CERME 10 Paper

with Reviews

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Rejected, also as a poster, even if recommended by both reviewers

> "Papers may be accepted for presentation in the conference but not for publication if, for example, they do not achieve academic quality adequate for publication, but nevertheless raise interesting or novel ideas that are relevant for work group discussions. For this reason and due to the policy of inclusion, a rejection of a paper for presentation should be an exception."
>
> (CERME 10 Review Guidelines, second page)

Cupcounting and calculus in preschool and in special needs education

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To improve PISA results, institutional skepticism rethinks mathematics education to search for hidden alternatives to choices institutionalized as nature. Rethinking preschool and primary school mathematics uncovers cup-counting in bundles less than ten; as well as re-counting to change the unit, later called proportionality, and next-to addition, later called integration. As to ICT, information and communications technology, a calculator can predict re-counting results before being carried out manually. By allowing overloads and underloads when re-counting in the same unit, cup-writing takes the hardness out of addition, subtraction, multiplication and division. This offers preschool students a good start and special needs students a new start when entering or reentering ordinary classes only allowing ten-counting and on-top addition to take place.

Keywords: Numbers, numeracy, addition, calculus, elementary school mathematics.

Decreased PISA performance despite increased research

Being highly useful to the outside world, mathematics is one of the core parts of institutionalized education. Consequently, research in mathematics education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 year since 1969. Likewise, funding has increased as witnessed e.g. by the creation of a National Center for Mathematics Education in Sweden. However, despite increased research and funding, the former model country Sweden has seen its PISA result in mathematics decrease from 509 in 2003 to 478 in 2012, the lowest in the Nordic countries and significantly below the OECD average at 494. This caused OECD to write the report 'Improving Schools in Sweden' describing the Swedish school system as being 'in need of urgent change' (OECD, 2015).

Created to help students cope with the outside world, schools institutionalize subjects as inside means to outside goals. To each goal there are many means, to be replaced if not leading to the goal; unless a means becomes a goal itself, thus preventing looking for alternative means that could lead to the real goal if difficult to access. So we can ask: Does mathematics education have a goal-means exchange seeing inside mathematics as the goal and the outside world as a means?

Once created as a means to solve an outside problem, not solving the problem easily becomes a means to necessitate the institution. So to avoid a goal/means exchange, an institution must be reminded constantly about its outside goal. Institutional skepticism is created to do precisely that.

Institutional skepticism

The ancient Greek sophists saw enlightenment as a means to avoid hidden patronization by Plato philosophy presenting choices as nature. Inspired by this, institutional skepticism combines the skepticism of existentialist and postmodern thinking. The 1700 Enlightenment century created two republics, one in North America and one in France. In North America, the sophist warning against hidden patronization is kept alive by American pragmatism, symbolic interactionism and Grounded theory (Glaser & Strauss, 1967), the method of natural research resonating with Piaget's principles of natural learning (Piaget, 1970). In France, skepticism towards our four fundamental institutions, words and sentences and cures and schools, is formulated in the poststructuralist thinking of Derrida, Lyotard, Foucault and Bourdieu warning against institutionalized categories, correctness, diagnoses, and education all presenting patronizing choices as nature (Lyotard, 1984; Tarp, 2004).

Building on Kierkegaard, Nietzsche and Heidegger, Sartre defines existentialism by saying that to existentialist thinkers 'existence precedes essence, or (..) that subjectivity must be the starting point' (Marino, 2004, p. 344). Kierkegaard was skeptical to institutionalized Christianity seen also by Nietzsche as imprisoning people in moral serfdom until someone 'may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.' (Marino, 2004, pp. 186–187). Inspired by Heidegger, Arendt divided human activity into labor and work aiming at survival and reproduction, and action focusing on politics, creating institutions to be treated with utmost care to avoid the banality of evil by turning totalitarian (Arendt, 1963).

Since one existence gives rise to many essence-claims, the existentialist distinction between existence and essence offers a perspective to distinguish between one goal and many means.

Mathematics as essence

In ancient Greece the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas. With astronomy and music as independent knowledge areas, today mathematics is a common label for the two remaining activities, geometry and algebra (Freudenthal, 1973) both rooted in the physical fact Many through their original meanings, 'to measure earth' in Greek and 'to reunite Many' in Arabic.

Then the invention of the concept SET allowed mathematics to be a self-referring collection of 'well-proven' statements about 'well-defined' concepts, i.e. as 'MetaMatics', defined top-down as examples from abstractions instead of bottom-up as abstractions from examples. However, by looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false: If $M = \{A \mid A \notin A)\}$ then $M \in M \Leftrightarrow M \notin M$. The Zermelo–Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract essence. Thus SET transformed grounded mathematics into a self-referring 'MetaMatism', a mixture of MetaMatics and 'MatheMatism' true inside a classroom but not outside where claims as '1 + 2 IS 3' meet counter-examples as e.g. 1 week + 2 days is 9 days. And, as expected, teaching numbers without units and meaningless self-reference creates learning problems.

Mathematics as existence

Chosen by the Pythagoreans as a common label, mathematics has no existence itself, only its content has, algebra and geometry. Algebra contains four ways to unite as shown when writing out fully the total T = 342 = 3*B^2 + 4*B + 2*1 = 3 bundles of bundles and 4 bundles and 2 unbundled singles = 3 blocks. Here we see that we unite by using on-top addition, multiplication, power and next-to addition, called integration, each with a reversing splitting operation. So, with a human need to describe the physical fact Many, algebra was created as a natural science about Many. To deal with Many, we count by bundling and stacking. But first we rearrange sticks in icons. Thus five ones becomes one five-icon 5 with five sticks if written less sloppy. In this way we create icons for numbers until ten since we do not need an icon for the bundle-number as show when counting in fives: one, two, three, four, bundle, one bundle and one, one bundle and two etc.

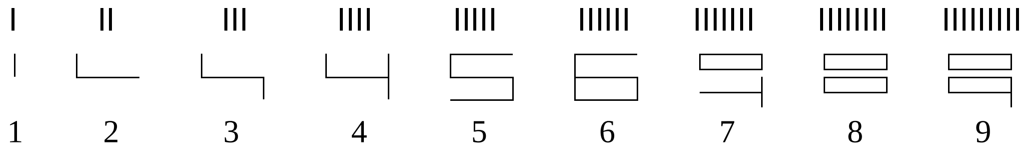
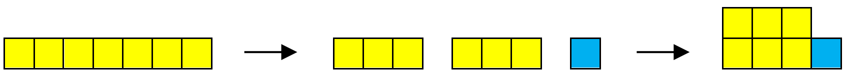


Figure 1: Digits as icons containing as many sticks as they represent

Holding 4 fingers together 2 by 2, a 3year old child will say ‘That is not 4, that is 2 2s. This inspires ‘cup-counting’ bundling a total in icon-bundles. Here a total T of 7 1s can be bundled in 3s as T = 2 3s and 1 where the bundles are placed in a bundle-cup with a stick for each bundle, leaving the unbundled outside. Then we describe by icons, first using ‘cup-writing’, T = 2)1, then using ‘decimal-writing’ with a decimal point to separate the bundles from the unbundled, and including the unit 3s, T = 2.1 3s. Moving a stick outside or inside the cup changes the normal form to overload or underload form. Also, we can use plastic letters as B and C for the bundles.

T = 7 = IIIIIII → III III I → II) I → 2)1 3s = 1)4 3s = 3)-2 3s or BBI → 2BI

Using squares or LEGO blocks or an abacus, we can stack the 3-bundles on-top of each other with an additional stack of unbundled 1s next-to, thus showing the total as a double stack described by a cup-number or a decimal number, T = 7 = 2 3s & 1 = 2)1 3s = 2.1 3s.



We live in space and in time. To include both when counting, we introduce two different ways of counting: in space, geometry-counting, and in time, algebra-counting. Counting in space, we count blocks and report the result on a ten-by-ten abacus in geometry-mode, or with squares. Counting in time, we count sticks and report the result on a ten-by-ten abacus in algebra-mode, or with strokes.

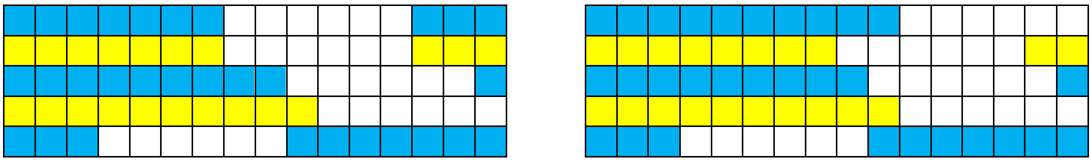


Figure 2: 7 counted in 3s on an abacus in geometry mode and in algebra mode

To predict the result we use a calculator. A stack of 2 3s is iconized as 2*3, or 2x3 showing a lift used 2 times to stack the 3s. As for the two icons for taking away, division shows the broom wiping away several times, and subtraction shows the trace left when taking away just once.

Thus by entering ‘7/3’ we ask the calculator ‘from 7 we can take away 3s how many times?’ The answer is ‘2.some’. To find the leftovers we take away the 2 3s by asking ‘7 – 2*3’. From the answer ‘1’ we conclude that 7 = 2)1 3s. Likewise, showing ‘7 – 2*3 = 1’, a display indirectly predicts that 7 can be recounted as 2 3s and 1, or as 2)1 3s.

7 / 3	2.some
7 – 2 * 3	1

A calculator thus uses a ‘recount-formula‘, T = (T/B)*B, saying that ‘from T, T/b times Bs can be taken away’; and a ‘restack-formula’, T = (T–B)+B, saying that ‘from T, T–B is left if B is taken away and placed next-to’. The two formulas may be shown by using LEGO blocks.

Re-counting in the same unit and in a different unit

Once counted, totals can be re-counted in the same unit, or in a different unit. Recounting in the same unit, changing a bundle to singles allows recounting a total of 4 2s as 3)2 2s with an outside overload; or as 5)-2 2s with an outside underload thus leading to negative numbers:

Letters	Sticks	Total T =	Calculator	
B B B B	II II II II	4)0 2s	4*2 – 4*2	0
B B B I I	II II II I I	3)2 2s	4*2 – 3*2	2
B B B B B <u>B</u>	II II II II II <u>II</u>	5)-2 2s	4*2 – 5*2	-2

Figure 3: Recounting 4 2s in the same unit creates overload or underload

To recount in a different unit means changing unit, also called proportionality or linearity. Asking ‘3 4s is how many 5s?’ we can use sticks or letters to see that 3 4s becomes 2)2 5s.

IIII IIII IIII → IIIII IIIII I I → 2)2 5s. With letters, C = BI so that BBB → BB IIII → CC II

A calculator can predict the result. Entering ‘3*4/5’ we ask ‘from 3 4s we take away 5s how many times?’ The answer is ‘2.some’. To find the leftovers we take away the 2 5s and ask ‘3*4 – 2*5’. Receiving the answer ‘2’ we conclude that 3 4s can be recounted as 2 5s and 2, or as 2)2 5s.

3 * 4 / 5	2.some
3 * 4 – 2 * 5	2

Double-counting creates proportionality as per-numbers

Counting a quantity in 2 different physical units gives a ‘per-number’ as e.g. 2$ per 3kg, or 2$/3kg. To answer the question ‘6$ = ?kg’ we use the per-number to recount 6 in 2s: 6$ = (6/2)*2$ = 3*3kg = 9kg. And vice versa: Asking ‘?$ = 12kg’, the answer is 12kg = (12/3)*3kg = 4*2$ = 8$.

Once counted, totals can be added on-top or next-to.

Asking ‘3 5s and 2 3s total how many 5s?’ we see that to be added on-top, the units must be the same, so the 2 3s must be recounted in 5s as 1)1 5s that added to the 3 5s gives a total of 4)1 5s.

IIIII IIIII IIIII III III → IIIII IIIII IIIII IIIII I → 4)1 5s. With letters: 3B + 2C = 3B III III = 4BI.

Using a calculator to predict the result, we use a bracket before counting in 5s: Asking ‘(3*5 + 2*3)/5’, the answer is 4.some. Taking away 4 5s leaves 1. Thus we get 4)1 5s.

(3 * 5 + 2 * 3)/ 5	4.some
(3 * 5 + 2 * 3) – 4 * 5	1

Since 3*5 is an area, adding next-to means adding areas called integration. Asking ‘3 5s and 2 3s total how many 8s?’ we use sticks to get the answer 2)5 8s.

IIIII IIIII IIIII III III → IIIII III IIIII III IIIII → 2)5 8s → 2.5 8s

Using a calculator to predict the result we include the two totals in a bracket before counting in 8s: Asking ‘(3*5 + 2*3)/8’, the answer is 2.some. Taking away the 2 8s leaves 5. Thus we get 2)5 8s.

(3 * 5 + 2 * 3)/ 8	2.some
(4 * 5 + 2 * 3) – 2 * 8	5

Reversing adding on-top and next-to

Reversed addition is called backward calculation or solving equations. Reversing next-to addition is called reversed integration or differentiation. Asking ‘3 5s and how many 3s total 2)6 8s?’, using sticks will give the answer 2)1 3s:

IIIII IIIII IIIII III III I ← IIIII III) IIIII III) IIIIII ← 2)6 8s

Using a calculator to predict the result the remaining is bracketed before counted in 3s. Adding or integrating two stacks next-to each other means multiplying before adding. Reversing integration means subtracting before dividing, as shown in the gradient formula y’ = Δy/t = (y2 – y1)/t.

(2 * 8 + 6 – 3 * 5)/ 3	2
(2 * 8 + 6 – 3 * 5) – 2 * 3	1

Primary schools use ten-counting only

In primary school numbers are counted in tens to be added, subtracted, multiplied and divided. This leads to questions as ‘3 4s = ? tens’. Using sticks to de-bundle and re-bundle shows that 3 4s is 1.2 tens. Using the recount- and restack-formula above is impossible since the calculator has no ten button. Instead it is programmed to give the answer directly in a short form that leaves out the unit and misplaces the decimal point one place to the right, strangely enough called a ‘natural’ number.

3 * 4	12

Recounting icon-numbers in tens is called doing times tables to be learned by heart. So from grade 1, 3*4 is not 3 4s any more but has to be recounted in tens as 1.2 tens, or 12 in the short form.

Recounting tens in icons by asking ‘38 = ? 7s’ is predicted by a calculator as 5.3 7s, i.e. as 5*7 + 3. Since the result must be given in tens, 0.3 7s must be written in fraction form as 3/7 and calculated as 0.428…, shown directly by the calculator, 38/7 = 5.428…

38 / 7	5.some
38 – 5 * 7	3

Without recounting, primary school labels the problem ‘38 = ? 7s’ as an example of a division, 38/7, which is hard to many, or as an equation ’38 = x*7’ to be postponed to secondary school.

Designing a micro-curriculum

With curriculum architecture as one of its core activities, the MATHeCADEMY.net was asked to design a micro-curriculum understandable and attractive to teachers stuck with division problems; and allowing special need students to return to their ordinary class. Two were designed.

In the ‘1 cup and 5 sticks’ micro-curriculum, 5 is cup-counted in 2s as 1)3 2s or 2)1 2s or 3)-1 2s to show that a total can be counted in 3 ways: overload, normal or underload with an inside and an outside for the bundles and singles. So to divide 336 by 7, 5 bundles are moved outside as 50 singles to recount 336 with an overload: 336 = 33)6 = 28)56, which divided by 7 gives 4)8 = 48.

Besides the ‘Cure Math Dislike by 1 cup and 5 sticks’, 8 extra micro-curricula were designed (mathecademy.net/preschool/) where cup-counting involves division, multiplication, subtraction and later next-to and on-top addition, in contrast to primary school that turns this order around and only allows on-top addition using carrying instead of overloads. Thus, if using cup-writing with overloads or underload instead of carrying, the order of operations can be turned around to respect that totals must be counted before being added.

<table>
<tr><th></th><th>Carry</th><th>Cup-writing</th><th>Words</th></tr>
<tr><td>Add</td><td>1
4 5
1 7
6 2</td><td>4)5
1)7
5)12
6)2 = 62</td><td>4 ten 5
1 ten 7
5 ten 12
5 ten 1 ten 2
6 ten 2 = 62</td></tr>
<tr><td>Subtract</td><td>1
4 5
1 7
2 8</td><td>4)5
1)7
3)-2
2)10-2 = 2)8 = 28</td><td>4 ten 5
1 ten 7
3 ten less2
2 ten 8 = 28</td></tr>
<tr><td>Multiply</td><td>4
2 6 * 7
1 8 2</td><td>7 * 2)6
14)42
18) 2 = 182</td><td>7 times 2 ten 6
14 ten 42
14 ten 4 ten 2
18 ten 2 = 182</td></tr>
<tr><td>Divide</td><td>2 4 rest 1
3 | 7 3
6
1 3
1 2
1</td><td>7)3 counted in 3s
6)13
6)12 + 1
2 3s)4 3s + 1
24 3s + 1
73 = 24*3 + 1</td><td>7ten3
6ten 13
6ten12 + 1
3 times 2ten4 + 1
3 times 24 + 1</td></tr>
</table>

Figure 4: Cup-writing with overloads and underloads instead of carrying

In the first micro-curriculum the learner uses sticks and a folding rule to build the number-icons up to nine; and uses strokes to draw them thus realizing there are as many sticks and strokes in the icon as the number it represents, if written less sloppy. In the second the learner counts a given total in icons by bundling sticks and using a cup for the bundles; and reporting first with cup-writing and decimal numbers with a unit; then by using an abacus in algebra and geometry mode. In the third the learner recounts a total in the same unit thus experiencing creating or removing overloads and underloads. In the fourth the learner recounts a total in a different unit. In the fifth the learner adds two icon-numbers on-top of each other. In the sixth the learner adds two icon-numbers next-to each other. In the seventh the learner reverses on-top addition. And in the eights, the learner reverses next-to addition. Finally, the learner sees how double-counting creates per-numbers.

	Examples	Calculator prediction	
M2	7 1s is how many 3s? I I I I I I I → III III I → 2)1 3s → 2.1 3s	7/3 7 – 2*3	2.some 1
M3	'2.7 5s is also how many 5s?' IIIII IIIII IIIIIII = V V V II = V V V V III 2)7 = 2+1)7–5 = 3)2 = 3+1)2–5 = 4)-3 So 2.7 5s = 3.2 5s = 4.-3 5s	(2*5+7)/5 (2*5+7) – 3*5 (2*5+7) – 4*5	3.some 2 -3
M4	2 5s is how many 4s?' IIIII IIIII = IIII I IIII I = IIII IIII I I So 2 5s = 2.2 4s	2*5 / 4 2*5 – 2*4	2.some 2
M5	'2 5s and 4 3s total how many 5s?' IIIII IIIII III III III III = V V V V I I So 2 5s + 4 3s = 4.2 5s	(2*5+4*3) /5 (2*5+4*3) – 4*5	4.some 2
M6	'2 5s and 4 3s total how many 8s?' IIIII IIIII III III III III = IIIIIIII IIIIIIII III III So 2 5s + 4 3s = 2.6 8s	(2*5+4*3) /8 (2*5+4*3) – 2*8	2.some 6
M7	'2 5s and ? 3s total 4 5s?' IIIII IIIII IIIII IIIII = IIIII IIIII III III III I so 2 5s + 3.1 3s = 4 5s	(4*5 – 2*5)/3 (4*5 – 2*5) – 3*5	3.some 1
M8	'2 5s and ? 3s total how 2.1 8s?' IIIIIIII IIIIIIII I = IIIII III IIIII III I so 2 5s + 2.1 3s = 2.1 8s	(2*8+1 – 2*5)/3 (2*8+1 – 2*5) – 2*8	2.some 1

Figure 5: A calculator predicts counting and adding results

One curriculum used silent education where the teacher demonstrates and guides through actions only, not using words; and one curriculum was carried out by a substitute teacher speaking a foreign language not understood by the learner. In both cases the abacus and the calculator quickly took over the communication. For further details watch www.youtube.com/watch?v=IE5nk2YEQIA.

After these micro-curricula a learner went back to her grade 6 class where proportionality created learning problems. The learner suggested renaming it to double-counting but the teacher insisted on following the textbook. However, observing that the class took over the double-counting method, he finally gave in and allowed proportionality to be renamed and treated as double-counting. When asked what she had learned besides double-counting both learners and the teacher were amazed when hearing about next-to addition as integration.

Thus cup-counting and a calculator for predicting recounting results allowed the learner to reach the outside goal, mastering Many, by following an alternative to the institutionalized means that because of a goal-means exchange had become a stumbling block to her; and performing and reversing next-to addition introduced her to and prepared her for later calculus classes.

Literature on cup-counting

No research literature on cup-counting was found. Likewise, it is not mentioned by Dienes (1964).

Conclusion and recommendation

As to theory, two genres exist; a master genre exemplifying existing theory, and a research genre developing new theory by including a question and a theoretical guidance to a valid answer based upon analyzing reliable data. To avoid indifference, this paper addresses the OECD report 'Improving schools in Sweden' by asking if mathematics education might have a goal-means exchange. As theoretical guidance, institutional skepticism allows using the existentialist existence-versus-essence distinction to distinguish outside goals from inside means, which leads to asking when mathematics is respectively existence and essence. Analyzing traditional math shows that by being set-based and by adding numbers without units, its concepts and statements are unrooted and little applicable to the outside world, thus being primarily essence. Then grounded theory helps showing how mathematics looks like if grounded in its physical root, Many. To tell the difference, two names are coined, 'ManyMatics' versus 'MetaMatism' mixing 'MetaMatics' defining concepts as examples of abstractions instead of as abstractions from examples, with 'MatheMatism' valid only inside classrooms. To validate its findings and again to avoid indifference, the paper includes a classroom test of a micro curriculum described in details to allow it to be tested in other classrooms. Its originality should welcome the paper for publishing since no literature on ManyMatics exists.

So, if a research conference fails to accept the paper for presentation or as a poster, an extra exchange can be added to help solving the paradox that the Swedish problems occur despite increased research and funding: Neglecting a genre analysis might exchange the master and research genres with the consequence that peer-review becomes unable to accept groundbreaking new paradigms. Such research conferences include master papers that, although career promoting, are unable to uncover alternative, hidden ways to guide solving problems in mathematics education.

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CERME10 Review Guidelines

Paper Submissions

The CERME10 Paper Review Form can be downloaded here. Make a copy and use it for all your Thematic Working Group paper reviews. The following rules apply to all types of paper submissions (for poster proposals, see later in this document).

1. Papers should be about research, significantly related to mathematics and to education, and conform to the aims of CERME (see the General Information on the website). They should be original: i.e., not have been published previously. Contributions need not be limited to completed research. On-going studies may be submitted, provided that theoretical framework and preliminary results are provided in the text submitted. Papers should be concise (maximum 8 pages in the specified format), but must contain all information necessary to inform both reviewers and other researchers. Two types of papers are suitable for CERME: (A) Reports of studies (involving empirical or developmental research) and (B) Theoretical and philosophical essays.

2. To be accepted, papers should meet the review criteria set out below.

3. Authors are requested to refer to related research presented at previous CERME Conferences, as well as other relevant published research. The authors should state what is new in their paper and how it builds on past research, or goes in new directions. Proposals too similar to papers previously presented at international conferences cannot be accepted. Proposals that represent new and significant contributions to research in any aspect of mathematics education are especially welcome.

4. Since all development and research is conducted in a specific theoretical, scientific and cultural context, the paper should shortly specify its context with an international audience in mind. For instance, while the use of English as a common language is often practical, it also necessitates a certain vigilance to avoid implicit generalization, or suppression, of various local conditions. The reviewer must check that the author(s) make explicit the key assumptions underlying the design of the study and explain why your work is relevant in your cultural context (e.g. pedagogy, research environment, ...).

5. Each participant may propose only ONE paper, although a group of authors may propose several papers, each one to be attributed to a different person attending the conference (whose name must be underlined in the paper). Each person can have her/his name underlined at most in one paper OR in one poster proposal.

6. Papers accepted for discussion in any group may be considered also for inclusion in the post-conference proceedings. See below for further details.

7. The format of submitted papers must be as specified in the Guidelines for Authors. Papers not using this format in full will not be accepted.

Timetable for reviews

CERME10 uses a submission website managed by Keynote PCO. Nevertheless the TWG Leader will organize an internal review in the TWG:

- TWG teams send papers to reviewers by 22nd September 2016.
- The reviewers send back (by e-mail) their reviews to TWG Leader by 20th October 2016.

Criteria for review of submitted papers

Papers should

1. Be relevant to the theme of the Thematic Working Group, and

2. Fit quality aspects that are outlined below.

Reviewers are asked to comment on the following aspects of submitted papers:

Reports of Studies (Empirical or Developmental)

Surveys, observational, ethnographic, experimental or quasi-experimental studies, case studies are all suitable. Papers should contain at least the following:

1. a statement about the focus of the paper: what is the question or the problem that is treated?

2. an indication of the theoretical framework of (or theoretical constructs used in) the study reported, including references to the related literature;

3. an indication of the methodology used (including problem, goals and/or research questions; criteria for the selection of participants or sampling; data collection instruments and procedures; data analysis procedures);

4. an indication on the scientific and cultural context in which this study is embedded (explaining crucial assumptions and the possible contingency of the relevance of the study for a specific cultural context);

5. results;

6. final remarks or conclusions, indicating the significance of the paper.

Theoretical and Philosophical Essays

These should include at least the following:

1. a statement about the focus of the paper and a rationale why the study is a relevant one;

2. an indication of the theoretical or philosophical framework within which the focus or theme of the paper is developed; CERME10 Review Guidelines

3. an indication of the scientific and cultural context in which this study is embedded (explaining crucial assumptions and the possible contingency of the relevance of the study for a specific cultural context);

4. reference to related literature;

5. a clearly articulated statement of the author's position on the focus or theme and of the arguments that support this position;

6. implications for the existing research in the area.

Presentation and Publication

Reviewers will make a clear recommendation on each paper, choosing one of the four points of view referring to the acceptance of a paper for the conference:

1. ACCEPT for presentation without further modification

2. ACCEPT for presentation subject to modification as detailed below

3. REJECT but resubmit the paper as a poster

4. REJECT

Papers may be accepted for presentation in the conference but not for publication if, for example, they do not achieve academic quality adequate for publication, but nevertheless raise interesting or novel ideas that are relevant for work group discussions. For this reason and due to the policy of inclusion, a rejection of a paper for presentation should be an exception.

The reviewers are asked to indicate possible changes that should be done before the conference.

The TWG leader team will decide after the conference (based on the work in the group) whether an (the) author(s) should be given the opportunity to revise the 8 pages paper (based on specific modifications to be done) in view of publication in the proceedings, or if the author(s) should instead be offered to submit a shortened, two page version for inclusion in the proceedings (pending final approval of the TWG leaders, as for all other contributions).

Note that all accepted papers will be made available on the CERME website prior to the conference. This process constitutes the paper presentation as a preliminary to its consideration within the relevant Thematic Working Group at the conference.

Poster Proposals

The following rules apply to poster proposals.

1. Poster Presentations are suggested for those whose work is more suitably communicated in a pictorial or graphical format or demonstration, rather than through a traditional written text. A time will be allotted within the conference, during which presenters will be available at their posters for informal discussion with conference participants. CERME10 Review Guidelines

2. Posters should be about research, significantly related to mathematics and to education, and conform to the aims of CERME. They must relate to one of the Thematic Working Groups of CERME10.

3. The format of poster proposals must be as specified in the Guidelines for Authors. Proposals not using this format in full will not be accepted.

4. Each participant may propose only ONE poster, although a group of authors may propose several posters, each one to be attributed to a different person attending the conference (whose name must be underlined in the paper Each person can have her/his name underlined at most in one paper OR in one poster proposal.

Criteria for review of poster proposals TWG Co-leaders will make their decisions about acceptance or rejection on the following aspects of submitted poster proposals:

1. a statement about the focus of the poster;

2. an indication of the theoretical framework (or constructs) of the study reported;

3. an indication of and justification for its content;

4. a statement about the format chosen by the author for the poster;

5. possible implications for the existing research in the area.

Presentation and Publication

For each Poster submitted (this concerns the two-pages proposal), reviewers will make a clear recommendation, choosing between:

1. ACCEPT for presentation without further modification

2. ACCEPT for presentation subject to modification as detailed below

3. REJECT

Concerning the digital posters proposal, the TWG co-leaders must keep in mind the guidelines for authors:

1. Use a presentation software like Powerpoint or similar software

2. The digital poster corresponds to one slide, in A0 size (portrait or landscape)

3. The digital poster must include a title, a short description of the research topic, the theoretical framework, the method, and, if possible, research results and implications based on the research results.

4. Remember, that poster is a visual medium. We recommend using diagrams, tables, and pictures in posters.

If the digital poster does not follow these guidelines, it will need to be adjusted by the authors, following the advice of the TWG co-leaders.

Posters may be followed by a publication in the proceedings. But posters may be accepted for presentation in the conference but not for publication if, for example, they do not achieve CERME10 Review Guidelines academic quality adequate for publication, but nevertheless raise interesting or novel ideas that are relevant for work group discussions.

The reviewers are asked to indicate possible changes that should be done before the conference, in particular concerning the digital version of the poster.

The TWG leader team will decide after the conference (based on the work in the group) whether an (the) author(s) should be given the opportunity to propose a contribution for the proceedings.

CERME10 PAPER REVIEW FORM 1 – empirical Study

For Theoretical/Philosophical Papers, please go to the second version of this form, below.

RETURN BY E-MAIL as soon as possible, but no later than *20th October 2016*

TO: (group leader) …………………………………………………

Thematic Working Group Number and Title:……..……………………………………………………

Title of the paper proposal:……………………………………………………………………………..

Name of the reviewer:……………………………………………….. E-mail address: ……………………….

YOUR REVIEW

Please comment in each of the following sections. In each section use as much space as required.

1. Focus and rationale of the paper

2. Theoretical framework and related literature

3. Methodology

3. Statement and discussion of results

4. Clarity

5. Relevance to this CERME10 TWG audience

My recommendations for presentation at CERME10 are as follows (mark one):

1. ACCEPT for presentation without further modification
2. ACCEPT for presentation subject to modification as detailed below
3. REJECT but resubmit the paper as a poster
4. REJECT

Detailed reasons for the recommendation and general comments (e.g. suggestions for improvement, reasons for rejection/unsuitability):

CERME10 PAPER REVIEW FORM 2 – Theoretical/Philosophical Paper

For Research Study Papers, please go to the first version of this form, above.

RETURN BY E-MAIL as soon as possible, but no later than *20th October 2016*

TO: (group leader) ………………………………………………

Thematic Working Group Number and Title: ……..……………………………………………………

Title of the paper proposal:……………………………………………………………………………..

Name of the reviewer:…………………………………… E-mail address: ……………………

YOUR REVIEW

Please comment in each of the following sections. In each section use as much space as required.

1. Focus and rationale of the paper

2. Theoretical or philosophical framework and related literature

3. Author’s position on the focus or theme

3. Implications for existing research in the area

4. Clarity

5. Relevance to this CERME10 TWG audience

My recommendations for presentation at CERME10 are as follows (mark one):

1. ACCEPT for presentation without further modification
2. ACCEPT for presentation subject to modification as detailed below
3. REJECT but resubmit the paper as a poster
4. REJECT

Detailed reasons for the recommendation and general comments (e.g. suggestions for improvement, reasons for rejection/unsuitability):

CERME10 PAPER REVIEW FORM 2 – Theoretical/Philosophical Paper

TO: (group leader) Rathgeb-Schnierer, Elisabeth

Thematic Working Group Number and Title: Arithmetic and Number Systems

Title of the paper proposal: Cupcounting and calculus in preschool and in special needs education

Name of the reviewer: GM

YOUR REVIEW

Please comment in each of the following sections. In each section use as much space as required.

Preliminary remark

I must confess that I am not sure about which type of paper this could be according to the CERME 10 Review Guideline. While it seems to be clear that the paper cannot be rated as an empirical study, neither does it, as far as I see, convincingly fit the criteria for a theoretical or philosophical essay. Since I had to choose between these two types, I selected the one that seems to be less inappropriate to me.

1. Focus and rationale of the paper

The paper starts with linking the decrease of Sweden's PISA results in mathematics with the worldwide increase in funding and research activities in the field of mathematics education, insinuating that the parallel existence of these two phenomena would indicate a failure of "*the* institution" (or "institutions"? or just "*the* school"?) and a "goal/means exchange" in the sense that schools, instead of preparing children to cope with the mathematics of the outside world, had transformed mathematics from a means into kind of a goal in itself.

What this exactly means and how it refers to the PISA results of Sweden remains unclear to me. Actually, I doubt that there has been a fundamental change in the ways mathematic is being taught in Swedish schools since the first PISA study, and the author himself doesn't claim such a change. But if the alleged "goal/means exchange" is not a new phenomenon (and, as I understand the author, not restricted to Sweden), how could it be responsible for the only recent decline in one nation's overall results in an international large-scale assessment?

The author continues with remarks on what he calls "institutional skepticism", which according to the paper is the adequate response to the development alleged by him in the first section. He claims that there is a long tradition of that very stance starting from the Greek sophists to French post-structuralism and existentialism. He then discerns between "mathematics as essence" and "mathematics as existence", and I have to admit that I am not able to follow his explanations thoroughly enough to report them in a compact form. However, the author seems to maintain that mathematics as it is taught in schools is misled by Zermelo and Frankel's set-theory into a "self-referring 'MetaMatism'"; this seems to be what he calls "mathematics as essence". As an example of this, he states that numbers would institutionally be taught "without units" and in "meaningless self-reference" which would result into children not understanding that 1 week + 2 days are not 3 but 9 days.

What follows under the subtitle "mathematics as existence" might be what the author proposes as an alternative way of teaching arithmetic in primary school, even if the didactical context is not explicitly stated. The author describes activities of bundling items into non-decimal units, what he calls "cup-counting", and creating diverse symbolic notations to record the results of these activities; of re-counting items in the same unit allowing overloads and underloads, leading to the use of negative numbers to record a certain quantity of items; of re-counting using a different unit, what he equates with “proportionality or linearity”; and some more activities including division, multiplication, subtraction and addition to be taught in that very order on the basis of “cup-counting” and “cup-writing with overloads and underloads”. The use of a calculator as a tool to “predict the result” of the bundling activities is mentioned, if not explicitly recommended, for each of these activities.

Finally, the author outlines some “micro-curricula” that implement the above mentioned ideas and are reported to have been tried out with some children. The paper lacks specifications about the nearer circumstances of that trials, e.g.: Number of children? Number of classes? Age and grade? How much time did the children spend with the single stages of the curricula? It is stated that one grade-6 learner who had participated in a trial of these curricula has overcome his “learning problems” in the field of proportionality. However, the paper does not specify what kind of problems this learner had nor what exactly is meant by the assertion that the treatment had allowed him “to reach the outside goal, mastering Many”.

Notably, at no point does the author make clear how and why the activities he describes might possibly help children “cope with the outside world” in a better way when compared to recommendations given by international literature on mathematic education as to how natural numbers, the decimal system, addition, subtraction, multiplication and division should be taught in primary school. In fact, literature and research on early arithmetic teaching and learning are totally ignored throughout the paper, including the existing literature on the assets and drawbacks of engaging primary grade pupils in non-decimal bundling as a means to better understand the base 10 system, which is in no way a new idea.

And, since the author does not clarify in adequate detail which kind of learning difficulties in the field of basic arithmetic he intends to overcome by what he calls “cup-counting”, it also remains unclear how these activities and the apparently demanding symbolic notations and transformations of notations that he suggests together with them could contribute to reducing learning difficulties.

Nevertheless, in his final chapter the authors claims that cup-counting as an “alternative to the ruling tradition” were fit to “soften strong primary school traditions” and thereby “to avoid goal-means exchanges”, once more leaving it up to the reader to fill these abstractions with meaning.

2. Theoretical or philosophical framework and related literature

As mentioned, as far as the paper deals with basic arithmetic education, it completely ignores theoretical as well as empirical work in this very field. When it comes to establishing “institutional skepticism”, the author invokes numerous philosophers from Plato and the Greek sophists over Kierkegaard and Nietzsche to Foucault and Bourdieau. The problem is that the paper doesn’t allow a reader who is not well acquainted with all those scholars to understand for him- or herself why their names are dropped.

3. Author’s position on the focus or theme

As described above, the author’s position remains vague in more than one aspect. Of course, he makes quite clear that he sets out for a fundamental critique on “the ruling tradition”, the “hidden patronization” by “institutionalized” mathematics education. However, neither does he present a precise and detailed account of what he refers to as the “ruling tradition”, to explain why this tradition leads to a “goal/means exchange”, and how this exchange is connected with learning difficulties and the unpleasant PISA results he uses as a starting point. Nor is he sufficiently explicit about what exactly he suggests as an alternative. It remains untold whether the micro-curricula that are shortly outlined are intended to be implemented in regular classrooms or in additional courses, at which grades, and based upon which requirements that children should already have acquired when they are exposed to this kind of instruction.

4. Implications for existing research in the area

On account of the aforementioned fundamental shortcomings of the paper, I doubt it could have any substantial implications for existing research in the area. To contribute to the solution of any theoretical or practical problem, research first of all has to be very clear in identifying and analyzing what is the problem that it intends to solve. I cannot see that the paper at hand meets this precondition. Of course, there has been and is international research – and still is a great need for further research – regarding the acquisition of a sound understanding of the base 10 number system. This might arguably involve research on the preliminary use of non-decimal bundling as a means to foster understanding of bundling as a main idea of arithmetic. However, the paper at hand does not seem to offer any new and promising points of departure for scientific research in this or any other field of mathematics education.

5. Clarity

The paper lacks clarity not in the sense of any linguistic deficiencies, but because of the author's remaining vague in the many aspects already outlined above.

6. Relevance to this CERME10 TWG audience

While the issue of teaching and learning of place value systems and the arithmetic main idea of bundling certainly are of high relevance to the TWG, I have severe doubts that the paper at hand can be the starting point of fruitful discussions for the working group due to the shortcomings outlined in detail above.

My recommendations for presentation at CERME10 are as follows:

REJECT but resubmit the paper as a poster

Detailed reasons for the recommendation and general comments (e.g. suggestions for improvement, reasons for rejection/unsuitability):

Due to the shortcomings already described above, I would definitely say that the paper does not achieve academic quality adequate for publication in a scientific magazine. However, the CERME guidelines are very clear in their plea for inclusion, and of course I am aware of the possibility that I myself am wrong or at least too strict in my judgement. What's more, even though in an inadequate manner, the paper deals with some relevant issues that certainly deserve to be discussed at a scientific conference. Hence inviting the author to resubmit the paper as a poster might be an appropriate solution, offering the author the opportunity to present his attack on the "ruling tradition" of mathematics education to an international scientific public, and at the same time allowing the members of the working group to decide for themselves whether to discuss the issues of the paper within the framework set out by the author or to adopt are more "traditional" stance by firstly clarifying the question and then seeking a solution on the basis of relevant published research and sound argumentation.

In the case of a resubmission I would recommend to change the title – or, if possible, to make clear how "calculus" is defined by the author and what the paper has to do with preschool and special needs education.

CERME10 PAPER REVIEW FORM 1 – empirical Study

For Theoretical/Philosophical Papers, please go to the second version of this form, below.

RETURN BY E-MAIL as soon as possible, but no later than *20th October 2016*

TO: (group leader) Prof. Dr. Elisabeth Rathgeb-Schnierer

Thematic Working Group Number and Title: TWG2 - Arithmetic and Number Systems

TITLE OF THE PAPER PROPOSAL:

Cupcounting and calculus in preschool and in special needs education

Name of the reviewer: KL

YOUR REVIEW

Please comment in each of the following sections. In each section use as much space as required.

1. Focus and rationale of the paper

The rationale and the motivation of the project within the cultural content become clear: The author and the government see a need for an 'urgent change' because of decreased PISA results.

The focus of the presented project stayed unclear to me. What does the researcher exactly want to find out?

What is the MAIN focus?:

- Does he want to find out anything about the mathematical content and the (new) methods (e.g. cup-counting)?
- Does he want to research and evaluate the designed micro-curriculum with certain goals?
- Or is his focus on how to support low archiving learners and preschool students (inside or outside the classroom?)?
- ... all combined?

2. Theoretical framework and related literature

Some theoretical background and related literature is cited concerning: Institutional skepticism, mathematical essence and existence. The author also points out that there is no research literature on cup-counting. Are there any related findings, which could be considerable and important as a theoretical background?

E.g. published research about:

- Teaching and learning of the arithmetical content
- Teaching and learning in general to design a micro-curriculum (Why for example does the teacher guide trough action without using any words or a foreign? Why is this decision made and which theory it is build on?)
- Teaching and learning of low archiving learners

3. Methodology

The developed micro-curriculum gets tested in school, which indicates a design research approach.

The following remains unclear to me:

- How many students/ learners/ teachers are involved (participants, sampling)?
- During which time period do the learners work with the micro-curriculum?
- How gets the data collected?
- How gets the data analysed?

- What is the precise focus (research questions)?

4. Statement and discussion of results

The examples, which are given, describe some observations, which indicate successful learning and therefor might indicate efficiency of the used approach.

It would be nice to get to know more (especially contrasting) examples and to put these results in relation to a theoretical background and other findings.

5. Clarity

Please see questions above, which I think need clarification.

6. Relevance to this CERME10 TWG audience

With more background knowledge about the theoretical framework, the focus and the methodology it could be an interesting approach, worth to discuss.

The focus on special needs education is in terms of the worldwide implementation of inclusion a very present topic.

My recommendations for presentation at CERME10 are as follows (mark one):

5. ACCEPT for presentation without further modification
6. ACCEPT for presentation subject to modification as detailed below
7. REJECT but resubmit the paper as a poster
8. REJECT

Detailed reasons for the recommendation and general comments (e.g. suggestions for improvement, reasons for rejection/unsuitability):

Please see above.

Corresponding Author: Allan Tarp

Title: Cupcounting and calculus in preschool and in special needs education

Thank you for your proposal of a paper for CERME 10 TWG 2. The paper has been read by two other proposers, and all Group 2 leaders (resp. Elli Rathgeb-Schnierer). The two reviews of the other proposers are attached with this summary overview.

Decision regarding **presentation at the conference:**

ACCEPT for presentation without further modification

ACCEPT for presentation subject to modification as detailed below

REJECT and RESUBMIT as a poster

REJECT

Your paper suggests a new approach on learning number and operation, and states that this approach is helpful to create more arithmetic expertise in preschool and special needs eds. Although, the paper generally tackles a topic that is very interesting and important to discuss TWG2, the leader team decided on rejection because of several very critical aspects which were also pointed out by both reviewers (see attached reviews).

The critical aspects are for instance:

- Generally, the paper doesn't meet the CERME requirements for submission since it cannot be clearly related to one of the categories: it is neither an empirical study nor a theoretical or philosophical essay.
- The theoretical constructs are not clearly described, and the reader does not understand what is meant. There are a lot of abstract and general statements which are neither filled with meaning nor related to literature.
- The paper lacks clarity in various ways since the author remains vague in many aspects (e.g. the aim of the project, the micro-curriculum, the concrete benefit of the approach, the connection of the findings to the theoretical background and other findings etc.).
- The paper is full of statements that are neither based on literature nor empirical findings.
- The paper focusses on basic arithmetic education but ignores the vast body of theoretical and empirical work that exists in this field.

We are sorry, but because of all critical aspects mentioned by the reviewers and described above the article barely meets international scientific standards, and cannot be accepted for presentation at the conference.

Best regards,

Elli