

To Cure Math Dislike replace MetaMatism with ManyMath.

From a TopDown Modern to a
BottomUp PostModern Curriculum

Outlining an *Improved Curriculum* for Sweden

Curriculum Architect

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Designed as a VIRUSeCADEMY
to Teach Teachers to Teach MatheMatics as **ManyMath**
- a Natural Science about the physical fact **Many**

September 2016

Improving Schools in
Sweden:
An OECD Perspective



A Language House with two Languages

To describe the world we need 2 languages: a **Word-** and a **Number-Language**. Both are part of a two floor **Language House** that describes the world by a language - and that describes the language by a **meta-language**, a grammar.

In the WordLanguage, language comes before its **BottomUp** grammar.
 In the NumberLanguage, **Top-Down** Modern Math teaches language after grammar.
 And grammar before language means huge learning problems.

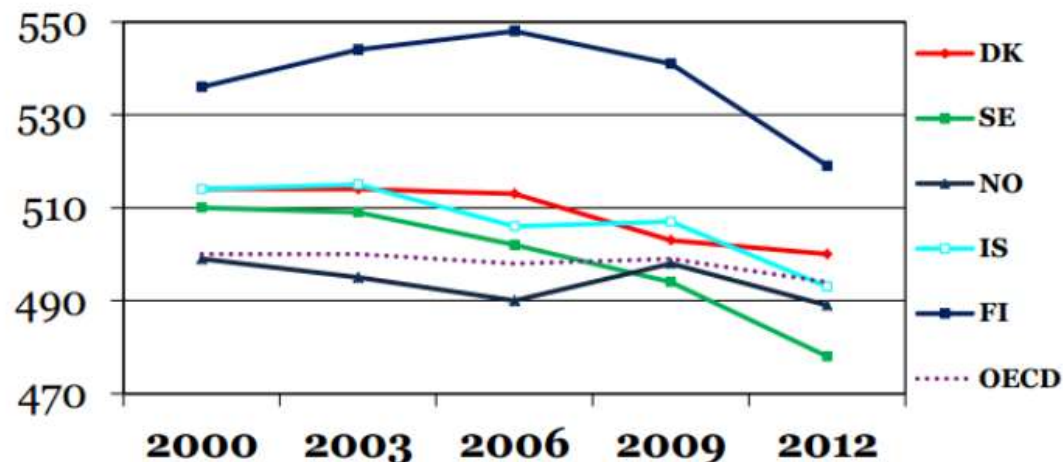
	WordLanguage	NumberLanguage
MetaLanguage	The apple is a subject	T is a function
Language	The apple is green <i>(opinion)</i>	$T = 2+3*x$ <i>(prediction)</i>
	Qualities	Quantities

The World

Maybe it is TopDown ModernMath causing a MeltDown of Swedish PISA results in spite of Increased Funding?

www.uvm.dk/~media/UVM/Filer/Udd/Folke/PDF13/Dec/131203%20PISA%20Resultatnotat.pdf

Figur 2. Udvikling i matematikresultaterne i nordiske lande (2000-2012).



Ser man bort fra Finland (519 point), er Danmark det eneste af de nordiske lande, som er placeret i gruppen, der ligger signifikant over det internationale gennemsnit. Eleverne i Island (493 point) og i Norge (489 point) præsterer omkring gennemsnittet, mens den svenske score (478 point) er signifikant lavere end gennemsnittet. I tabel 1 nedenfor vises tallene bag figur 2.

Tabel 1. Gennemsnit for nordiske lande 2003-2012

	2003	2006	2009	2012	2012-2009	2012-2003
Finland	544	548	541	519	-22	-25
Danmark	514	513	503	500	-3	-14
Island	515	506	507	493	-14	-22
Norge	495	490	498	489	-9	-6
Sverige	509	502	494	478	-16	-31
OECD	500	498	499	494	-5	-6

All melt down, but as to the OECD average, Finland & Denmark are significantly above, Iceland & Norway are on level, only Sweden is significantly below

Schools Exclude 1 of 4 Socially

“PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life.” (page 3)



<http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm>

Let's help Sweden Improve Math Education

To find a cure, we need a research method.

One is inspired by the ancient Greek Sophist warning:

“Know **nature** from **choice** - to avoid being patronized by choice presented as nature”.

PostModern: Skeptical towards nature-claims. To unmask false nature, simply **discover** hidden alternatives to choice presented as nature.

PostModern Discovery Research, Contingency Research, or Cinderella Research: The **cure** for the Prince's **broken heart** was outside the consensus.



A Goal/Means Confusion in Math Education?

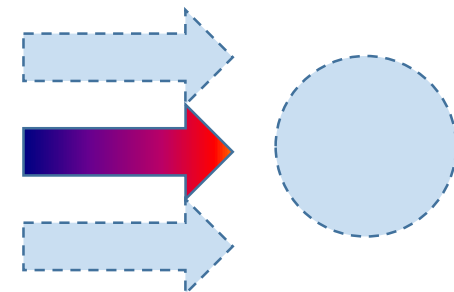
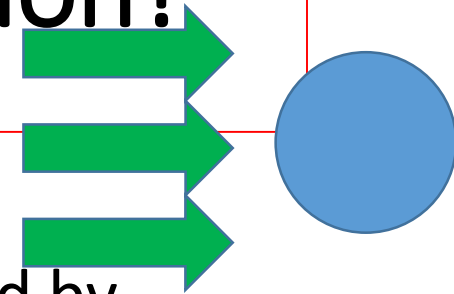
Use Occam's Razor principle: First look for a simple explanation.

An educational subject always has an outside GOAL to be reached by several inside MEANS. But, if seen as mandatory, an INSIDE means becomes a goal hiding its alternatives, thus becoming false nature keeping learners from reaching the original OUTSIDE goal.

So, if neglecting its outside goal, **Mastering Many**, Mathematics Education becomes an undiagnosed 'cure', forced upon 'patients', showing a natural resistance against an unwanted and unneeded 'treatment'.

Thus, to explain the meltdown in Swedish PISA results we ask:

*Is there a **Goal/Means Confusion** in (Swedish) Math Education?*



Defining MatheMatics

According to Freudenthal, the Pythagoreans used the Greek word for knowledge, mathematics, as a common label for their 4 knowledge areas: *astronomy* and *music* and *geometry* and *arithmetic*.

With astronomy and music as independent subjects, today only the two other activities remain, both rooted in the physical fact **Many**:

- **Geometry**, meaning to measure earth in Greek
- **Algebra**, meaning to reunite numbers in Arabic

Then **SET** created ModernMath, as an independent, self-rooted subject.


Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht-Holland: D. Reidel Publ. Comp.



An Observation: Five Questions to be Answered (please discuss)

<i>This is true</i>	Always	Never	Sometimes
$2 + 3 = 5$			
$2 \times 3 = 6$			
$\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$			
$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$			
<u>C1:</u> a FUNCTION is <u>C2:</u> - <i>or both</i>	an example of a <u>set relation</u> where first component identity gives second component identity		
	for example $2+x$, but not $2+3$ i.e. a name for a <u>calculation</u> with an unspecified number		

Five Questions Answered

<i>This is true</i>	Always	Never	Sometimes
2 + 3 = 5	2weeks + 3days = 17days; only with the same unit x		
2 x 3 = 6	x	2x3 is 2 3s III III that can always be recounted as 6 1s	
$\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$	 x 1 red of 2 apples + 2 of 3 apples is 3 of 5 apples, and not 7 of 6		
$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$	x Only if taken of the same total		
<u>C1:</u> a FUNCTION is	an example of a set relation where		(after SET , 1900)
	first component identity implies second component identity		
<u>C2:</u> - or both	for example 2+x, but not 2+3 i.e. a name for a calculation with an unspecified number		(before SET , 1750-1900)

Based upon these observation we define:
MetaMatism = MetaMatics + MatheMatism

Meta-Matics is defining a concept, not as a ~~BottomUp~~ abstraction from ~~many examples~~ but as a TopDown example of an abstraction, derived from the meta-physical abstraction **SET**, made **meaningless** by self-reference as shown by Russell's version of the liar paradox: If M does, it does not, belong to the set of sets not belonging to itself (and vice versa).

$$\text{With } \mathbf{M} = \{ \mathbf{A} \mid \mathbf{A} \notin \mathbf{A} \} : \quad \mathbf{M} \in \mathbf{M} \Leftrightarrow \mathbf{M} \notin \mathbf{M}$$

Mathe-Matism is a statement that is correct inside, but seldom outside a classroom , as e.g. adding numbers without units as 2+3 = 5, where e.g. 2w+3d=17d. In contrast to 2x3 = 6 saying that 2 **3s** can be recounted as 6 **1s**.



ModernMath teaches MetaMatism from day one

MetaMatics: Cardinality is linear. Each point has a number-name to be learned by heart. Counting "twenty-nine, **twenty-ten**" diagnoses you with DisCalculia excluding you from class to be cured by specialists.

MatheMatism: Numbers are added **without units**.
And units must not be introduced to help students with problems in multiplication or division.

Repeat: $2+3$ **IS** 5



Yes, Math Ed has a Goal/Meaning Confusion

As a common label for its two activities, Geometry & Algebra, math has two outside goals: to measure Earth and to reunite Many.

Transformed to self-referring TopDown MetaMatism, it became its own goal blocking the way to the outside goals, reduced to applications of mathematics to be taught, 'of course', after mathematics itself has been taught and learned.

So, to reach the outside goal, **mastering of Many**, we must look for a different alternative way, a **ManyMath**, built as a BottomUp Grounded Theory, a Natural Science, about the physical fact Many.

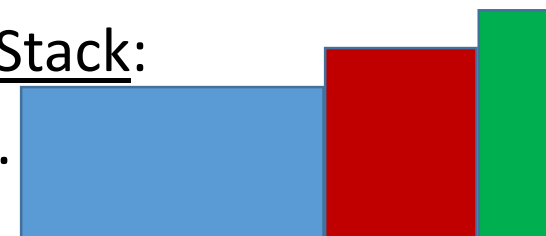
ManyMath, created to Master Many, and respecting the Child's own NumberLanguage

2

To tell nature from choice, we ask: How will math look if grounded as a Natural Science about the physical fact Many, i.e. as a ManyMath?

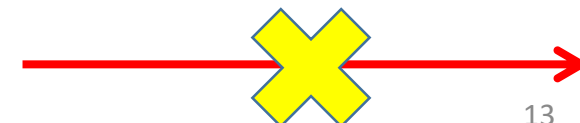
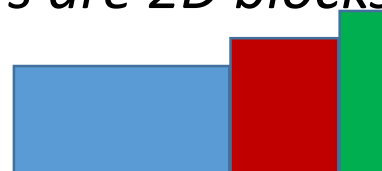
- Take 1: To master Many, we math! *Oops, math is a label, not an action word.*
- Take 2: To master Many, we act. Asking 'How Many?', we Bundle & Stack:

456 = 4 x BundleBundle + 5 x Bundle + 6 x 1 = three stacks of bundles.
















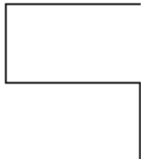



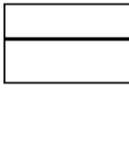
All numbers have units - as recognized by children when showing 4 fingers held together 2 by 2 makes a 3-year-old child say: 'No, that is not 4, that is 2 **2s**.'

So natural numbers are 2D blocks - not a 1D Cardinality-line.



1. Creating Icons: → → →

Counting in ones means naming the different degrees of Many.
Counting in icons means changing **four ones** to **one fours**
rearranged as a **4-icon** with four sticks or strokes. So an icon
contains as many strokes as it represents - if written less sloppy.

one	two	three	four	five	six	seven	eight	nine
								
								
1	2	3	4	5	6	7	8	9

Counting Sequences



Being counted as 1B, the Bundle number needs no icon. So counting a dozen we say:

	I	I	I	I	I	I	I	I	I	I	I	I
4s	1	2	3	B	1B1	1B2	1B3	2B	2B1	2B2	2B3	3B
7s	1	2	3	4	5	6	B	1B1	1B2	1B3	1B4	1B5
tens	1	2	3	4	5	6	7	8	9	B	1B1	1B2

As to number names, eleven and twelve come from ‘one left’ and ‘two left’ in Danish, (en / tve levnet), again showing that counting takes place by taking away bundles.

2. CupCounting in Icons: $9 = ? \text{ 4s}$

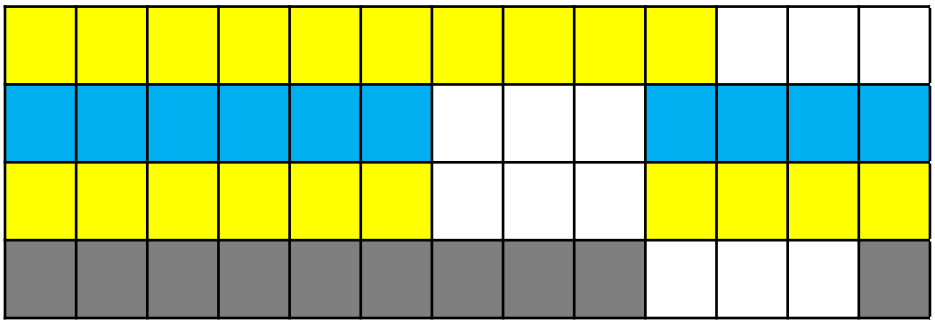
$9 = \text{|||||} = \text{||| |||} = \text{II}| = 2)1 \text{ 4s} = 2 \text{ Bundles \& } 1 \text{ 4s}$

To count, we bundle & use a bundle-cup with 1 stick per bundle.
 We report with **cup-writing** 2)1 4s or **decimal-writing** 2.1 4s
 where the decimal point separates the bundles from the singles.

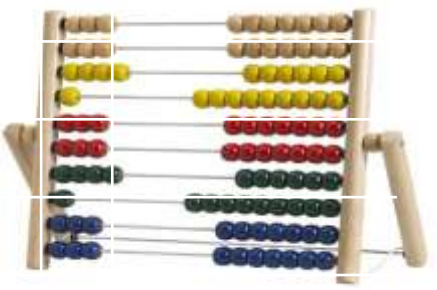


Shown on a western **ABACUS** in

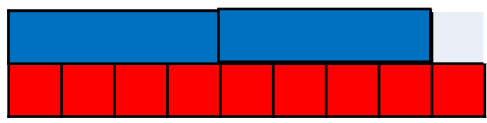
Geometry/space mode



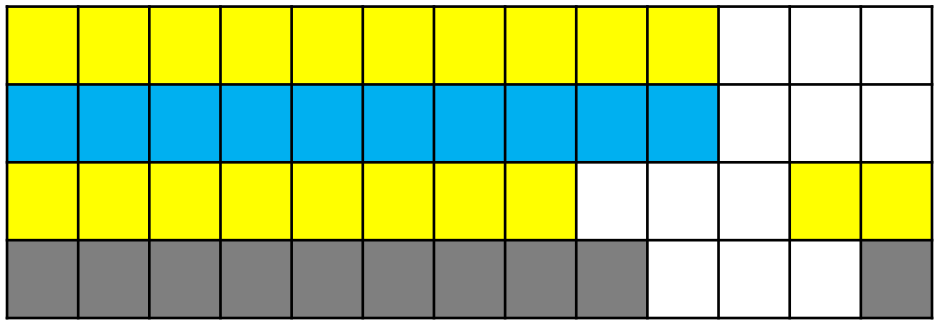
or



LEGO blocks:



Algebra/time mode



Counting creates Division & Multiplication & Subtraction - also as Icons

‘From 9 take away **4s**’ we write 9/4
iconizing the sweeping away by a broom, called division.

‘2 times stack **4s**’ we write 2x4
iconizing the stacking up by a lift called multiplication.

‘From 9 take away 2 **4s**’ to look for un-bundled we write 9 – 2x4
iconizing the dragging away by a trace called subtraction.

So counting includes division and multiplication and subtraction:

Finding the bundles: $9 = 9/4$ **4s**. Finding the un-bundled: $9 - 2 \times 4 = 1$.



Counting creates Two Counting Formulas

*As the Sentences of the NumberLanguage, **Formulas Predict***

Bundling & stacking create two counting formulas (re-bundle and re-stack):

$T = (T/b) \times b$	from a total T, T/b times, bs is taken away and stacked
$T = (T-b) + b$	from a total T, T-b is left when b is taken away and placed next-to

With the counting formulas, a calculator predicts the counting result $9 = 2)1 \text{ } 4s$

$9/4$
 $9 - 2 \times 4$

$2.\text{some}$
 1



3. ReCounting in the Same Unit creates Overload & Underload (Negative Numbers)

$$\begin{aligned} T &= 3)0 \quad 2s \\ &= 2)2 \quad 2s \\ &= 4)-2 \quad 2s \end{aligned}$$

ReCounting 3 2s in 2s:

Sticks	Calculator	Cup-writing	3 ways
≡ ≡ ≡		3) 0 2s	Normal
≡ ≡ ≡ ≡	3x2 – 2x2 2	2) 2 2s	Overload
≡ ≡ ≡ ≡ ≡	3x2 – 4x2 -2	4) -2 2s 4)-2 = 4 less 2	Underload

So a total can be ReCounted in 3 ways: Normal, Overload or Underload.

Or as a **2digit** Number if using Bundles of Bundles:

	=	≡ ≡ ≡	=	<u>≡ ≡</u> ≡
6	=	3 B	=	1 BB 1 B
6	=	3)0 2s	=	1) 1)0 2s = 11)0 2s

4. ReCounting in a Different Unit

$$3 \text{ } 4\text{s} = ? \text{ } 5\text{s}$$

$$3 \text{ } 4\text{s} = \text{||||} \text{ ||||} \text{ ||||} = \text{||||} \text{ ||||} \text{ ||||} = \text{||||} \text{ ||||} \text{ ||} = 2 \text{ } 2 \text{ } 5\text{s}$$

CALCULATOR-prediction:

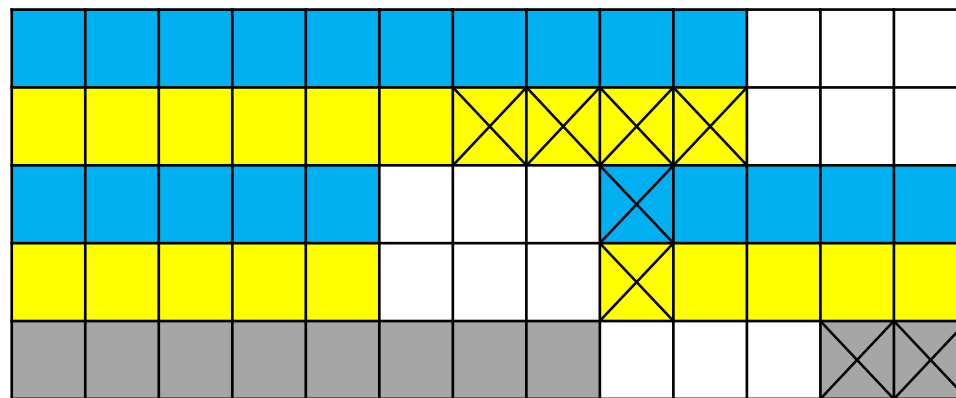
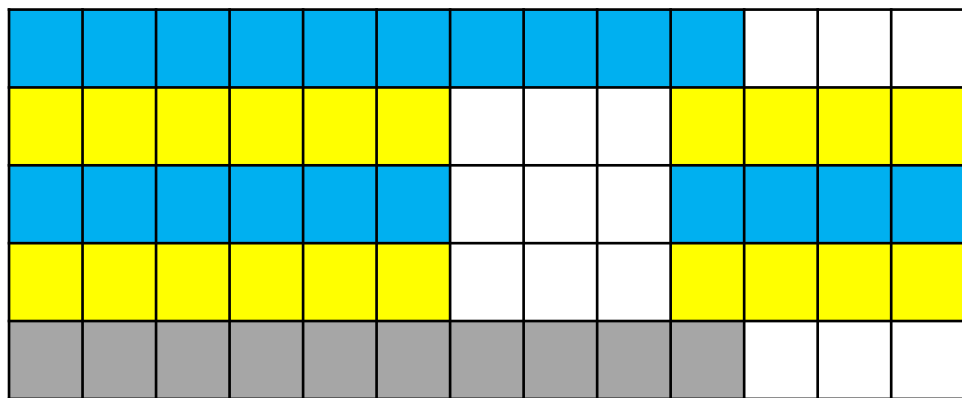
$$3 \times 4 / 5$$

2.some

$$3 \times 4 - 2 \times 5$$

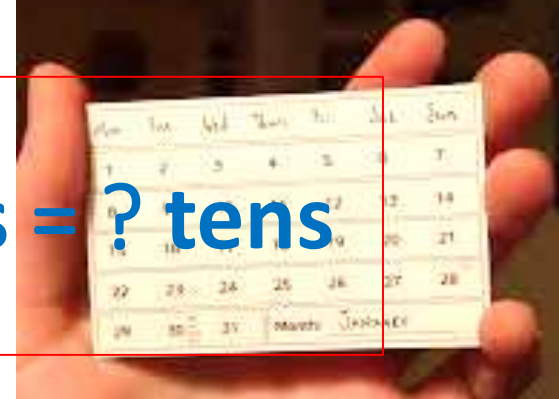
2

Abacus in Geometry mode



Change Unit = **Proportionality, Core Math**

5. ReCounting in Tens (Multiplication) 3 7s = ? tens



$$3 \text{ 7s} = \text{|||||||} \text{ |||||} \text{ |||||} = \text{|||||||} \text{ |} \text{ |||||} = 2)1 \text{ tens}$$

CALCULATOR-prediction: The calculator has no ten icon.

The calculator gives the answer directly

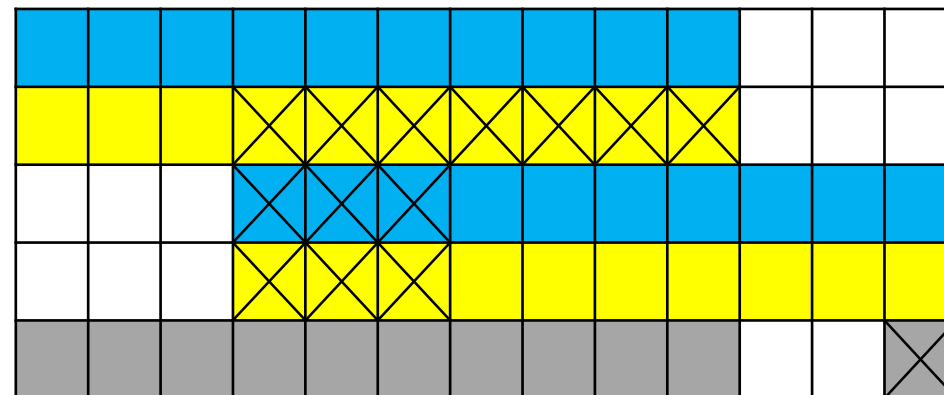
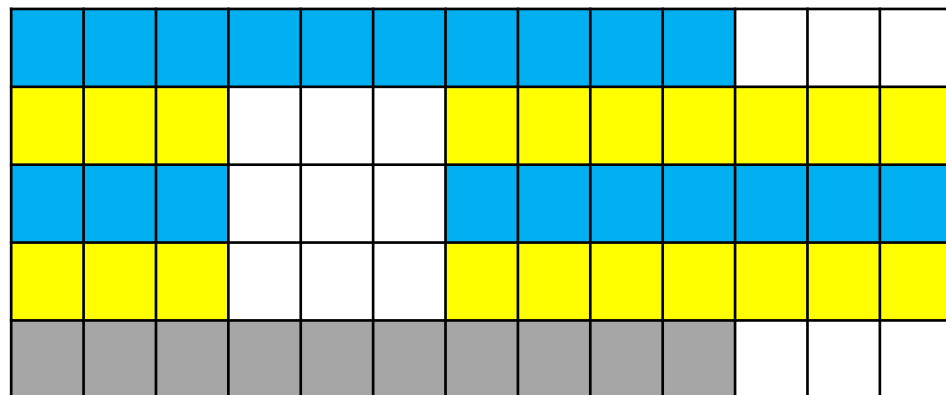
- but **without unit** and with **misplaced** decimal point

3x7

21

A Natural Number ???

Abacus in Geometry mode



So $T = 21 = 2.1$ tens is not a **1D length** on a number line, it is a **2D block** of tens

6. ReCounting from Tens (Division)

$29 = ? \text{ } 6\text{s}$

$29 = ? \text{ } 6\text{s} = \text{|||||} \text{|||||} \text{|||||} = \text{|||||} \text{|||||} \text{|||||} \text{|||||} \text{|||||} = 4\text{)5 } 6\text{s}$

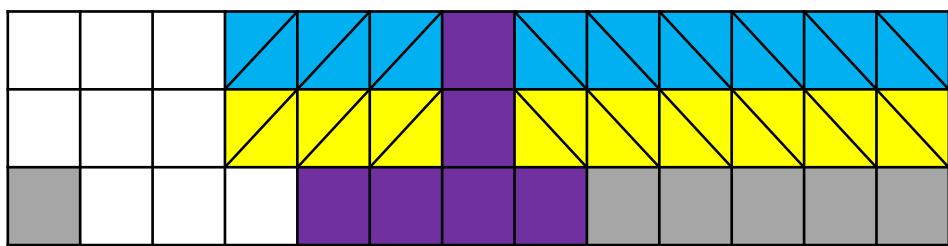
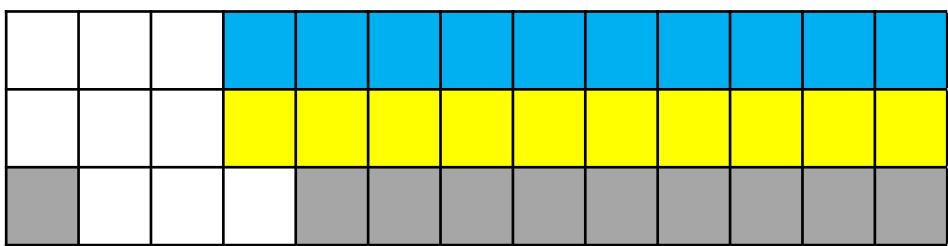
CALCULATOR-prediction:

$29/6$	4.some
$29 - 4 \times 6$	5

Reversed calculation (Equation): $? \times 6 = 29 = (29/6) \times 6$, so $? = 29/6 = 4\text{)5}$

Opposite Side & Sign method: if $u \times 6 = 29$ then $u = 29/6$

Abacus in Geometry mode



ReCounting from tens = Division = Solving an Equation = Core Math



ReCounting large Numbers in or from Tens

Same number-area, but New form

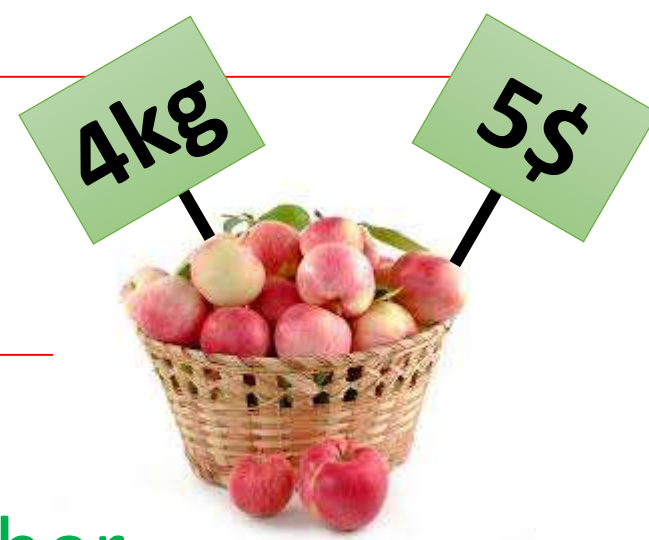
Recounting 6 **47s**

Recounting 476 in **7s**

*Using CupWriting to seprate **INSIDE** bundles from **OUTSIDE** 1s*

$T = 6 \times 47 = 6 \times \begin{array}{l} 40 \\ 7 \end{array}$  $= 240 + 42$ $= 280 + 2$ $= 282$	$T = 476 = \begin{array}{l} 400 \\ 70 \\ 6 \end{array}$  $= 420 + 56$ $= 6 \times 70 + 8 \times 7$ $= 68 \times 7$
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7. DoubleCounting creates **PerNumbers** creating Fractions & Proportionality



With **4kg = 5\$** we have

4kg per 5\$ = $4\text{kg}/5\$ = 4/5 \text{ kg}/\$ =$ a **PerNumber**

$$4\$/100\$ = 4/100 = 4\%$$

Questions:

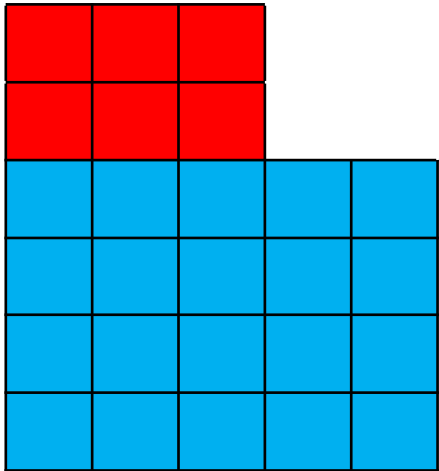
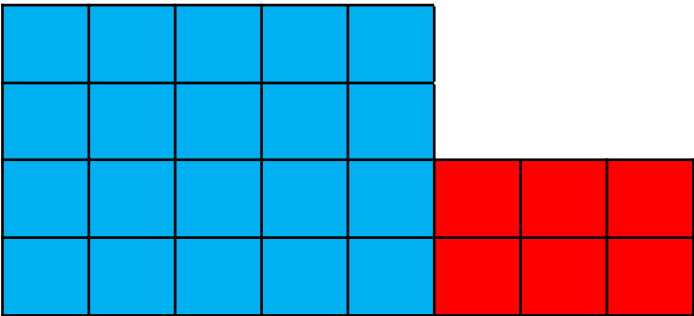
7kg = ?\$	8\$ = ?kg
$7\text{kg} = (7/4)*4\text{kg}$ $= (7/4)*5\$ = 8.75\$$	$8\$ = (8/5)*5\$$ $= (8/5)*4\text{kg} = 6.4\text{kg}$

Answer: *Recount in the **PerNumber***

(RegulaDeTri)

8. Once Counted & ReCounted, Totals are Added, BUT NextTo or OnTop?

NextTo ➡	OnTop ⬆
$4\ 5s + 2\ 3s = 3)2\ 8s$	$4\ 5s + 2\ 3s = 4\ 5s + 1)1\ 5s = 5)1\ 5s$
The areas are integrated <i>Integrate areas = Integration</i>	The units are changed to be the same <i>Change unit = Proportionality</i>

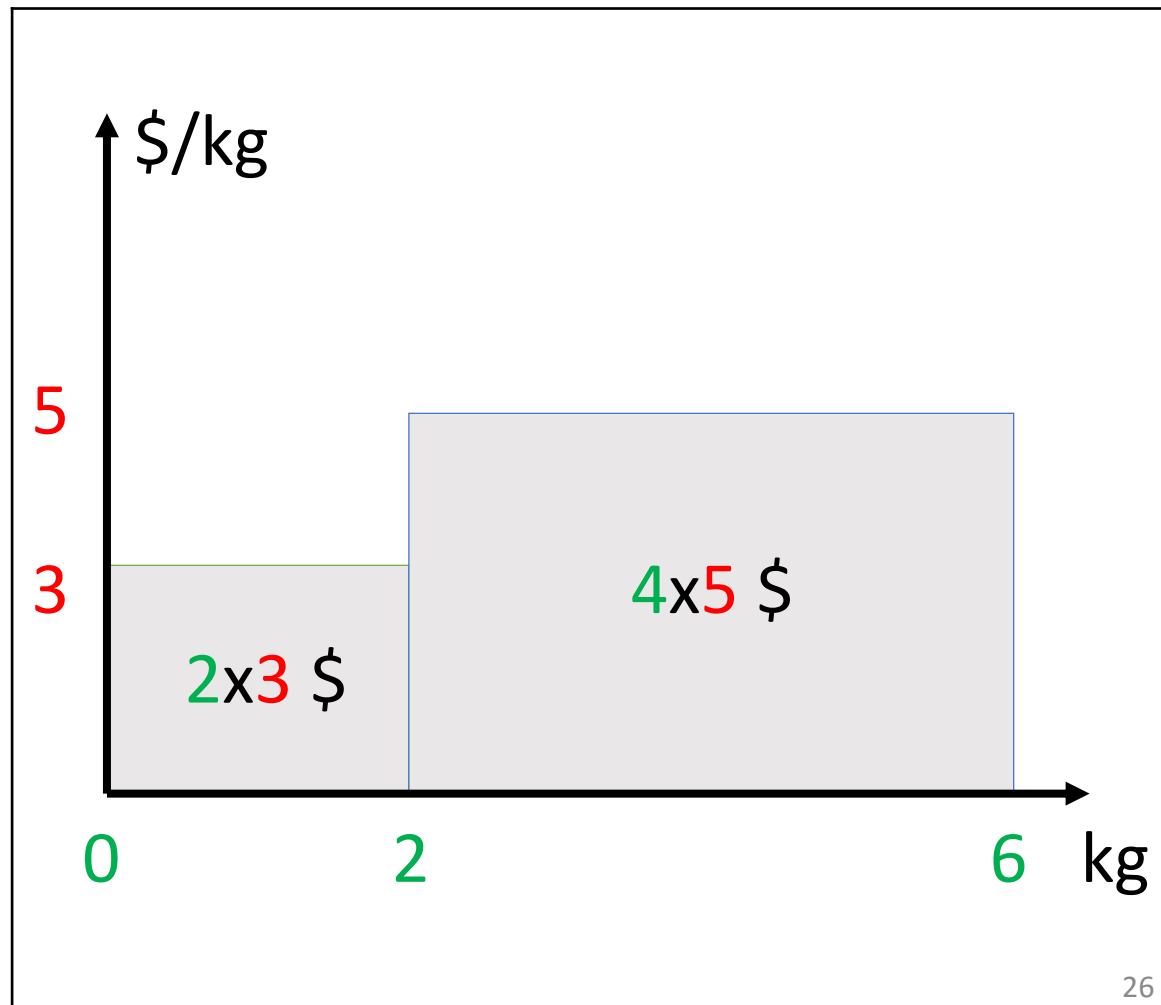


9. Adding PerNumbers as Areas (Integration)

$$\begin{array}{rcl}
 2 \text{ kg} & \text{at} & 3 \text{ \$/kg} \\
 + 4 \text{ kg} & \text{at} & 5 \text{ \$/kg} \\
 \hline
 (2+4) \text{ kg} & \text{at} & \frac{2 \times 3 + 4 \times 5}{2+4} \text{ \$/kg}
 \end{array}$$

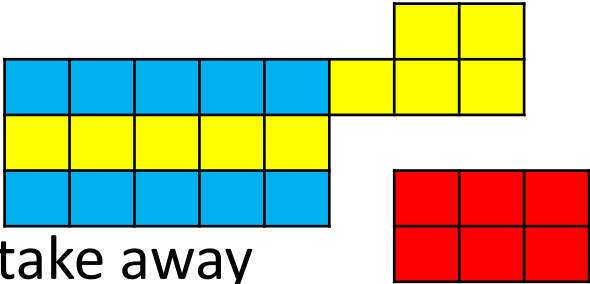
Unit-numbers add on-top.

Per-numbers add next-to as **areas** under the per-number graph, i.e. as **integration**.



10. Reversing Addition, or Solving Equations

OnTop	Opposite Side & Sign	NextTo
$2 + ? = 8 \quad = (8-2) + 2$	$2 \times ? = 8 \quad = (8/2) \times 2$	$2.3s + ? 5s = 3.2 8s$
$? = 8-2$ <i>Solved by re-stacking</i>	$? = 8/2$ <i>Solved by re-bundling</i>	$? = (3.2 8s - 2.3s)/5$ <i>Solved by differentiation: $(T-T1)/5 = \Delta T/5$</i>

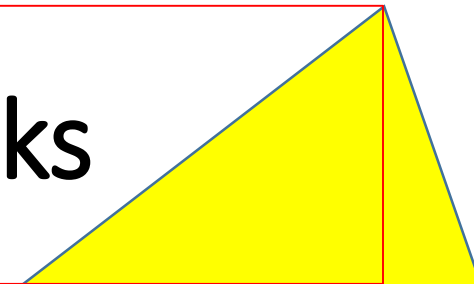


Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Geometry: Measuring Earth in HalfBlocks



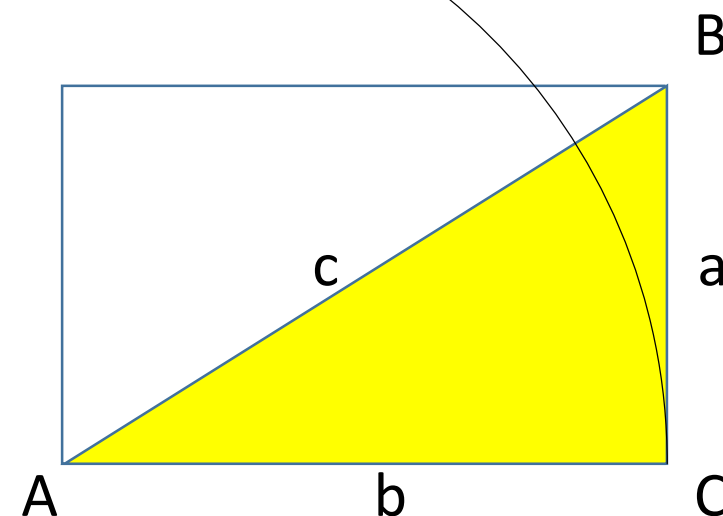
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras theorem. And connected with the angles by formulas recounting the sides in sides or in the diagonal:

$$A+B+C = 180$$

$$a*a + b*b = c*c$$

$$\tan A = a/b = \Delta y / \Delta x = \text{gradient}; \sin A = a/c; \cos A = b/c$$

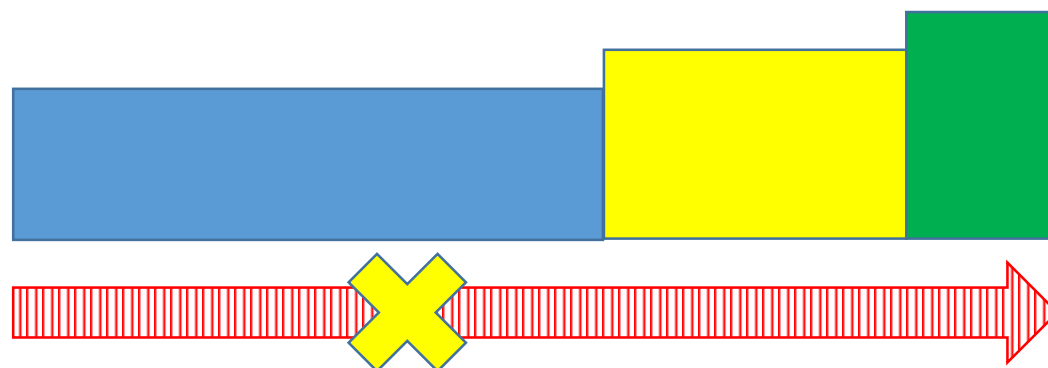
$$\text{Circle: circum./diam.} = \pi = n * \tan(180/n) \text{ for } n \text{ large}$$



Defining ManyMath: To master Many, we Recount in Blocks that add NextTo or OnTop

In ManyMath,

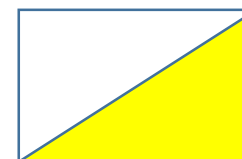
- Numbers are 2D blocks
- not on a 1D line



- Algebra: to (re)unite blocks next-to or on-top



- Geometry: to measure half-blocks





Is ManyMath Different from ModernMath

<i>Same Question</i>	ManyMath	ModernMath
Digits	Icons, different from letters	Symbols like letters
Natural numbers	T = 2.3 tens	23
Order of operations	/ x - +	+ - x /
Operations	Icons for counting the process: sweep, stack, drag & connect	Mappings from a set-product to a set
Addition	On-top and next-to	Only on-top
Fractions	Per-numbers, not numbers but operators needing a number to give a number	Rational numbers
Per-numbers	Double-counting	Not accepted

Same Question – Different Answers

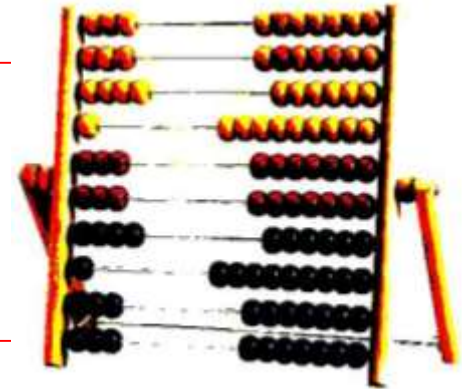
	ManyMath	ModernMath
A formula	A calculation with numbers & letters	An example of a function
A function $f(x)$	A placeholder for an unspecified formula with x as unspecified number. Thus $f(4)$ is a meaningless syntax error.	An example of a set relation where first component identity implies second component identity
An equation	A name for a reversed calculation. Solved by moving to the opposite Side with opposite Sign.	An example of an equivalence relation between two number-names solved by neutralizing using associative & commutative laws and abstract group theory
Integration	Preschool: Next-to addition, for all. Middle school: Adding piece-wise constant per-numbers, for all. High school: Adding locally constant per-numbers, for almost all.	Last year in high school, for the few

Yes, ModernMath & ManyMath are Different

	ManyMath	ModernMath
Algebra	Re-unite constant and variable unit-numbers and per-numbers	A search for patterns
The root of Mathematics	The physical fact Many	The metaphysical invention SET
A concept	An abstraction from examples 	An example of an abstraction derived from SET (MetaMatics) 
How true is $2+3 = 5$ & $2 \times 3 = 6$	$2 \times 3 = 6$ is true by nature since 2 3s can be recounted as 6 1s . $2+3 = 5$ is true inside but seldom outside a class: $2\mathbf{w}+3\mathbf{d} = 17\mathbf{d}$, etc.	Both true by nature (MatheMatism) MetaMatism = MetaMatics + MatheMatism

ModernMath versus ManyMath

Primary School Curriculum

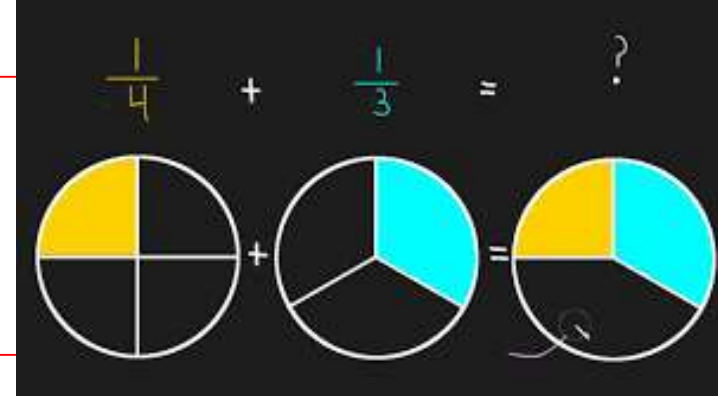


4

ModernMath	ManyMath
<p>1dim. Number-line with number-names</p> <p>Addition & Subtraction before Multiplication & Division</p>	<p>2dim. Number-blocks with units.</p> <p>Multiplication & Division before Subtraction & Addition</p>
<ul style="list-style-type: none"> • One and two digit numbers • Addition • Subtraction • Multiplication • Division • Simple fractions 	<ul style="list-style-type: none"> • CupCount Many in BundleCups • ReCount Many in same Unit & in new Unit (Proportionality) • ReCount: In Tens & From Tens (Multiplication & Division) • Calculator Prediction: ReCountFormula • Addition: NextTo (Integration) & OnTop • Reversed addition: Equations

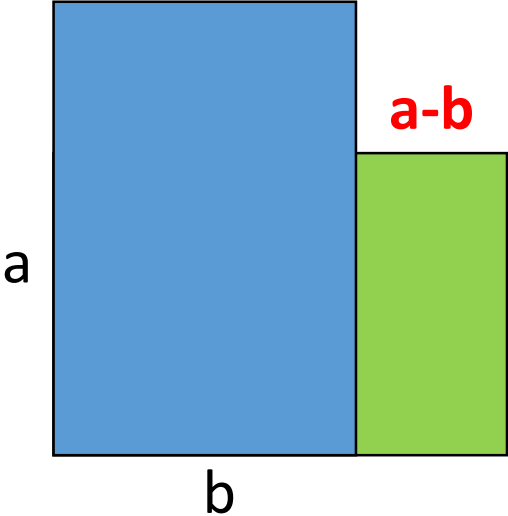
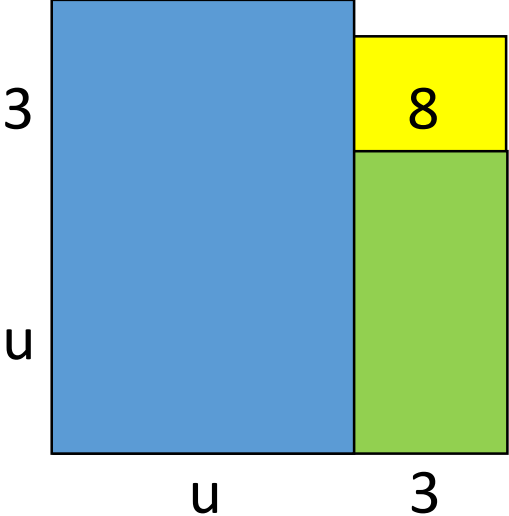
ModernMath versus ManyMath

Middle School Curriculum



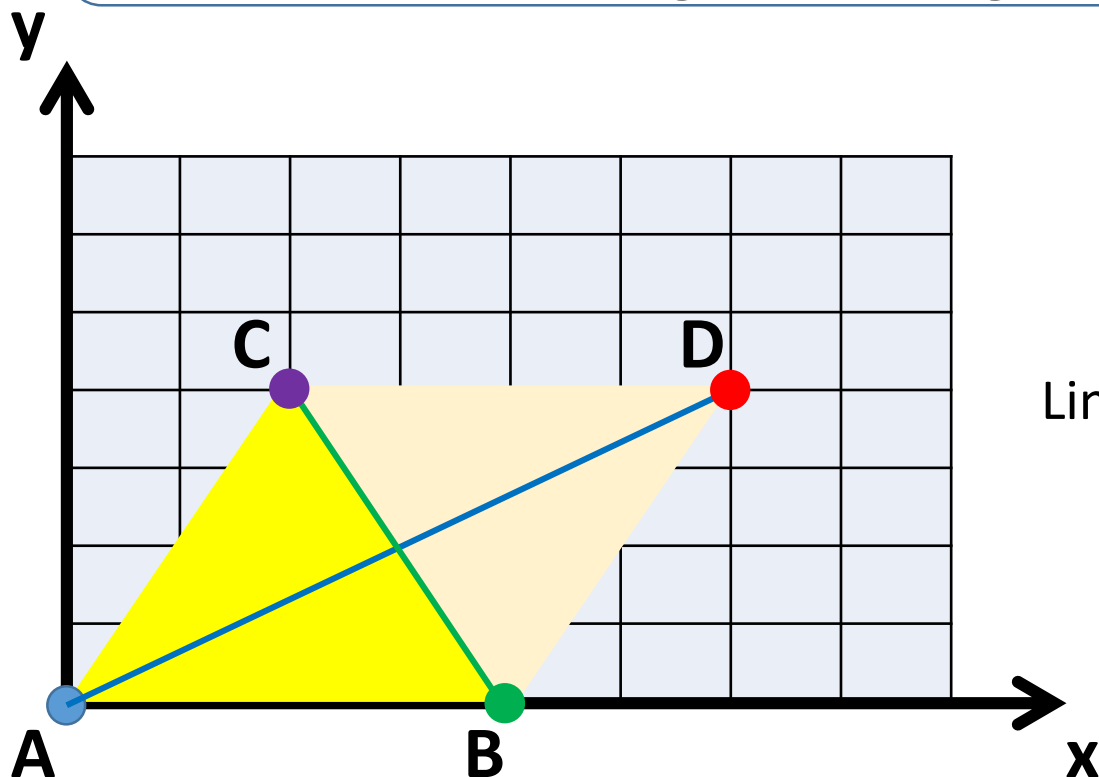
ModernMath	ManyMath
Fractions are numbers that can be added without units	Fractions are PerNumbers (operators needing a number to become a number) and added by areas (integration)
<ul style="list-style-type: none"> • Negative numbers • Fractions • Percentages & Decimals • Proportionality • LetterNumbers • Algebraic fractions • Solve a linear equation • Solve 2 equations w. 2 unknowns 	<ul style="list-style-type: none"> • DoubleCounting produces PerNumbers & PerFives (fractions) & PerHundreds (%) • Geometry and algebra go hand in hand when working with letter-numbers and letter-formulas; and with lines and forms • The coordinate system coordinates geometry and algebra so that length can be translated to Δ-change, and vice versa

Geometry helps Algebra, going Hand in Hand

Quadratic Rule with 2 Cards	Quadratic Equations with 3 Cards
	
<p>Corner = $(a-b)^2 = a^2 - 2 \text{ cards} + b^2$</p> <p>So $(a-b)^2 = a^2 - 2 \times a \times b + b^2$</p>	<p>$u^2 + 6u + 8 = 0$</p> <p>$(u+3)^2 = u^2 + 6u + 8 + 1$</p> <p>$(u+3)^2 = 0 + 1$</p> <p>$u = -3 \pm 1$</p> <p><u>$u = -4 \text{ \& } u = -2$</u></p>

Algebra helps Geometry, going Hand in Hand

A triangle ABC with $A(0,0)$ and $B(4,0)$ and $C(2,4)$ is extended to a parallelogram ABCD to the right. Find D and the intersection point between the two diagonals using both **Geometry & Algebra**.



From A to B $\Delta x = x_2 - x_1 = 4 - 0 = 4$,
So, also from C to D $\Delta x = 4$; $D(2+4,4) = D(6,4)$

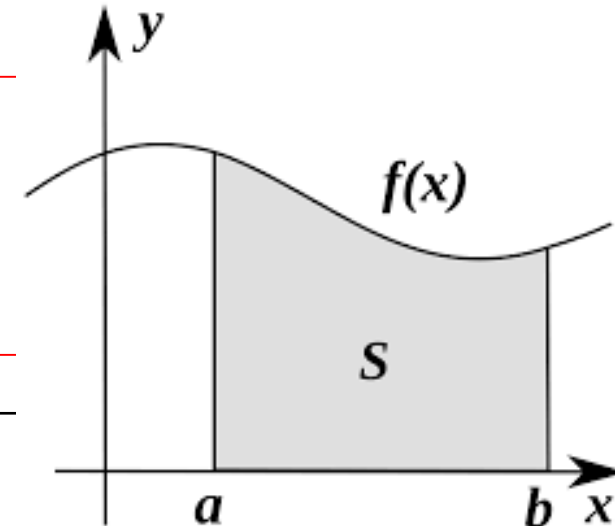
Line AD : $\Delta y / \Delta x = 4/6$ & Line CB : $\Delta y / \Delta x = -4/2$
Line AD : $(y-0)/(x-0) = 4/6$ & Line CB : $(y-0)/(x-4) = -2$
Line AD : $y = 4/6 * x$ and Line CB : $y = -2 * (x-4)$

Intersection: $x = 3$ and $y = 2$

Tested by geometrical construction

ModernMath versus ManyMath

High School Curriculum

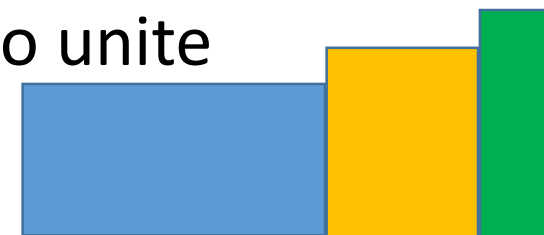


ModernMath	ManyMath
Functions are set-relations	Functions are formulas with two variables
<ul style="list-style-type: none"> • Squares and square roots • Solve quadratic equations • Linear functions • Quadratic functions • Exponential functions • Logarithm • Differential Calculus • Integral Calculus • Statistics & propability 	<ul style="list-style-type: none"> • Integral Calculus as adding PerNumbers • Change & Global/Piecewise/Local Constancy • Root/log as finding/counting change-factors • Constant change: Proportional, linear, quadratic, exponential, power • Simple and compound interest • Predictable Change: Integral Calculus & Differential Calculus • Unpredictable Change: Stat. & prop.

ManyMath Includes Algebra's 4 ways to ReUnit

456 = 4 x Bundle² + 5 x Bundle + 6 x 1 shows the 4 ways to unite

- Addition / *Subtraction* unites / *splits into* Variable Unit-numbers
- Multiplication / *Division* unites / *splits into* Constant Unit-numbers
- Power / *Root&Log* unites / *splits into* Constant Per-numbers
- Integration / *Differentiation* unites / *splits into* Variable Per-numbers



Operations unite / <i>split into</i>	Variable	Constant
Unit-numbers <i>m, s, \$, kg</i>	$T = a + n$ $T - a = n$	$T = a \times n$ $T/n = a$
Per-numbers <i>m/s, \$/kg, m/(100m) = %</i>	$T = \int a \, dn$ $dT/dn = a$	$T = a^n$ $\log_a T = n, \sqrt[n]{T} = a$

Primary, Middle & HighSchool Core Curriculum

To lead to its outside goal, a **NumberLanguage Mastering Many**,
 a math curriculum must be based on basic Algebra, reuniting Many

Operations unite <i>split into</i>	Variable	Constant
Unit-numbers <i>m, s, \$, kg</i>	$T = a + n$ $T - a = n$	$T = a \times n$ $T/n = a$
Per-numbers <i>m/s, \$/kg, m/(100m) = %</i>	$T = \int a \, dn$ $dT/dn = a$	$T = a^n$ $\log_a T = n, \sqrt[n]{T} = a$

Primary
CoreCur

Middle
CoreCur

High
CoreCur

Main Points of a ManyMath Curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage

- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- CupCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: $/ \times - +$

Middle school – integrating algebra and geometry, the content of the label math

- DoubleCounting produces PerNumbers as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE

- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

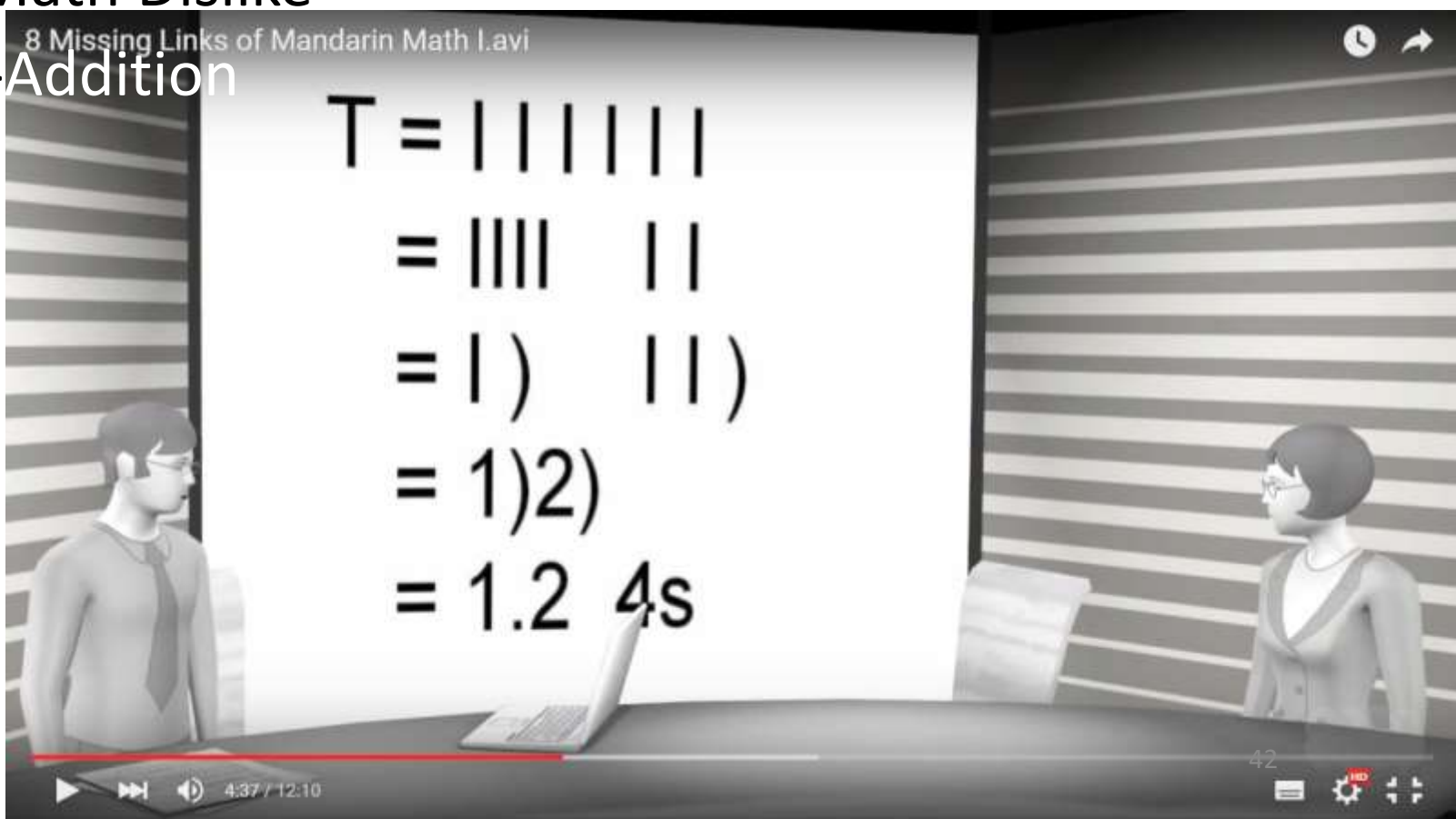
ManyMath is Different – but does it make a Difference? Try it out.

- Watch some MrAlTarp YouTube videos
- Try the **CupCount before you add** Booklet
- Try a 1day free Skype seminar **How to Cure Math Dislike**
- Try Action Learning and Action Research, e.g. **1Cup, 5Sticks**
- Collect data and Report on its 8 **MicroCurricula**, M1-M8
- Try a 1year online InService TeacherTraining at the MATHeCADEMY.net using PYRAMIDeDUCATION to teach teachers to teach MatheMatics as **ManyMath**, a Natural Science about the root of mathematics, **Many**

Some MrAlTarp YouTube Videos

Screens & Scripts on MATH^eCADEMY.net

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



CupCount 'fore you Add Booklet, free to Download

My many Math Tears will not Stay – if I Cup the Stray Away

CupCount 'fore you Add

MathDislike Cured by 1 Cup & 5 Sticks

$$5 = \text{||||} = \text{1 Cup} = 1)3 \text{ 2s}$$

$$5 = \text{||||} = \text{2 Cups} = 2)1 \text{ 2s}$$

$$5 = \text{||||} = \text{3 Cups} = 3)-1 \text{ 2s}$$

CupCount 7 in 3s: $7 = 2)1 \text{ 3s} = 1)4 \text{ 3s} = 3)-2 \text{ 3s}$

NO, 4×7 is not 28, it is $4 \text{ 7s} = 2)8 = 1)18 = 3)-2 \text{ tens}$

NO, $30/6$ is not 30 divided by 6, it is 30 counted in 6s

CupWrite to tell InSide Bundles from OutSide 1s:

- $65 + 27 = 6)5 + 2)7 = 8)12 = 9)2 = 92$
- $65 - 27 = 6)5 - 2)7 = 4)-2 = 3)8 = 38$
- $7 \times 48 = 7 \times 4)8 = 28)56 = 33)6 = 336$
- $336 / 7 = 33)6 / 7 = 28)56 / 7 = 4)8 = 48$

MatheMatics as ManyMath
- a Natural Science about Many

Makes Math Potentials Blossom
in Children, Adults & Migrants

Allan.Tarp

MATHeCADEMY.net

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03. CupCounting in Icons

Job		Do	Calculator
9 in 5s	Line	T =	9/5 1.some
	Count	1, 2, 3, 4, 5, 10, 11, 12, 13, 14	9 - 1*5 4
	Bundle	T =	
	Stack		9 - 0*5 9
	Answer	T = 9 = 1.4 5s	9 - 2*5 -1
9 in 4s	Line	T =	9/4 2.some
	Count	1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15	9 - 2*4 1
	Bundle	T =	
	Cup	T = 2)1 4s = 1)5 4s = 3)-3 4s	9 - 1*4 5
	Answer	T = 9 = 2.1 4s	9 - 3*4 -3
9 in 3s	Line		
	Count		
	Bundle		9/
	Cup		9 -
	Stack		
8 in 4s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		
8 in 3s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		

1day free Skype Seminar: To Cure Math Dislike, **CupCount** before you **Add**

Action Learning based on the Child's own 2D NumberLanguage

09-11. Listen and Discuss the PowerPointPresentation

To Cure MathDislike, replace MetaMatism with ManyMath

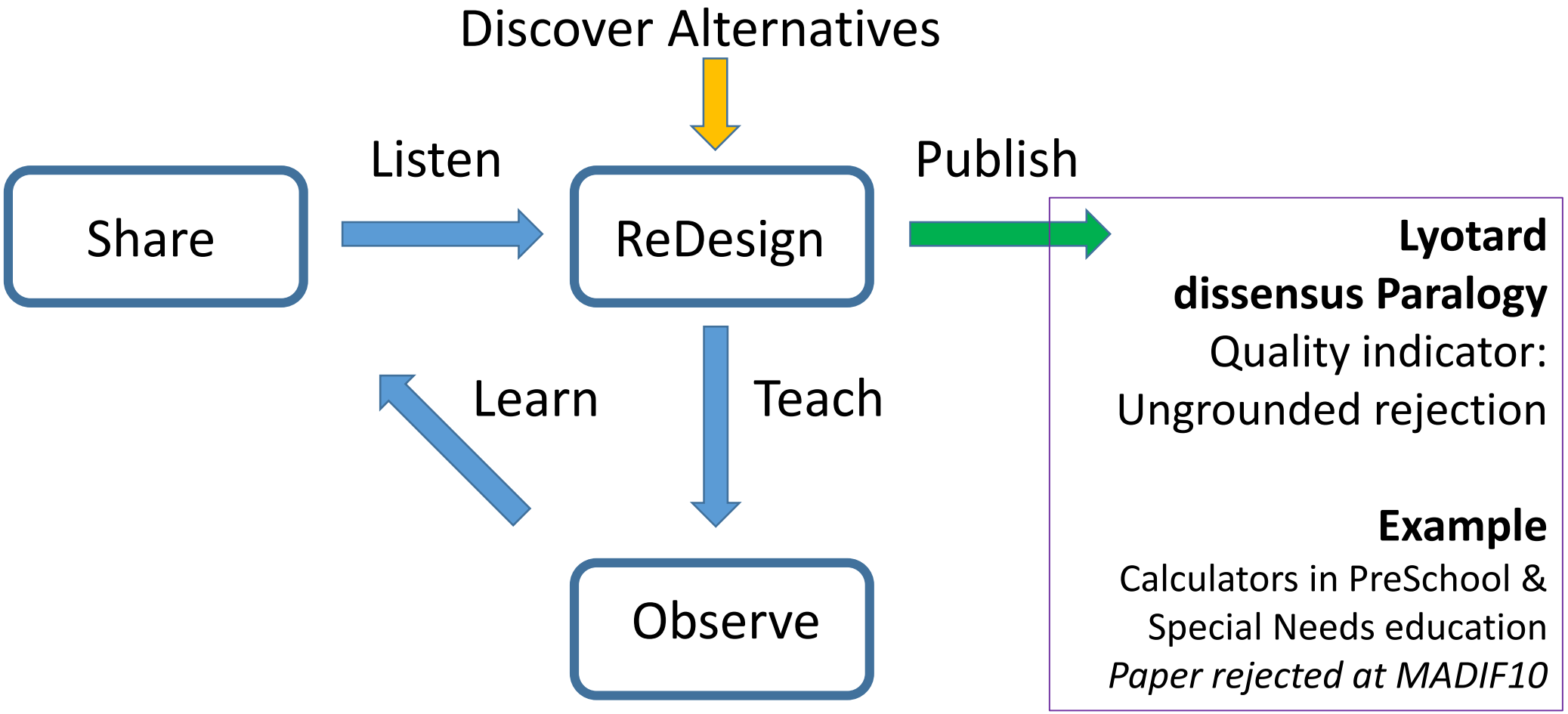
- **MetaMatism** = MetaMatics + MatheMatism
- **MetaMatics** presents a concept TopDown as an example instead of BottomUp as an abstraction
- **MatheMatism** is true inside but rarely outside classrooms
- **ManyMath**, a natural science about Many mastering Many by CupCounting & Adding NextTo and OnTop.

11-13. Skype Conference. Lunch.

13-15. Do: Try out the CupCount before you Add booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in CupCounting & NextTo Addition.

15-16. Coffee. Skype Conference.

Action Learning & Action Research



A Primary School Test Curriculum, before Math Dislike CURED by 1 Cup & 5 Sticks




$$336/7 =$$

? ? ?



Having problems in a division class, the teacher says: "Timeout, class. Next week no division, instead we take a field trip back to day 1 to learn CupCounting"

Let's recount 5 in 2s by bundling, using a cup for the bundles:

5 =	=		= 1)3 2s = 1 Bundle & 3 2s	overload
5 =	=		= 2)1 2s = 2 Bundles & 1 2s	normal
5 =	=		= 3)-1 2s = 3 Bundles less 1 2s	underload

Now we know that numbers can be ReCounted in 3 ways:

Normal, overload or underload if we move a stick **OUTSIDE** or **INSIDE**.

Now CupCount 7 in 3s:

$$7 = ||||| || = 2)1 \text{ 3s} = 1)4 \text{ 3s} = 3)-2 \text{ 3s}$$

A Primary School Test Curriculum, after Math Dislike CURED by 1 Cup & 5 Sticks

$$\begin{array}{l}
 336/7 \\
 = 33)6 /7 \\
 = 28)56 /7 = 4)8 \\
 \text{😊} \quad \text{✌️} \quad \text{👉}
 \end{array}$$

When counting in TENS, before calculating, we cup-write the number to separate the **INSIDE** bundles from the **OUTSIDE** singles. Later we recount.

- $65 + 27 = 6)5 + 2)7 = 8)12 = 9)2 = 92$
- $65 - 27 = 6)5 - 2)7 = 4)-2 = 3)8 = 38$
- $7 \times 48 = 7 \times 4)8 = 28)56 = 33)6 = 336$
- $336 /7 = 33)6 /7 = 28)56 /7 = 4)8 = 48$

With 336 we have 33 **INSIDE**, so to get 28, so we move 5 **OUTSIDE** as 50.

Now try 456 / 7.

- $456 /7 = 45)6 /7 = 42)36 /7 = 6)5 + 1 = 65 \text{ } 1/7$

8 MicroCurricula for Action Learning & Research

- C1. Create Icons
- C2. Count in Icons (Rational Numbers)
- C3. ReCount in the Same Icon (Negative Numbers)
- C4. ReCount in a Different Icon (Proportionality)
- A1. Add OnTop (Proportionality)
- A2. Add NextTo (Integrate)
- A3. Reverse Adding OnTop (Solve Equations)
- A4. Reverse Adding NextTo (Differentiate)

4 Counted in 3s

Sticks

G-counting		A-counting	
	lay out		lay out
	bundle		bundle
	stack	① ①	cups
T = 1.1 3s	Total	1) 1)	cup-writing
		T = 1.1 3s	Total

4

Round it up & Color it

Clap, Sing, Walk, Act & Letter it

Unite it

Split it

Reward: Stickers, each counting two

Abacus

mode	A-mode

Calculator

4 / 3	1.some
4 - 1 x 3	1
T = 4 = 1.1 3s	

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ManyMath: CupCount before you Add

Teach **Multiplication** before Addition & Add **NextTo** before OnTop

[FREE 1day SKYPE Cure Math Dislike Teacher Seminar](#)

[CupCount & ReCount](#) * [KopTae](#) & [OmTae](#) * [ICME13 Papers](#)

PPP: [Existentialism in Math Ed](#) * [From MetaMatism to ManyMath](#)

Hire MrAITarp: 2 weeks (Free), or 2 months

I	II	III	IIII	IIII	IIII	IIII	IIII	IIII
1	2	3	4	5	6	7	8	9

We ACT to deal with the outside world.

We MATH to deal with the natural fact MANY ???

Oops, sorry, math is not an action word!

We COUNT & ADD to deal with MANY.

• CupCount & ReCount:

$T = \text{I} \text{I} \text{I} \text{I} \text{I} \text{I} = \text{III} \text{ III} \text{ I} = \text{II} \text{ I} = 2 \text{ I} = 2.1 \text{ 3s}$

$T = 2 \text{ I} 1 \text{ 3s} = 1 \text{ I} 4 \text{ 3s} = 3 \text{ I} -2 \text{ 3s}$ (Overload or Deficit)

$T = 2 \text{ I} 1 \text{ 3s} = 1 \text{ I} 2 \text{ 5s} = 3 \text{ I} 1 \text{ 2s} = 11 \text{ I} 1 \text{ 2s}$

$T = 3 \times 8 = 3 \text{ 8s} = 2.6 \text{ 9s} = 2.4 \text{ tens}$, or the sloppy version 24

Counting gives a decimal number with a unit (a natural number).

Adding OnTop, a Total may be ReCounted to shift the unit.

Adding NextTo, means Integration of areas.

• Add OnTop & Add NextTo:

Teaches Teachers to Teach
MatheMatics as **ManyMath**,
a Natural Science about **MANY**.
The **CATS** method: To learn Math
Count & **Add** in **Time** & **Space**

Teacher Training in CATS ManyMath Count & Add in Time & Space

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C1 COUNTING MANY

Questions	Answers
How to count Many?	By bundling and stacking the total T predicted by $T = (T/b) \times b$.
How to recount 8 in 3s, $T = 8 = 7 \times 3$	$T = 8 = 7 \times 3 = 21$, $T = 8 = (8/3) \times 3 = 2 \times 3 + 2 = 2 \times 3 + 2 = 2 \times 3 + 2$
How to recount 6 kg in 5: $T = 6 \text{ kg} = 75$	If $4 \text{ kg} = 25$ then $6 \text{ kg} = (6/4) \times 25 = (6/4) \times 25 = 37.5$
How to count in standard bundles?	Bundling bundles gives a multiple stack, a stock or polynomial: $T = 423 = 4 \text{ Bundles} + 2 \text{ Bundles} = 4 \text{ tens} + 2 \text{ tens} = 4 \times 10 + 2 \times 10 = 40 + 20 = 60$

1 REPETITION BECOMES MANY

Question: How can repetition in time be represented in space?

Answer: By iconisation: put a finger to the throat and add a match or a stroke for each beat of the heart.

Example: $\dots \rightarrow \text{|||||}$

Exercise: Find other examples of spatial representation of temporal repetition.

2 MANY BECOMES BUNDLES

Question: How can we organise Many?

Answer: By bundling: line up the total and divide it into bundles.

Examples: $\text{|||||} \rightarrow \text{|||||}$ or $\text{|||||} \rightarrow \text{|||||}$ or $\text{|||||} \rightarrow \text{|||||}$ or ...

Exercise: Take a lot of matches and bundle them in 2s, then in 3s, then in 4s, etc.

3 BUNDLES BECOME ICONS

Question: How can we arrange the different degrees of Many?

Answer: By iconisation: the strokes of the different degrees of Many are rearranged as icons, realising that there would be four strokes in the number-icon 4, etc., if written in a less sloppy way.

Example:

1	2	3	4	5	6	7	8	9
/	2	3	4	5	6	7	8	9

Exercise: Find other ways to build icons for the numbers above. Invent icons for ten, eleven and twelve.

4 MANY IS COUNTED AS A STACK OR AS A STOCK

Question: How can we arrange the different degrees of Many?

Answer: By counting, by bundling and by stacking: First the total is lined up, then it is bundled and equal bundles are stacked and finally the height is counted as e.g. $T = 3 \times 4 = 3 \times 4$.

Examples:

Leftovers are arranged in a separate stack creating a stock:

Or the 3 leftovers are counted in 4s: $3 = 3/4 \times 4$.

We count in 4s by taking away 4s.

The process 'from T take away 4' may be iconized as 'T/4' and worded as 'T minus 4'.

The 4 taken away does not disappear, they are just put aside so the original total T is divided into two totals, one containing T/4 and the other containing 4:

9 = (9/4) + 4 = 2 + 4 as predicted by the 'restack-equation' or 'readd-equation' $T = (T/b) \times b$.

The repeated process 'from T take away 4s' may be iconized as 'T/4' and worded as 'T counted in 4s'. So the 'recount-equation' or 'rebundle-equation' $T = (T/4) \times 4$ predicts the result of recounting the total T in 4-bundles: $T = (T/4) \times 4 = 3 \times 4 + 3 \times 1 = 3 \times 4 + 3 = 12 + 3 = 15$. T/4 is called a per-number, T a stock-number or a total, and 4 a base.

Exercise: Take a lot of matches and count and stack them in 2s, then in 3s, then in 4s, etc.

GRASP by grasping - the LAB approach

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A1 ADDING MANY

Question	Answer
How to add stacks concretely?	By restacking overloads predicted by the restack-equation $T = (T/b) \times b$.
$T = 27 = 16 = 2 \text{ tens} + 1 \text{ ten} = 2 \text{ tens} + 1 \text{ ten} = 2 \times 10 + 1 \times 10 = 2 \times 10 + 1 \times 10 = 20 + 10 = 30$	$2 \text{ tens} + 1 \text{ ten} = 2 \times 10 + 1 \times 10 = 20 + 10 = 30$
How to add stacks abstractly?	Vertical calculation uses carrying. Horizontal calculation uses FOIL.

1 STACKS ARE SOLD

Question: How can we sell more from a stack than we have?

Answer: Create an overload by recounting and doing internal trade.

Example: From the stock $T = 3 \times 5 + 2 \times 1$ we want to sell 3 1s, but we only have 2 1s in stock. However we can perform an 'internal trade' between the 5-stack and the 1-stack trading 1 5s to 5 1s:

After the matches we use cups and internal trade to write $T = 3 \times 5 = 3 \times (2 \times 5) = 3 \times 10 = 30$ and $2 \times 1 = 2$ so $T = 30 + 2 = 32$.

Or: $T = 3 \times 5 = 3 \times 5 = 15$ and $2 \times 1 = 2$ so $T = 15 + 2 = 17$.

In case of ten we have $T = 3 \times 5 = 15$ and $2 \times 1 = 2$ so $T = 15 + 2 = 17$.

Exercise 1: Sell 3 from 41. Sell 34 from 421. Sell 342 from 4231. Count in fives. First use matches, then write.

Exercise 2: Sell 3 from 41. Sell 34 from 421. Sell 342 from 4231. Count in tens. First use matches, then write.

2 STACKS ARE BOUGHT

Question: How can stacks be added?

Answer: Remove the overload by recounting and doing internal trade.

Example: To the stock $T = 2 \times 5 + 4 \times 1$ we add the stock $T' = 1 \times 5 + 3 \times 1$. After adding the 1s we are able to recount 7 1s to 1 5s + 2 1s, as predicted by the restack-equation: $T = 7 \times (7/5) + 5 = 1 \times 5 + 2$.

After the matches we use cups and internal trade to write $T = 24 + 13 = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

Or: $T = 24 + 13 = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

In case of ten we have $T = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

Exercise 1: Add 3 to 24. Add 43 to 34. Add 241 to 444. Count in fives. First use matches, then write.

Exercise 2: Add 8 to 24. Add 79 to 34. Add 879 to 444. Count in tens. First use matches, then write.

3 STOCKS ARE SPLIT

Question: How can stocks be split?

Answer: Create an overload by recounting and doing internal trade.

Example: The stock $T = 3 \times 5 + 4 \times 1$ is split in two parts.

After the matches we use cups and internal trade to write $T = 24 + 13 = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

Or: $T = 24 + 13 = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

In case of ten we have $T = 24 + 13 = 37$ and $3 \times 5 = 15$ so $T = 37 + 15 = 52$ and $4 \times 2 = 8$ so $T = 52 + 8 = 60$.

Exercise 1: Split 43 in 2. Split 43 in 3. Split 34 in 4. Split 43 in 5. Count in fives. First use matches, then write.

Exercise 2: Split 43 in 2. Split 43 in 3. Split 34 in 4. Split 43 in 5. Count in tens. First use matches, then write.

GRASP by grasping - the LAB approach

MATHECADEMY.net

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T1 COUNT&ADD IN TIME

Question	Answer
How can counting & adding be reversed?	By calculating backward moving a number to the other side reversing its calculation sign.
Counting '3s' and adding '2' gives 14.	$3 \times 3 + 2 = 14$ is reversed to $14 = (14/3) \times 3$.
Can all calculations be reversed?	Yes: $10 \times 10 = 100$ is reversed to $100 = 10 \times 10$, 10×10 is reversed to $10 = 100/10$, 10×10 is reversed to $10 = 100/10$.

1 REVERSED CODING

Question: How can we decode a coded number?

Answer: Use reversed calculations, also called solving equations.

Example:

Coding hides the bundle-size: $T = 3 \times 3 + 1 \rightarrow T = 3 \times 3 + 1$. A table can be used to guess the Total when coded. The table can be drawn as a graph.

x	0	1	2	3	4	5
T = 3x + 1	1	4	7	10	13	16

A decoding can take place in three steps:

- First the coding $x + 3 = 5$ is decoded by restacking: From the 5-stack we take away 3 to a new stack leaving $5 - 3 = 2$ in the original stack as predicted by the restack-equation $T = (T - 3) \times 3$. $T = 5 = (5 - 3) \times 3 = 2 \times 3$.
- Next the coding $2 \times x = 6$ is decoded by recounting: The 6 is recounted to 3 2s and overturned to 2 3s as predicted by the recount-equation $T = (T/2) \times 2$. $T = 6 = (6/2) \times 2 = 3 \times 2$.
- Finally the coding $2 \times x = 1$ is decoded. First we restack 7 by taking away 1: $7 = (7 - 1) \times 1 = 6 \times 1$. Then the 6 is recounted in 2s and overturned.

After the result is predicted by applying both the restack-equation and the recount-equation.

Remark: The recount-equation and the restack-equation show directly that equations are solved when moving a number to the other side of the equation sign reversing its calculation sign:

Equation	Restacking	Recounting
$T = 3 \times 3 + 1$	$T = (T - 3) \times 3$	$T = (T/2) \times 2$
$T = 3 \times 3 + 1$	$T = (T - 3) \times 3$	$T = (T/2) \times 2$

Exercise 1: Decode $2 \times x = 10$, $3 \times x = 17$, $4 \times x = 19$, $5 \times x = 21$. First use matches, then write.

Exercise 2: Decode $2 \times x = 6$, $3 \times x = 7$, $4 \times x = 11$, $5 \times x = 13$. First use matches, then write.

Exercise 3: Decode $x \times 2 = 6$, $x \times 3 = 7$, $x \times 2 = 6$, $x \times 3 = 7$, $x \times 2 = 6$, $x \times 3 = 7$. First use matches, then write.

GRASP by grasping - the LAB approach

MATHECADEMY.net

8,27 x 11,70 in

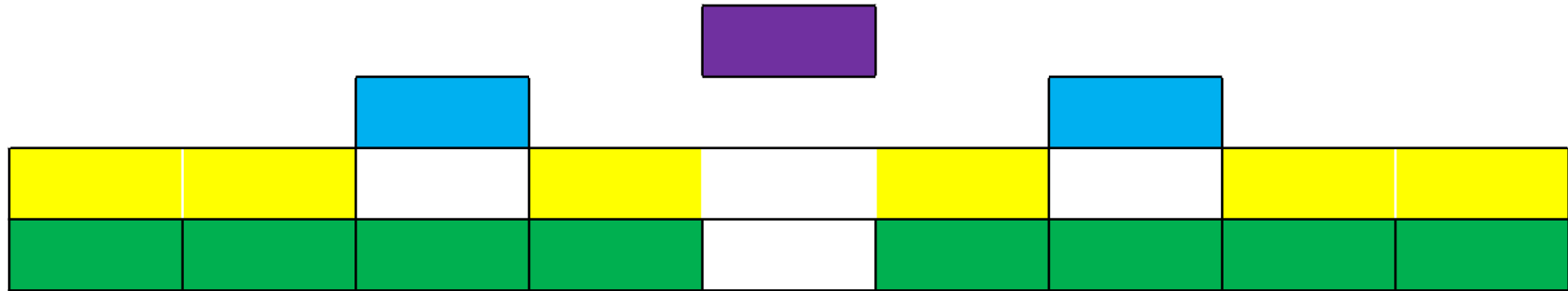
PYRAMIDeDUCATION

To learn MATH: **C**ount&**A**dd MANY
Always ask Many, not the Instructor
 MATHeCADEMY.net - a VIRUS**e**CADEMY

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve **C**ount&**A**dd problems.
- The coach assists the instructors when instructing their team and when correcting the **C**ount&**A**dd assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
 2 Instructors
 3 Pairs
 2 Teams



When using Theory, Beware of Disagreements

	TopDown	BottomUp
Philosophy	Plato essentialism	Sartre existentialism
Psychology	Vygotsky essence-teaching	Piaget existence-meeting
Sociology	German institutional idealism	French/American institutional skepticism
Research	MetaPhysical theory exemplification	Physical grounded theory creation

Main Point: **Think Things** - don't **Echo Essence**

- No, 5×7 is not 35. It is 5 **7s**, that might be recounted as 4.3 **8s** or as 3.5 **tens**.
- No, $65/7$ is not 65 split between 7. It is 6.5 **tens** recounted in **7s** which of course makes the block-number thinner and higher.
- No, $1/3$ is not a number. It is an operator needing a number to become a number, e.g. $1/3$ of 6.
- No, 5 is not a number. It is an operator needing a number to become a number, e.g. 5 **7s**.
- Don't teach children 1D numbers. They already know 2D numbers.

Main Main Point:

CupCount before you Add, Respect the Child's own 2D Numbers





To Improve Math Education

BEWARE of Goal-Means Confusions
UNITE its roots: Algebra & Geometry
RESPECT & Develop the Child's own 2D Numbers

CupCount before you **Add**
Calculus before OnTop **Addition**

ByeBye to MetaMatism
Welcome to ManyMath

Thank You for Your Time

Allan.Tarp@MATH**e**CADEMY.net
Free 1Day Skype Teacher Seminar
Free Uni Franchise

Solving Equations BottomUp or TopDown

ManyMath

$2 + u = 5 = (5-2) + 2$	Solved by re-stacking 5	$2 \times u = 5 = (5/2) \times 2$	Solved by re-bundling 5
$u = 5-2 = 3$	Test: $2 + 3 = 5$ OK	$u = 5/2 = 2\frac{1}{2}$	Test: $2 \times 3 = 6$ OK

MatheMatics

$2 + u = 5$	Addition has 0 as its neutral element, and 2 has -2 as its inverse element
$(2 + u) + (-2) = 5 + (-2)$	Adding 2's inverse element to both number-names
$(u + 2) + (-2) = 3$	Applying the commutative law to $u + 2$, 3 is the short number-name for $5+(-2)$
$u + (2 + (-2)) = 3$	Applying the associative law
$u + 0 = 3$	Applying the definition of an inverse element
$u = 3$	Applying the definition of a neutral element. <i>With arrows a test is not needed.</i>

No ReCounting: Bye to Golden Math Opportunities

No Icon Creation	So, as letters, digits are just symbols to be learned by heart
Only Counting in tens	T = 2.3 tens = 23; oops, no unit & misplaced decimal point
No ReCounting in the Same Icon	So 37 is no more 2)17 or 4.-3
No ReCounting in a Different Icon	No more 3 x 5 is 3 5s , but 15, postponed to Multiplication No more 24 = ? 3s . Instead we ask 24/3, postponed to Division
No Adding NextTo	Postponed to Integral Calculus
No Reversed Adding NextTo	Postponed to Differential Calculus, made difficult by being taught before Integral Calculus
Only Adding OnTop	No CupWriting: $24 + 58 = 7)12.$ Only Carrying: $7^12 = 82$ No CupWriting: $74 - 39 = 4)-5 = 35.$ Only Carrying: $74 = 6^{10}4$
No Reversed Adding OnTop	Postponed to Solving Equations

Dienes on Place Value and MultiBase Blocks

“The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. (..)

In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..)

Educators today use the “multibase blocks”, but most of them only use the base ten, yet they call the set “multibase”. These educators miss the point of the material entirely.”

(What is a base?, <http://www.zoltandienes.com/academic-articles/>)

Yes, Recounting looks like Dienes Blocks, but ...

Dienes teaches the 1D place value system with 3D, 4D, etc. blocks to illustrate the importance of the power concept.

- ManyMath teaches decimal numbers with units and stays with 2D to illustrate the importance of the block concept and adding areas.

Dienes wants to bring examples of abstractions to the classroom

- ManyMath wants to build abstractions from outside examples

Dienes teaches top-down 'MetaMatics' derived from the concept Set

- ManyMath teaches a bottom-up natural science about the physical fact Many; and sees Set as a meaningless concept because of Russell's set-paradox.

1D Roman Numbers and 2D Arabic Numbers

To see the difference we write down a total T of **six scores** and a **dozen**:

- $T = \text{XX XX XX XX XX XX} + \text{XII} = \text{CXXXII}$,
- $T = 6 \text{ 20s} + 1 \text{ 12s} = 1 * \text{BB} + 3 * \text{B} + 2 * 1 = 132$, where Bundle = ten

Both systems use bundling to simplify.

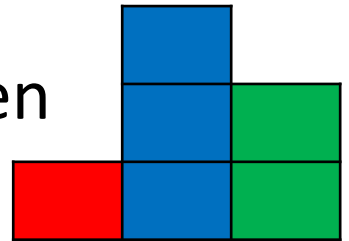
The Roman uses a 1D juxtaposition of different bundle sizes.

The Arabic uses one bundle size only.

More bundles are described by multiplication: $3 * \text{B}$, i.e. as 2D areas.

Bundle-of-bundles are described by power: $1 * \text{BB} = 1 * \text{B}^2$.

Totals are described by next-to addition of 2D area blocks (integration).



Creating or Curing Dislike/DysCalCulia

Having problems learning mathematics has many names: Difficulty, disability, disorder, dislike, deficiency, low attainment, low performance or DysCalCulia.

How to Create it	How to Cure it
<ul style="list-style-type: none"> ● Teach 1D LineNumbers as '8' ● No Counting before Adding ● Adding before Multiplying ● Adding without Units: $2+3=5$ 	<ul style="list-style-type: none"> ● Teach 2D BlockNumbers as '2 4s' ● CupCounting before Adding ● Multiplying before Adding ● Adding with Units: $2\mathbf{w}+3\mathbf{d}=17\mathbf{d}$

Scholastic, Patronizing & Grounded Mathematics Education Research

Scholastic research hides alternatives through discourse-protection and self-reference thus presenting its **choice** as nature.

Patronizing research sees the institution as rational and the agent as irrational. Thus math education problems lies with the agents.

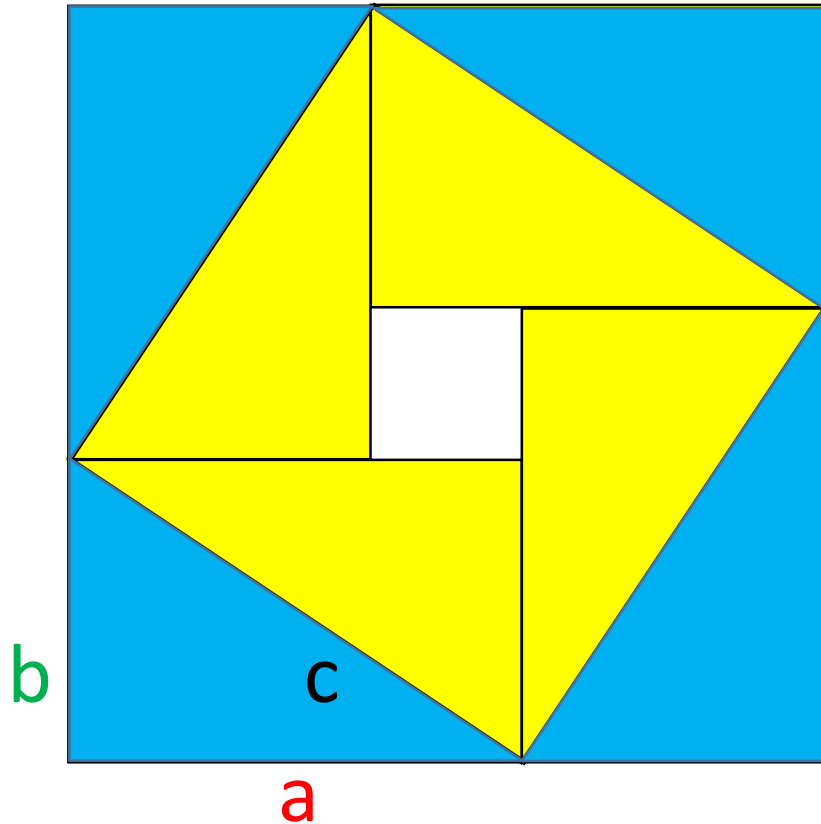
Grounded research sees the problems lying with the institutions

- North America: Focusing on the agents, look for hidden rationality behind apparent irrationality
- France: Focusing on the institutions, look for hidden irrationality behind apparent rationality

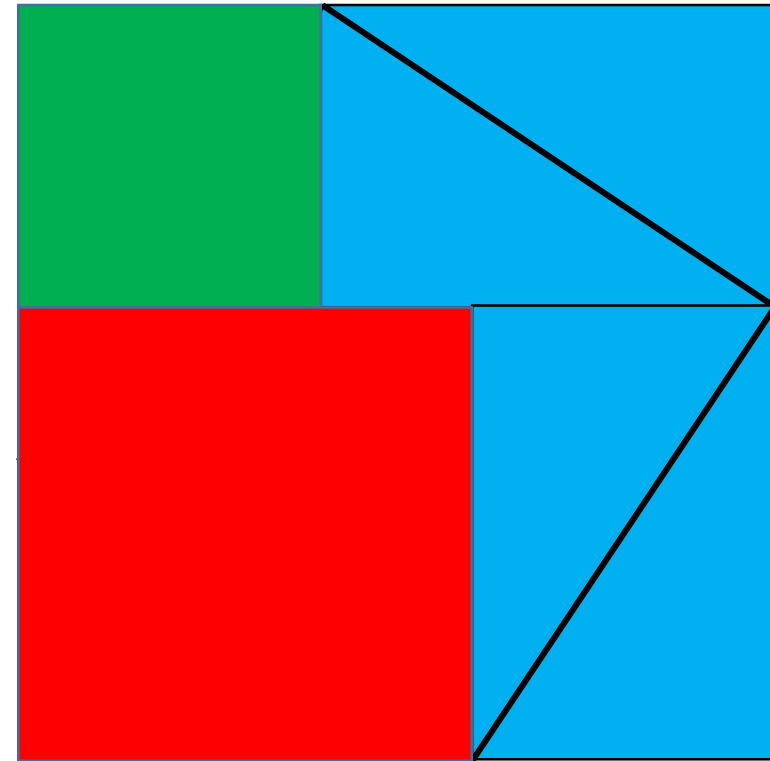
MatheMatics: Unmask Yourself, Please

- In Greek you mean 'knowledge'. You were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic. Later only 2 activities remained: Geometry and Algebra
- Then self-referering Set transformed you from a Natural Science about the physical fact Many to a metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism
- So please, unmask your true identity, and tell us how you would like to be presented in education: Self-referring MetaMatism for the few - or grounded ManyMath for the many

Pythagoras shown by 4 Cards with Diagonals



$$c^2 + 4 \frac{1}{2} \text{cards}$$



$$a^2 + b^2 + 2 \text{ cards}$$