

# Existentialism

## in Mathematics Education

*from Essence to Existence*

Allan Tarp

MATHeCADEMY.net

a Virus-ACADeMY

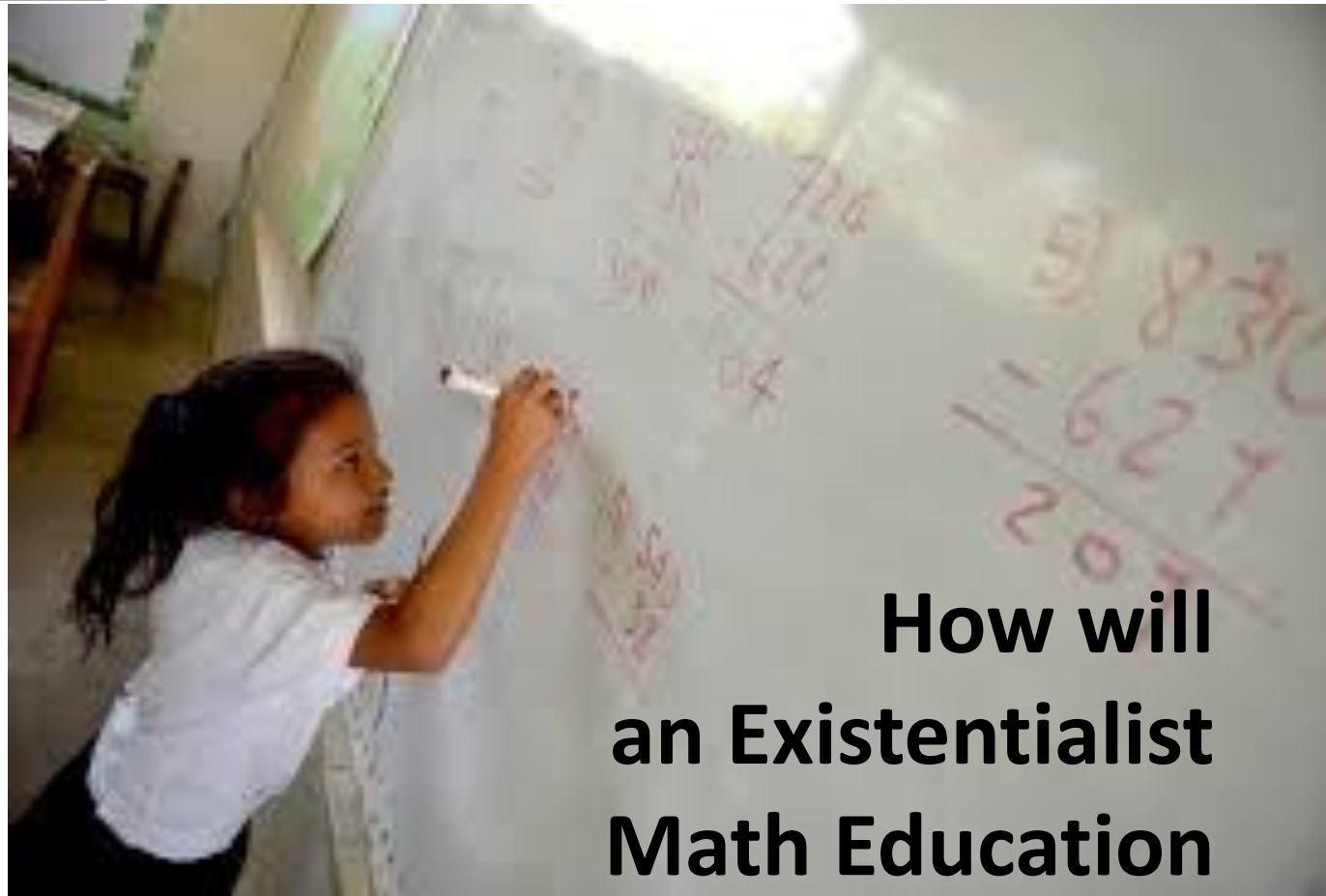
Teaching Teachers to Teach  
Mathematics as ManyMatics,  
a Natural Science about Many

ICME13 2016

# Abstract

In math and its education, the difference between **essence** and **existence** is undiscussed, although central to existentialist thinking.

So we ask:

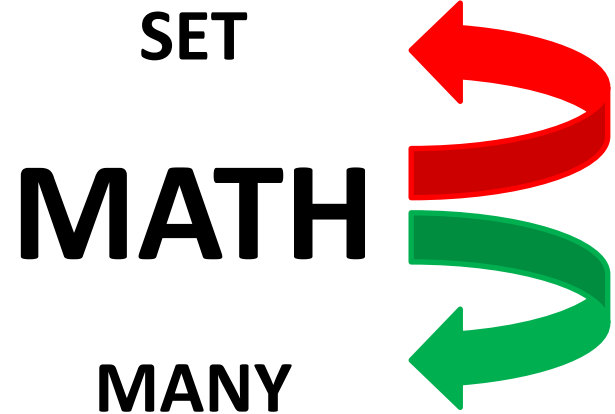


**How will  
an Existentialist  
Math Education  
look like?**

# Is Math Rooted Above or Below?

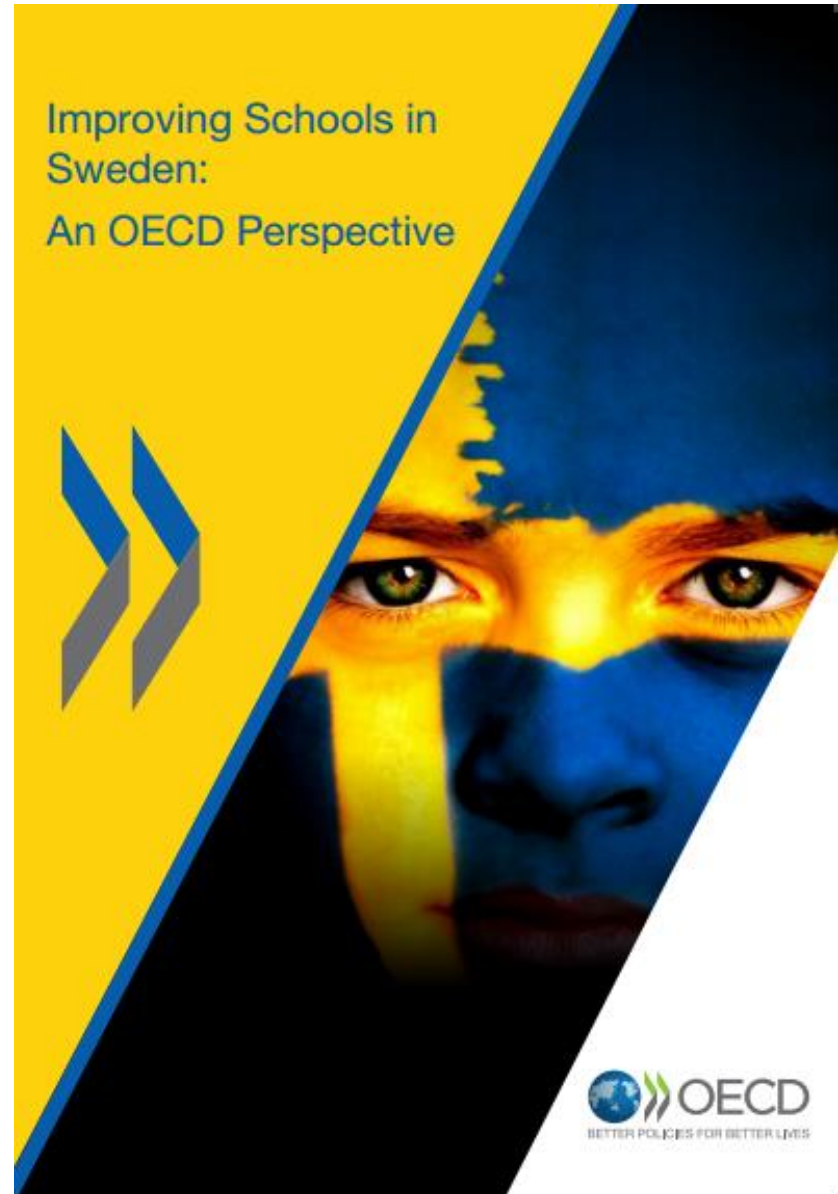
So we close the door to the library's self-referring math and go outside to rebuild math from its roots, the physical fact **Many**.

Likewise, we ask if math is learned by exposure to its outside roots, or to its inside essence claims?



# Background

Institutionalized education has math as a core subject. OECD **PISA** studies evaluate its success. In spite of huge funding meant to improve math education research, the 2015 OECD report 'Improving Schools in Sweden' shows 'a stark **decline** in the performance of 15-year-old ... with more than one out of four students not even achieving ... competencies to actively **participate in life.**'

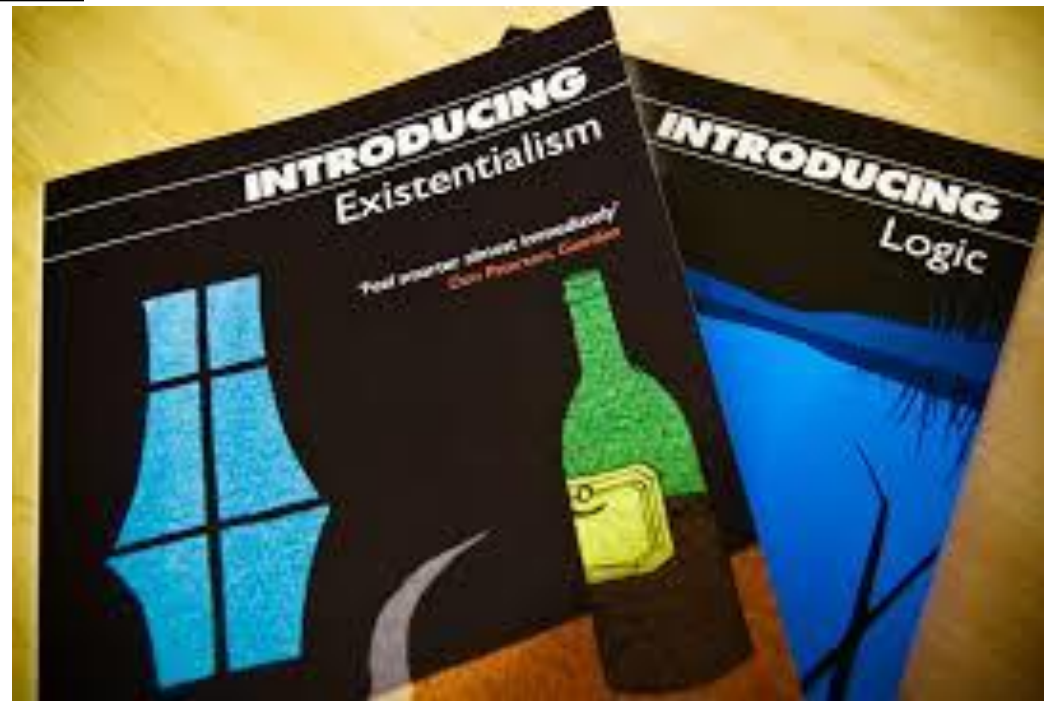


# An Existentialist Hypothesis

With more funding meaning more problems we ask:  
Is Math Education teaching Math or something else?

To answer, we ask:  
*How will an **existentialist math education** look like?*

The purpose is not to replace one tradition with another but to **uncover hidden alternatives to choices presented as nature.**



# Mathematics is just a mere Label

The Pythagoreans labeled their four knowledge areas by a Greek word for knowledge, mathematics. With astronomy and music as independent areas, today mathematics is a common label for the two remaining activities, **Geometry** meaning to measure earth in Greek, and **Algebra** meaning to reunite numbers in Arabic and replacing Greek arithmetic (Freudenthal 1973).



# Sophists and Philo-Sophists

The Greeks used the word 'sophy' for men of knowledge, the sophists and the philosophers, disagreeing on the nature of knowledge. Seeing **democracy** with information and debate and choice as the natural state-form, the sophists emphasized knowing nature from choice to prevent patronization by choices presented as nature. Seeing **autocracy** patronized by themselves as the natural state-form, the philosophers saw choice as an illusion since to them physical existence was but examples of metaphysical essence only visible to the philosophers educated at the Plato academy having as entrance sign '**Let no one ignorant of geometry enter.**'



# Sophist Skepticism

Today, the **sophist skepticism** towards false is-claims is carried on by the skepticism of the two Enlightenment republics, in North America as **pragmatism**, and in France as **existentialism** and **post-structuralism**

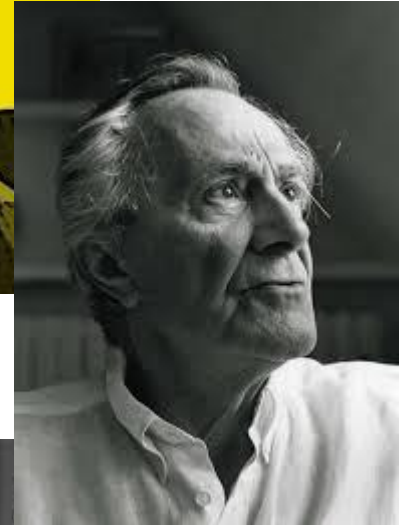




# PostStructuralism

French post-structuralism identifies hidden patronization in our most fundamental **institutions**:

- **Words** (Derrida)
- **Correctness** (Lyotard)
- **Cure** (Foucault)
- **Knowledge** (Bourdieu)



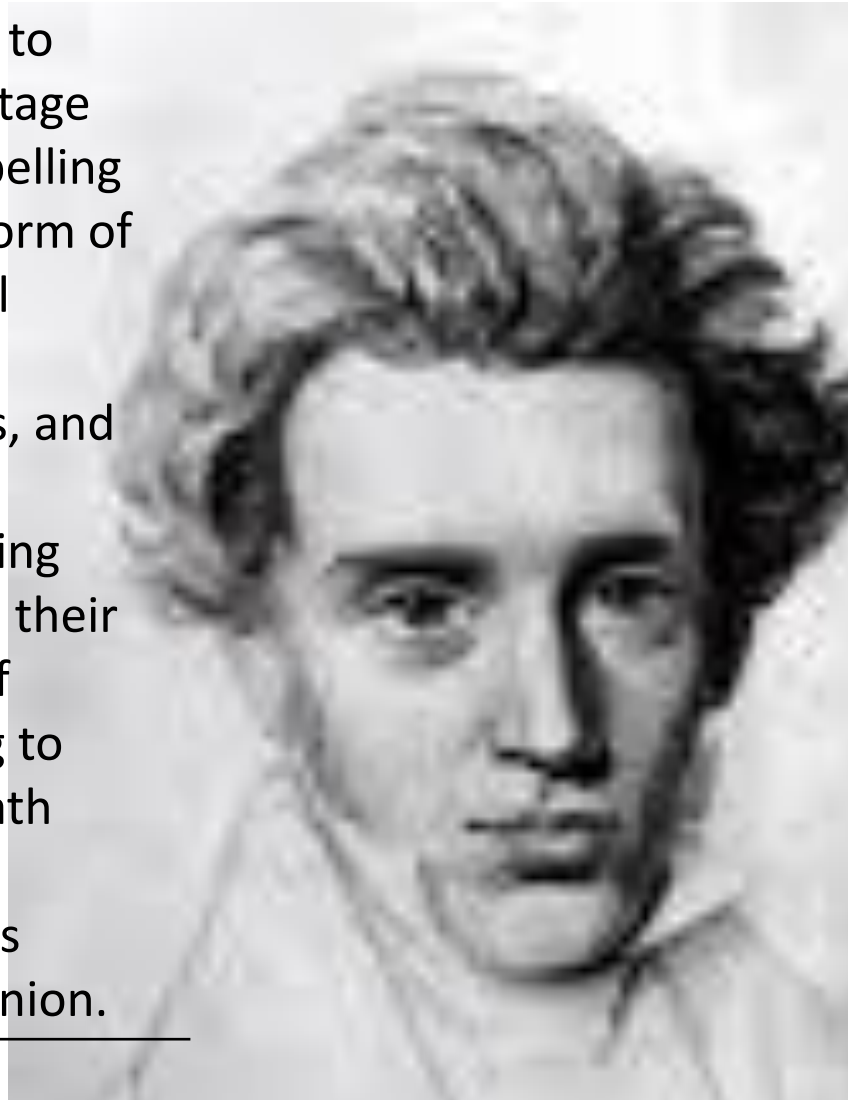
# Existentialism

**Existentialism** builds on the work of Kierkegaard, Nietzsche and Heidegger; and Sartre, defining existentialism as holding that '**existence precedes essence**, or (..) that subjectivity must be the starting point'



# Christianity YES, Christendom NO

In Denmark, a heritage allowed **Kierkegaard** to publish whatever he wrote. At the end, shortage forced him to shift to flying papers when rebelling against institutionalized **Christianity** in the form of **Christendom**. Focusing on the three classical virtues **Truth** and **Beauty** and **Goodness**, Kierkegaard left truth to the natural sciences, and argued that to change from a person to a personality the individual should stop admiring beauty created by others and instead realize their own existence through individual choices. Of course, angst is a consequence when fearing to choose the bad instead of the good, and death might follow, but so will forgiveness and resurrection to a new life in real existence, as promised by Christianity in the Holy Communion.



# Yes, I'm the great *Redeemer*

In Germany, **Nietzsche** saw institutionalized Christendom as the creator of moral **is-statements** that prevented individuals from realizing their true existence through individual choices and action. To end this serfdom he hoped that someday we will see a '*redeeming man* (..) whose isolation is misunderstood by the people as if it were flight *from* reality – while it is only his absorption, immersion, penetration *into* reality, so that (..) he may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.'



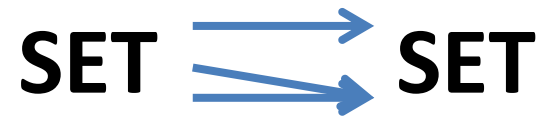
# IS-ism gives Banality of Evil

Likewise in Germany, Heidegger saw that to avoid traditional essence-claims, **is**-statements must be replaced by **has**-statements so that being is characterized by what it has, 'Dasein'. Arendt carried his work further by dividing human activity into **labor** and **work** focusing on the private sphere, and **action** focusing on the political sphere thus accepting as the first philosopher political action as a worthy human activity, creating **institutions** that should be treated with care to avoid '**the banality of evil**' if turning **totalitarian** by seeing itself as a **goal** and not a **means**.

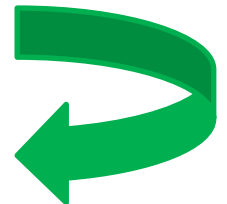
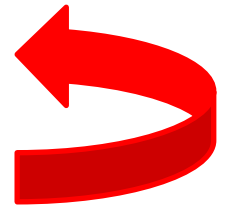


# Mathematics as Essence 01

Within **math**, the existentialist distinction is shown by the **function** concept, defined by Euler as labeling the existence of calculations combining known and unknown numbers, but defined today as set-relations where first component-identity implies second-component identity thus becoming pure essence through **self-reference**. The set-concept transformed MatheMatics to **MetaMatics**, a self-referring collection of well-proven statements about well-defined concepts, defined as **examples from internal abstractions instead of as abstractions from external examples**.



*function*



$$\cancel{y=2+3*4}, y=2+3*x$$

# Mathematics as Essence 02

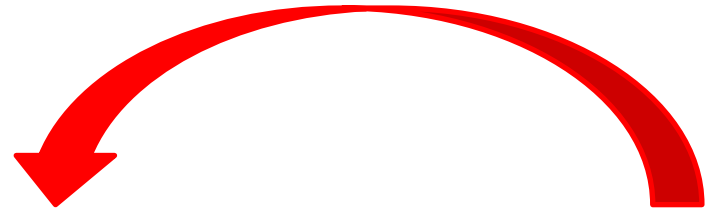
Looking at the set of sets not belonging to itself allowed **Russell** to show that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false.

The Zermelo–Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by its inability to separate outside examples from inside abstractions. That institutionalized education ignores this can be seen as an example of '**symbolic violence**' used to protect the privileges of today's 'knowledge nobility' (Bourdieu 1977).

**If**  $M = \{A \mid A \notin A\}$   
**then**  $M \in M \Leftrightarrow M \notin M$

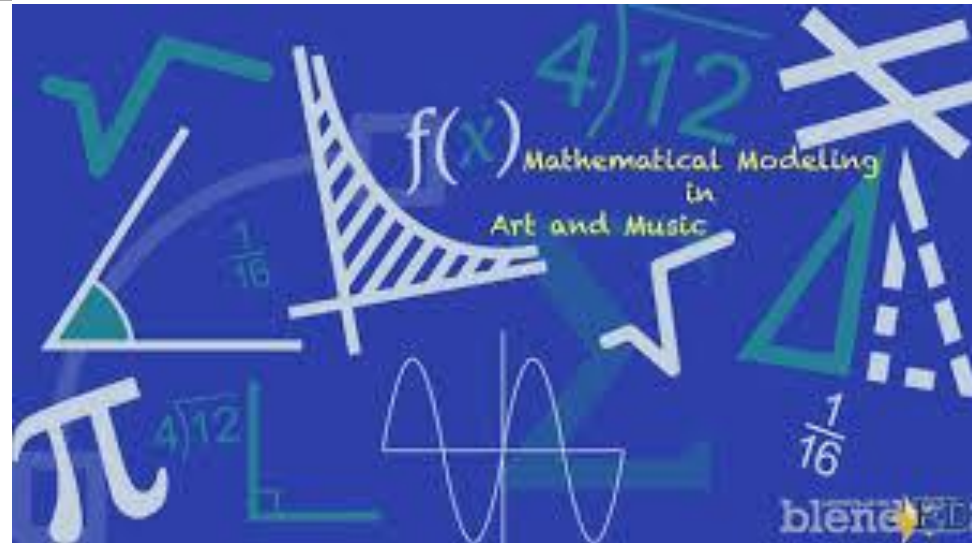


**This Sentence is False**



# Mathematics as Essence 03

Behind colorful illustrations, self-referring **MetaMatics** is taught through a gradual presentation of different number types from natural numbers to real numbers, together with the four basic operations, addition and subtraction and multiplication and division, where especially division and letter fractions create learning problems. Equations are introduced as equivalent number names to be changed by identical operations. In pre-calculus polynomial functions are introduced as a basis for calculus presenting differential before integral calculus.



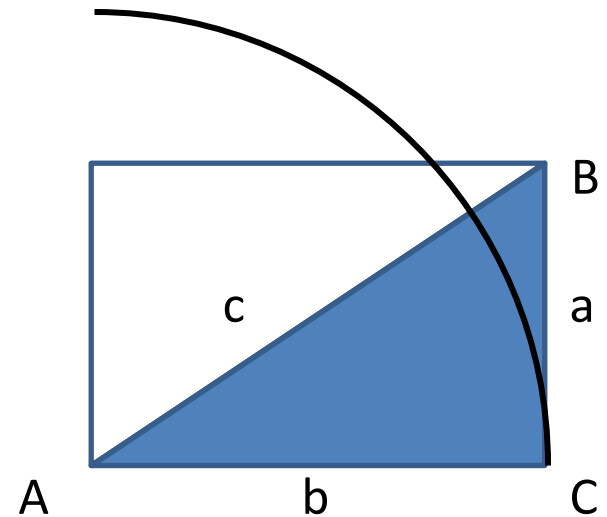


# Mathematics as Existence 01

Chosen by the Pythagoreans as a common label, **mathematics** has no existence itself, only its content has, **geometry** and **algebra**, both rooted as natural sciences about the physical fact **Many**.

The root of **geometry** is the standard form, a rectangle, that halved by a diagonal becomes two right-angled triangles where the sides and the angles are connected by three laws.

Being filled from the inside by such triangles, a circle with radius  $r$  gets its circumference formula.



$$A+B+C = 180,$$
$$a^2+b^2 = c^2$$
$$\sin A = a/c.$$

$$\text{Circumference} = 2 \cdot \pi \cdot r$$

$$\pi = n \cdot \sin(180/n)$$

for  $n$  sufficiently large

# Mathematics as Existence 02

Meeting Many we ask 'How Many Total?'  
**Counting & Adding** gives the answer.

We count by **bundling** and stacking as seen when writing a total T in its full form:

This shows Algebra's four ways to unite:  
**OnTop addition** unites variable numbers  
**Multiplication** unites constant numbers  
**Power** unites constant factors  
**NextTo addition**, also called integration, unites variable blocks.

As indicated by its name, uniting can be reversed to split a total into parts predicted by the reversed operations: **subtraction** and **division** and **root & logarithm** and **differentiation**.



*How Many Total?*

$$T = 345 = 3 \cdot B^2 + 4 \cdot B + 5 \cdot 1$$



$T = a + b$ $T - a = b$	$T = a * b$ $\frac{T}{a} = b$
$T = \int f dx$ $\frac{dT}{dx} = f$	$T = a^b$ $\sqrt[b]{T} = a$ $\log_a(t) = b$

# CupCounting kills **Math Dislike**

**Counting** is a forgotten art that leads directly to core math as **proportionality & integration**. **CupCounting** allows a total to be counted also with overloads and underloads to make multiplication & division easy.

Thus, with CupCounting, a natural number is a **decimal number with a unit** where the decimal point separates the bundled from the singles.

$$\begin{aligned}
 5 &= ||||| = \text{Cup} | | | = 1)3 \mathbf{2s} \\
 5 &= |||| = \text{Cup} | = 2)1 \mathbf{2s} \\
 5 &= ||||| = \text{Cup} | = 3)-1 \mathbf{2s}
 \end{aligned}$$

*CupWriting tells InSide Bundles from OutSide 1s:*

- $65 + 27 = 6)5 + 2)7 = 8)2 = 9)2 = 92$
- $65 - 27 = 6)5 - 2)7 = 4)-2 = 3)8 = 38$
- $7 \times 48 = 7 \times 4)8 = 28)56 = 33)6 = 336$
- $336 / 7 = 33)6 / 7 = 28)56 / 7 = 4)8 = 48$

$$\begin{aligned}
 7 &= 3)1 \mathbf{2s} = 2)1 \mathbf{3s} = 1)3 \mathbf{4s} = 1)2 \mathbf{5s} \\
 7 &= 3.1 \mathbf{2s} = 2.1 \mathbf{3s} = 1.3 \mathbf{4s} = 1.2 \mathbf{5s}
 \end{aligned}$$

# ReCounting kills **Table Dislike**

**ReCounting 3 4s** in another unit allows children to practise changing units between **icon-numbers** 2-9 or to or from ten. So multiplication is really ReCounting icon-numbers in tens, and division is really ReCounting tens in icon-numbers.

**NO**

$3 \times 5$  IS NOT 15, but 3 **5s** to be ReCounted as 2.1 **7s**, 1.5 **tens**, or ...

**NO**

$24/8$  is not 24 split among 8 but 2.4 tens ReCounted in **8s**

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIII
	L	L	L	S	S	S	S	S
1	2	3	4	5	6	7	8	9

# ReCounting Predicted by a Calculator

**ReCounting 2 4s in 5s** or in tens the answer is predicted by a **ReCount-Formula  $T = (T/b) \times b$**  telling that from a total  $T$ ,  $b$ s can be taken away  $T/b$  times.

So the natural order of operations is division to bundle, multiplication to stack, subtraction to look for unbundle; and addition at last.

$$T = 2 \text{ 4s} = ? \text{ 5s} = 1.3 \text{ 5s}$$

since we get by using the **ReCount-Formula**

$$T = (T/B) \times B \quad \text{i.e. } T = T/B \text{ Bs}$$

& a **calculator**

$2 \times 4 / 5$	1.some
$2 \times 4 - 1 \times 5$	3

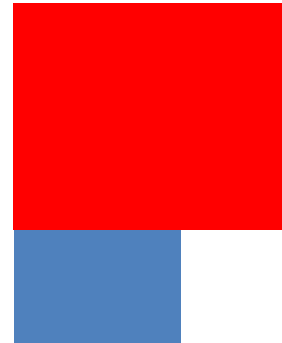
# Addition is not Well Defined

Once Counted or ReCounted, totals can be added, **but how, OnTop or NextTo?**

To add OnTop, the units must be the same rooting

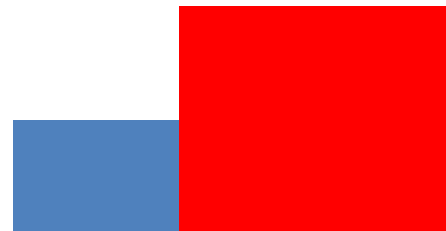
ReCounting & **Proportionality**

To add NextTo, the totals are added as blocks by their areas, rooting **Integration**



$$2 \text{ } 3s + 4 \text{ } 5s = 1.1 \text{ } 5s + 4 \text{ } 5s = 5.1 \text{ } 5s$$

$$2 \text{ } 3s + 4 \text{ } 5s = 3.2 \text{ } 8s$$



# DoubleCounting roots PerNumbers

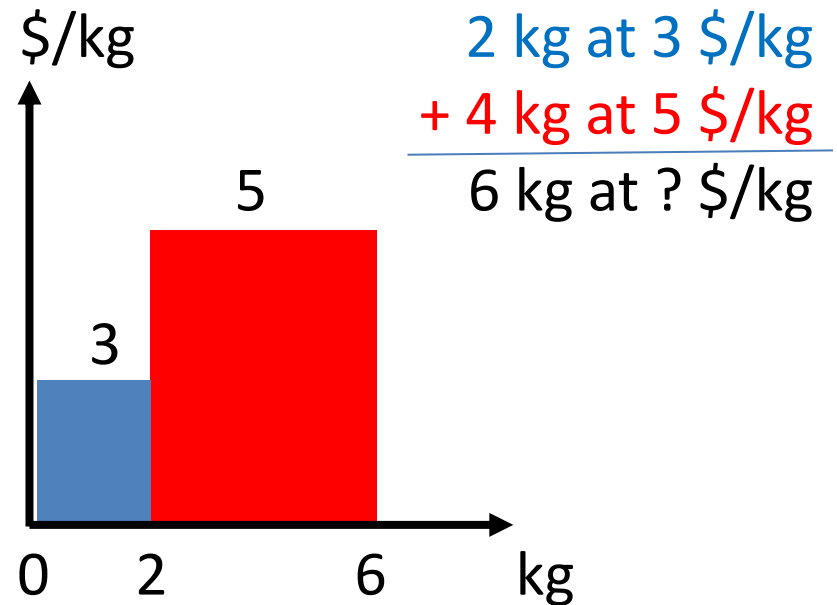
With physical units, the need for changing units roots per-numbers as  $3\$/4\text{kg}$  serving as bridges when recounting  $\$$ s in 3s or kgs in 4s.

As **per-numbers**, fractions are not numbers, but operators needing a number to become a number.

So rational numbers are pure ghostly essence without any existence behind.

**Adding per-numbers**, they are multiplied to unit-numbers and added as areas, called **integration**.

$$\begin{aligned} \text{With } 3\$/4\text{kg, } 15\$ &= ? \text{ kg} \\ 15\$ &= (15/3) \cdot 3\$ \\ &= (15/3) \cdot 4\text{kg} = \mathbf{20\text{kg}} \end{aligned}$$



# Learning as Preaching ESSENCE

Constructivist learning theory has a European social Vygotskian and a North American radical Piagetian version, seeing learning taking place through teaching or exposure respectively.

*The question, however, is what is to be learned?*

**Vygotsky** sees learning as a **top-down** adaption to the ruling essence-claims even if self-reference makes them meaningless; and sees teaching as implanting them in the learner's mind using outside artefacts as means.





# LEARNING as Meeting EXISTENCE

**Piaget** sees learning as a **bottom-up** adaptation to the outside world, and sees teaching as asking guiding questions to outside existence brought inside the classroom to allow learners construct individual schemata to be accommodated through exposure and communication.



# EXISTENCE preceding ESSENCE

To let **existence precede essence**, Piaget is useful to mediate learning through inside exposure to outside existence. Vygotsky is useful for a quick mediation of inside essence-claims rooted in outside existence.

However, uncritically mediating the ruling self-referring claims involves a high risk for Vygotsky practicing *the banality of evil*.



# Institutionalized Education as ESSENCE and EXISTENCE, east

Post-primary education has a line- and a block-organized version. To get Napoleon out of Berlin, Humboldt created a European **line-organized office-directed** education that concentrates teenagers in age-groups, and force them to follow the same schedule. To meet the international norm that 95% of an age-group finish high school, dropout rates are lowered by low passing grades and by a strict retention policy.



# Institutionalized Education as ESSENCE and EXISTENCE, west

In the North American republics, middle and high schools teachers teach their major subject in their own classroom where they welcome teenagers with recognition:

‘Inside, you carry a **talent** and it is our mutual task to uncover and develop your **personal talent** through **daily lessons in self-chosen half-year blocks**. If successful I say ‘**Good job**, you have a **talent**, you need more’. If not I say ‘**Good try**, you have **courage** to try uncertainty, now try something else that might be your **talent**.’

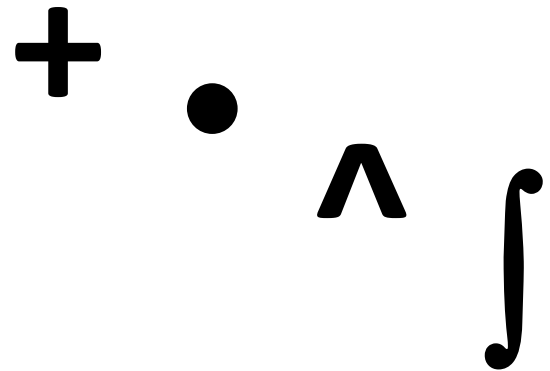
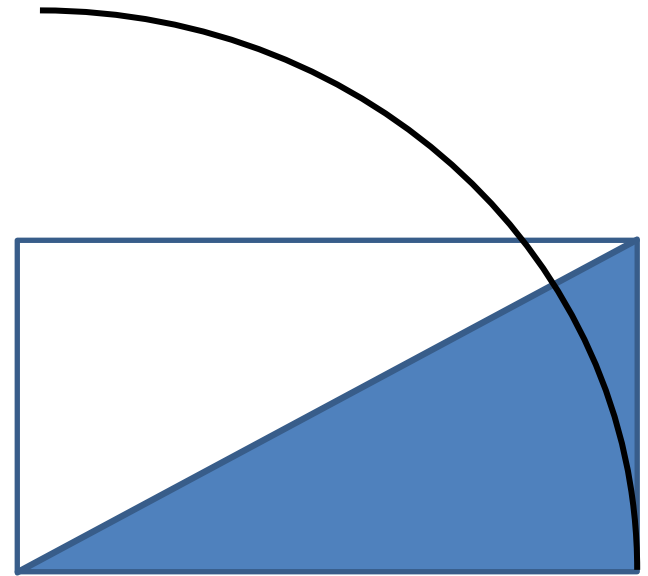


# Conclusion 01

An existentialist view replacing essence with existence exposes today's mathematics as **self-referring essence** with no existence behind.

What has existence is **Many**, waiting to be united by bundling and stacking into a decimal number with a unit, presented geometrically or algebraically as a row of blocks or digits.

Thus mathematics exists as a common **label** for **geometry** measuring forms divided into triangles, and **algebra** reuniting numbers by 4 uniting techniques: addition, multiplication, power and integration, each with a reverse splitting technique.



# Conclusion 02

Teaching self-referring essence means teaching MatheMatics as

**MetaMatism**, a mixture of **MetaMatics** defining concepts as top-down examples of abstractions instead of as bottom-up abstractions from examples; and

**MatheMatism** true inside, but not outside the classroom as e.g. claiming that 2+3 IS 5 in spite of countless counter-examples:

2 weeks + 3 days = 17 days,

2 m + 3 cm = 203 cm; and in spite of

**The Fraction Paradox:**

$$T = \frac{1}{2} + \frac{2}{3} = ?$$

**Preaching Essence:**

$$T = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$$

**Observing Existence:**

Counting apples in fruits:

$$T = \frac{1}{2} \text{ of } 2 + \frac{2}{3} \text{ of } 3 = \frac{3}{5} \text{ of } 5$$

*How can it be 7 of 6?*

# Conclusion 03

As to learning, mediating essence should be replaced by guided exposure to what exists, the physical fact. Many, the roots of mathematics, thus replacing Vygotsky with Piaget.

**ESSENCE**

5 **IS** the follower of 4



**EXISTENCE**

IIII = 5  
= 1)2 3s  
= 2)1 2s = 1)3 2s = 3)-1 2s



# Final Conclusion

And institutionalized education, using camps to concentrate teenagers in age-groups obliged to follow forced schedules, should be labeled as such to allow mathematics education avoid the banality of evil when making a **goal-means confusion** seeing the outside as a means to inside goals. Christianity's Holy Communion offers forgiveness to individuals, not to institutions. Instead institutionalized force should be limited to provide teenagers with daily lessons in self-chosen half-year blocks to uncover and develop their individual talent, as would be the case if the North American Enlightenment republics replaced essence with existence in algebra and geometry.





# Sociology of Mathematics & Education

Michael Foucault: “It seems to me that the real political task in a society such as ours is to criticize the working of institutions, which appear to be neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (..)

If one fails to recognize these points of support of class power, one risks allowing them to continue to exist; and to see this class power reconstitute itself even after an apparent revolutionary process.”

Allan Tarp: “To cure, be sure, the diagnose is not self-referring.”

# Kuhn: A Powerful Research Paradigm

## Nine ICME13 Papers by Allan Tarp:

01. Proportionality and Integration in Preschool through IconCounting and NextTo Addition	01
02. Calculators and IconCounting and CupWriting in PreSchool and in Special Needs Education	05
03. As PerNumbers, Fractions are Operators, not Numbers	09
04. From Essence to Existence in Geometry Education	13
05. Preschool, Middle school and High school Calculus	17
06. Essence and Existence in Conflicting Cognitive Theories	21
07. From Essence to Existence in Curriculum Development	25
08. From Essence to Existence in Math Teacher Education	29
09. From Essence to Existence in Mathematics Education	33

# References

- Arendt, H.** (1963). *Eichmann in Jerusalem, a Report on the Banality of Evil*. London: Penguin Books.
- Bourdieu, P.** (1977). *Reproduction in Education, Society and Culture*. London: Sage.
- The Chomsky-Foucault Debate on Human Nature***. New York: The new Press. 2006
- Freudenthal, H.** (1973). *Mathematics as an Educational Task*. Dordrecht-Holland: D. Reidel Publ. Company.
- Marino, G.** (2004). *Basic Writings of Existentialism*. New York: Modern Library.
- MrALTarp** (2012). *8 Missing Links in Mandarin Math*.  
<https://www.youtube.com/watch?v=sTJiQEOTpAM>
- OECD.** (2015). *Improving Schools in Sweden: An OECD Perspective*.  
<http://www.oecd.org/edu/school/improving-schools-in-sweden-an-oecd-perspective.htm>.