p. 51)

“The reforms of recent years are important, but evidence suggests they are also somewhat piecemeal, and simply too few, considering the serious situation of the Swedish school system.” (p. 55)

“Sweden faces a serious deterioration in the quality and status of the teaching profession that requires immediate system-wide attention. This can only be accomplished by building capacity for teaching and learning through a long-term human resource strategy for the school sector.” (p. 112)



# A Goal/Mean Confusion in Math Education?

Occam's Razor principle: First look for a simple explanation.

An educational subject always has an outside goal to be reached by inside means. But, if seen as mandatory, an inside means becomes a new goal that might hinder learners reaching the original outside goal.

Not accepting its outside goal, Mastering of Many, Mathematics Education is an undiagnosed cure, forced upon patients showing natural resistance against an unwanted treatment.

So to explain the meltdown in Swedish/Nordic PISA results we can ask:

*Is there a Goal/Mean Confusion  
in Swedish and Nordic Mathematics Education?*

# Exemplifying Good & Bad & Evil

**Evil** if I hide alternatives and present my **choice** as nature

**Bad** if I force my own **choice** upon others

**Good** if I perform all 3 parts of a democratic **choice** process

- Tell nature from **choice** and uncover all alternatives
- Analyze and discuss the alternatives
- Make a flexible **choice** - to change if new information should appear

# Defining MatheMatics

According to Freudenthal (1973) the Pythagoreans used the Greek word for mastering, mathematics, as a common label for their four master areas.

With astronomy and music as independent subjects, today only the two other activities remain, both rooted in the physical fact Many:

- Geometry, meaning to measure earth in Greek
- Algebra, meaning to reunite in Arabic

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht-Holland: D. Reidel Publ. Comp.

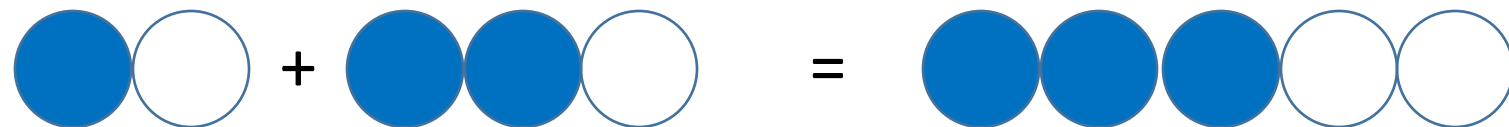


# Defining MatheMatism , **Bad** MatheMatics

**MatheMatism** is a statement that is correct inside, but seldom outside a classroom , thus being bad by forcing its choice upon others.

**Ex1.** Adding numbers without units as  $2+3 = 5$ , where e.g.  $2\mathbf{w}+3\mathbf{d}=17\mathbf{d}$ . In contrast to  $2 \times 3 = 6$  observing that 2 **3s** can be recounted as 6 **1s**.

**Ex2.** ‘The fraction paradox’: The textbook and the teacher insist that  $1/2 + 2/3$  **IS**  $7/6$  even if the students protest: counting cokes,  $1/2$  of 2 bottles and  $2/3$  of 3 bottles gives  $3/5$  of 5 as cokes and never 7 cokes of 6 bottles:



# Defining MetaMatics, **Evil** MatheMatics

**MetaMatics** is defining a concept, not as an abstraction from many examples but as an example of an abstraction, derived from the meta-physical abstraction SET, made meaningless by self-reference as shown by Russell's version of the liar paradox: If M does, it does not (and vice versa) belong to the set of sets not belonging to itself: If  $M = \{ A \mid A \notin A \}$  then  $M \in M \Leftrightarrow M \notin M$ . By hiding its outside grounded alternative, self-referring MetaMatics is evil.

**Ex.** What is a **function**?

From inside, an example of a **set relation** where 1component identity implies 2component identity. From outside, a **name** for a formula with 2 unknowns.

# Teaching MatheMatics as MetaMatism Means Trouble

**MetaMatism** = MetaMatics + MatheMatism, so by its self-reference, MetaMatism only provides inside definitions and inside proofs of its concepts and statements, thus hiding outside roots and validity.

## Two Claims

- The Swedish and Nordic PISA meltdown is caused by teaching MatheMatics as **MetaMatism** instead of as **ManyMatics**
- As Evil and Bad MatheMatics, **MetaMatism** creates DysCalCulia - that can be avoided by teaching **ManyMatics** instead.

# ManyMatics, created to Master Many

To tell nature from choice, we ask: How will math look if grounded as a Natural Science about the physical fact Many, i.e. as a ManyMatics?

- Take 1: To master Many, we math! *Oops, math is a label, not an action word.*
- Take 2: To master Many, we act. Asking 'How Many?', we Bundle & Stack:  
 $456 = 4 \times \text{BundleBundle} + 5 \times \text{Bundle} + 6 \times 1 = \text{three stacks of bundles.}$



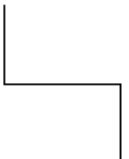
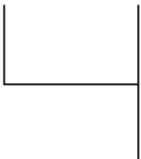
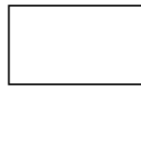
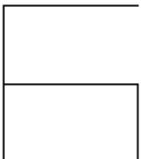
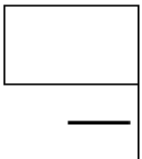

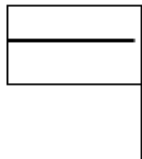
All numbers have units - as recognized by children when showing 4 fingers held together 2 by 2 makes a 3-year-old child say: 'No, that is not 4, that is 2 **2s**.'

*So natural numbers are 2D blocks, not placed on a 1D line.*



# Creating Icons: → →

Counting in ones means naming the different degrees of Many.  
We stop at nine since when bundling in tens, ten becomes 1 Bundle, needing no icon of its own. Counting in icons means changing **four ones** to **one fours** rearranged as a **4-icon** with four sticks or strokes. So an icon contains as many strokes as it represents if written less sloppy.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIII
								
1	2	3	4	5	6	7	8	9

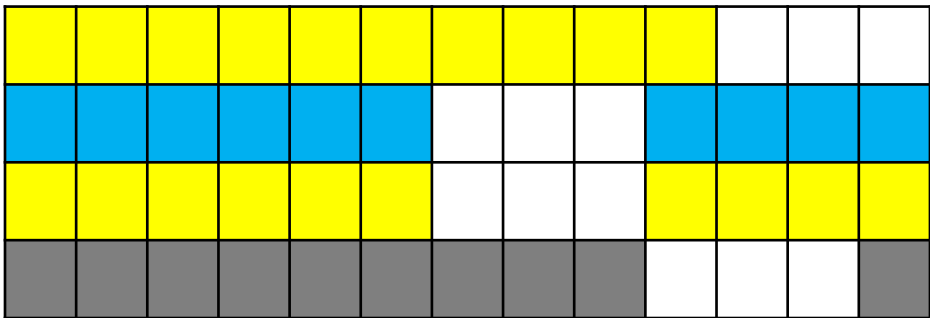
# Counting in Icons: 9 = ? 4s

$$9 = \text{|||||} = \text{||||} \text{ |||} \text{ |} = \text{II}| = 2)1 = 2.1 \text{ 4s}$$

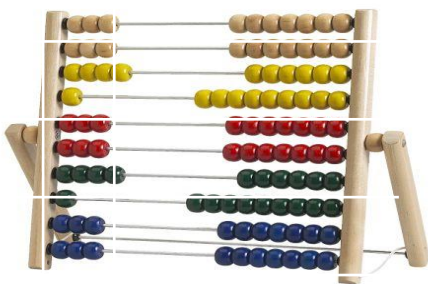
To count, we bundle & use a bundle-cup with 1 stick per bundle.  
We report with **cup-writing** or **decimal-writing** where the decimal point separates the bundles from the singles.

Shown on a western **ABACUS** in

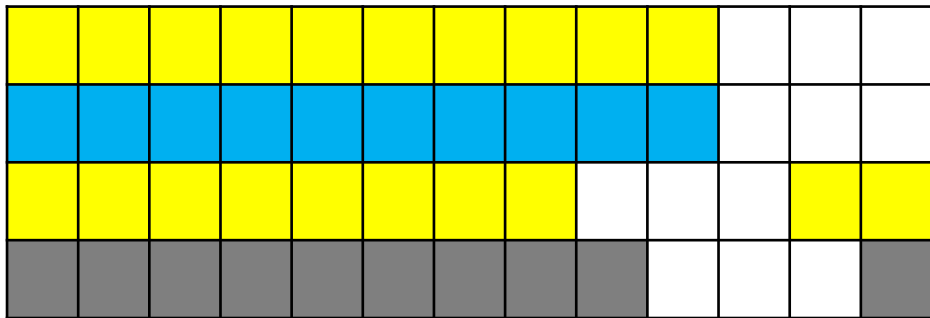
**Geometry/space mode**



or



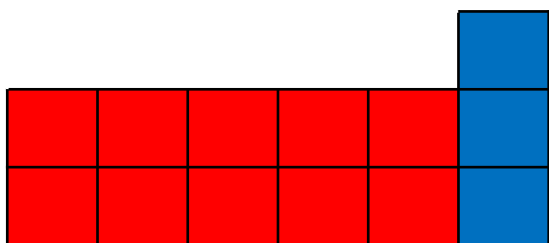
**Algebra/time mode**





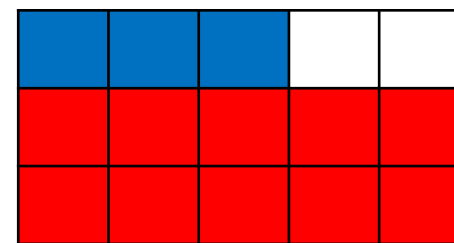
When counting by bundling and stacking,  
the unbundled singles can be placed

**NextTo** the stack  
counted as a stack of **1s**



**T = 2.3 5s**  
A decimal number

## OnTop of the stack counted as a bundle



T = 2 3/5  
A fraction

# Counting creates Division & Multiplication & Subtraction - also as Icons

‘From 9 take away **4s**’ we write  $9/4$

iconizing the sweeping away by a broom, called division.

‘2 times stack **4s**’ we write  $2 \times 4$

iconizing the lifting up by a jack called multiplication.

‘From 9 take away 2 **4s**’ to look for un-bundled we write  $9 - 2 \times 4$

iconizing the dragging away by a stroke called subtraction.

The counting process includes division, multiplication and subtraction:

Finding the bundles:  $9 = 9/4$  **4s**.      Finding the un-bundled:  $9 - 2 \times 4 = 1$ .

# Counting creates Two Counting Formulas

Bundling & stacking create two counting formulas (re-bundle and re-stack):

<b><math>T = (T/b) \times b</math></b>	from a total T, T/b times, bs is taken away and stacked
<b><math>T = (T-b) + b</math></b>	from a total T, T – b is left when b is taken away and placed next-to

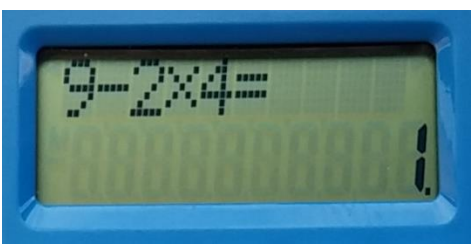
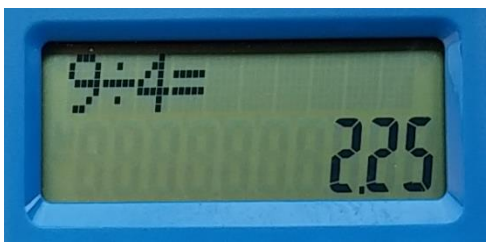
The two counting formulas allow a calculator to predict the counting result

9/4

2.some

9 – 2x4

1



# Counting Sequences

Being counted as 1B, the Bundle number needs no icon.

	I	I	I	I	I	I	I	I	I	I	I	I
5s	1	2	3	4	<b>B</b>	1B1	1B2	1B3	1B4	2B	2B1	2B2
7s	1	2	3	4	5	6	<b>B</b>	1B1	1B2	1B3	1B4	1B5
tens	1	2	3	4	5	6	7	8	9	<b>B</b>	1B1	1B2

3 4s counted	in <b>5s</b> as	$T = 2B2 = 2 \times 5 + 2 = 2.2$	<b>5s</b>
	in <b>7s</b> as	$T = 1B5 = 1 \times 7 + 5 = 1.5$	<b>7s</b>
	in <b>tens</b> as	$T = 1B2 = 1 \times \text{ten} + 2 = 1.2$	<b>tens</b>

As to number names, eleven and twelve come from ‘one left’ and ‘two left’ in Danish, (en / tve levnet), again showing that counting takes place by taking away bundles.

# ReCounting in the Same Unit creates Overloads & Negative Numbers

T = 3.0 2s

= 2.2 2s

= 4.-2 2s

ReCounting 3 2s in 2s:

Sticks	Calculator	Total T	Cup-Writing
⌘ ⌘ ⌘		3.0 2s	3) 2s
⌘ ⌘	3x2 – 2x2                      2	2.2 2s	2) 2 2s
⌘ ⌘ ⌘ ⌘	3x2 – 4x2                      -2	4.-2 2s	4) -2 2s

(4.-2 = 4 less 2 )

And 2digit Numbers if using Bundles of Bundles:

|||||

=

⌘ ⌘ ⌘

=

⌘ ⌘ ⌘

6

=

3 B

=

1 BB 1 B

6

=

3)

=

1) 1)

=

11.0 2s = 11 2s

# ReCounting in a Different Unit

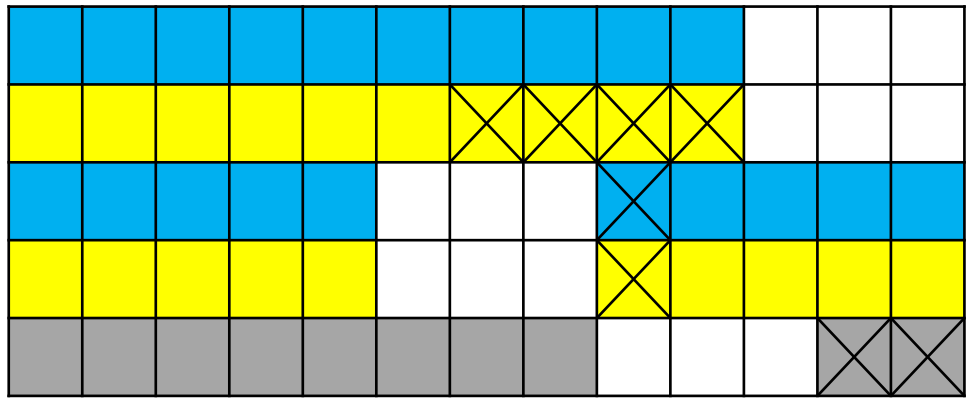
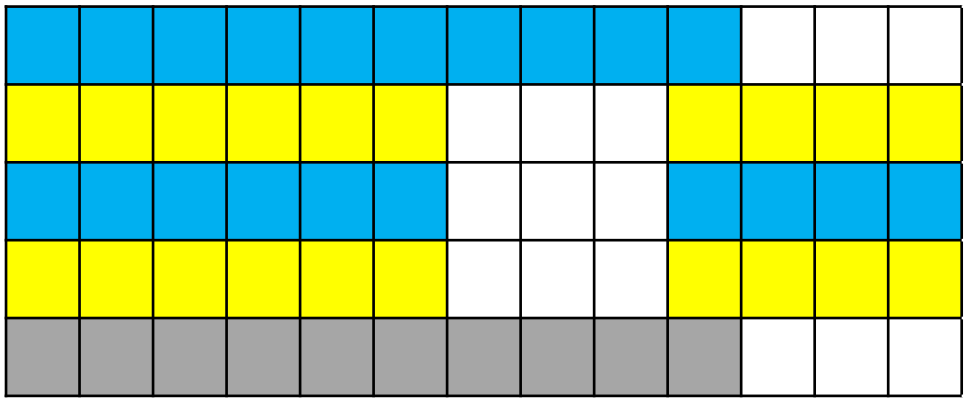
$$3 \text{ } 4\text{s} = ? \text{ } 5\text{s}$$

$$3 \text{ } 4\text{s} = \text{||||} \text{ ||||} \text{ ||||} = \text{||||} \text{ ||||} \text{ ||||} = \text{||||} \text{ ||||} \text{ ||} = 2.2 \text{ } 5\text{s}$$

CALCULATOR-prediction:

$3 \times 4 / 5$	2.some
$3 \times 4 - 2 \times 5$	2

Abacus in Geometry mode



Change Unit = **Proportionality, Core Math**



# ReCounting in Tens

**3 7s = ? tens**

3 7s = ||||| ||||| ||||| = ||||| ||||| ||||| = 2.1 tens

CALCULATOR-prediction: The calculator has no ten icon.

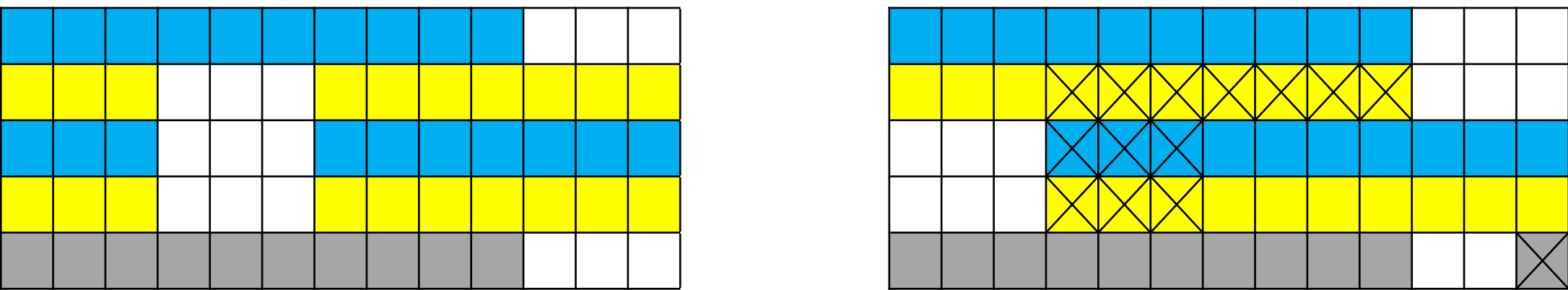
The calculator gives the answer directly

- but **without unit** and with **misplaced** decimal point

3x7	21
-----	----

A Natural Number ???

Abacus in Geometry mode



ReCounting to tens = **Multiplication Tables, Core Math**

# ReCounting from Tens

$$29 = ? \text{ 6s}$$

$$29 = ? \text{ 6s} = \text{|||||} \text{|||||} \text{|||||} \text{|||||} = \text{|||||} \text{|||||} \text{|||||} \text{|||||} \text{|||||} = 4.5 \text{ 6s}$$

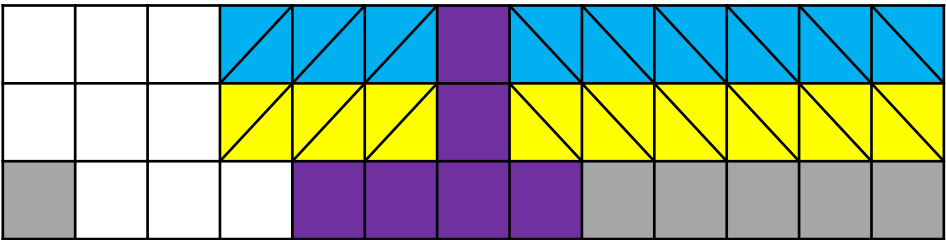
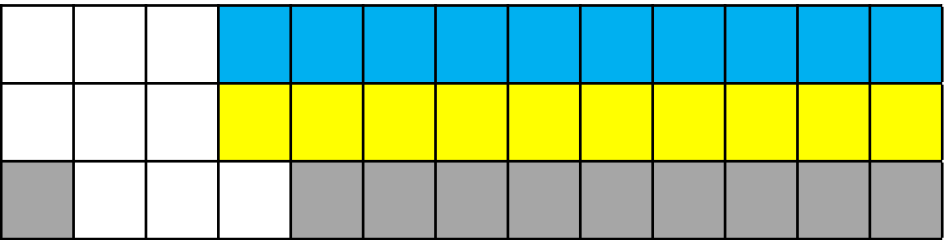
CALCULATOR-prediction:

$29/6$	4.some
$29 - 4 \times 6$	5

Reversed calculation (Equation):  $? \times 6 = 29 = (29/6) \times 6$ , so  $? = 29/6 = 4 + 5/6$

‘Opposite side with opposite sign’ method: if  $u \times 6 = 29$  then  $u = 29/6$

Abacus in Geometry mode



ReCounting from tens = Solving an Equation, Core Math

# ReCounting big Numbers in Tens (Multiplication)

Recounting 6 **47s**

$$\begin{aligned} T &= 6 \times 47 = 6 \times \begin{array}{r} 4) \ 7 \\ = 24)42 \\ = 28) \ 2 \\ = 282 \end{array} \end{aligned}$$

$$\begin{aligned} T &= 6 \times 47 = 6 \times \begin{array}{r} 5) \ -3 \\ = 30) \ -18 \\ = 28) \ 2 \\ = 282 \end{array} \end{aligned}$$

Recounting 36 **47s**

$$\begin{aligned} T &= 36 \times 47 = 36 \times \begin{array}{r} 4) \ 7 \\ = 144)252 \\ = 169) \ 2 \\ = 1692 \end{array} \end{aligned}$$

$$\begin{aligned} T &= 36 \times 47 = 36 \times \begin{array}{r} 5) \ -3 \\ = 180) \ -108 \\ = 169) \ 2 \\ = 1692 \end{array} \end{aligned}$$

# ReCounting big Numbers in Icons (Division)

Recounting a total T of 478 in **7s**

Recounting a total T of 374 in **12s**

$$\begin{aligned}
 T = 478 &= 47) \quad 8 \\
 &= 42) \quad 58 \\
 &= 42) \quad 56 + 2 \\
 &= 6 \times 7) \quad 8 \times 7 + 2 \\
 &= 68 \times 7 + 2
 \end{aligned}$$

$$\begin{aligned}
 T = 374 &= 37) \quad 4 \\
 &= 36) \quad 14 \\
 &= 36) \quad 12 + 2 \\
 &= 3 \times 12) \quad 1 \times 12 + 2 \\
 &= 31 \times 12 + 2
 \end{aligned}$$

$$T = 478 = 68 \times 7 + 2$$

$$478 / 7 = 68 + 2/7$$

$$T = 374 = 31 \times 12 + 2$$

$$374 / 12 = 31 + 2/12$$

# DoubleCounting creates PerNumbers (Proportionality)

With **4kg = 5\$** we have  $4\text{kg}/5\$ = 4/5 \text{ kg}/\$ = \text{a per-number}$

$$4\$/100\$ = 4/100 = 4\%$$

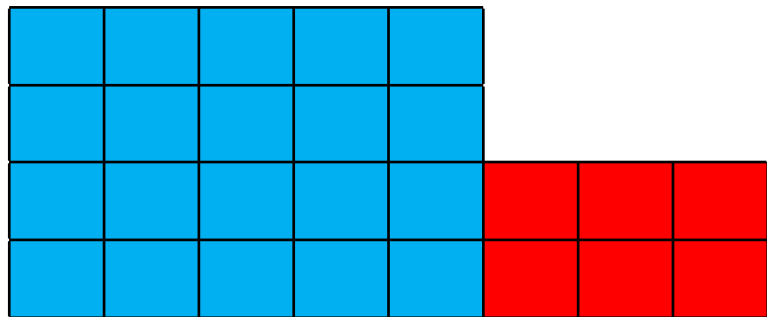
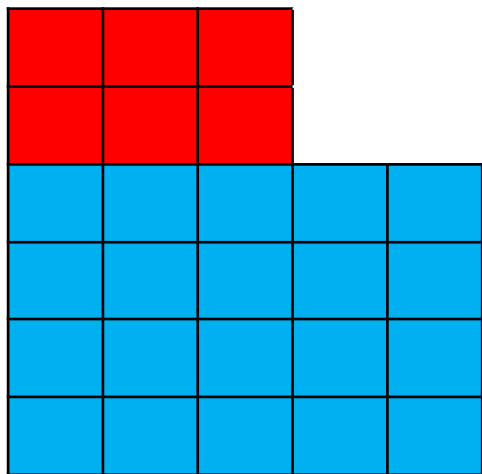
## Questions:

<b>7kg = ?\$</b>	<b>8\$ = ?kg</b>
$7\text{kg} = (7/4)*4\text{kg}$ $= (7/4)*5\$ = 8.75\$$	$8\$ = (8/5)*5\$$ $= (8/5)*4\text{kg} = 6.4\text{kg}$

**Answer:** *Recount in the per-number*

# Once Counted & ReCounted, Totals can be Added

OnTop	NextTo
$2 \text{ 3s} + 4 \text{ 5s} = 1.1 \text{ 5s} + 4 \text{ 5s} = 5.1 \text{ 5s}$	$2 \text{ 3s} + 4 \text{ 5s} = 3.2 \text{ 8s}$
The units are changed to be the same. <i>Change unit = Proportionality</i>	The areas are integrated. <i>Integrate areas = Integration</i>





# Adding PerNumbers as Areas (Integration)

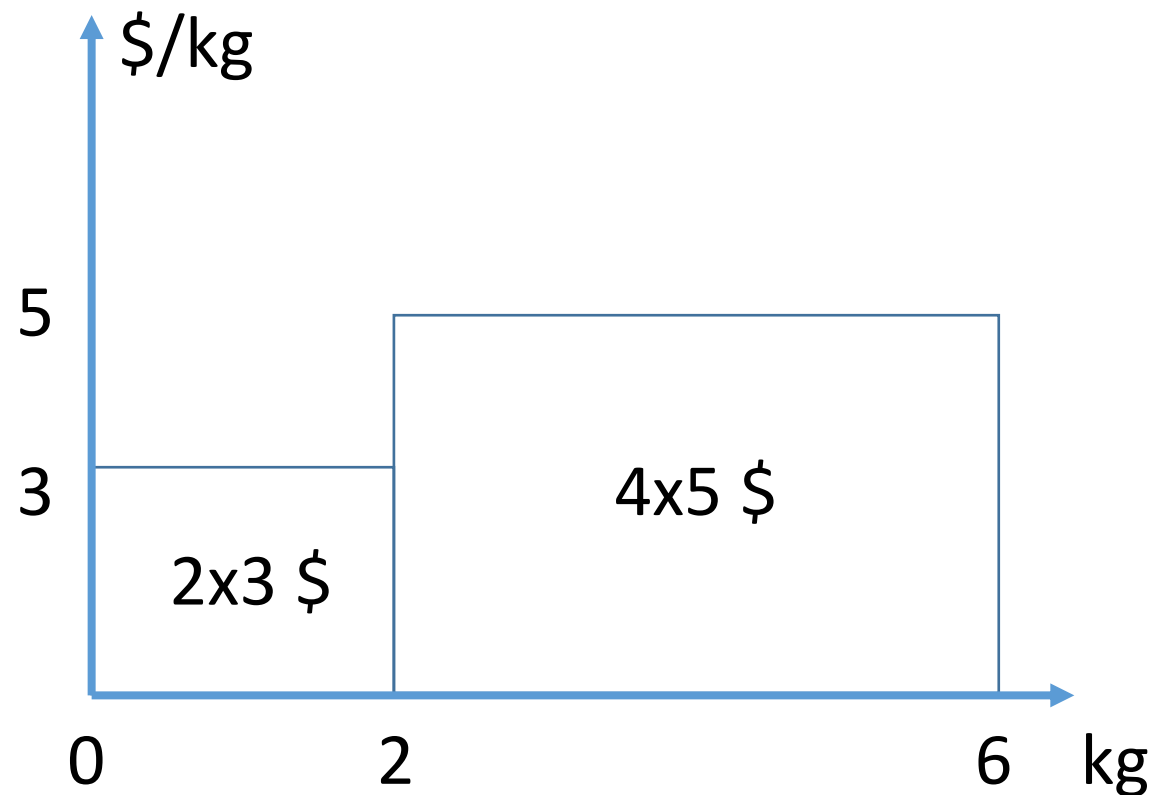
2 kg at 3 \$/kg  
 + 4 kg at 5 \$/kg  


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 (2+4)kg at  $(2 \times 3 + 4 \times 5) / (2+4)$  \$/kg

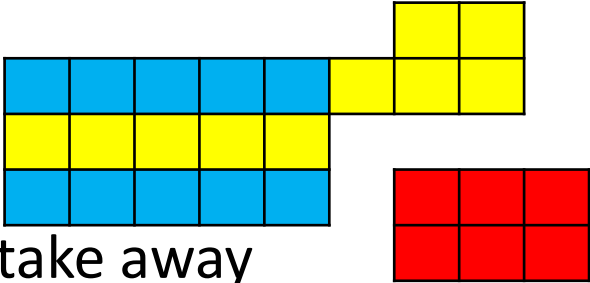
Unit-numbers add on-top.

Per-numbers add next-to as **areas** under the per-number graph.



# Reversing Addition, or Solving Equations

OnTop	Opposite Side & Sign	NextTo
$2 + ? = 8 \quad = (8-2) + 2$	$2 \times ? = 8 \quad = (8/2) \times 2$	$2.3s + ? 5s = 3.2 8s$
$? = 8-2$ <i>Solved by re-stacking</i>	$? = 8/2$ <i>Solved by re-bundling</i>	$? = (3.2 8s - 2.3s)/5$ <i>Solved by differentiation: <math>(T-T1)/5 = \Delta T/5</math></i>



## Hymn to Equations

Equations are the best we know,  
they are solved by isolation.

But first, the bracket must be placed  
around multiplication.

We change the sign and take away  
and only x itself will stay.

We just keep on moving, we never give up.  
So feed us equations, we don't want to stop!

# Geometry: Measuring HalfBlocks

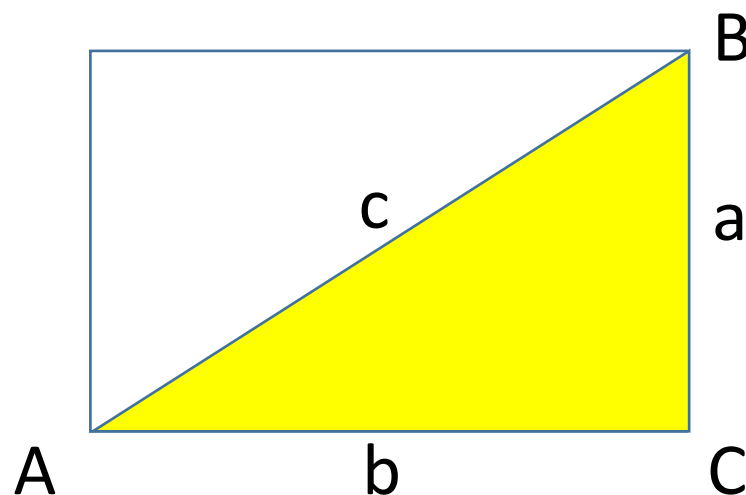
Geometry means to measure earth in Greek

The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base  $b$ , the height  $a$  and the diagonal  $c$  connected by the Pythagoras theorem  $a^2 + b^2 = c^2$ . And connected with the angles by formulas recounting the sides in diagonals:

$$a = (a/c) \times c = \sin A \times c$$

$$b = (b/c) \times c = \cos A \times b$$

$$a = (a/b) \times b = \tan A \times b$$

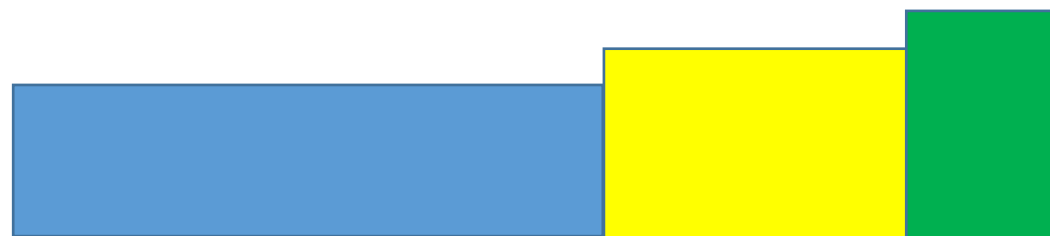


# Defining ManyMatics: To master Many, we Recount in Blocks to add OnTop or NextTo

In ManyMatics,

Numbers are 2D blocks,

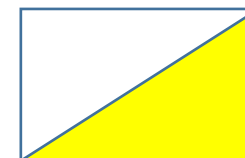
- not on a 1D line



Algebra: to (re)unite blocks on-top or next-to



Geometry: to measure half-blocks

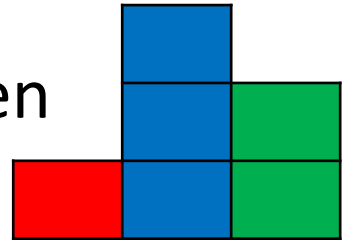


# 1D Roman Numbers and 2D Arabic Numbers

To see the difference we write down a total T of **six scores** and a **dozen**:

- $T = \text{XX XX XX XX XX XX} + \text{XII} = \text{CXXXII}$  ,
- $T = 6 \text{ 20s} + 1 \text{ 12s} = 1 * \text{BB} + 3 * \text{B} + 2 * 1 = 132$  , where Bundle = ten

Both systems use bundling to simplify.



The Roman uses a 1D juxtaposition of different bundle sizes.

The Arabic uses one bundle size only.

More bundles are described by multiplication:  $3 * \text{B}$ , i.e. as 2D areas.

Bundle-of-bundles are described by power:  $1 * \text{BB} = 1 * \text{B}^2$ .

Totals are described by next-to addition of 2D area blocks (integration).

# Counting Numbers Add, Cardinal Numbers do Not

In the Total  $T = 2 \text{ } 3\text{s}$

- 2 is a counting number (an operator)
- 3 is a bundle, base, unit or cardinal number.
- Counting numbers add **if** the units are the same
- Unit or cardinal numbers **do not add**, they stay



# ReCounting roots Algebra's 4 ways to ReUnite

Addition / *Subtraction* unites / *splits* into Variable Unit-numbers

Multiplication / *Division* unites / *splits* into constant Unit-numbers

Power / *Root&Log* unites / *splits* into constant Per-numbers

Integration / *Differentiation* unites / *splits* into variable Per-numbers

Operations <b>unite</b> / <i>split into</i>	Variable	Constant
Unit-numbers <i>m, s, \$, kg</i>	$T = a + n$ $T - a = n$	$T = a \times n$ $T/n = a$
Per-numbers <i>m/s, \$/kg, m/(100m) = %</i>	$T = \int a \, dn$ $dT/dn = a$	$T = a^n$ $\log_a T = n, \sqrt[n]{T} = a$

# Is ManyMatics Different from 'MatheMatics'

<i>Same Questions</i>	<b>ManyMatics</b>	<b>MatheMatics</b>
<b>Digits</b>	Icons, different from letters	Symbols like letters
<b>Natural numbers</b>	2.3 tens	23
<b>Fractions</b>	Operators (per-numbers) needing a number to produce a number	Rational numbers
<b>Per-numbers</b>	Double-counting	Not accepted
<b>Operations</b>	Icons for the counting process	Mappings from a set-product to a set
<b>Order of operations</b>	/, x, -, +	+, -, x, /

# Same Questions – Different Answers

Addition	On-top and next-to	Only on-top
Integration	<p>Preschool: Next-to addition, for all</p> <p>Middle school: Adding piece-wise constant per-numbers, for all</p> <p>High school: Adding locally constant per-numbers, for almost all</p>	<p>Last year in high school, for the few</p>
A formula	<p>A stand-by calculation with numbers and letters</p>	<p>An example of a function that is an example of a relation in a set-product where first component identity implies second component identity</p>
Algebra	<p>Re-unite constant and variable unit-numbers and per-numbers</p>	<p>A search for patterns</p>

# Yes, ManyMatics is Different

<b>An equation</b>	A name for a reversed operation	An example of an equivalence relation between two number-names
<b>The root of Mathematics</b>	The physical fact Many	The metaphysical invention SET
<b>A concept</b>	An abstraction from many examples	An example of an abstraction derived from SET (MetaMatics)
<b>How true is <math>2+3 = 5</math>   &amp;   <math>2 \times 3 = 6</math></b>	$2 \times 3 = 6$ is true by nature since 2 <b>3s</b> can be recounted as 6 <b>1s</b> . $2+3 = 5$ is true inside but seldom outside a class: $2\mathbf{w}+3\mathbf{d} = 17\mathbf{d}$ , etc.	Both true by nature (MatheMatism)

# Can Education be Different

I

From secondary school, continental Europe uses **line-organized** education with forced classes and forced schedules making teenagers stay together in age groups even if girls are two years ahead in mental development.

The classroom belongs to the class. This forces teachers to change room and to teach several subjects outside their training in lower secondary school.

Tertiary education is also **line-organized** preparing for offices in the public or private sector. This makes it difficult to change line in the case of unemployment, and it forces the youth to stay in education until close to 30 making reproduction fall to 1.5 child/family so European (child)population will decrease to 25% in 100 years.

# Yes, Education can also be Different

II

Alternatively, North America uses **block-organized** education saying to teenagers: “Welcome, you carry a talent! Together we will uncover and develop your personal talent through daily lessons in self-chosen half-year blocks. If successful the school will say ‘**good job**, you have a **talent**, you need some more’. If not, the school will say ‘**good try**, you have **courage**, now try something else’”.

The classroom belongs to the teacher teaching only one subject. Likewise, college is **block-organized** easy to supplement with additional blocks in the case of unemployment. At the age of 25, most students have an education, a job and a family with three children, one for mother, one for father and one for the state to secure reproduction.

*But why does Europe choose MetaMatism & lines instead of ManyMatics & blocks?*

# Sociology of Mathematics & Education

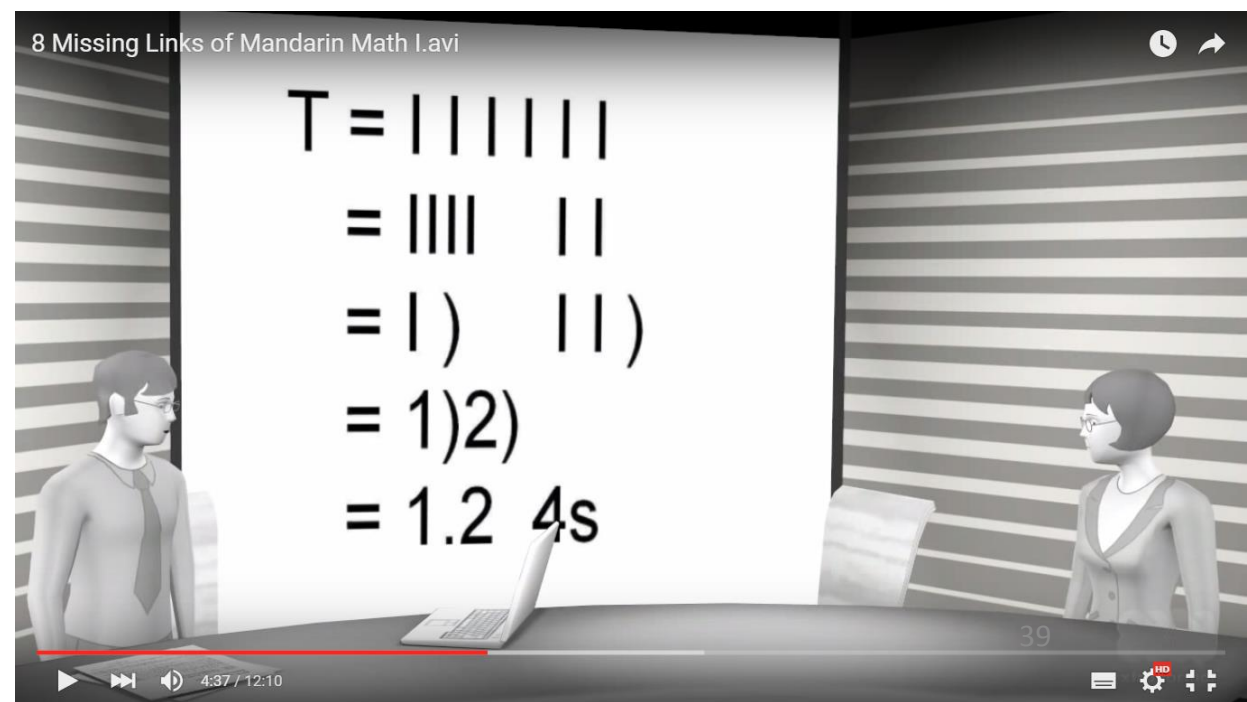
I

According to Pierre Bourdieu, Europe has replaced a blood-nobility with a knowledge-nobility using the Chinese mandarin technique to monopolize knowledge by making education so difficult that only their children get access to the public offices in the Bildung-based administration created in Berlin in 1807 to get Napoleon out.

The Mandarins used the alphabet.  
EU's knowledge-nobility uses math  
and lines as means to their goal.

Bourdieu, P. (1977). *Reproduction in Education, Society and Culture*. London: Sage.

MrAltarp: [youtube.com/watch?v=sTJiQEOTpAM](https://youtube.com/watch?v=sTJiQEOTpAM)



# Sociology of Mathematics & Education

## II

Michael Foucault: “It seems to me that the real political task in a society such as ours is to criticize the working of institutions, which appear to be neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (..)

If one fails to recognize these points of support of class power, one risks allowing them to continue to exist; and to see this class power reconstitute itself even after an apparent revolutionary process.”

*The Chomsky-Foucault Debate on Human Nature.* New York: The new Press. 2006



# Sociology of Mathematics & Education

## III

Inspired by Heidegger, Hannah Arendt divides human activity into labor and work both focusing on the private sphere; and action focusing on the political sphere creating institutions to be treated with care to avoid 'the banality of evil' present for all employees: You must follow orders in the private & in the public sector, both obeying necessities 'compete or die' & 'conform or die'.

Refusing to follow orders, in the private sector you just find a competing company, in the public sector you loose your job.

*Arendt, H. (1963). Eichmann in Jerusalem, a Report on the Banality of Evil. London: Penguin Books.*

# Philosophy of Mathematics & Education

Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines Existentialism by saying that to existentialist thinkers

‘**Existence** precedes **Essence**’.

Kierkegaard was skeptical towards institutionalized Christianity, seen also by Nietzsche as imprisoning people in moral serfdom until someone ‘may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.’

The existentialist distinction between **Existence** and **Essence** allows a distinction between **outside** and **inside** goals to be made and discussed.

Marino, G. (2004). *Basic Writings of Existentialism*. New York: Modern Library.

# Psychology: The Piaget - Vygotsky Conflict

They disagree profoundly as to the importance of teaching.

Vygotsky: Teach them more, and they will learn better.

Piaget: Teach them less, instead arrange meetings with the object.

From an existentialist viewpoint, distinguishing between Existence and Essence there is a danger that a textbook reflects only essence.

Seeing the textbook as the goal, Vygotskian theory has difficulties discussing goal/means confusions; in opposition to Piagetian theory pointing out that too much teaching will prevent this discussion.

*<http://www.azquotes.com/>*

# Yes, Math Ed has a Goal/Mean Confusion

Chosen as a common label for its two remaining activities, Geometry & Algebra, mathematics has two outside goals: to measure Earth and to reunite Many.

Transformed to self-referring MetaMatism, it became its own goal blocking the way to the outside goals, reduced to applications of mathematics to be taught, 'of course', after mathematics itself has been taught and learned.

In this way 'mathematics education' becomes an undiagnosed cure against a self-referring need, unneeded by patients showing natural resistance.

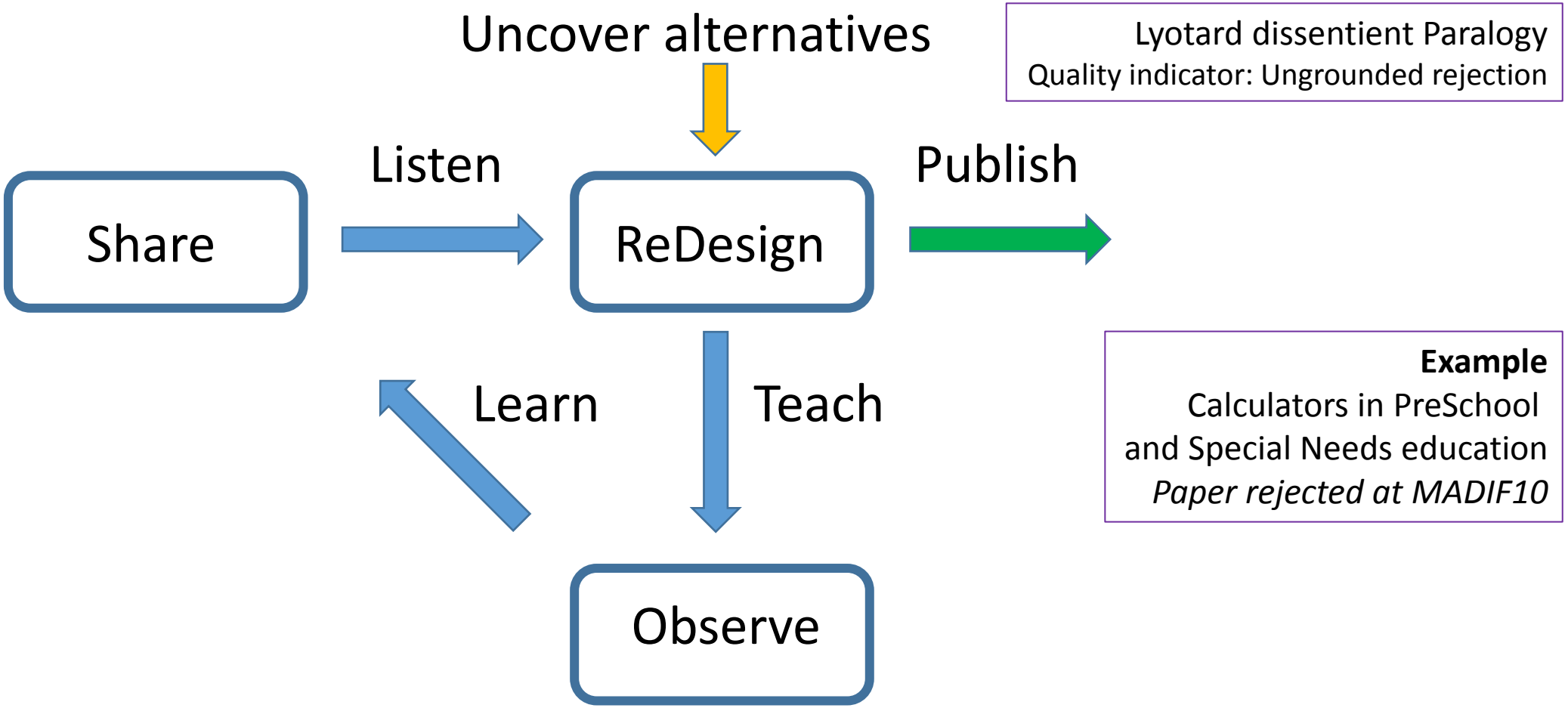
So, to reach the outside goal, mastering of Many, we must look for a different alternative way, as e.g. ManyMatics, built as a grounded theory, a Natural Science, about the physical fact Many.

# Will the Difference make a Difference?

Action Learning and Research will show:

- Try out the 'Recount – don't add' booklet
- Try out the 1day seminar 'How to avoid DysCalCulia'
- Try out a 1year teacher training at the  
MATHeCADEMY.net using PYRAMIDeDUCATION  
to teach teachers to teach MatheMatics as ManyMatics,
- Collect data and Report on its 8 MicroCurricula, M1-M8
- Finally, try out Block Organized secondary schools

# Action Learning & Action Research



# 1day Skype Seminar:

## To avoid DysCalCulia, **ReCount- don't Add**

Action Learning on the child's own 2D NumberLanguage as observed when showing 4 fingers together 2 by 2 makes a 3-year-old child say 'No, that is not 4, that is 2 2s.'

### 09-11. Listening and Discussing: **Good & Bad & Evil** MatheMatics

**Bad MatheMatism** is true inside but rarely outside classrooms.

**Evil MetaMatics** presents a concept TopDown as an example instead of BottomUp as an abstraction.

**Good ManyMatics**, a natural science Many mastering Many by ReCounting & adding OnTop/NextTo.

2D Block Numbers with units as a hidden alternative to the traditional 1D Line Numbers without

*Adding 1D Line Numbers without units may create Dyscalculia.*

### 11-13. Skype Conference. Lunch.

**13-15. Doing: Trying out the 'ReCount – don't Add' booklet** to experience proportionality & calculus & solving equations as golden LearningOpportunities in ReCounting and NextTo Addition.

### 15-16. Coffee. Skype Conference.

# 8 MicroCurricula for Action Learning & Research

C1. Create Icons

C2. Count in Icons (Rational Numbers)

C3. ReCount in the Same Icon (Negative Numbers)

C4. ReCount in a Different Icon (Proportionality)

A1. Add OnTop (Proportionality)

A2. Add NextTo (Integrate)

A3. Reverse Adding OnTop (Solve Equations)

A4. Reverse Adding NextTo (Differentiate)

## 4 Counted in 3s

Sticks

G-counting	A-counting
<i>lay out</i>	<i>lay out</i>
<i>bundle</i>	<i>bundle</i>
<i>stack</i>	① ① <i>cups</i>
T = 1.1 3s <i>Total</i>	1) 1) <i>cup-writing</i>
	T = 1.1 3s <i>Total</i>

## 4

Round it up & Color it

Clap, Sing, Walk, Act & Letter it

Unite it

Split it

Reward: Stickers, each counting two

MATHeCADEMY.net

## Abacus

mode	A-mode

## Calculator

4 / 3	1.some
4 - 1 x 3	1
T = 4 = 1.1 3s	
MATHeCADEMY.net	



# ReCount – don't Add Booklet, free to Download

## ReCount don't Add

MatheMatics as ManyMatics  
for NewComers & LateComers & Migrants  
to Avoid DysCalCulia



The Direct Way to Core Mathematics:  
Proportionality & Fractions & Calculus & Solving Equations

Allan.Tarp  
MATHeCADEMY.net

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### 03. ReCounting in Icons

Q?		Do	Calculator
9 in 5s	Line	T =	
	Count	1, 2, 3, 4, 8, 181, 182, 183, <u>184</u>	
	Bundle	T =	9/5      1.some
	Stack		9 - 1*5      4
	Cup	T =      1)4	
	Answer	T = 9 = 1.4 5s	
9 in 4s	Line	T =	
	Count	1, 2, 3, 8, 181, 182, 183, 28, <u>281</u>	
	Bundle	T =	9/4      2.some
	Cup	T = 2)1	9 - 2*4      1
	Stack		
	Answer	T = 9 = 2.1 4s	
9 in 3s	Line		
	Count		
	Bundle		
	Cup		9/
	Stack		9 -
	Answer		
8 in 4s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		
	Answer		
8 in 3s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		
	Answer		

# MATHeCADEMY.net

Teaches Teachers to Teach  
MATHEmatics as **MANY**matics,  
a Natural Science about **MANY**.  
The **CATS** method: To learn Math  
**Count & Add** in **Time & Space**

**MATHeCADEMY.net**  
MATHEmatics as MANYmatics, a Natural Science about MANY – the CATS approach: Count & Add in Time & Space

HOME INTRO COUNT ADD TIME SPACE DK VIDEOS PAPERS PRESCHOOL **VARIOUS** BOOK

**ManyMatics: ReCount – don't Add.**

Teach **Multiplication** before Addition & Add **NextTo** before OnTop

We ACT to deal with the outside world. [ReCounting Seminars](#)  
 We MATH to deal with the natural fact MANY ??? [Rejected Paper](#)  
 Oops, sorry, math is not an action word! [Avoid DysCalCulia](#)  
 We COUNT & ADD to deal with MANY. [ReCount – don't Add Booklet](#)

- Count & ReCount:

$T = \text{|||||} = \text{||| ||} \text{ } 1 = \text{||} \text{ } 1 = 2 \text{ } 1 = 2.1 \text{ } 3s$   
 $T = 2.1 \text{ } 3s = 1.4 \text{ } 3s = 3.-2 \text{ } 3s$  (Overload or Deficit)  
 $T = 2.1 \text{ } 3s = 1.2 \text{ } 5s = 3.1 \text{ } 2s = 11.1 \text{ } 2s$   
 $T = 3 \times 8 = 3 \text{ } 8s = 2.6 \text{ } 9s = 2.4 \text{ } tens$ , or the sloppy version 24

I	II	III	IIII	IIII	IIII	IIII	IIII	IIII
1	2	3	4	5	6	7	8	9

# Teacher Training in CATS ManyMatics

[illegible]

# PYRAMIDeDUCATION

*To learn MATH:  
Count&Add MANY  
But ReCount before you Add*

In PYRAMIDeDUCATION a group of 8 learners are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

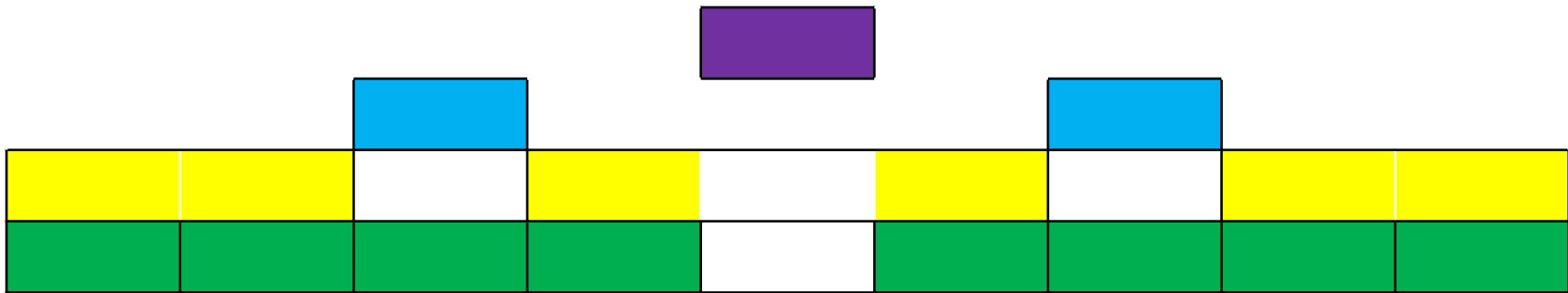
- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each learner pays by coaching a new group of 8 learners.

1 Coach

2 Instructors

3 Pairs

2 Teams



# MatheMatics: Unmask Yourself, Please

- In Greek you mean ‘mastering’. You were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic. Later only 2 activities remained: Geometry and Algebra
- Then Set transformed you from a Natural Science about the physical fact Many to a metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism
- So please, unmask your true identity, and tell us how you would like to be presented in education:
- MetaMatism for the few - or ManyMatics for the many

*But to Improve Schools: Don't preach **Essence**, teach **Existence***



# Creating or Avoiding DysCalCulia

Having problems learning mathematics has many names:

Difficulty, disability, deficiency, disorder, low attainment, low performance or DysCalCulia.

How to Create it	How to Avoid it
<ul style="list-style-type: none"> <li>● Teach 1D LineNumbers as '8'</li> <li>● No Counting before Adding</li> <li>● Adding before Multiplying</li> <li>● Adding without Units: <math>2+3=5</math></li> </ul>	<ul style="list-style-type: none"> <li>● Teach 2D BlockNumbers as '2 4s'</li> <li>● ReCounting before Adding</li> <li>● Multiplying before Adding</li> <li>● Adding with Units: <math>2\mathbf{w}+3\mathbf{d}=17\mathbf{d}</math></li> </ul>

# Good, Bad & Evil Math Education Research

**Evil** research hides alternatives through discourse-protection and self-reference thus presenting its **choice** as nature

**Bad** research sees the institution as rational and the agent as irrational. Thus math education problems lies with the agents.

**Good** research sees the problems lying with the institutions

- North America: Focusing on the agents, look for hidden rationality behind apparent irrationality
- France: Focusing on the institutions, look for hidden irrationality behind apparent rationality





# No ReCounting: Bye to Golden Math Opportunities

No Icon Creation	So, as letters, digits are just symbols to be learned by heart
Only Counting in tens	T = 2.3 <b>tens</b> = 23; oops, no unit & misplaced decimal point
No ReCounting in the Same Icon	So 37 is no more 2)17 or 4.- <b>3</b>
No ReCounting in a Different Icon	No more 3 x 5 is 3 <b>5s</b> , but 15, postponed to Multiplication No more 24 = ? <b>3s</b> . Instead we ask 24/3, postponed to Division
No Adding NextTo	Postponed to Integral Calculus
No Reversed Adding NextTo	Postponed to Differential Calculus, made difficult by being taught before Integral Calculus
Only Adding OnTop	No CupWriting: $24 + 58 = 7)12.$ Only Carrying: $7^12 = 82$ No CupWriting: $74 - 39 = 4)-5 = 35.$ Only Carrying: $74 = 6^{10}4$
No Reversed Adding OnTop	Postponed to Solving Equations

# Dienes on Place Value and MultiBase Blocks

“The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. (..)

In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..)

Educators today use the “multibase blocks”, but most of them only use the base ten, yet they call the set “multibase”. These educators miss the point of the material entirely.”

(What is a base?, <http://www.zoltandienes.com/academic-articles/>)

## Yes, Recounting looks like Dienes Blocks, but ...

Dienes teaches the 1D place value system with 3D, 4D, etc. blocks to illustrate the importance of the power concept.

- ManyMatics teaches decimal numbers with units and stays with 2D to illustrate the importance of the block concept and adding areas.

Dienes wants to bring examples of abstractions to the classroom

- ManyMatics wants to build abstractions from outside examples

Dienes teaches top-down 'MetaMatics' derived from the concept Set

- ManyMatics teaches a bottom-up natural science about the physical fact Many; and sees Set as a meaningless concept because of Russell's set-paradox.



ByeBye to LineOrganized MetaMatism  
Welcome to BlockOrganized ManyMatics

# Thank You for Your Time

Allan.Tarp@MATHeCADEMY.net  
Free Uni Franchise

# Solving Equations BottomUp or TopDown

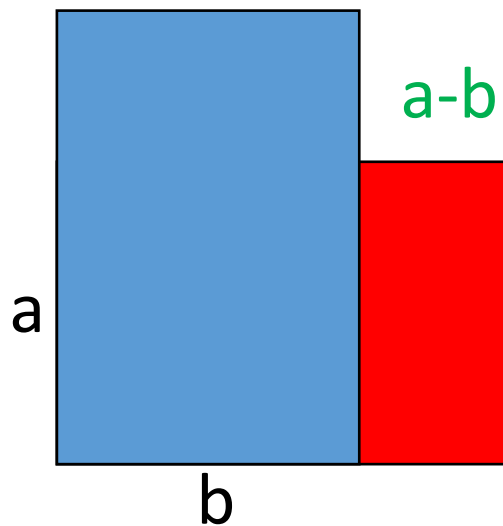
## ManyMatics

$2 + u = 5 = (5-2) + 2$	Solved by re-stacking 5	$2 \times u = 5 = (5/2) \times 2$	Solved by re-bundling 5
$u = 5-2 = 3$	Test: $2 + 3 = 5$ OK	$u = 5/2 = 2\frac{1}{2}$	Test: $2 \times 3 = 6$ OK

## MatheMatics

↕	$2 + u = 5$	Addition has 0 as its neutral element, and 2 has -2 as its inverse element
↕	$(2 + u) + (-2) = 5 + (-2)$	Adding 2's inverse element to both number-names
↕	$(u + 2) + (-2) = 3$	Applying the commutative law to $u + 2$ , 3 is the short number-name for $5+(-2)$
↕	$u + (2 + (-2)) = 3$	Applying the associative law
↕	$u + 0 = 3$	Applying the definition of an inverse element
↕	$u = 3$	Applying the definition of a neutral element. <i>With arrows a test is not needed.</i>

# Quadratic Rule with 2 Cards



$$\text{Corner} = (a-b)^2 = a^2 - 2 \text{ cards} + b^2$$

$$\text{So } (a-b)^2 = a^2 - 2 \times a \times b + b^2$$

# Quadratic Equations with 3 Cards

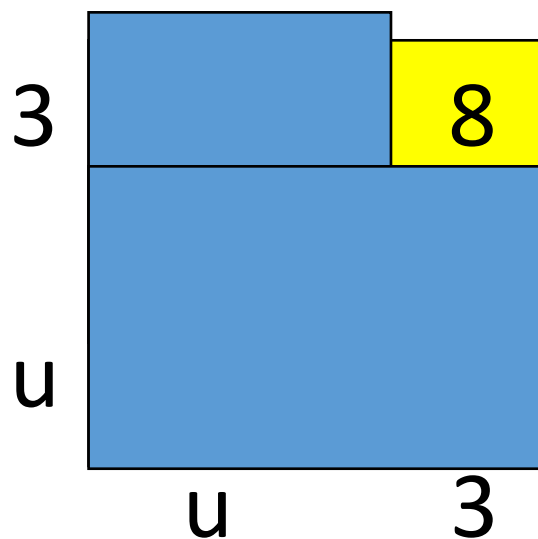
$$u^2 + 6u + 8 = 0$$

$$(u+3)^2 = u^2 + 6u + 8 + 1$$

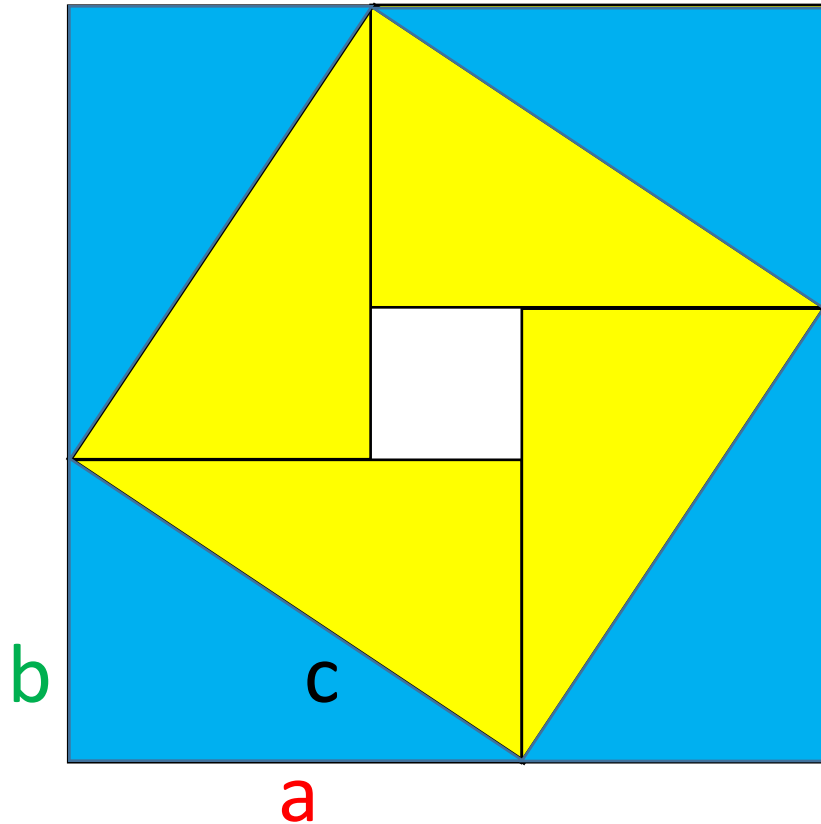
$$(u+3)^2 = 1$$

$$u = -3 \pm 1$$

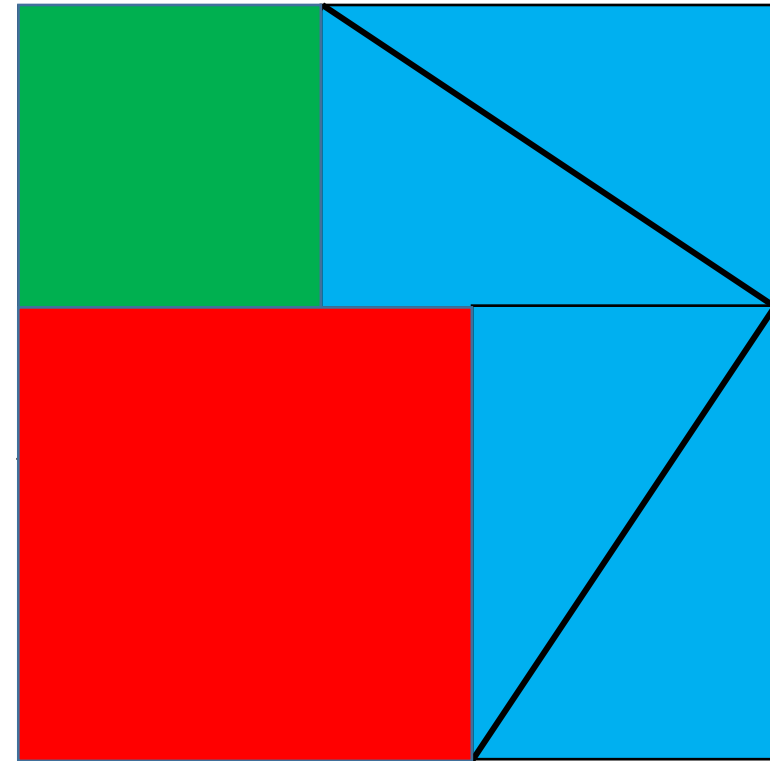
$$u = -4, u = -2$$



# Pythagoras shown by 4 Cards with Diagonals



$$c^2 + 4 \frac{1}{2} \text{cards}$$



$$a^2 + b^2 + 2 \text{ cards}$$