
# To Cure Math Dislike
## do not Teach Mathematics

*ReInvent Numbers & Algebra & Geometry &*
**Teach Algebra & Geometry Hand-In-Hand**

From TopDown Modern **MetaMatism**
to BottomUp PostModern **ManyMath**

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Designed as a VIRUSeCADEMY
to Teach Teachers to Teach MatheMatics as **ManyMath**
- a Natural Science about the physical fact **Many**

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# Contents

# A Language House with two Languages

To describe the world we need 2 languages: a **Word-** and a **Number-Language.**
Both are part of a two floor **Language House** that describes the world by a language - and that describes the language by a **meta-language**, a grammar.

In the WordLanguage, language comes before its BottomUp grammar.
In the NumberLanguage, Top-Down Modern Math teaches language after grammar.
And grammar before language means huge learning problems.

| | WordLanguage | NumberLanguage |
|---|---|---|
| MetaLanguage | The apple is a subject | T is a function |
| Language | The apple is green *(opinion)* | T = 2+3*x *(prediction)* |
| | *Qualities* | *Quantities* |

**The World**

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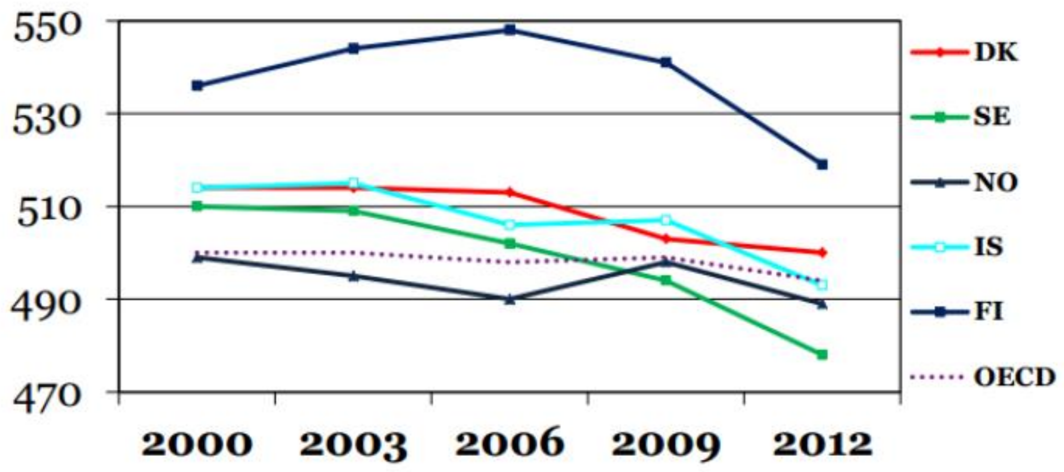
# Maybe it is TopDown ModernMath causing a MeltDown of Swedish PISA results in spite of Increased Funding?

www.uvm.dk/~/media/UVM/Filer/Udd/Folke/PDF13/Dec/131203%20PISA%20Resultatnotat.pdf

**Figur 2. Udvikling i matematikresultaterne i nordiske lande (2000-2012).**

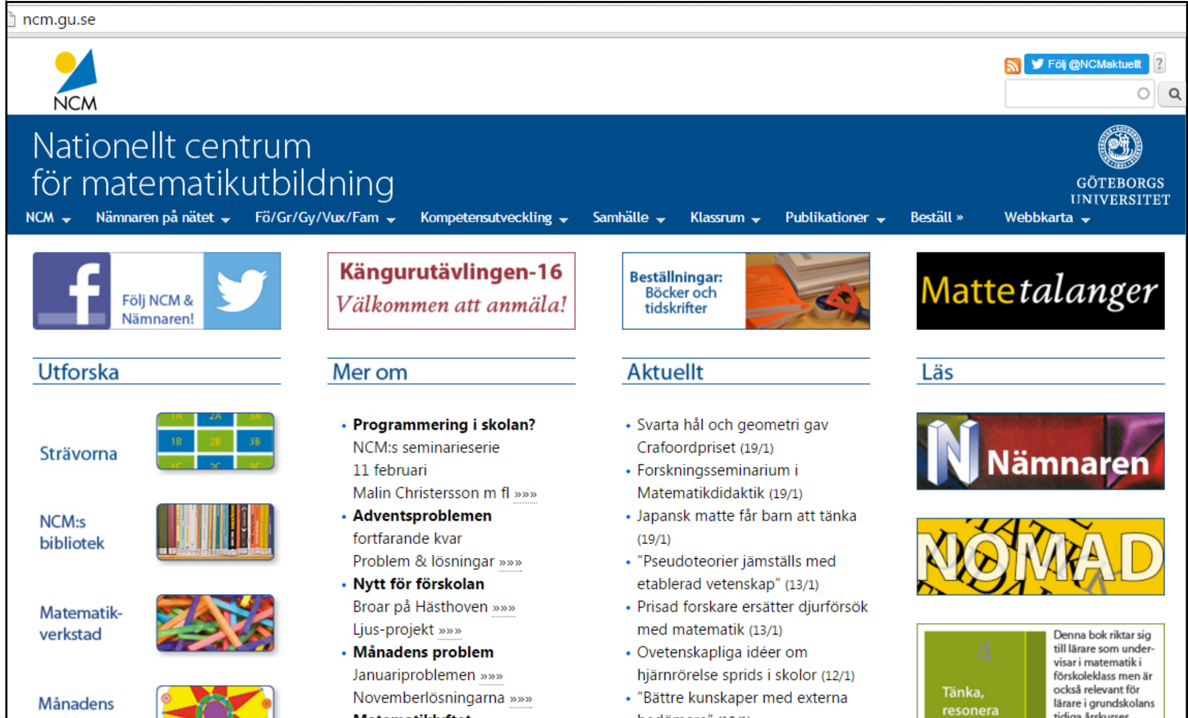



Ser man bort fra Finland (519 point), er Danmark det eneste af de nordiske lande, som er placeret i gruppen, der ligger signifikant over det internationale gennemsnit. Eleverne i Island (493 point) og i Norge (489 point) præsterer omkring gennemsnittet, mens den svenske score (478 point) er signifikant lavere end gennemsnittet. I tabel 1 nedenfor vises tallene bag figur 2.

**Tabel 1. Gennemsnit for nordiske lande 2003-2012**

| | 2003 | 2006 | 2009 | 2012 | 2012-2009 | 2012-2003 |
|---|---|---|---|---|---|---|
| Finland | 544 | 548 | 541 | 519 | -22 | -25 |
| **Danmark** | 514 | 513 | 503 | 500 | -3 | -14 |
| Island | 515 | 506 | 507 | 493 | -14 | -22 |
| Norge | 495 | 490 | 498 | 489 | -9 | -6 |
| Sverige | 509 | 502 | 494 | 478 | -16 | -31 |
| OECD | 500 | 498 | 499 | 494 | -5 | -6 |

All melt down, but as to the OECD average,
Finland & Denmark are significantly above,
Iceland & Norway are on level,
only Sweden is significantly below



# Schools Exclude 1 of 4 Socially

"PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (page 3)


http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm

# A Need for Urgent Reforms and Change

*Part I: A school system in need of urgent change. (p. 11)*

“Sweden has the highest percentage of students arriving late for school among all OECD countries, especially among socio-economically disadvantaged and immigrant students, and the lack of punctuality has increased between 2003 and 2012. There is also a higher-than-average percentage of students in Sweden who skip classes, in particular among disadvantaged and immigrant students. Arriving late for school and skipping classes are associated highly negatively with mathematics performance in PISA and can have serious adverse effects on the lives of young people, as they can cut into school learning and also distract other students.” (p. 69)

(Note: Male immigrants make Sweden beat China with 123 boys/100 girls of the 16-17 years old)

# Serious Situation and Serious Deterioration

"If serious shortcomings are identified in a school, the Schools Inspectorate can determine that the deficient school should be closed for up to six months until the deficiencies are corrected. However, this is very much a last resort and has rarely been applied." (p. 51)

"The reforms of recent years are important, but evidence suggests they are also somewhat piecemeal, and simply too few, considering the serious situation of the Swedish school system." (p. 55)

"Sweden faces a serious deterioration in the quality and status of the teaching profession that requires immediate system-wide attention. This can only be accomplished by building capacity for teaching and learning through a long-term human resource strategy for the school sector." (p. 112)

# Let's help Sweden Improve Math Education

To find a **cure**, we need a research method.

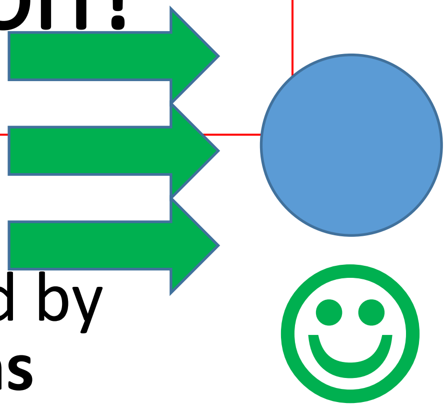
One is inspired by the ancient Greek Sophist warning:

"Know **nature** from **choice** - to avoid being patronized by choice presented as nature".

**PostModern:** Skeptical towards nature-claims. To unmask false nature, simply **discover** hidden alternatives to choice presented as nature.

PostModern Discovery Research, Contingency Research, or Cinderella Research: The **cure** for the Prince's broken heart was outside the consensus.

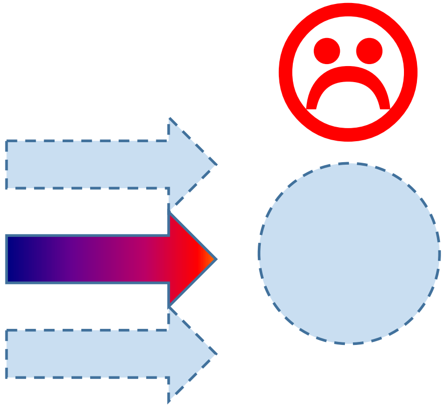
# A Goal/Means Exchange in Math Education?



Use Occam's Razor principle: First look for a simple explanation.

An educational subject always has an outside GOAL to be reached by several inside MEANS. But, if seen as mandatory, an INSIDE **means becomes a goal hiding its alternatives**, thus becoming false nature keeping learners from reaching the original OUTSIDE goal.

So, if neglecting its outside goal, **Mastering Many**, Mathematics Education becomes an undiagnosed 'cure', forced upon 'patients', showing a natural resistance against an unwanted and unneeded 'treatment'.



Thus, to explain the meltdown in Swedish PISA results we ask:

*Is there a **Goal/Means Exchange** in (Swedish) Math Education?*

# Defining MatheMatics

According to Freudenthal, the Pythagoreans used the Greek word for knowledge, mathematics, as a common label for their 4 knowledge areas: *astronomy* and *music* and *geometry* and *arithmetic*.

With astronomy and music as independent subjects, today only the two other activities remain, both rooted in the physical fact **Many**:

- **Geometry**, meaning to measure earth in Greek
- **Algebra**, meaning to reunite numbers in Arabic

Then **SET** created ModernMath, as an independent, self-rooted subject.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht-Holland: D. Reidel Publ. Comp.

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# An Observation:
# Five Questions to be Answered (please discuss)

| *This is true* | Always | Never | Sometimes |
|---|---|---|---|
| **2 + 3 = 5** | | | |
| **2 x 3 = 6** | | | |
| $\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$ | | | |
| $\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$ | | | |

| | |
|---|---|
| *C1:* a **FUNCTION** is | an example of a set relation where first component identity gives second component identity |
| *C2:* - *or both* | for example 2+x, but not 2+3 i.e. a name for a calculation witn an unspecified number |

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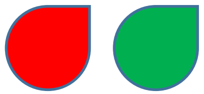
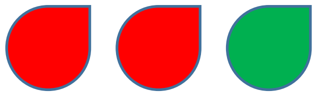
# Five Questions Answered

| *This is true* | Always | Never | Sometimes |
|---|---|---|---|
| **2 + 3 = 5** | 2weeks + 3days = 17days; only with the same unit | | **x** |
| **2 x 3 = 6** | **x** | 2x3 is 2 **3s** **III III** that can always be recounted as 6 **1s** | |
| $\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$ |  1 red of 2 apples + 2 of 3 apples is 3 of 5 apples, and not 7 of 6 |  | **x** |
| $\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$ | | | **x** Only if taken of the same total |
| *C1:* a **FUNCTION** is | an example of a set relation where first component identity implies second component identity | | (after **SET**, 1900) |
| *C2:* *- or both* | for example 2+x, but not 2+3 i.e. a name for a calculation witn an unspecified number | | (before **SET**, 1750-1900) |

# Based upon these observation we define:
# MetaMatism = MetaMatics + MatheMatism

**Meta-Matics** is defining a concept, not as a ~~BottomUp abstraction from many examples~~ but as a TopDown example of an abstraction, derived from the meta-physical abstraction **SET**, made meaningless by self-reference as shown by Russell's version of the liar paradox: If M does, it does not, belong to the set of sets not belonging to itself (and vice versa).

With $\mathbf{M} = \{\mathbf{A} \mid \mathbf{A} \notin \mathbf{A}\}$ : $\mathbf{M} \in \mathbf{M} \Leftrightarrow \mathbf{M} \notin \mathbf{M}$

**Mathe-Matism** is a statement that is correct inside, but seldom outside a classroom , as e.g. adding numbers without units as 2+3 = 5, where e.g. 2**w**+3**d**=17**d**. In contrast to 2x3 = 6 saying that 2 **3s** can be recounted as 6 **1s**.

# ModernMath teaches MetaMatism from day one

**MetaMatics**: Cardinality is linear. Each point has a number-name to be learned by heart. Counting ”twenty-nine, twenty-ten” diagnoses you with DisCalculia excluding you from class to be cured by specialists.

**MatheMatism**: Numbers are added without units. And units must not be introduced to help students with problems in multiplication or division.

Repeat: 2+3 IS 5

# Yes, Math Ed has a Goal/Means Exchange

As a common label for its two activities, Geometry & Algebra, math has two outside goals: to measure Earth and to reunite Many.

Transformed to self-referring TopDown MetaMatism, it became its own goal blocking the way to the outside goals, reduced to applications of mathematics to be taught, 'of course', after mathematics itself has been taught and learned.

So, to reach the outside goal, **mastering of Many**, we must look for a different alternative way, a **ManyMath**, built as a BottomUp Grounded Theory, a Natural Science, about the physical fact Many.

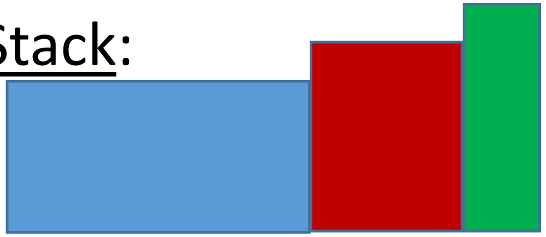
2

# ManyMath, created to Master Many, and respecting the Child's own NumberLanguage

To tell nature from choice, we ask: How will math look if grounded as a Natural Science about the physical fact Many, i.e. as a <u>ManyMath?</u>

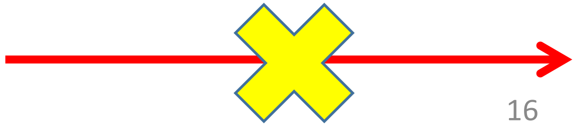
- Take 1: To master Many, we math! *Oops, math is a label, not an action word.*
- Take 2: To master Many, we act. Asking 'How Many?', we <u>Bundle</u> & <u>Stack</u>:

456 = 4 x BundleBundle + 5 x Bundle + 6 x 1 = three stacks of bundles.



All numbers have units - as recognized by children when showing 4 fingers held together 2 by 2 makes a 3-year-old child say: 'No, that is not 4, that is 2 **2s**.'

*So natural numbers are 2D blocks - not a 1D Cardinality-line.*

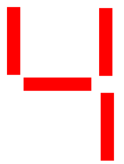


# Counting Sequences

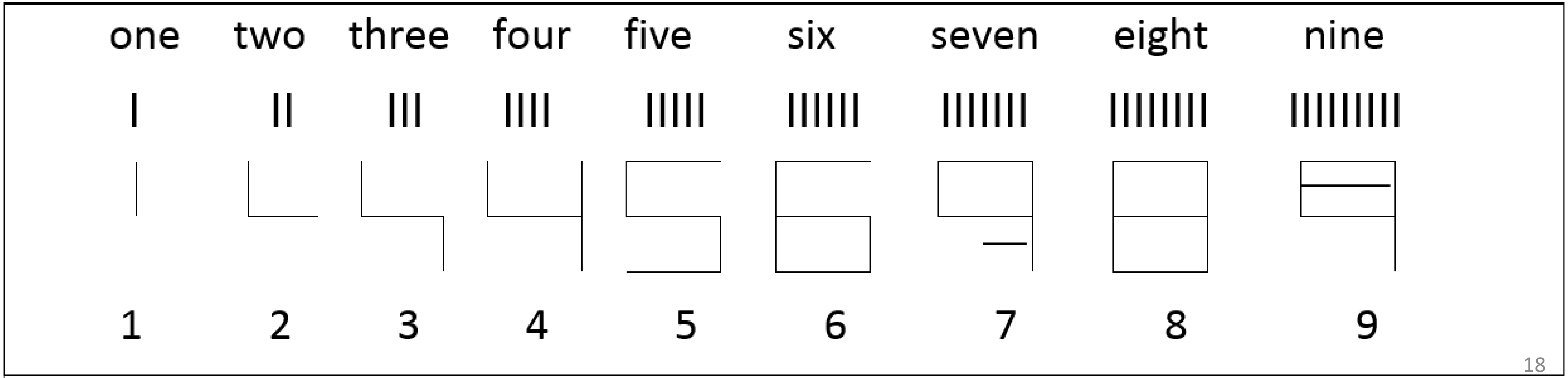
Being counted as 1B, the Bundle number needs no icon. So counting a dozen we say:

| | I | I | I | I | I | I | I | I | I | I | I | I |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **4s** | 1 | 2 | 3 | **B** | 1B1 | 1B2 | 1B3 | 2B | 2B1 | 2B2 | 2B3 | 3B |
| **7s** | 1 | 2 | 3 | 4 | 5 | 6 | **B** | 1B1 | 1B2 | 1B3 | 1B4 | 1B5 |
| **tens** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | **B** | 1B1 | 1B2 |

As to number names, eleven and twelve come from 'one left' and 'two left' in Danish, (en / twe levnet), again showing that counting takes place by taking away bundles.

# 1. Creating Icons: I I I I → IIII →  → 4

Counting in ones means naming the different degrees of Many. Counting in icons means changing four ones to one fours rearranged as a 4-icon with four sticks or strokes. So an icon contains as many strokes as it represents - if written less sloppy.



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# 2. CupCounting in Icons: 9 = ? **4s**

9 = I I I I I I I I I = IIII IIII I = **II**)I = 2)1 **4s** = 2 Bundles & 1 **4s**

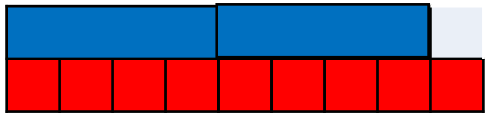
To count, we bundle & use a bundle-cup with 1 stick per bundle.

We report with **cup-writing** 2)1 **4s** or **decimal-writing** 2.1 **4s**

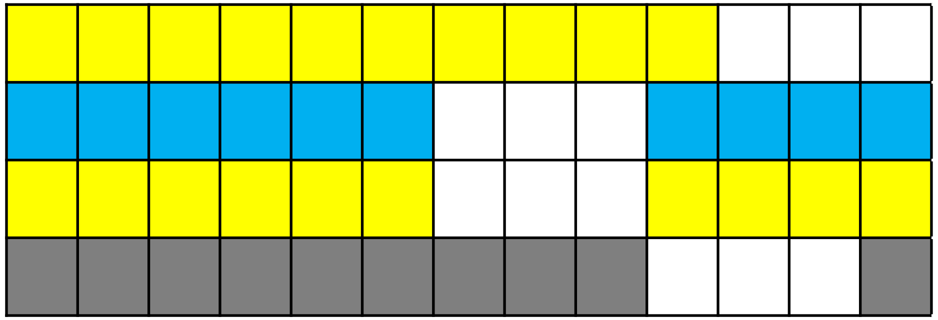
where the decimal point separates the bundles from the singles.

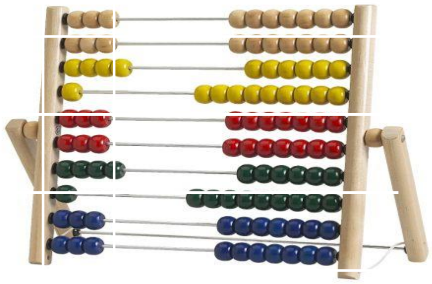
Shown on a western **ABACUS** in

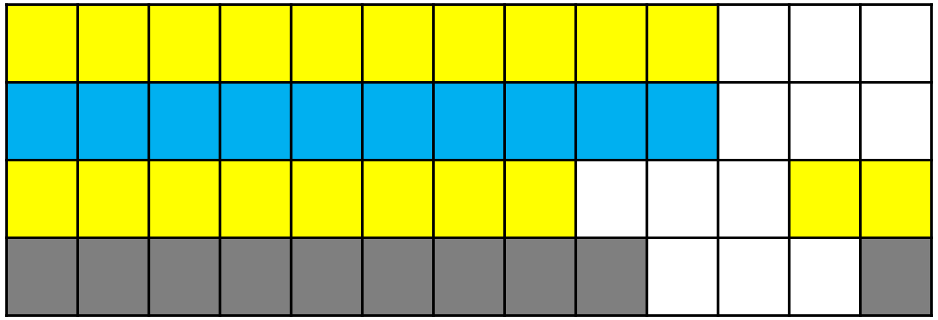
LEGO blocks:



**G**eometry/space mode or **A**lgebra/time mode







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# Counting creates Division & Multiplication & Subtraction - also as Icons

‘From 9 take away **4s**’ we write 9/4
iconizing the sweeping away by a broom, called division.

‘2 times stack **4s**’ we write 2x4
iconizing the stacking up by a lift called multiplication.

‘From 9 take away 2 **4s**’ to look for un-bundled we write 9 – 2x4
iconizing the dragging away by a trace called subtraction.

So counting includes division and multiplication and subtraction:

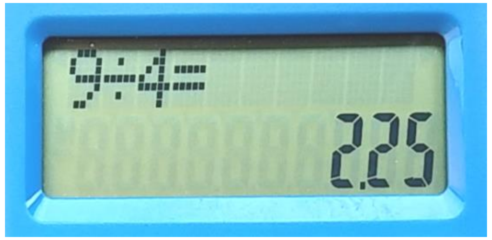
Finding the bundles: 9 = 9/4 **4s**. Finding the un-bundled: 9 – 2x4 = 1.

# Counting creates Two Counting Formulas

| | |
|---|---|
| *ReCount*<br>**T = (T/b) x b** | from a total T, **T/b** times,<br>bs is taken away and stacked |
| *ReStack*<br>**T = (T–b) + b** | from a total T, **T–b** is left when<br>b is taken away and placed next-to |

With formulas, a calculator can **predict** the counting result 9 = 2)1 **4s**

| | |
|---|---|
| **9/4** | **2.some** |
| **9 – 2x4** | **1** |



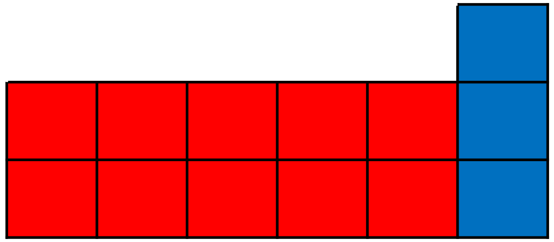


As the Sentences of the NumberLanguage, **Formulas Predict**

# The UnBundled become Decimals or Fractions
# 0.3 **5s** or 3/5

When counting by bundling and stacking, the unbundled singles can be placed

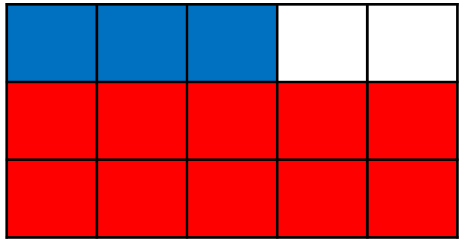
**NextTo** the stack counted as a stack of **1s**



T = 2)3 **5s** = 2.3 **5s**

*A decimal number*

**OnTop** of the stack counted as a bundle



T = 2 3/5 **5s**

*A fraction*

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# 3. ReCounting in the Same Unit creates Overload & Underload (Negative Numbers)

T = 3)0 2s
= 2)2 2s
= 4)-2 2s

ReCounting 3 **2s** in **2s**:

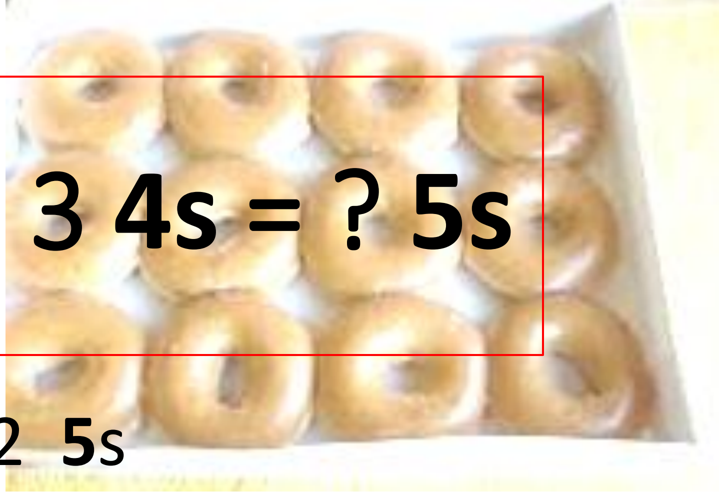
| Sticks | Calculator | | Cup-writing | | 3 ways |
|---|---|---|---|---|---|
| Ⅱ Ⅱ Ⅱ | | | 3) 0 **2s** | | Normal |
| Ⅱ Ⅱ I I | 3x2 – 2x2 | 2 | 2) 2 **2s** | | Overload |
| Ⅱ Ⅱ Ⅱ Ⅱ | 3x2 – 4x2 | -2 | 4) -2 **2s** | *4)-2 = 4 less 2* | Underload |

So a total can be ReCounted in 3 ways: Normal, Overload or Underload.

Or as a **2digit** Number if using Bundles of Bundles:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| I I I I I I | = | Ⅱ Ⅱ Ⅱ | = | Ⅱ Ⅱ | Ⅱ | | | |
| 6 | = | 3 **B** | = | 1 **BB** | 1 **B** | | | |
| 6 | = | 3)0 **2s** | = | 1) | 1)0 **2s** | = | 11)0 **2s** | |

# 4. ReCounting in a Different Unit 3 4s = ? 5s

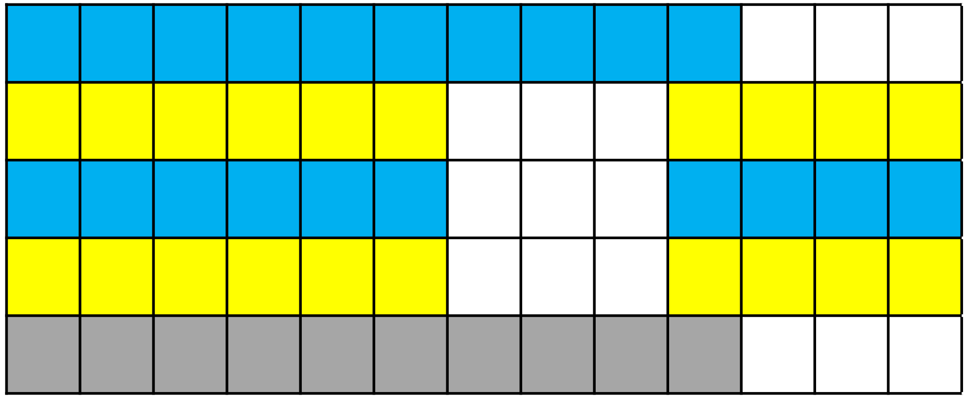


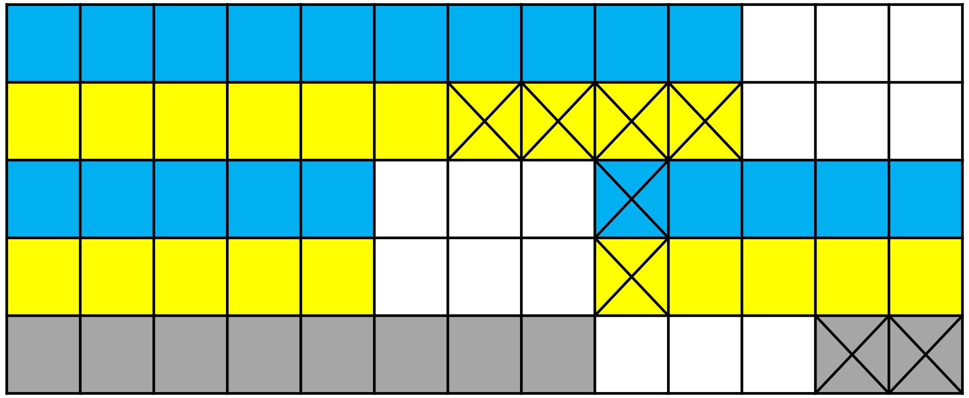
3 **4**s = IIII IIII IIII = I I I I I I I I I I I I = IIIII IIIII II = 2)2 **5**s

CALCULATOR-prediction:

| | |
|---|---|
| **3x4/5** | **2.**some |
| **3x4 – 2x5** | **2** |

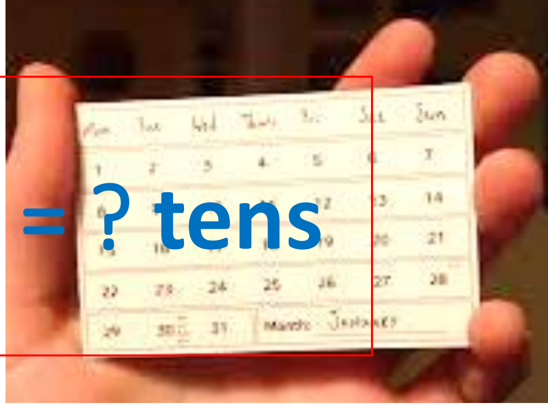
Abacus in Geometry mode





**Change Unit = Proportionality, Core Math**

# 5. ReCounting in Tens (Multiplication) 3 7s = ? tens



3 **7**s = IIIIIII IIIIIII IIIIIII = IIIIIIIIII I IIIIIIIIII = 2)1 **tens**

CALCULATOR-prediction: The calculator has no ten icon.

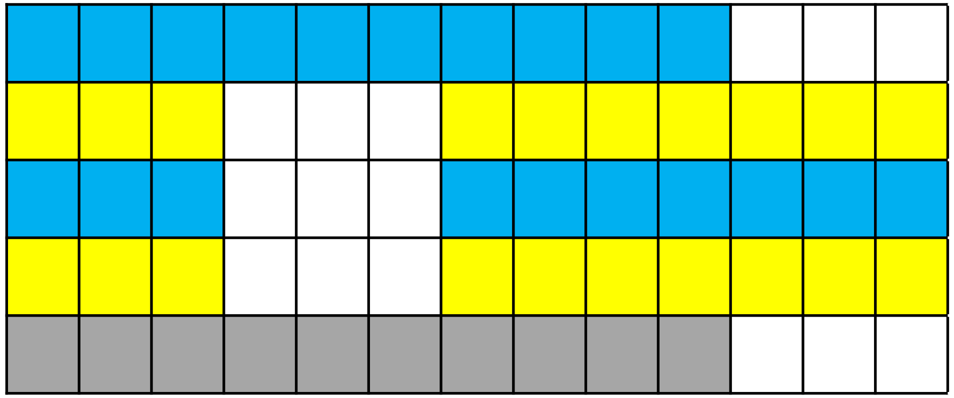
The calculator gives the answer directly

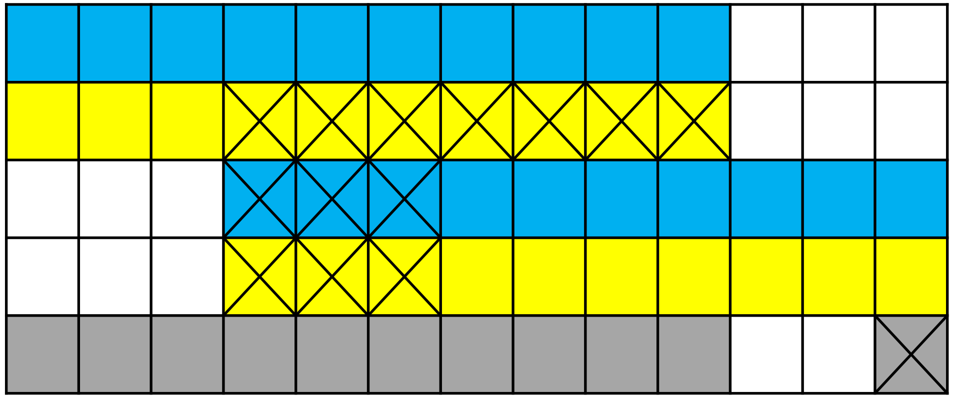
- but without unit and with misplaced decimal point

A Natural Number ???

| 3x7 | 21 |
|---|---|

Abacus in Geometry mode





So T = 21 = 2.1 tens is not a **1D length** on a number line, it is a **2D block** of tens

# 6. ReCounting from Tens (Division) 29 = ? 6s

29 = ? **6s** = IIIIIIIIII IIIIIIIIII IIIIIIIII = IIIIII IIIIII IIIIII IIIIII IIIII = 4)5 **6s**

CALCULATOR-prediction:

| | |
|---|---|
| **29/6** | **4.**some |
| **29 – 4x6** | **5** |

Reversed calculation (Equation): ? x 6 = 29 = (29/6) x 6, so ? = 29/6 = 4)5

OppoSIte SIde & SIgn method: if **u x 6 = 29** then **u = 29/6**

Abacus in Geometry mode

**ReCounting from tens = Division = Solving an Equation = Core Math**

# ReCounting large Numbers in or from Tens
## *Same number-area, but New form*

Recounting 6 **47s** Recounting 476 in **7s**

*Using CupWriting to seprate* ***INSIDE*** *bundles from* ***OUTSIDE*** *1s*

| | |
|---|---|
| T = 6 x 47 = 6 x 4) 7 | T = 476 = 47) 6 |
| = 24)42 | = 42) 56 |
| = 28) 2 | = 6x7) 8x7 |
| = 282 | = 68 x 7 |

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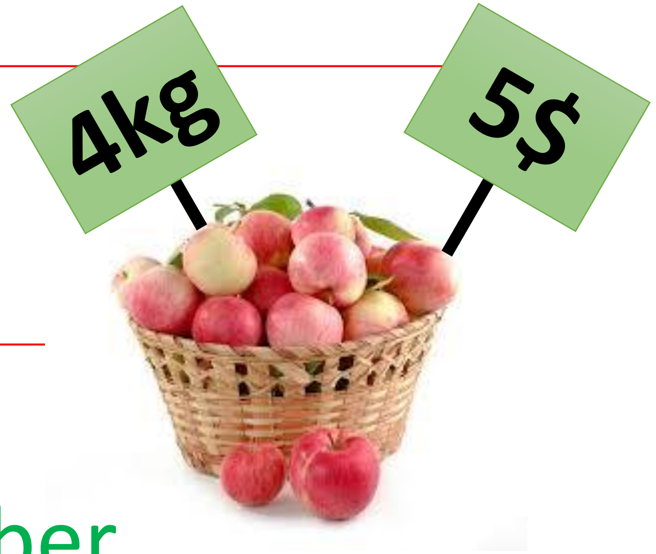
# 7. DoubleCounting creates **PerNumbers** creating Fractions & Proportionality




With **4kg = 5$** we have

4kg per 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber

*4$/100$ = 4/100 = 4%*

**Questions**:

| **7kg = ?$** | **8$ = ?kg** |
|---|---|
| 7kg = (7/4)*4kg<br>= (7/4)*5$ = 8.75$ | 8$ = (8/5)*5$<br>= (8/5)*4kg = 6.4kg |

**Answer**: *Recount in the PerNumber* *(RegulaDeTri)*

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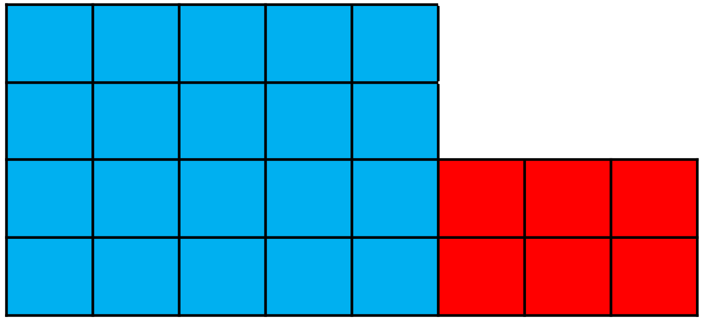
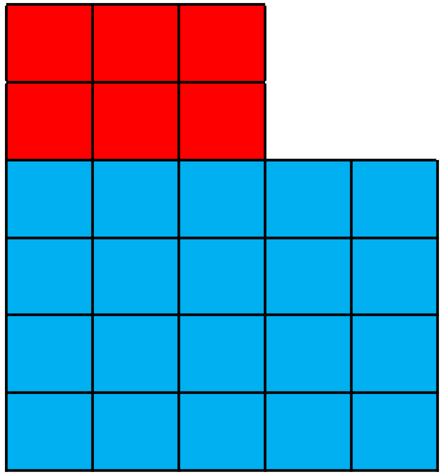
# 8. Once Counted & ReCounted, Totals are Added, BUT NextTo or OnTop?

| NextTo → | OnTop ↑ |
|---|---|
| 4 5s + 2 3s = 3)2 8s | 4 5s + 2 3s = 4 5s + 1)1 5s = 5)1 5s |
| The areas are integrated<br>*Integrate areas = Integration* | The units are changed to be the same<br>*Change unit = Proportionality* |

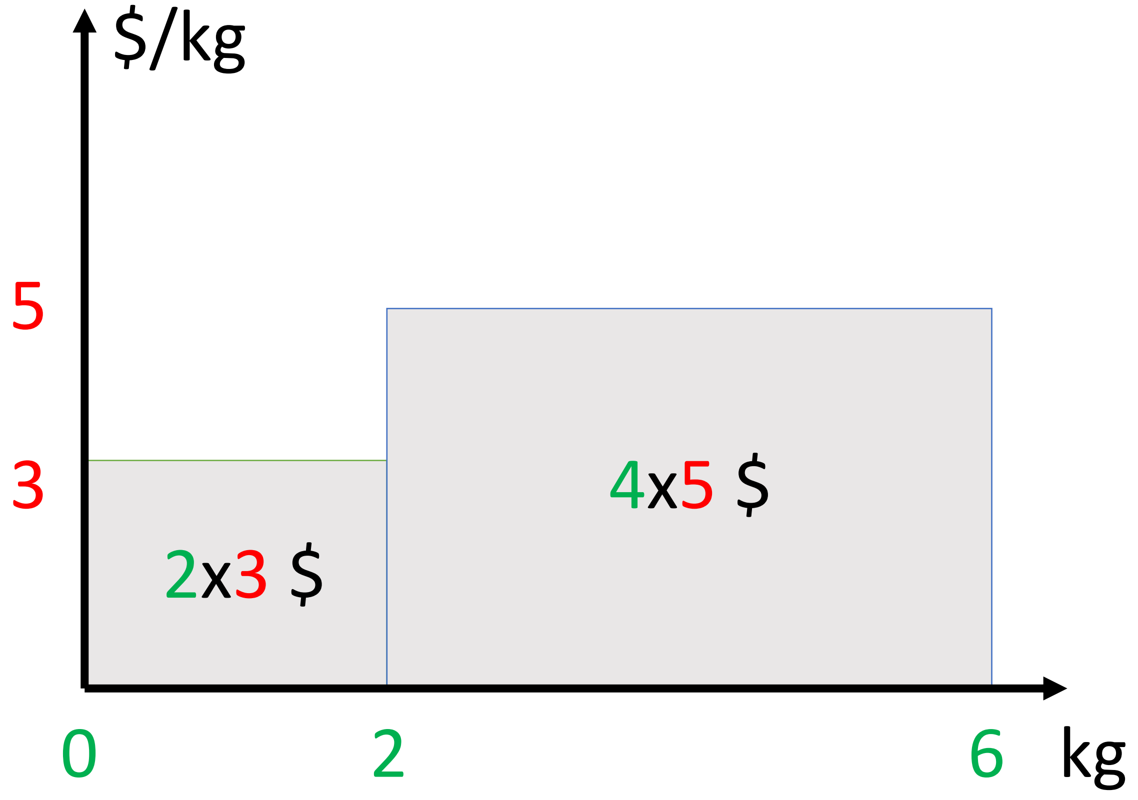




# 9. Adding PerNumbers as Areas (Integration)

2 kg at 3 $/kg
\+ 4 kg at 5 $/kg

$$(2+4)\text{kg at } \frac{2\times3 + 4\times5}{2+4} \text{ \$/kg}$$

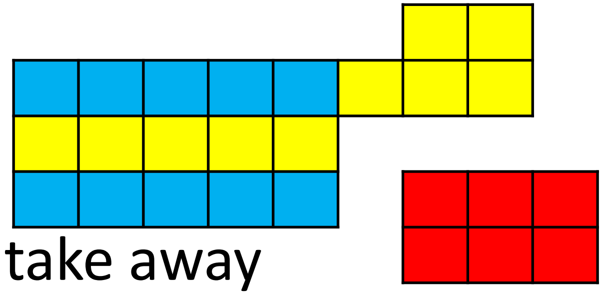



Unit-numbers add on-top.

Per-numbers add next-to as **areas** under the per-number graph, i.e. as **integration**.

# 10. Reversing Addition, or Solving Equations

| **OppoSIte SIde with OppoSIte SIgn** | | **NextTo** |
|---|---|---|
| **2 x ? = 8** = (8/2) x 2 | **2 + ? = 8** = (8-2) + 2 | **2 3s + ? 5s = 3.2 8s** |
| ? = 8/2<br>*Solved by ReCounting* | ? = 8-2<br>*Solved by ReStacking* | ? = (3.2 8s – 2 3s)/5<br>*Solved by differentiation: (T-T1)/5 = ΔT/5* |



**<u>Hymn to Equations</u>**

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

# Geometry: Measuring Earth in HalfBlocks

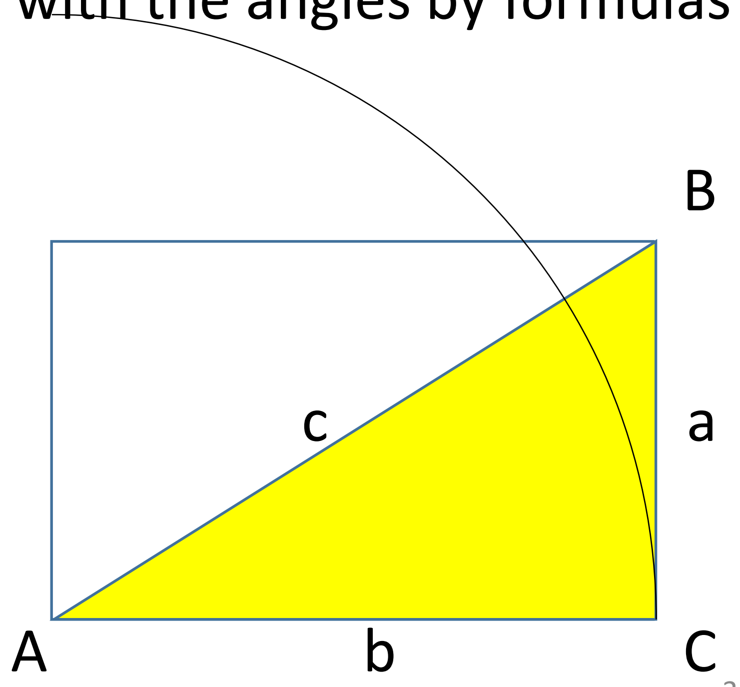
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras theorem. And connected with the angles by formulas recounting the sides in sides or in the diagonal:

A+B+C = 180

a*a + b*b = c*c

tanA = a/b = Δy/Δx = gradient; sinA = a/c; cosA = b/c

Circle: circum./diam. = **π = n*tan(180/n)** for n large

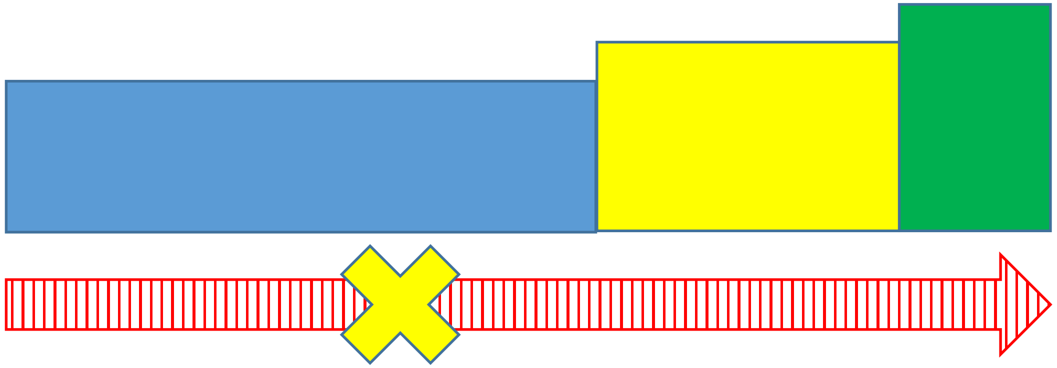


# Defining ManyMath: To master Many, we Recount in Blocks that add NextTo or OnTop

In ManyMath,

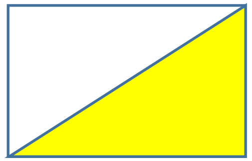
- Numbers are 2D blocks

- not on a 1D line



- Algebra: to (re)unite blocks next-to or on-top

- Geometry: to measure half-blocks



# Is ManyMath Different from ModernMath

3

| *Same Question* | **ManyMath** | **ModernMath** |
|---|---|---|
| **Digits** | Icons, different from letters | Symbols like letters |
| **Natural numbers** | T = 2.3 **tens** | 23 |
| **Order of operations** | / x - + | + - x / |
| **Operations** | Icons for counting the process: sweep, stack, drag & connect | Mappings from a set-product to a set |
| **Addition** | On-top and next-to | Only on-top |
| **Fractions** | Per-numbers, not numbers but operators needing a number to give a number | Rational numbers |
| **Per-numbers** | Double-counting | Not accepted |

# Same Question – Different Answers

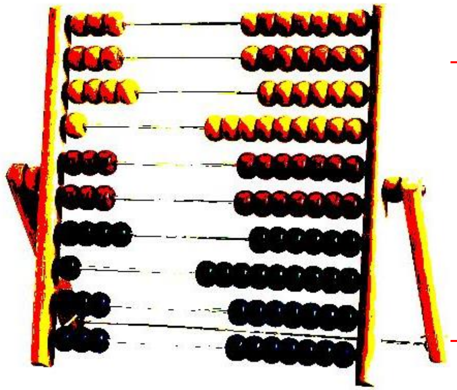
| | ManyMath | ModernMath |
|---|---|---|
| **A formula** | A calculation with numbers & letters | An example of a function |
| **A function f(x)** | A placeholder for an unspecified formula with x as unspecified number. Thus f(4) is a meaningless syntax error. | An example of a set relation where first component identity implies second component identity |
| **An equation** | A name for a reversed calculation. Solved by moving to the oppo**SI**te **SI**de with oppo**SI**te **SI**gn. | An example of an equivalence relation between two number-names solved by neutralizing using associative & commu-tative laws and abstract group theory |
| **Integration** | Preschool: Next-to addition, for all. Middle school: Adding piece-wise constant per-numbers, for all. High school: Adding locally constant per-numbers, for almost all. | Last year in high school, for the few |

# Yes, ModernMath & ManyMath are Different

| | **ManyMath** | **ModernMath** |
|---|---|---|
| **Algebra** | Re-unite constant and variable unit-numbers and per-numbers | A search for patterns |
| **The root of Mathematics** | The physical fact Many | The metaphysical invention SET |
| **A concept** | An abstraction from examples ↑ | An example of an abstraction derived from SET (MetaMatics) ↓ |
| **How true is 2+3 = 5 & 2x3 = 6** | 2x3 = 6 is true by nature since 2 **3s** can be recounted as 6 **1s**. 2+3 = 5 is true inside but seldom outside a class: 2**w**+3**d** = 17**d**, etc. | Both true by nature (MatheMatism) **MetaMatism** = MetaMatics + MatheMatism |

MATHeCDMY

# ModernMath versus ManyMath Primary School Curriculum

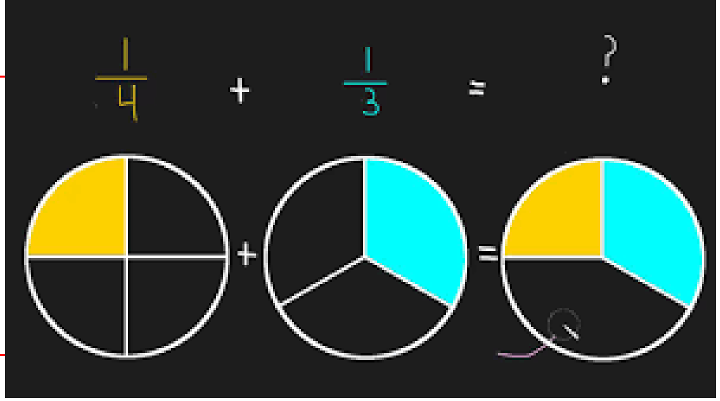


4

| ModernMath | ManyMath |
|---|---|
| 1dim. Number-line with number-names<br>Addition & Subtraction before<br>Multiplication & Division | 2dim. Number-blocks with units.<br>Multiplication & Division before<br>Subtraction & Addition |
| • One and two digit numbers<br>• Addition<br>• Subtraction<br>• Multiplication<br>• Division<br>• Simple fractions | • CupCount Many in BundleCups<br>• ReCount Many in same Unit & in new Unit (Proportionality)<br>• ReCount: In Tens & From Tens (Multiplication & Division)<br>• Calculator Prediction: RecountFormula<br>• Addition: NextTo (Integration) & OnTop<br>• Reversed addition: Equations |

MATHeCDMY

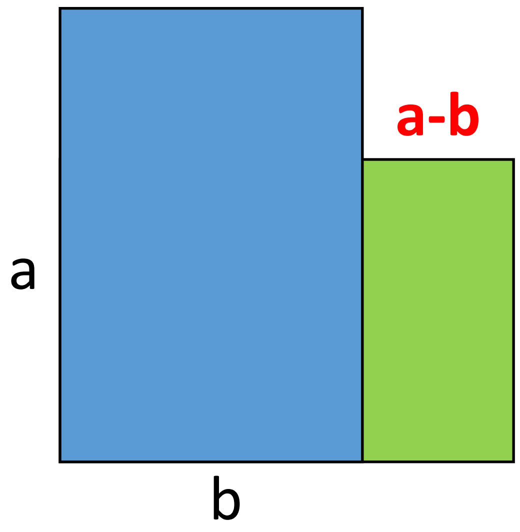
# ModernMath versus ManyMath Middle School Curriculum




| ModernMath | ManyMath |
|---|---|
| Fractions are numbers that can be added without units | Fractions are PerNumbers (operators needing a number to become a number) and added by areas (integration) |
| • Negative numbers<br>• Fractions<br>• Percentages & Decimals<br>• Proportionality<br>• LetterNumbers<br>• Algebraic fractions<br>• Solve a linear equation<br>• Solve 2 equations w. 2 unknowns | • DoubleCounting produces PerNumbers & PerFives (fractions) & PerHundreds ( %)<br>• Geometry and algebra go hand in hand when working with letter-numbers and letter-formulas; and with lines and forms<br>• The coordinate system coordinates geometry and algebra so that length can be translated to Δ-change, and vice versa |

# Geometry helps Algebra, going Hand in Hand

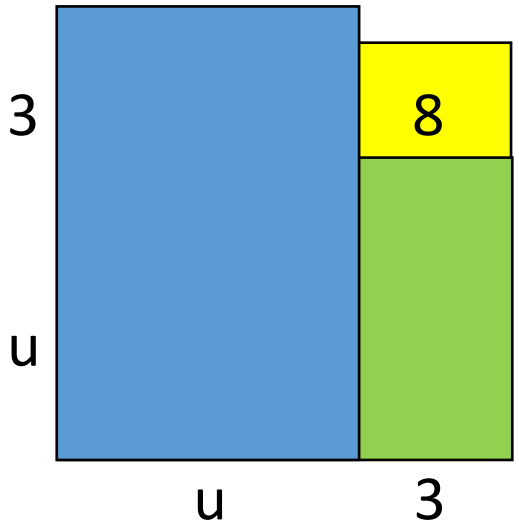
## Quadratic Rule with 2 Cards




Corner = (a-b)^2 = a^2 – 2 cards + b^2

So (a-b)^2 = a^2 – 2 x a x b + b^2

## Quadratic Equations with 3 Cards




**u^2 + 6u + 8 = 0**

(u+3)^2 = u^2 + 6u + 8 + 1

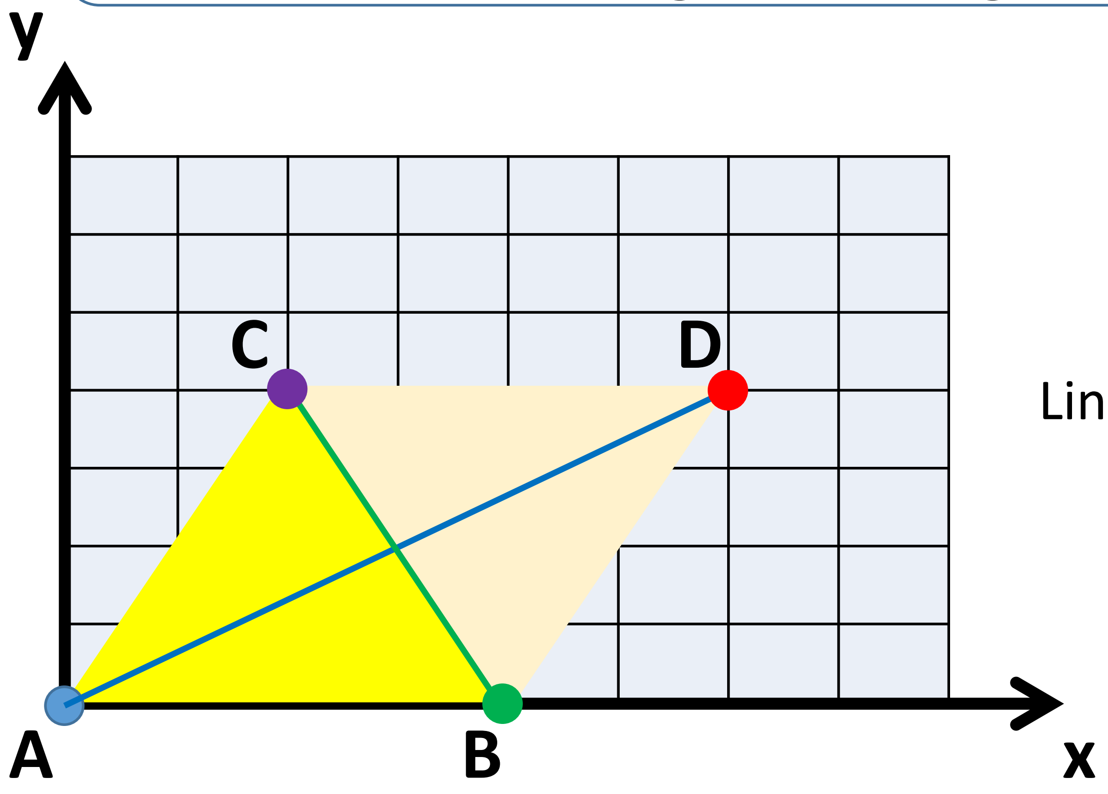
(u+3)^2 = 0 + 1

u = -3 ± 1

**u = -4 & u = -2**

# Algebra helps Geometry, going Hand in Hand

A triangle ABC with A(0,0) and B(4,0) and C(2,4) is extended to a parallelogram ABCD to the right. Find D and the intersection point between the two diagonals using both **Geometry & Algebra**.




From A to B Δx = x2-x1 = 4-0 = 4,

So, also from C to D Δx = 4; D(2+4,4) = D(6,4)

Line AD: Δy/Δx = 4/6 & Line CB: Δy/Δx = -4/2

Line AD: (y-0)/(x-0) = 4/6 & Line CB: (y-0)/(x-4) = -2

Line AD: y = 4/6*x and Line CB: y = -2*(x-4)

Intersection: **x = 3 and y = 2**

*Tested by geometrical construction*

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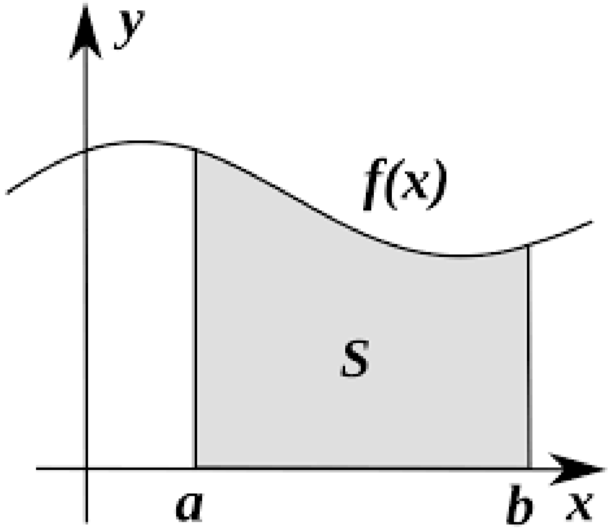
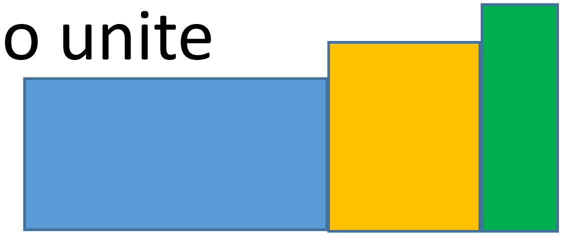
# ModernMath versus ManyMath High School Curriculum




| ModernMath | ManyMath |
|---|---|
| Functions are set-relations | Functions are formulas with two variables |
| • Squares and square roots<br>• Solve quadratic equations<br>• Linear functions<br>• Quadratic functions<br>• Exponential functions<br>• Logarithm<br>• Differential Calculus<br>• Integral Calculus<br>• Statistics & propability | • Integral Calculus as adding PerNumbers<br>• Change & Global/Piecewice/Local Constancy<br>• Root/log as finding/counting change-factors<br>• Constant change: Proportional, linear, quadratic, exponential, power<br>• Simple and compound interest<br>• Predictable Change: Integral Calculus & Differential Calculus<br>• Unpredictable Change: Stat. & prop. |

# ManyMath Includes Algebra's 4 ways to ReUnite

456 = 4 x Bundle^2 + **5 x Bundle** + 6 x 1 shows the 4 ways to unite



- Addition / *Subtraction* unites / *splits into* Variable Unit-numbers
- Multiplication / *Division* unites / *splits into* Constant Unit-numbers
- Power / *Root&Log* unites / *splits into* Constant Per-numbers
- Integration / *Differentiation* unites / *splits into* Variable Per-numbers

| **Operations unite** / *split into* | **Variable** | **Constant** |
|---|---|---|
| **Unit-numbers**<br>*m, s, $, kg* | **T = a + n**<br>*T – a = n* | **T = a x n**<br>*T/n = a* |
| **Per-numbers**<br>*m/s, $/kg, m/(100m) = %* | **T = ∫ a dn**<br>*dT/dn = a* | **T = a^n**<br>$log_a T = n$, $\sqrt[n]{T} = a$ |

# Primary, Middle & HighSchool **Core** Curriculum

To lead to its outside goal, **a NumberLanguage Mastering Many**,
a math curriculum must be based on basic Algebra, reuniting Many

| **Operations unite** *split into* | **Variable** | **Constant** |
|---|---|---|
| **Unit-numbers** *m, s, $, kg* | **T = a + n** *T – a = n* | **T = a x n** *T/n = a* |
| **Per-numbers** *m/s, $/kg, m/(100m) = %* | **T = ∫ a dn** *dT/dn = a* | **T = a^n** $\log_a T = n$, $\sqrt[n]{T} = a$ |

Primary CoreCur

Middle CoreCur

High CoreCur

# Main Points of a ManyMath Curriculum

**Primary School – respecting and developing the Child's own 2D NumberLanguage**

- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- CupCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: / x - +

**Middle school – integrating algebra and geometry, the content of the label math**

- DoubleCounting produces PerNumbers as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always so length becomes change and vv.

**High School – integrating algebra and geometry to master CHANGE**

- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

# Can Education be Different I

From secondary school, continental Europe uses **line-organized** education with forced classes and forced schedules making teenagers stay together in age groups even if girls are two years ahead in mental development.

The classroom belongs to the class. This forces teachers to change room and to teach several subjects outside their training in lower secondary school.

Tertiary education is also **line-organized** preparing for offices in the public or private sector. This makes it difficult to change line in the case of unemployment, and it forces the youth to stay in education until close to 30 making reproduction fall to 1.5 child/family so European (child)population will decrease to 25% in 100 years.

# Yes, Education can also be Different II

Alternatively, North America uses **block-organized** education saying to teenagers:

"Welcome, you carry a talent! Together we will uncover and develop your personal talent through daily lessons in self-chosen half-year blocks. If successful the school will say '**good job**, you have a **talent**, you need some more'. If not, the school will say '**good try**, you have **courage**, now try something else'".

The classroom belongs to the teacher teaching only one subject. Likewise, college is **block-organized** easy to supplement with additional blocks in the case of unemployment. At the age of 25, most students have an education, a job and a family with three children, one for mother, one for father and one for the state to secure reproduction.

*But why does Europe choose MetaMatism & lines instead of ManyMatics & blocks?*

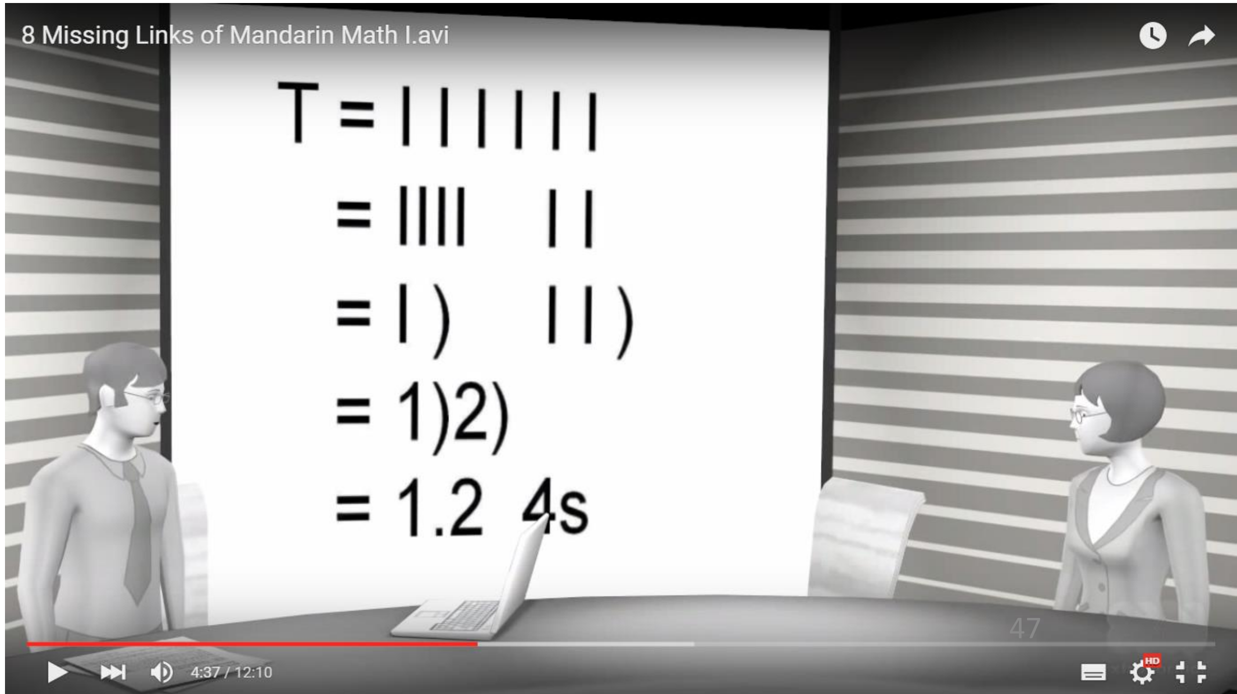
# Sociology of Mathematics & Education

According to Pierre Bourdieu, Europe has replaced a blood-nobility with a knowledge-nobility using the Chinese mandarin technique to monopolize knowledge by making education so difficult that only their children get access to the public offices in the Bildung-based administration created in Berlin in 1807 to get Napoleon out.

The Mandarins used the alphabet.
EU's knowledge-nobility uses math and lines as means to their goal.

Bourdieu, P. (1977). *Reproduction in Education, Society and Culture.* London: Sage.

*MrAlTarp: youtube.com/watch?v=sTJiQEOTpAM*



# Sociology of Mathematics & Education II

Michael Foucault: “It seems to me that the real political task in a society such as ours is to criticize the working of institutions, which appear to be neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (..)

If one fails to recognize these points of support of class power, one risks allowing them to continue to exist; and to see this class power reconstitute itself even after an apparent revolutionary process.”

*The Chomsky-Foucault Debate on Human Nature.* New York: The new Press. 2006

# Sociology of Mathematics & Education III

Inspired by Heidegger, Hannah Arendt divides human activity into labor and work both focusing on the private sphere; and action focusing on the political sphere creating institutions to be treated with care to avoid ‘the banality of evil‘ present for all employees: You must follow orders in the private & in the public sector, both obeying necessities ‘compete or die’ & ‘conform or die’. Refusing to follow orders, in the private sector you just find a competing company, in the public sector you loose your job.

*Arendt, H. (1963*). *Eichmann in Jerusalem, a Report on the Banality of Evil.* London: Penguin Books.

# Philosophy of Mathematics & Education

Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines Existentialism by saying that to existentialist thinkers

'**Existence** precedes **Essence**'.

Kierkegaard was skeptical towards institutionalized Christianity, seen also by Nietzsche as imprisoning people in moral serfdom until someone 'may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.'

The existentialist distinction between **Existence** and **Essence** allows a distinction between **outside** and **inside** goals to be made and discussed.

Marino, G. (2004). *Basic Writings of Existentialism.* New York: Modern Library.

# Psychology: The Piaget - Vygotsky Conflict

They disagree profoundly as to the importance of teaching.
Vygotsky: Teach them more, and they will learn better.
Piaget: Teach them less, instead arrange meetings with the object.

From an existentialist viewpoint, distinguishing between Existence and Essence there is a danger that a textbook reflects only essence.

Seeing the textbook as the goal, Vygotskyan theory has difficulties discussing goal/means exchanges; in opposition to Piagetian theory pointing out that too much teaching will prevent this discussion.

*http://www.azquotes.com/*

# ManyMath is Different – but does it make a Difference? Try it out.

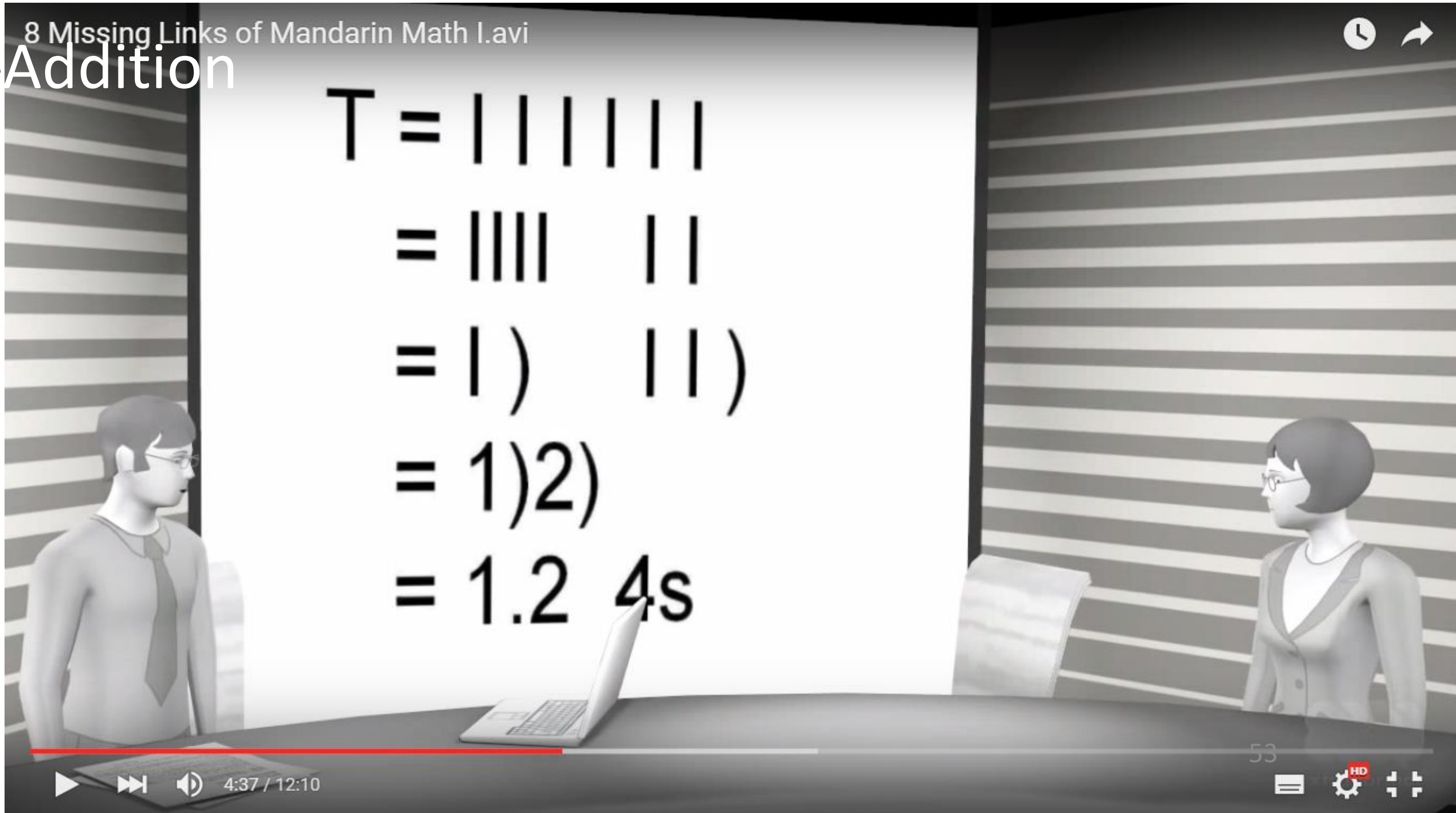


- Watch some MrAlTarp YouTube videos
- Try the **CupCount before you add** Booklet
- Try a 1day free Skype seminar **How to Cure Math Dislike**
- Try Action Learning and Action Research, e.g. **1Cup, 5Sticks**
- Collect data and Report on its 8 **MicroCurricula**, M1-M8
- Try a 1year online InService TeacherTraining at the MATHeCADEMY.net using PYRAMIDeDUCATION to teach teachers to teach MatheMatics as **ManyMath**, a Natural Science about the root of mathematics, **Many**

# Some MrAlTarp YouTube Videos

*Screens & Scripts on MATHeCADEMY.net*

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



MATHeCDMY

# CupCount 'fore you Add Booklet, free to Download

*My many Math Tears will not Stay – if I Cup the Stray Away*

## CupCount 'fore you Add

**MathDislike Cured by 1 Cup & 5 Sticks**

5 = II I I I = [cup: I] I I I = 1)3 2s

5 = II II I = [cup: II] I = 2)1 2s

5 = II II I II = [cup: III] I = 3)-1 2s

CupCount 7 in 3s: 7 = 2)1 3s = 1)4 3s = 3)-2 3s

NO, 4x7 is not 28, it is 4 7s = 2)8 = 1)18 = 3)-2 tens

NO, 30/6 is not 30 divided by 6, it is 30 counted in 6s

CupWrite to tell InSide Bundles from OutSide 1s:

- 65 + 27 = 6)5 + 2)7 = 8)12 = 9)2 = 92
- 65 – 27 = 6)5 – 2)7 = 4)-2 = 3)8 = 38
- 7x 48 = 7x 4)8 = 28)56 = 33)6 = 336
- 336 /7 = 33)6 /7 = 28)56 /7 = 4)8 = 48

**MatheMatics as ManyMath**
- a Natural Science about Many

*Makes Math Potentials Blossom in Children, Adults & Migrants*

Allan.Tarp

MATHeCADEMY.net

Contents



03. CupCounting in Icons

| Job | | Do | Calculator |
|---|---|---|---|
| 9 in 5s | Line | T = I I I I I I I I I | 9/5 1.some |
| | Count | 1, 2, 3, 4, B, 1B1, 1B2, 1B3, 1B4 | 9 – 1*5 4 |
| | Bundle | T = IIIII I I I I | |
| | Stack | [stack of 5 red and 4 white squares] | 9 – 0*5 9 |
| | Cup | T = 1)4 5s = 0)9 5s = 2)-1 5s | 9 – 2*5 -1 |
| | Answer | T = 9 = 1.4 5s | |
| 9 in 4s | Line | T = I I I I I I I I I | 9/4 2.some |
| | Count | 1, 2, 3, B, 1B1, 1B2, 1B3, 2B, 2B1 | 9 – 2*4 1 |
| | Bundle | T = IIII IIII I | |
| | Cup | T = 2)1 4s = 1)5 4s = 3)-3 4s | 9 – 1*4 5 |
| | Stack | [stack of 2×4 red squares and 1 white square] | 9 – 3*4 -3 |
| | Answer | T = 9 = 2.1 4s | |
| 9 in 3s | Line | | |
| | Count | | |
| | Bundle | | 9/ |
| | Cup | | 9 – |
| | Stack | | |
| | Answer | | |
| 8 in 4s | Line | | |
| | Count | | |
| | Bundle | | 8 |
| | Cup | | 8 |
| | Stack | | |
| | Answer | | |
| 8 in 3s | Line | | |
| | Count | | |
| | Bundle | | 8 |
| | Cup | | 8 |
| | Stack | | |
| | Answer | | |

# 1day free Skype Seminar:
# To Cure Math Dislike, CupCount before you Add

## Action Learning based on the Child's own 2D NumberLanguage

**09-11. Listen and Discuss the PowerPointPresentation**
To Cure MathDislike, replace MetaMatism with ManyMath

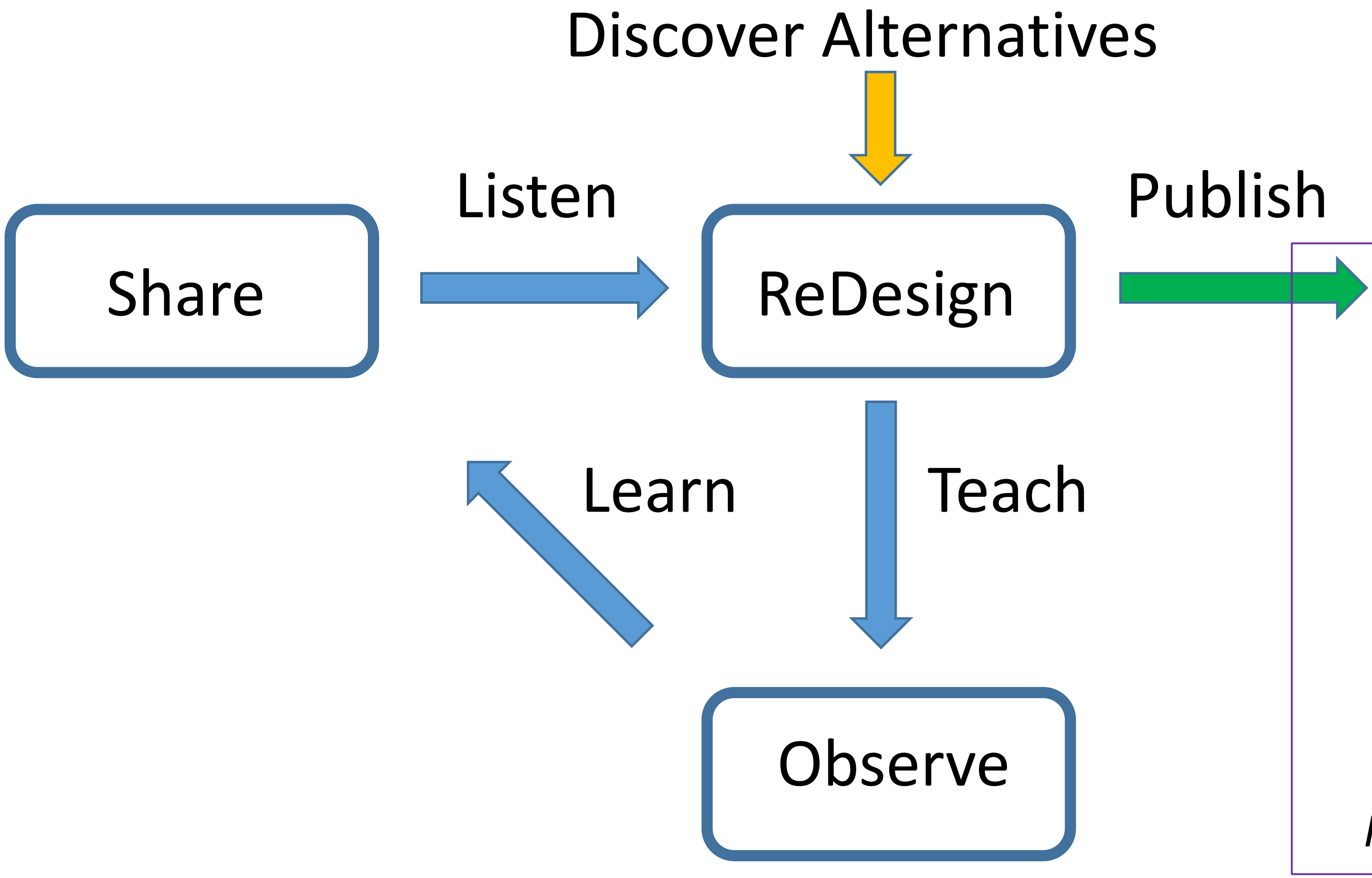
- **MetaMatism** = MetaMatics + MatheMatism
- **MetaMatics** presents a concept TopDown as an example instead of BottomUp as an abstraction
- **MatheMatism** is true inside but rarely outside classrooms
- **ManyMath**, a natural science about Many mastering Many by CupCounting & Adding NextTo and OnTop.

**11-13. Skype Conference. Lunch.**

**13-15. Do: Try out the CupCount before you Add** booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in CupCounting & NextTo Addition.

**15-16. Coffee. Skype Conference.**

# Action Learning & Action Research




**Lyotard**

**dissensus Paralogy**

Quality indicator:

Ungrounded rejection

**Example**

Calculators in PreSchool &
Special Needs education
*Paper rejected at MADIF10*

MATHeCDMY

# A Primary School Test Curriculum, before
## *Math Dislike CURED by 1 Cup & 5 Sticks*

336/7 =
? ? ?
☹ 💣 ☠

Having problems in a division class, the teacher says: “Timeout, class. Next week no division, instead we take a field trip back to day 1 to learn CupCounting”

Let’s recount 5 in 2s by bundling, using a cup for the bundles:

**5** = II I I I = [I] I I I = 1)3 **2s** = 1 Bundle & 3 **2s** *overload*

**5** = II II I = [II] I = 2)1 **2s** = 2 Bundles & 1 **2s** *normal*

**5** = II II II = [III] I = 3)-1 **2s** = 3 Bundles less 1 **2s** *underload*

Now we know that numbers can be ReCounted in 3 ways:
Normal, overload or underload if we move a stick **OUTSIDE** or **INSIDE**.

Now CupCount 7 in **3s**:

**7** = I I I I I I I = 2)1 **3s** = 1)4 **3s** = 3)-2 **3s**

MATHeCDMY

# A Primary School Test Curriculum, after
## *Math Dislike CURED by 1 Cup & 5 Sticks*

336/7
= 33)6 /7
= 28)56 /7 = 4)8
☺ ✌ 🏳

When counting in TENS, before calculating, we cup-write the number to separate the **INSIDE** bundles from the **OUTSIDE** singles. Later we recount.

- **65 + 27** = 6)5 + 2)7 = 8)12 = 9)2 = **92**
- **65 – 27** = 6)5 – 2)7 = 4)-2 = 3)8 = **38**
- **7x 48** = 7x 4)8 = 28)56 = 33)6 = **336**
- **336 /7** = 33)6 /7 = 28)56 /7 = 4)8 = **48**

*With 336 we have 33 **INSIDE**, so to get 28, so we move 5 **OUTSIDE** as 50.*
*Now try 456 / 7.*

- **456 /7** = 45)6 /7 = 42)36 /7 = 6)5 + 1 = **65 1/7**

# 8 MicroCurricula for Action Learning & Research

C1. Create Icons

C2. Count in Icons (Rational Numbers)

C3. ReCount in the Same Icon (Negative Numbers)

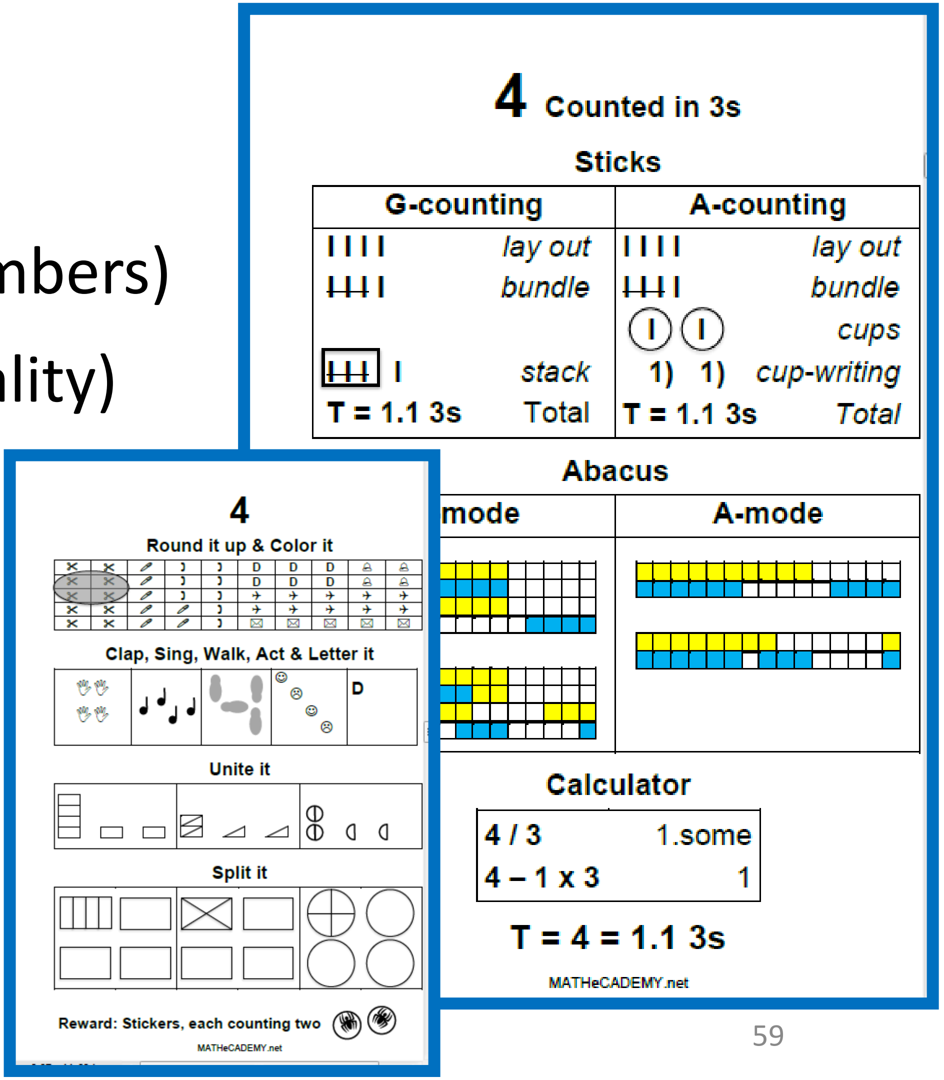
C4. ReCount in a Different Icon (Proportionality)

A1. Add OnTop (Proportionality)

A2. Add NextTo (Integrate)

A3. Reverse Adding OnTop (Solve Equations)

A4. Reverse Adding NextTo (Differentiate)



# MATHeCADEMY.net

a VIRUSeCADEMY Teaching Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY – the CATS approach: Count & Add in Time & Space



## ManyMath Cures Mathematics Dislike: CupCount before you Add

Teach **Calculus** before Addition by adding **NextTo** before OnTop

**FREE 1day SKYPE Teacher Seminar: Cure Math Dislike**

CupCount & ReCount * KopTæl & OmTæl * ICME13 Papers

**PPP: Existentialism in Math Ed * Curing Math Dislike**

Hire MrAlTarp: 2 weeks (Free), or 2 months

| I | II | III | IIII | IIIII | IIIIII | IIIIIII | IIIIIIII | IIIIIIIII |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

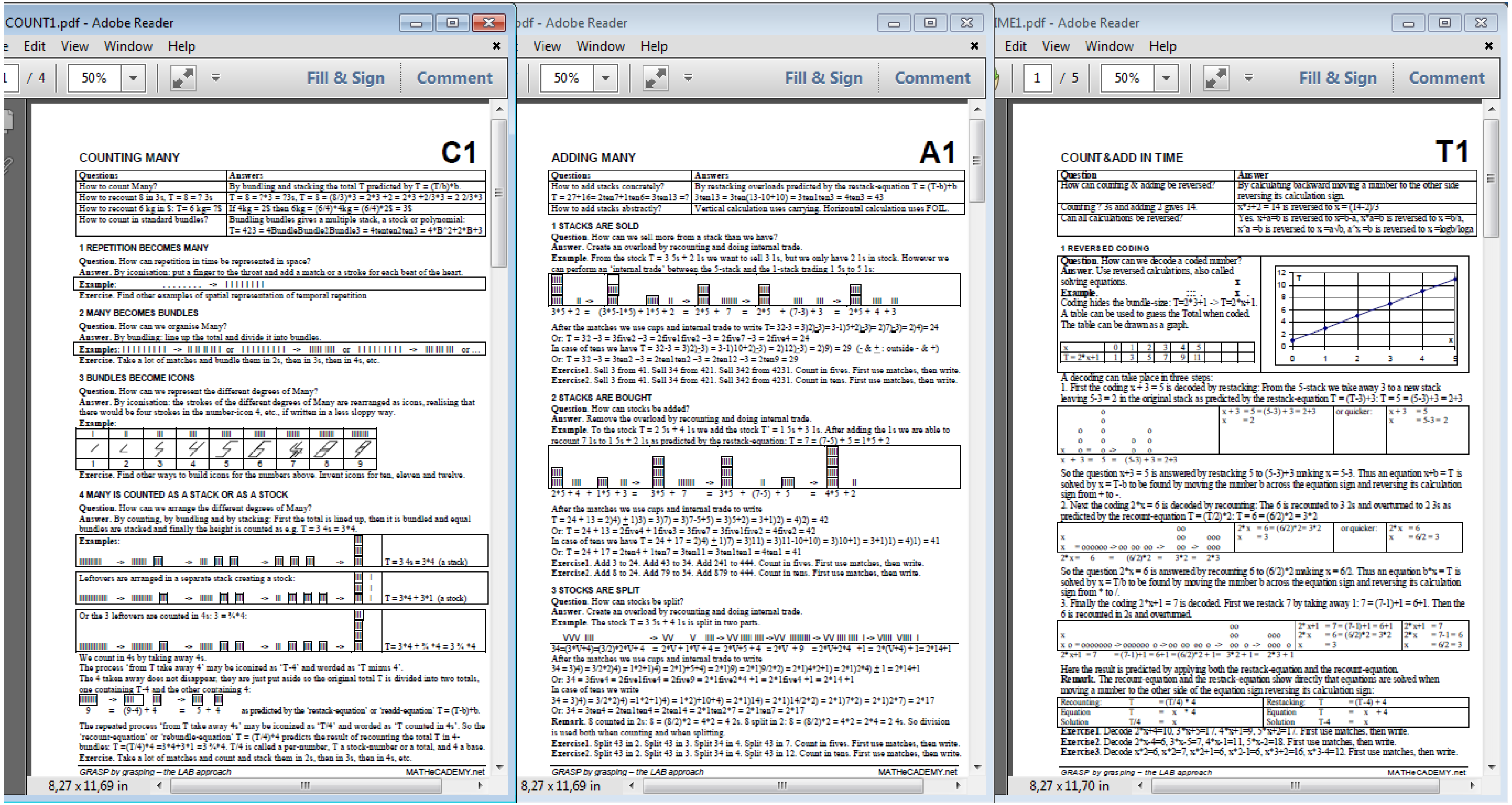
We ACT to deal with the outside world.
We MATH to deal with the natural fact MANY ???
Oops, sorry, math is not an action word!
We COUNT & ADD to deal with MANY.

- CupCount & ReCount:

T = 7 = I I I I I I I = III III I = II) I = 2)1 = 2.1 **3s**
T = 2)1 3s = 1)4 **3s** = 3)-2 **3s** (Overload or Underload)
T = 2)1 **3s** = 1)2 **5s** = 3)1 **2s** = 11)1 **2s**

Teaches Teachers to Teach MatheMatics as **ManyMath**, a Natural Science about **MANY**. The **CATS** method: To learn Math **C**ount & **A**dd in **T**ime & **S**pace

# Teacher Training in CATS ManyMath
# Count & Add in Time & Space



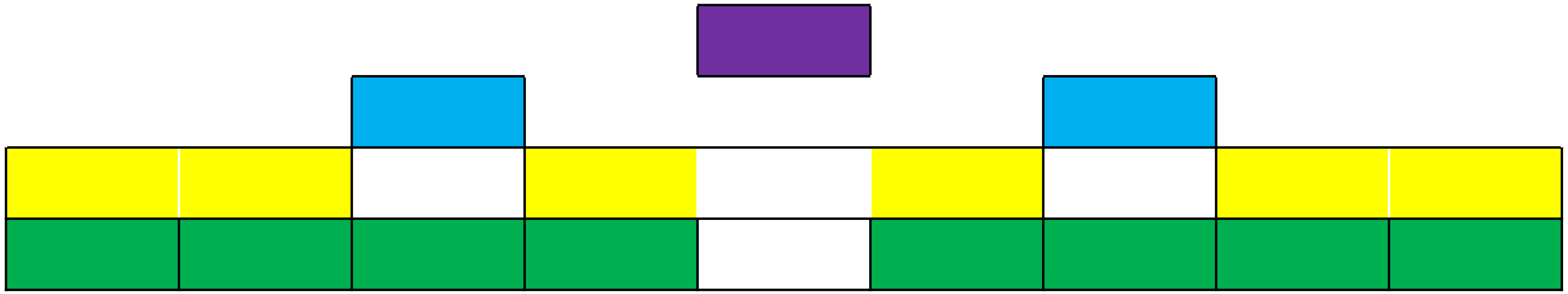
# PYRAMIDeDUCATION

To learn MATH: Count&Add MANY
*Always ask Many, not the Instructor*
MATHeCADEMY.net - a VIRUSeCADEMY

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams



# When using Theory, Beware of Disagreements

| | **TopDown** | **BottomUp** |
|---|---|---|
| **Philosophy** | Plato essentialism | Sartre existentialism |
| **Psychology** | Vygotsky<br>essence-teaching | Piaget<br>existence-meeting |
| **Sociology** | German<br>institutional idealism | French/American<br>institutional skepticism |
| **Research** | MetaPhysical<br>theory exemplification | Physical grounded<br>theory creation |

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# Main Point: Think Things - don’t Echo Essence

- No, 5x7 is not 35. It is 5 **7s**, that might be recounted as 4.3 **8s** or as 3.5 **tens**.
- No, 65/7 is not 65 split between 7. It is 6.5 **tens** recounted in **7s** which of course makes the block-number thinner and higher.
- No, 1/3 is not a number. It is an operator needing a number to become a number, e.g. 1/3 of 6.
- No, 5 is not a number. It is an operator needing a number to become a number, e.g. 5 **7s**.
- Don’t teach children 1D numbers. They already know 2D numbers.

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# Main Main Point:

## CupCount before you Add, Respect the Child's own 2D Numbers

# To **Improve** Math Education

**BEWARE** of Goal-Means Exchanges
**UNITE** its roots: Algebra & Geometry
**RESPECT & Develop** the Child's own 2D Numbers

**CupCount before you Add**
**Calculus before OnTop Addition**

**ByeBye to MetaMatism**
**Welcome to ManyMath**

**Thank You** for Your Time

Allan.Tarp@MATHeCADEMY.net
*Free 1Day Skype Teacher Seminar*
*Free Uni Franchise*

# Solving Equations BottomUp or TopDown

## ManyMath

| | |
|---|---|
| **2 + u = 6** = (6-2) + 2 | Solved by re-stacking 6 |
| u = 6-2 = 4 | Test: 2 + 4 = 6 OK |

| | |
|---|---|
| **2 x u = 6** = (6/2) x 2 | Solved by re-bundling 6 |
| u = 6/2 = 3 | Test: 2 x 3 = 6 OK |

## MatheMatics

| | | |
|---|---|---|
| | **2 + u = 6** | Addition has 0 as its neutral element, and 2 has -2 as its inverse element |
| ↕ | (2 + u) + (-2) = 6 + (-2) | Adding 2’s inverse element to both number-names |
| ↕ | (u + 2) + (-2) = 4 | Applying the commutative law to u + 2, 4 is the short number-name for 6+(-2) |
| ↕ | u + (2 + (-2)) = 4 | Applying the associative law |
| ↕ | u + 0 = 4 | Applying the definition of an inverse element |
| ↕ | u = 4 | Applying the definition of a neutral element. *With arrows a test is not needed.* |

# No ReCounting: Bye to Golden Math Opportunities

| | |
|---|---|
| No Icon Creation | So, as letters, digits are just symbols to be learned by heart |
| Only Counting in tens | T = 2.3 **tens** = 23; oops, no unit & misplaced decimal point |
| No ReCounting in the Same Icon | So 37 is no more 2)17 or 4.-3 |
| No ReCounting in a Different Icon | No more 3 x 5 is 3 **5s**, but 15, postponed to Multiplication<br>No more 24 = ? **3s.** Instead we ask 24/3, postponed to Division |
| No Adding NextTo | Postponed to Integral Calculus |
| No Reversed Adding NextTo | Postponed to Differential Calculus, made difficult by being taught before Integral Calculus |
| Only Adding OnTop | No CupWriting: 24 + 58 = 7)12. Only Carrying: $7^{1}2$ = 82<br>No CupWriting: 74 – 39 = 4)-5 = 35. Only Carrying: 74 = $6^{10}4$ |
| No Reversed Adding OnTop | Postponed to Solving Equations |

# Dienes on Place Value and MultiBase Blocks

“The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. (..)

In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..)

Educators today use the “multibase blocks”, but most of them only use the base ten, yet they call the set “multibase”. These educators miss the point of the material entirely.”

(What is a base?, http://www.zoltandienes.com/academic-articles/)

# Yes, Recounting looks like Dienes Blocks, but …

Dienes teaches the 1D place value system with 3D, 4D, etc. blocks to illustrate the importance of the power concept.

- ManyMath teaches decimal numbers with units and stays with 2D to illustrate the importance of the block concept and adding areas.

Dienes wants to bring examples of abstractions to the classroom

- ManyMath wants to build abstractions from outside examples

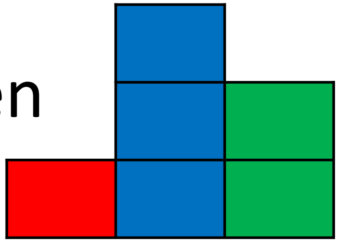
Dienes teaches top-down ‘MetaMatics‘ derived from the concept Set

- ManyMath teaches a bottom-up natural science about the physical fact Many; and sees Set as a meaningless concept because of Russell’s set-paradox.

# 1D Roman Numbers and 2D Arabic Numbers

To see the difference we write down a total T of **six scores** and **a dozen**:

- T = XX XX XX XX XX XX + XII = CXXXII ,
- T = 6  20s + 1  12s = 1*BB + 3*B + 2*1 = 132 , where Bundle = ten



Both systems use bundling to simplify.

The Roman uses a 1D juxtaposition of different bundle sizes.

The Arabic uses one bundle size only.

More bundles are described by multiplication: 3*B, i.e. as 2D areas.

Bundle-of-bundles are described by power: 1*BB = 1*B^2.

Totals are described by next-to addition of 2D area blocks (integration).

# Creating or Curing Dislike/DysCalCulia

Having problems learning mathematics has many names:

Difficulty, disability, disorder, dislike, deficiency, low attainment, low performance or DysCalCulia.

| How to Create it | How to Cure it |
|---|---|
| ● Teach 1D LineNumbers as '8'<br>● No Counting before Adding<br>● Adding before Multiplying<br>● Adding without Units: 2+3=5 | ● Teach 2D BlockNumbers as '2 **4s**'<br>● CupCounting before Adding<br>● Multiplying before Adding<br>● Adding with Units: 2**w**+3**d**=17**d** |

# Scholastic, Patronizing & Grounded Mathematics Education Research

Scholastic research hides alternatives through discourse-protection and self-reference thus presenting its **choice** as nature.

Patronizing research sees the institution as rational and the agent as irrational. Thus math education problems lies with the agents.

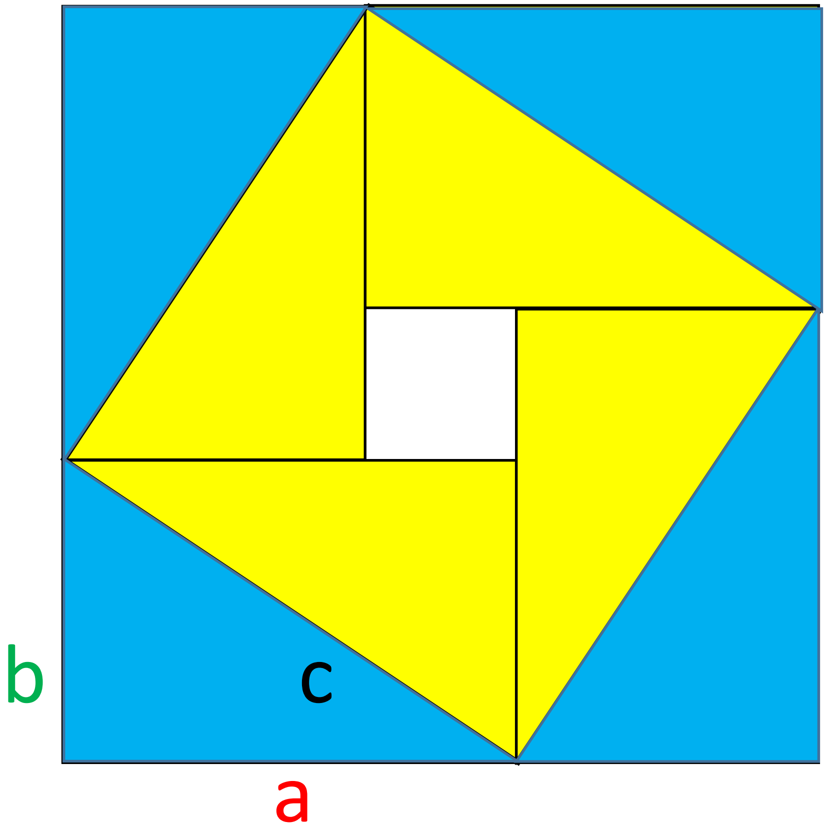
Grounded research sees the problems lying with the institutions

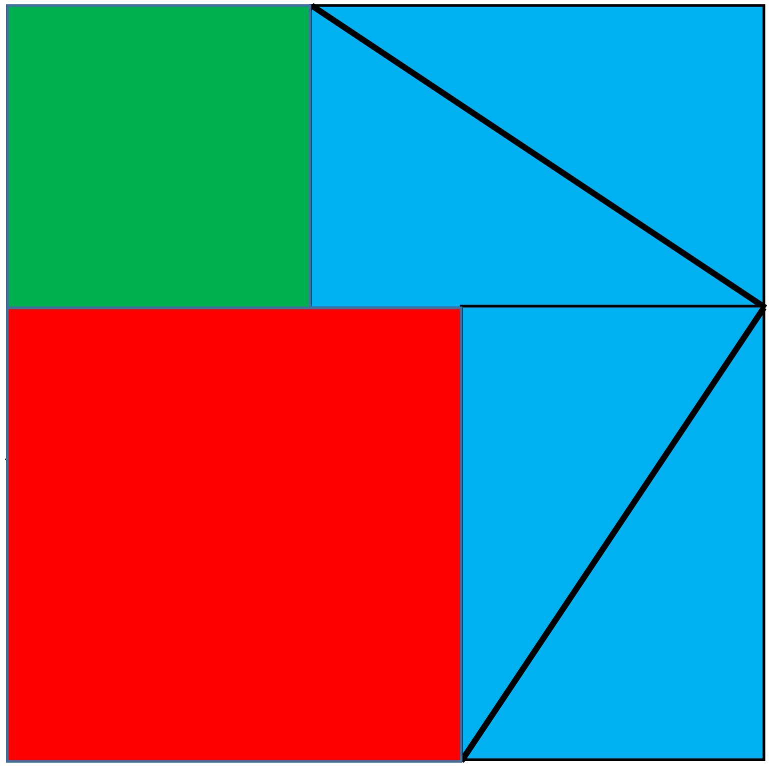
- North America: Focusing on the agents, look for hidden rationality behind apparent irrationality
- France: Focusing on the institutions, look for hidden irrationality behind apparent rationality

# MatheMatics: Unmask Yourself, Please

- In Greek you mean 'knowledge'. You were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic. Later only 2 activities remained: Geometry and Algebra
- Then self-referering Set transformed you from a Natural Science about the physical fact Many to a metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism
- So please, unmask your true identity, and tell us how you would like to be presented in education: Self-referring MetaMatism for the few - or grounded ManyMath for the many

# Pythagoras shown by 4 Cards with Diagonals



**c^2** + 4 ½cards

**a^2 + b^2** + 2 cards