

# THE MES 9 CONTRIBUTIONS

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# **A 1YEAR PRE-ENGINEER COURSE FOR YOUNG MIGRANTS, A JOB FOR CRITICAL OR CIVILIZED MATH EDUCATION**

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*UN population forecasts predict a continuing migrant flow to Europe to benefit from its socialist welfare and educational systems. But a critical question could ask: Is European education ready to benefit from the engineer potential in young migrants allowing them to build up welfare and education in their own country? Is critical socialist thinking able to reform its European line-organized office directed education dating back to the Napoleon wars? A recent OECD report saying that Sweden should urgently reform its school system to improve quality and equity suggests that a solution might instead be provided by the civilized thinking of the North American Enlightenment republics, historically created to receive and integrate migrants through its half-year block-organized talent developing education.*

## **BACKGROUND AND QUESTION**

According to the numbers of hours spend there, education is by far the most extensive public intervention in private life; and with the basic human need for a word- and a number-language for communication, mathematics is one of its core subjects. Consequently, research in math education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 year since 1969. Likewise, funding has increased as witnessed e.g. by the creation of a National Centre for Mathematics Education in Sweden that with its positive attitude to receiving male migrants now beats China with 123 boys/ 100 girls of the 16-17 years old. However, despite increased research and funding, Sweden has seen its PISA result in mathematics decrease from 509 in 2003 to 478 in 2012, the lowest in the Nordic countries and significantly below the OECD average at 494. This caused OECD to write the report 'Improving Schools in Sweden' describing the Swedish school system as being 'in need of urgent change' (OECD, 2015).

PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (p. 3)

In the report OECD writes

Sweden has the highest percentage of students arriving late for school among all OECD countries, especially among socio-economically disadvantaged and immigrant students, and the lack of punctuality has increased between 2003 and 2012. There is also a higher-than-average percentage of students in Sweden who skip classes, in particular among disadvantaged and immigrant students. Arriving late for school and skipping classes are associated highly negatively with mathematics performance in PISA and can have serious

adverse effects on the lives of young people, as they can cut into school learning and also distract other students. (p. 69) The reforms of recent years are important, but evidence suggests they are also somewhat piecemeal, and simply too few, considering the serious situation of the Swedish school system. (p. 55) Sweden faces a serious deterioration in the quality and status of the teaching profession that requires immediate system-wide attention. This can only be accomplished by building capacity for teaching and learning through a long-term human resource strategy for the school sector. (p. 112)

Inspired by the OECD report we can ask: How to improve mathematics and its education to better serve the population and migrants? And more specifically: How to design a 1 year pre-engineer course for young migrants beginning from scratch?

Critical and civilized thinking provide two kinds of answers.

## **CRITICAL AND CIVILIZED THINKING**

As to the content of critical thinking, the Oxford Dictionary of Philosophy writes:

The title is specifically applied to the philosophical approach of the Frankfurt school. This owed its philosophical background to Hegel and to Marx, seeing social and cultural imperfections as defects of rationality, and comparing them with an ideal to which the progress of reason, embodied in pure and undistorting social arrangements, would ideally tend (pp. 88-89)

Civilized thinking mixes existentialism, seeing existence as preceding essence, with the thinking of the two Enlightenment republics, American pragmatism being skeptical towards any philosophical is-claim, and French post-structuralism warning against hidden patronization in choices presented as nature. But to more clearly see the difference between the two we need to go back in history.

## **A HISTORICAL BACKGROUND**

The distance from its energy source allows water in all three forms: solid, liquid and gas. Thus a continuous flow of incoming high order energy from the sun and outgoing low order waste energy to space during the night allows green cells to store energy to be exploited by grey cells coming in three forms: reptiles, mammals and humans. That by standing up allowed the brain to develop language by remembering sounds given to what the forelegs transformed to hands was grasping. Thus meeting the two fundamental needs shown by the holes in the head: to supply the stomach and the lungs and the brain with food and oxygen and information.

When humans left Africa some went east to the fertile river valleys, some went west to the mountains. Trade took place exchanging eastern silk and pepper with western silver. Its silver mines allowed ancient Greece to develop a culture where men could leave the daily routine work to women and slaves to discuss social matters as ‘can adults live together on equal terms or is patronization needed as with children?’

Social theory thus has human interaction as its main focus. As to communication, the most basic interaction, Berne (1964) has developed a transactional analysis describing three different ego-states called Parent and Adult and Child to reflect the social fact

that human interaction can be patronized and non-democratic, or it can be non-patronized and democratic. In a family the interaction between children and parents will typically be one of patronization. In a society adult interaction typically will be non-patronized, unless the society is a non-democratic autocracy where patronization is carried on into adulthood. In this way Berne describes the main problem in human interaction, the choice between patronization and self-determination or 'Mündigkeit'. The fact that the German word 'Mündigkeit' does not have an English equivalent indicates that social interaction is quite different outside continental Europe and inside where the presence of and resistance against patronization created the label 'Mündigkeit'.

The debate on patronization runs all the way through the history of social theory (Russell, 1945). In ancient Greece the sophists warned against hidden patronization coming from choices presented as nature. Hence to practice the three ingredients of a democracy, information and debate and decision, a population should be enlightened to tell choice from nature. Seeing the physical as examples of metaphysical forms only visible to philosophers from his academy, Plato labelled democratic debate as ignorance. Instead social power should be given to the philosophers who could make wise decisions based upon information coming from insight and knowledge, thus needing neither debate nor democracy. In this way Plato instituted the patronization that Foucault calls 'pastoral power' to be continued first by the Christian church and later by modern universities still using the scholastic research method of only allowing late opponents to already defended texts to be accepted as researchers.

The Greek silver mines lasted about hundred years. Then the Romans took over, financing their empire by silver mines in Spain, eventually captured by the Vandals and by the Arabs. The lack of silver made Europe descend into the dark Middle Age. Here the patronization question reappeared in the controversy on universals between the realists and the nominalists. The realist took the Plato standpoint by renaming his metaphysical forms to universals claimed to have independent existence and to be exemplified in the physical world, and consequently waiting to be discovered by philosophers. In contrast to this the nominalist saw universals as names invented to facilitate human interaction.

Then German silver transported to Italy reopened east-west trade financing the Renaissance, seeing a protestant uprising against the patronization of the Roman Catholic Church resulting in the bloody 30year war from 1618. To avoid the chaos of war, Hobbes in his book 'Leviathan' argued that to protect themselves against their natural egoistic state, humans would have a much better life if accepting the patronization of an autocratic monarch.

Seeing the laboratory as preceding the library, Brahe retrieved data for the motion of planets, which together with Kepler's interpretation allowed Newton to discover that the moon doesn't move among the stars, instead it falls towards the earth as does the apple, both following their own physical will and not the will of a metaphysical patronizer. This inspired Locke to argue against patronization. His chief work, the 'An

Essay Concerning Human Understanding', was highly inspirational in the Enlightenment 1700-century, which resulted in two democracies being installed, one in the US and one in France. American sociology sees human interaction as based upon enlightenment and freed from patronization. Its 'it is true if it works' pragmatism expressed by Peirce and James leads on to symbolic interactionism and to the natural observation rooted research paradigm Grounded Theory resonating with the principles of natural learning expressed by Piaget. In harmony with this, the US enlightenment school, being organized in half-year blocks and aiming at developing the talent of the individual has set the international standard followed worldwide outside Europe.

Inside Europe counter-enlightenment came from Germany where Hegel reinstalled metaphysical patronization in the form of a Spirit expressing itself through the history of the people. Trying to end the French Republic by war resulted in French occupation of Berlin. To get Napoleon out, the king realized that as the French he could no more depend on the blood nobility. So he asked Humboldt to use Hegel to design a line-organized Bildung education with three goals: Bildung must not enlighten to keep the population from demanding democracy as in France; instead, by imposing upon it a feeling of nationalism, Bildung should transform the population into a people following the will of the Spirit by fighting other people, especially the French. Finally Bildung should use the Spirit expressing itself in romanticism to sort out a knowledge nobility among the people for a central administration (Berglar, 1970).

Opposing Hegel, Nietzsche argued that only by freeing itself from metaphysical philosophical hegemony, western individuals would be able to realize their full potentials. Following Hegel, Marx claimed that until a socialist utopia has been established, a socialist party serving the interest of the working people should patronize people through a dictatorship of the proletariat. Once in power, Hegel-based socialism saw no reason to replace the Hegel-based counter-enlightening line-organized education with the enlightening block-organized education of the American republics. Marxist thinking developed into the critical theory of the Frankfurter school infiltrating the 1968 student revolt to secure that Europe's Bildung education could carry on its Hegel-based patronization.

Today, the sophist warning against unrooted is-claims is carried on by the existentialism of Kierkegaard and Nietzsche and Heidegger and Sartre, defining existentialism as holding that 'existence precedes essence, or (...) that subjectivity must be the starting point' (Marino, 2004: 344); and by French post-structuralism with Derrida and Lyotard and Foucault and Bourdieu showing skepticism towards hidden patronization in our most fundamental institutions: words, correctness, cures and education (Lyotard, 1984), (Tarp, 2004, 2). Foucault thus says:

It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (Chomsky & Foucault, 2006: 41)

In Germany, Arendt carried his Heidegger's work further by dividing human activity into labor and work focusing on the private sphere and action focusing on the political sphere thus accepting as the first philosopher political action as a worthy human activity creating institutions that should be treated with care to avoid 'the banality of evil' if turning totalitarian by the sheer banality of just following orders (Arendt, 1963). Likewise, Bauman points out that by following authorized routines modernity can create both gas turbines and gas chambers (Baumann, 1989).

As to their meanings, the word 'critical' comes from Greek 'kritike' meaning to pass judgement; and civilized comes from latin 'civis' meaning a free citizen. So civilized thinking means republican thinking always being skeptical towards false is-claims; and critical thinking means passing judgements; but to pass a judgement you must be elected judge by a democratic process, or have insight in the difference between right and wrong as e.g. believing in the Hegel assumption that instead of being free to create their own history, humans are puppets on a string playing out the manuscript of the Spirit. So basically the contradiction between critical and civilized thinking is a replay of the ancient controversy between the Greek philosophers and sophists.

## **CRITICAL VERSUS CIVILIZED MATHEMATICS EDUCATION**

The difference between critical and civilized mathematics education is seen in a paper describing how to deal with teaching and learning problems in a Brazilian math class (Tarp, 2004, 1)

In Brazil there is a research group, which has focused on issues related to new technologies and mathematics education. This research group has developed software and work with students at different levels and with teachers. A teacher from a nearby school approached the group (..) From the teacher perspective, she had some tough problems to face and she foresaw that the computers would be able to help her. The teacher was teaching a class of 5th graders, which in her view was really problematic. The kids were older (15 years average) than the expected age for this grade: 11. The kids felt humiliated somehow as they were put in a school with kids much younger than them and they had flunked many times, and at several instances they had to repeat all the subjects of a given school year because their 'failure in mathematics'. The students transformed this humiliation into violence in class. The teacher was in fact considering the possibility of just quitting the job since she could not work with those kids in a way she found effective. (..) The teacher was enthusiastic about a software, which deals with rational numbers. (..) both researchers and teacher had the 'intuition' that the computer might have a positive effect in this class and maybe could avoid that the students had to repeat this grade again. (Sec. 2, par. 2-4)

The teacher is supposed to teach rational numbers to a class with a mixture of 11 year old students and 15 year old repeaters having given up rational numbers and turning to violence. The research group could have asked critical questions as 'is rational numbers defined from below as an abstraction from concrete examples or from above as an example of an abstraction?' and 'why teach addition when it is meaningless to add fractions without units?' Instead the group uncritically assumed 'that the computer might have a positive effect'.

The paper also describes how civilized thinking would work differently:

The research group is working halftime in classrooms and halftime at the university. It focuses on the concerns of typical classrooms as expressed by students, teachers in their stories of complaints. The teacher complains about the violence in the class tempting her to quit the job since she cannot work in a way she finds effective. And the students complain about having to repeat the class because they don't want to learn about fractions, since the teacher by just echoing the textbook is unable to explain to the students, why they shall learn fractions, and what they are useful for. Asked to comment this, the teacher says that mathematics education means education in mathematics, and since rational numbers is part of the mathematics textbook it must be taught and learned. Mathematics is difficult to learn, so the students have to work harder, or be supported by computers. Hence the problems will not disappear before schools can afford computers, or the students decide to become more engaged in mathematics.

Based upon the motto "echo-phrasing is freezing, re-phrasing is freeing" postmodern thinking sees modern institutions frozen in echo-phrasings, that have to be discovered and rephrased. Since the teacher is echoing the textbook, the echoes can be found here. The textbook presents fractions as examples of rational numbers, being example of number sets, being examples of sets. This is the typical way of presentation within modern set-based mathematics explaining concepts as examples of more abstract concepts. This phrasing conflicts with the student demand for explanations relating fractions to their use.

So instead of developing software to supplement, and thus support the existing top-down echo-phrasing of fractions, the group begins to look for alternative bottom-up approaches in journals, other textbooks, other countries, and in other time periods. Also they use their imagination by accessing the silent part of their 'knowledge-iceberg' developed through years of classrooms experience as mathematics educators. Using curriculum architecture they design examples of micro curricula, where fractions emerges from dividing problems, that can be introduced into the ordinary classroom as e.g. games, where students work in pairs throwing dices and splitting the profit, or loss, proportional to their stakes shown by their dice-numbers.

This 'proportional splitting' approach leads to (and thus shows the authenticity and necessity of) fractions, and multiplication of fractions and integers. (Sec. 5, par. 2-5)

So where critical thinking shows no criticism towards the actual mathematics education tradition, civilized thinking asks if this tradition is nature or choice presented as nature and thus hiding alternatives.

## **CRITICIZING AND CIVILIZING RATIONAL NUMBERS**

In ancient Greece the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas. With astronomy and music as independent knowledge areas, today mathematics is a common label for the two remaining activities, geometry and algebra, (Freudenthal, 1973) both rooted in the physical fact Many through their original meanings, 'to measure earth' in Greek and 'to reunite Many' in Arabic.

Meeting Many we ask ‘how many?’ Counting and adding gives the answer. We count by bundling and stacking as seen when writing a total  $T$  in its block form:  $T = 354 = 3 \cdot B^2 + 5 \cdot B + 4 \cdot 1$  where the bundle  $B$  is ten typically. This illustrates the four ways to unite: On-top addition unites variable numbers, multiplication unites constant numbers, power unites constant factors and per-numbers, and next-to addition, also called integration, unites variable blocks. As indicated by its name, uniting can be reversed to split a total into parts predicted by the reversed operations: subtraction, division, root & logarithm and differentiation.

Operations <b>unite</b> /split Totals in	Variable	Constant
Unit-numbers m, s, kg, \$	$T = a + b$ $T - b = a$	$T = a \cdot b$ $T/b = a$
Per-numbers m/s, \$/kg, \$/100\$ = %	$T = \int a \cdot db$ $dT/db = a$	$T = a^b$ $b\sqrt[b]{T} = a, \log_a T = b$

**Table 1:** The four way to unite variable and constant unit- and per-numbers.

Although presented as nature, ten-bundling is a choice. Bundling Many in a ‘icon-bundles’ less than ten means asking e.g. ‘ $T = 7 = ? \text{ 4s}$ ’. The answer is predicted on a calculator by two formulas, a recount-formula ‘ $T = (T/B) \cdot B$ ’ telling that from a total  $T$ ,  $T/b$  times  $B$  s can be taken away, and a restack-formula ‘ $T = (T-B) + B$ ’ telling that from a total  $T$ ,  $T-B$  is left when  $B$  is taken away and placed next-to. First  $T = 7/4$  gives 1.some. Then  $T = 7 - 1 \cdot 4$  leaves 3. So the prediction is  $T = 7 = 1 \text{ 4s} \& 3 = 1.3 \text{ 4s} = 1 \frac{3}{4} \text{ 4s}$ . Thus with icon-counting, a natural number is a decimal number with a unit where the decimal point separates singles from bundles (Tarp, 2016)

Double-counting physical units creates per-numbers as  $3\$/4\text{kg}$ . With this, units can be changed by recounting \$s in 3s or kgs in 4s:  $15\$ = (15/3) \cdot 3\$ = (15/3) \cdot 4\text{kg} = 20\text{kg}$ . So as per-numbers, fractions are not numbers, but operators, needing a number to become a number. To add, per-numbers must be multiplied to unit-numbers, thus adding as areas, called integration:  $\frac{1}{2}$  of  $4 + \frac{2}{3}$  of  $3 = (\frac{1}{2} \cdot 4 + \frac{2}{3} \cdot 3)$  of  $(4+3) = 4$  of  $7$ .

The root of geometry is the standard form, a rectangle, that halved by a diagonal becomes two right-angled triangles with sides and angles connected by three laws,  $A+B+C = 180$ ,  $a^2+b^2 = c^2$  and  $\tan A = a/b$ . Being filled from the inside by triangles, a circle with radius  $r$  gets the circumference  $2 \cdot \pi \cdot r$  where  $\pi = n \cdot \tan(180/n)$  for  $n$  large.

Thus, as a label for algebra and geometry, mathematics is a natural science about the physical fact Many. However, the invention of the concept SET allowed mathematics to become a self-referring collection of ‘well-proven’ statements about ‘well-defined’ concepts, i.e. as ‘MetaMatics’, defined from above as examples from abstractions instead of from below as abstractions from examples. But, by looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox ‘this sentence is false’ being false if true and true if false: If  $M = \{ A \mid A \notin A \}$  then  $M \in M \Leftrightarrow M \notin M$ . The Zermelo–Fraenkel set-theory tries to avoid self-reference



by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract essence. To avoid self-reference Russell introduced a type theory allowing reference only to lower degree types. Consequently fractions cannot be numbers since they refer to numbers in their modern definition: In a set-product of integers, a fraction is an equivalence set created by the equivalence relation  $a/b \sim c/d$  if  $a \cdot d = b \cdot c$ .

Thus SET transformed grounded mathematics into a self-referring ‘MetaMatism’, a mixture of MetaMatics and ‘MatheMatism’ true inside a classroom but seldom outside where claims as ‘1 + 2 IS 3’ meet counter-examples as e.g. 1 week + 2 days is 9 days.

So rational numbers is pure MetaMatism by also being MatheMatism: Inside a classroom,  $1/2 + 2/3 = 7/6$ . Outside 1 coke out of 2 bottles and 2 cokes out of 3 bottles add up to 3 cokes out of 5 bottles, and not 7 cokes out of 6 bottles as taught inside.

Not criticizing rational numbers shows that critical thinking has taboos and that it lacks self-criticism by showing no criticism towards its own un-criticalness.

### **‘PRESCHOOL CALCULUS AND MULTIPLICATION BEFORE ADDITION’ AS A 1YEAR PRE-ENGINEER MATH COURSE**

As a label, mathematics has no content itself, only its ingredients have, algebra and geometry both rooted in the physical fact Many. To deal with Many we count & add. By counting a total T in bundles, cup-counting creates numbers as blocks of bundles and unbundled occurring in three different ways, normal and overload and underload as in  $T = 2)1 \text{ } 3s = 1)4 \text{ } 3s = 3)-2 \text{ } 3s$  when recounted in the same unit. Recounted in a different unit roots proportionality through the recount formula  $T = (T/B) \cdot B$  allowing a calculator to predict the result. Recounting in and from tens means resizing blocks where the height and the base are inversely proportional as in  $3 \text{ } 7s = 2)1 \text{ tens or } 4 \text{ tens} = 5 \text{ } 8s$ . Reversed addition is called equations solved by recounting:  $2 \cdot x = 8 = (8/2) \cdot 2$  so  $x = 8/2$ , showing the solving method ‘move to opposite side with opposite sign’. With counting before adding, division and multiplication comes before addition.

Once counted, totals can be added on-top if the units are made the same by proportionality, and next-to as areas also called integration. A composite area always changes with the last block added: change in Area = height\* change in base, or  $\Delta A = h \cdot \Delta b$  or  $h = \Delta A / \Delta b$ . So areas can be found by developing  $\Delta / \Delta x$ -calculations, also called differentiation in the case of replacing interval changes with local changes:  $y' = dy/dx = \Delta y / \Delta x$  for  $\Delta x$  arbitrarily small; as when the per-number is neither globally nor piecewise but locally constant (continuous) (Tarp, 2013).

Finally, double counting a physical quantity in two different units creates pre-numbers or fractions as  $2\$/3\text{kg} = 2/3 \text{ } \$/\text{kg}$  that must be multiplied to areas before being added.

The difference between a full critical and civilized mathematics education curriculum is illustrated in the appendix.

## DISCUSSION AND CONCLUSION

We asked: wanting to design a 1 year pre-engineer course for migrants beginning from scratch, should we use critical and civilized thinking?

Investigating its theoretical background shows that critical thinking is based on Marx, again based on Hegel counter-enlightenment going back to Greek Plato philosophy resonating with the Greek meaning of the word 'kritike', to pass judgement. For Plato, that was precisely what the philosophers were able to do since to them all physical was but examples of metaphysical forms only visible to them. Hegel replaced the forms with a Spirit expressing itself through the history of different people thinking they can decide their future themselves, but in reality just being puppets on a string playing out the masterplan of the Spirit. To Marx, the means to the Spirit's goal, a socialist society, was a proletarian dictatorship with a democracy in the form of a representative pyramid where the top central committee decided the correctness code that justified the judgement passed by critical thinking. Consequently, rational numbers cannot be criticized if part of this code. Likewise, criticizing Hegel-based line-organized office directed education is out of the question. With its lack of self-criticism and dependence on the will of a metaphysical Spirit, critical thinking reminds of a totalitarian religion preaching political correctness instead of teaching enlightenment.

Being skeptical towards ungrounded is-claims, civilized thinking unmasks false nature by uncovering hidden alternatives to choices presented as nature. So categories and correctness are grounded in the outside world; and as means avoiding the banality of evil, its institutions accommodate to resistance from the outside goals they are created to meet. Consequently, mathematics is ManyMath, a natural science accommodating to the physical fact Many; and education must be organized in flexible half-year blocks aiming at uncovering and developing the talent of the individual learner.

So as a 1 year pre-engineer course for migrants from scratch we will get to different answers. Uncritically accepting mathematics as meaningless MetaMatism, critical thinking will say it is impossible to learn a pre-engineer background in one year since mathematics is difficult to learn thus taking many hours of hard dedicated work.

Civilized thinking welcomes a course showing that while MetaMatism is difficult, ManyMath is quickly learned: To deal with many, we count and recount and double-count before performing next-top and on-top addition and reversed addition. First we count in ones to produce icons, then we cup-count in normal, overload and underload form by recounting in the same unit thus realizing that numbers are 2dimensional blocks and not names to the points on a 1dimensional cardinality line as claimed by MetaMatism. Then we recount in a new unit to proportional numbers. Then we recount in and from tens to resize the number blocks. Then we double-count to create per-numbers and fractions. Then to add on-top we must change the unit by proportional recounting; and to add per-numbers we must add next-to as areas where a composite area changes with the last block added. And finally reversed addition leads to solving equations presenting 'opposite side with opposite sign' as a natural method.

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## APPENDIX: A CRITICAL AND A CIVILIZED MATH CURRICULUM

### Primary school

Critical mathematics curriculum	Civilized mathematics curriculum (Tarp, 2016)
1dim. Number-line with number-names No counting, only adding and nextto Addition & Subtraction before Multiplication & Division Multiplication tables to be memorized No calculator	2dim. Number-blocks with units Counting before adding, nxto before ontop Multiplication & Division before Subtraction & Addition Multiplication tables recount to & from tens Calculator from the start as predictors
One and two digit numbers Addition Subtraction Multiplication Division Simple fractions	CupCount Many in BundleCups ReCount Many in same Unit & in new Unit (Proportionality) ReCount: In Tens & From Tens (Multiplication & Division) Calculator Prediction: RecountFormula Addition: NextTo (Integration) & OnTop Reversed addition: Equations

### Middle school

Fractions are numbers that can be added without units. Letter-fractions must be factorized before added	Fractions are PerNumbers (operators needing a number to become a number) and added by areas (integration)
Negative numbers Fractions Percentages & Decimals Proportionality LetterNumbers Algebraic fractions Solve a linear equation Solve 2 equations w. 2 unknowns	DoubleCounting produces PerNumbers & PerFives (fractions) & PerHundreds ( %) Geometry and algebra go hand in hand when working with letter-numbers and letter-formulas; and with lines and forms The coordinate system coordinates geometry and algebra so that length can be translated to D-change, and vice versa

### High school

Functions are set-relations	Functions are formulas with two variables
Squares and square roots Solve quadratic equations Linear functions Quadratic functions Exponential functions Logarithm Differential Calculus Integral Calculus Statistics & probability	Integral Calculus as adding PerNumbers Change & Global/Piecewise/Local constancy Root/log as finding/counting change-factors Constant change: Proportional, linear, quadratic, exponential, power Simple and compound interest Predictable Change: Integral Calculus & Differential Calculus Unpredictable Change: Statistics & probability

# ONLINE TEACHER TRAINING FOR CURING MATH DISLIKE: CUP&RE-COUNTING & MULTIPLICATION BEFORE ADDITION

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*Set transformed Mathematics from a mere label for Algebra and Geometry into a self-referring subject changing the two from example-containers to examples of set, causing massive learning problems as shown by PISA. Re-rooting mathematics in the physical fact Many, the MATeCADEMY.net offers an alternative teacher training.*

## BACKGROUND

In spite of increased mathematics education research, Swedish PISA results decrease as witnessed by the OECD 2015 report 'Improving Schools in Sweden'. Mathematics seems to be hard, but we could ask: Maybe it is not mathematics that is taught, and maybe there is a hidden mathematics that rooted in the outside world becomes meaningful? And if so, where can teachers learn about it? Existentialist thinking might provide an answer. Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines existentialism as holding that 'existence precedes essence' (Marino, 2004 p. 344). But how does essence-math differ from existence-math?

## A CASE: PETER, STUCK IN DIVISION AND FRACTIONS

Being a mathematics teacher in a class of ordinary students and repeaters flunking division and fractions, Peter is about to give up teaching when he learns about the '1 cup & 5 sticks' method to cure mathematics dislike by watching 'CupCount and ReCount before you Add' (<https://www.youtube.com/watch?v=IE5nk2YEQIAxx>).

Here 5 sticks are CupCounted in 2s using a cup for bundles. He sees that a total can be recounted in the same unit in 3 different forms: overload, standard and underload:

$$T = 5 = \text{|||||} = \text{||} \text{|||} = 1)3 \text{ 2s} = \text{||} \text{||} \text{||} = 2)1 \text{ 2s} = \text{||} \text{||} \text{||} \text{||} = 3)1 \text{ 2s}$$

So counted in bundles, a total has an inside number of bundles and an outside number of singles; and moving a stick out or in creates an over-load or an under-load.

When multiplying,  $7 \times 48$  is cup-written as  $7 \times 4)8$  resulting in 28 inside and 56 outside as an overload that can be recounted:  $T = 7 \times 4)8 = 28)56 = 33)6 = 336$ .

And when dividing,  $336/7$  is cup-written as  $33)6 /7$  recounted to 28 inside and 56 outside according to the multiplication table. So  $33)6 /7 = 28)56 /7 = 4)8 = 48$ .

To try it himself, Peter downloads the 'CupCount & ReCount Booklet'. He gives a copy to his colleagues and they decide to arrange a free 1day Skype seminar.

In the morning they watch the PowerPoint presentation 'Curing Math Dislike', and discuss six issues: first the problems of modern mathematics, MetaMatism; next the potentials of postmodern mathematics, ManyMath; then the difference between the

two; then a proposal for a ManyMath curriculum in primary and middle and high school; then theoretical aspects; and finally where to learn about ManyMath.

Here MetaMatism is a mixture of MatheMatism, true inside a classroom but rarely outside where ' $2+3 = 5$ ' is contradicted by  $2\text{weeks}+3\text{days} = 17\text{days}$ ; and MetaMatics, presenting a concept TopDown as an example of an abstraction instead of BottomUp as an abstraction from many examples: A function IS an example of a set-product.

In the afternoon the group works with an extended version of the CupCount & ReCount Booklet where Peter assists newcomers. At the seminar there are two Skype sessions with an external instructor, one at noon and one in the afternoon.

Bringing ManyMath to his classroom, Peter sees that many difficulties disappear, so he takes a 1year distance learning education at the MATHeCADEMY.net teaching teachers to teach MatheMatics as ManyMath, a natural science about Many. Peter and 7 others experience PYRAMIDeDUCATION where they are organised in 2 teams of 4 teachers choosing 3 pairs and 2 instructors by turn. An external coach assists the instructors instructing the rest of their team. Each pair works together to solve count&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation. In a pair each teacher corrects the other's routine-assignment. Each pair is the opponent on the essay of another pair. Each teacher pays by coaching a new group of 8 teachers.

At the academy, the 2x4 sections are called CATS for primary and secondary school inspired by the fact that to deal with Many, we Count & Add in Time & Space.

At the academy, primary school mathematics is learned through educational sentence-free meetings with the sentence subject developing tacit competences and individual sentences coming from abstractions and validations in the laboratory, i.e. through automatic 'grasp-to-grasp' learning.

Secondary school mathematics is learned through educational sentence-loaded tales abstracted from and validated in the laboratory, i.e. through automatic 'gossip-learning': Thank you for telling me something new about something I already knew.

## CONCLUSION

An existentialist distinction between essence and existence shows that what is taught in schools is not mathematics, but a self-referring MetaMatism turning mathematics upside down and containing some statements that do not apply outside. As a common label for Algebra and Geometry meaning reuniting Many and measuring Earth in Arabic and Greek, mathematics should let existence precede essence and become ManyMatics, a natural science about how to divide the earth and its Many products.



## REFERENCE

Marino, G. (2004). *Basic Writings of Existentialism*. New York: Modern Library.

# Cure MathDislike: CupCount 'fore you Add

## 1Day Skype Seminar on CupCounting, ReCounting & CupWriting

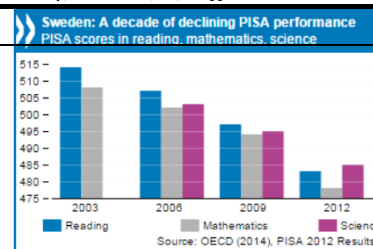
Action Learning on the child's own 2D NumberLanguage as observed when holding 4 fingers together 2 by 2 makes a 3-year-old child say 'No, that is not 4, that is 2 2s.'

09-11	<b>Listening and Discussing: Curing Math Dislike, a PowerPointPresentation</b> To master Many, we Math?? No, first we Count, then we Add. Math is a label, not an action word.	
	1. The problems of <b>Modern MatheMatics</b> , or <b>MetaMatism</b> 2. The potentials of <b>PostModern MatheMatics</b> , or <b>ManyMath</b> 3. The Difference between <b>MetaMatism</b> and <b>ManyMath</b> 4. A <b>ManyMath Curriculum</b> for Primary and Middle and High school 5. Theoretical aspects, and 6. Where to learn about <b>ManyMath</b> ? <b>Bad Math: MatheMatism</b> , true inside but rarely outside classes: $2+3$ IS 5, but $2\text{weeks}+3\text{days} = 17\text{d}$ ? Adding 1D Line Numbers without units may create MathDislike. <b>Evil Math: MetaMatics</b> , presenting a concept TopDown as an example of an abstraction instead of BottomUp as an abstraction from many examples: A function IS an example of a set-product. <b>Good Math: ManyMatics</b> , a natural science about Many mastering Many by CupCounting, ReCounting & CupWriting: $T = 5 = \text{I I I I I} = \text{II I I I} = 1)3 \text{ 2s} = \text{II II I} = 2)1 \text{ 2s} = \text{II II II I} = 3)-1 \text{ 2s}$ .	
11-13	Skype Conference. Lunch	
13-15	<b>Doing:</b> Trying out the <b>CupCount 'fore you Add</b> booklet to see proportionality and calculus and solving equations as golden LearningOpportunities in Cup- & Re-Counting and NextTo Addition.	
	<b>RECOUNTING</b> , in the same unit creates over- or under-load, in a new unit creates <i>proportionality</i> <b>Question:</b> $T = 2.1 \text{ 3s} = ? \text{ 3s}$ . <b>Answer:</b> $T = 2.1 = 2)1 = 1)4 = 3)-2 \text{ 3s}$ <b>Q:</b> $T = 2 \text{ 3s} = ? \text{ 4s}$ <b>A:</b> $T = 2 \text{ 3s} = \text{III III} = \text{IIII II} = 1)2 \text{ 4s} = 1)1 \text{ 5s} = 3) \text{ 2s} = 1)1) \text{ 2s} = 11.0 \text{ 2s}$	
	<b>CalculatorPrediction. Q:</b> $T = 2 \text{ 4s} = ? \text{ 5s}$ . <b>A:</b> $T = 1.3 \text{ 5s}$ since $2*4/5$ 1.some <i>RecountFormula</i> $T = (T/B)*B$ says 'From T, T/B times, Bs can be taken away' $2*4 - 1*5$ 3	
	<b>RECOUNTING</b> in and from Tens resizes blocks meaning teaching <i>multiplication before addition</i> : <b>Q:</b> $T = 3 \text{ 7s} = ? \text{ tens}$ . <b>A:</b> $T = 3*7 = 21 = 2.1 \text{ tens}$ . <b>Q:</b> $T = 47 = ? \text{ 6s}$ . <b>A:</b> $T = (47/6)*6 = 7 \text{ 6s} \& 5$	
	<b>Multiply &amp; Divide with CupWriting</b> creating or removing overloads <b>Q:</b> $T = 7 * 463 = ?$ <b>A:</b> $T = 7 * 4)6)3 = 28)42)21 = 28)44)1 = 32)4)1 = 3241$ <b>Q:</b> $T = 3241 / 7 = ?$ <b>A:</b> $T = 32)4)1 / 7 = 28)44)1 / 7 = 28)42)21 / 7 = 4)6)3 = 463$	
	<b>ADD NextTo.</b> <b>Q:</b> $T = 2 \text{ 4s} + 3 \text{ 5s} = ? \text{ 9s}$ . <b>A:</b> $T = 2.5 \text{ 9s}$ (integration)  <b>ADD OnTop.</b> <b>Q:</b> $T = 2 \text{ 4s} + 3 \text{ 5s} = ? \text{ 5s}$ . <b>A:</b> $T = 1.3 \text{ 5s} + 3 \text{ 5s} = 1)3 + 3) = 4)3 = 4.3 \text{ 5s}$ 	
	<b>DoubleCounting</b> in two units creates <b>PerNumbers</b> <b>Q:</b> $T = 10\$ = ?\text{kg}$ with 4\$ per 5kg. <b>A:</b> $T = 10\$ = (10/4) * 4\$ = (10/4) * 5 \text{ kg} = 12.5 \text{ kg}$	
	<b>Reversed Addition:</b> Solving Equations by moving to Opposite Side with Opposite Sign	
	$2 \times ? = 8 = (8/2) \times 2$ $? = 8/2, \text{ ReCounting}$	$2 + ? = 8 = (8-2) + 2$ $? = 8-2, \text{ ReStacking}$
	$T = 2 \text{ 3s} + ? \text{ 5s} = 3.2 \text{ 8s}$ $? = (3.2 \text{ 8s} - 2 \text{ 3s})/5 = \Delta T/5, \text{ Differentiation}$	
15-16	Coffee. Skype Conference.	

### Background

The effect of MathDislike is seen in the 2015 OECD report *Improving Schools in Sweden*: 'PISA 2012, however, showed a stark decline in the performance of 15-year-old students with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life'.

MATHeCADEMY.net offers UK or DK online Teacher Training based upon Action Learning and Research papers on CupCounting published at the ICME 2004-2012 ([mathecademy.net/papers/icme-trilogy](http://mathecademy.net/papers/icme-trilogy)). More details on MrAITarp YouTube videos:



## SUMMARY OF THE 4 PRIMARY AND SECONDARY 4 STUDY UNITS AT THE MATHECADEMY.NET

	QUESTIONS	ANSWERS
<b>C1 COUNT</b>	How to count Many? How to recount 8 in 3s: $T = 8 = ? \text{ 3s}$ How to recount 6kg in \$: $T = 6\text{kg} = ?\$$ How to count in standard bundles?	By bundling and stacking the total T predicted by $T = (T/b)*b$ $T = 8 = ?*3 = ?\text{3s}$ , $T = 8 = (8/3)*3 = 2\text{ 2 3s} = 2.2 \text{ 3s} = 2*3 + 2 = 2 \text{ 2/3}*3$ If $4\text{kg} = 2\$$ then $6\text{kg} = (6/4)*4\text{kg} = (6/4)*2\$ = 3\$$ Bundling bundles gives a multiple stack, a stock or polynomial: $T = 423 = 4\text{BundleBundle} + 2\text{Bundle} + 3 = 4\text{ten} \text{ 2ten } 3 = 4*B^2 + 2*B + 3$
<b>C2 COUNT</b>	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval $8.2 \pm 4.6$ (average $\pm 2*\text{deviation}$ )
<b>A1 ADD</b>	How to add stacks concretely? $T = 27+16 = 2\text{ten}7+1\text{ten}6 = 3\text{ten}13 = ?$ How to add stacks abstractly?	By restacking overloads predicted by the restack-equation $T = (T-b)+b$ $T = 27+16 = 2 \text{ ten } 7+1 \text{ ten } 6 = 3 \text{ ten } 13 = 3 \text{ ten } 1 \text{ ten } 3 = 4 \text{ ten } 3 = 43$ Vertical calculation uses carrying. Horizontal calculation uses FOIL
<b>A2 ADD</b>	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: $10 = 2\text{fold}5$ . Prime-numbers cannot: $5 = 1\text{fold}5$ Per-numbers occur when counting, when pricing and when splitting. The \$/day-number a is multiplied with the day-number b before added to the total \$-number T: $T_2 = T_1 + a*b$
<b>T1 TIME</b>	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. $x*3+2 = 14$ is reversed to $x = (14-2)/3$ Yes. $x+a = b$ is reversed to $x = b-a$ , $x*a = b$ is reversed to $x = b/a$ , $x^a = b$ is reversed to $x = a\sqrt[b]{b}$ , $a^x = b$ is reversed to $x = \log b / \log a$
<b>T2 TIME</b>	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If $K_0 = 30$ and $\Delta K/n = a = 2$ , then $K_7 = K_0 + a*n = 30 + 2*7 = 44$ If $K_0 = 30$ and $\Delta K/K = r = 2\%$ , then $K_7 = K_0*(1+r)^n = 30*1.02^7 = 34.46$ By solving a variable change-equation: If $K_0 = 30$ and $dK/dx = K'$ , then $\Delta K = K - K_0 = \int K' dx$
<b>S1 SPACE</b>	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: $\sin A = a/c$ , $\cos A = b/c$ , $\tan A = a/b$
<b>S2 SPACE</b>	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If $P_0(x,y) = (3,4)$ and if $\Delta y/\Delta x = 2$ , then $P_1(8,y) = P_1(x+\Delta x, y+\Delta y) = P_1((8-3)+3, 4+2*(8-3)) = (8,14)$ Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)



# DEBATE ON HOW TO IMPROVE MATHEMATICS EDUCATION

Allan Tarp

MATHeCADEMY.net, Denmark

*In this symposium the author invites opponents to debate how to improve mathematics education inspired by the Chomsky-Foucault debate on human nature. The main question is: 'If research cannot improve Math education, then what can?'*

Bo: Today we discuss Mathematics education and its research. Humans communicate in languages, a word-language and a number-language. In the family, we learn to speak the word language, and we are taught to read and write in institutionalized education, also taking care of the number-language under the name Mathematics, thus emphasizing the three r's: Reading, Writing and Arithmetic. Today governments control education, guided by a growing research community. Still international tests show that the learning of the number language is deteriorating in many countries. This raises the question: If research cannot improve Mathematics education, then what can? I hope our two guests will provide some answers. I hope you will give both a statement and a comment to the other's statement before the floor will comment.

## 1. MATHEMATICS ITSELF

Bo: We begin with Mathematics. The ancient Greeks Pythagoreans used this word as a common label for what we know, which at that time was Arithmetic, Geometry, Astronomy and Music. Later Astronomy and Music left, and Algebra and Statistics came in. So today, Mathematics is a common label for Arithmetic, Algebra, Geometry and Statistics, or is it? And what about the so-called 'New Math' appearing in the 1960s, is it still around, or has it been replaced by a post New-Math, that might be the same as pre New-Math? In other words, has pre-modern Math replaced modern Math as post-modern Math? So, I would like to ask: 'What is Mathematics, and how is it connected to our number-language?'

## 2. EDUCATION IN GENERAL

Bo: Now let us talk about education in general. On our planet, life takes the form of single black cells, or green or grey cells combined as plants or animals. To survive, plants need minerals, pumped in water from the ground through their leaves by the sun. Animals instead use their heart to pump the blood around, and use the holes in the head to supply the stomach with food and the brain with information. Adapted through genes, reptiles reproduce in high numbers to survive. Feeding their offspring while it adapts to the environment through experiencing, mammals reproduce with a few children per year. Humans only need a few children in their lifetime, since transforming the forelegs to hands and fingers allows humans to grasp the food, and to share information through communication and education by developing a language when associating sounds to what they grasp. Where food must be split in portions, information can be shared. Education takes place in the family and in the workplace; and in institutions with primary, secondary and tertiary education for children, for

teenagers and for the workplace. Continental Europe uses words for education that do not exist in the English language such as *Bildung*, *unterricht*, *erziehung*, *didactics*, etc. Likewise, Europe still holds on to the line-organized office preparing education that was created by the German autocracy shortly after 1800 to mobilize the population against the French democracy, whereas the North American republics have block-organized talent developing education from secondary school. As to testing, some countries use centralized test where others use local testing. And some use written tests and others oral tests. So, my next question is ‘what is education?’

### **3. MATHEMATICS EDUCATION**

Bo: Now let us talk about education in Mathematics, seen as one of the core subjects in schools together with reading and writing. However, there seems to be a difference here. If we deal with the outside world by proper actions, it has meaning to learn how to read and how to write since these are action-words. However, you cannot Math, you can reckon. At the European continent reckoning, called ‘*Rechnung*’ in German, was an independent subject until the arrival of the so-called new Mathematics around 1960. When opened up, Mathematics still contains subjects as fraction-reckoning, triangle-reckoning, differential-reckoning, probability-reckoning, etc. Today, Europe only offers classes in Mathematics, whereas the North American republics offer classes in algebra and geometry, both being action words meaning to reunite numbers and to measure earth in Arabic and Greek. Therefore, I ask, ‘what is Mathematics education?’

### **4. THE LEARNER**

Bo: Now let us talk about at the humans involved in Mathematics education: Governments choose curricula, build schools, buy textbooks and hire teachers to help learners learn. We begin with the learners. The tradition sees learning taking place when learners follow external instructions from the teacher in class and from the textbook at home. Then constructivism came along suggesting that instead learning takes place through internal construction. Therefore, I ask ‘what is a learner?’

### **5. THE TEACHER**

Bo: Now let us talk about the teacher. It seems straightforward to say that the job of a teacher is to teach learners so that learning takes place, checked by written tests. However, continental Europe calls a teacher a ‘*Lehrer*’ thus using the same word as for learning. In addition, a *Lehrer* is supposed to facilitate ‘*unterrichtung* and *erziehung* and to develop qualifications and competences. In teacher education, the subject *didactics*, meant to determine the content of *Bildung*, is unknown outside the continent. And until lately, educating *lehrers* took place outside the university in special *lehrer*-schools. Thus, being a teacher does not seem to be that well-defined. Therefore, my next question is ‘what is a teacher?’

### **6. THE POLITICAL SYSTEM**

Bo: Now let us talk about governments. Humans live together in societies with different degrees of patronization. In the debate on patronization, the ancient Greek sophists

argued that humans must be enlightened about the difference between nature and choice to prevent patronization by choices presented as nature. In contrast, the philosophers saw choice as an illusion since physical phenomena are but examples of metaphysical forms only visible to philosophers educated at Plato's Academy who consequently should be accepted as patronizers. Still today, democracies come in two forms with a low and high degree of institutionalized patronization using block-organized education for individual talent developing or using line-organized education for office preparation. As to exams, some governments prefer them centralized and some prefer them decentralized. As to curricula, the arrival of new Mathematics in the 1960s integrated its subfields under the common label Mathematics. Likewise, constructivism meant a change from lists of concepts to lists of competences. However, these changes came from Mathematics and education itself. So my question is: 'Should governments interfere in Mathematics education?'

## **7. RESEARCH**

Bo: Now let us talk about research. Tradition often sees research as a search for laws built upon reliable data and validated by unfalsified predictions. The ancient Greek Pythagoreans found three metaphysical laws obeyed by physical examples. In a triangle, two angles and two sides can vary freely, but the third ones must obey a law. In addition, shortening a string must obey a simple ratio-law to create musical harmony. Their findings inspired Plato to create an academy where knowledge meant explaining physical phenomena as examples of metaphysical forms only visible to philosophers educated at his academy by scholasticism as 'late opponents' defending their comments on an already defended comment against three opponents. However, this method discovered no new metaphysical laws before Newton by discovering the gravitational law brought the priority back to the physical level, thus reinventing natural science using a laboratory to create reliable data and test library predictions. This natural science inspired the 18th century Enlightenment period, which again created counter-enlightenment, so today research outside the natural sciences still uses Plato scholastics. Except for the two Enlightenment republics where American Pragmatism used natural science as an inspiration for its Grounded Theory, and where French post-structuralism has revived the ancient Greek sophist skepticism towards hidden patronization in categories, correctness and institutions that are ungrounded. Using classrooms to gather data and test predictions, Mathematics education research could be a natural science, but it seems to prefer scholastics by researching, not Math education, but the research on Math education instead. To discuss this paradox I therefore ask, 'what is research in general, and within Mathematics education specifically?'

## **8. CONFLICTING THEORIES**

Bo: Of course, Mathematics education research builds upon and finds inspiration in external theories. However, some theories are conflicting. Within Psychology, constructivism has a controversy between Vygotsky and Piaget. Vygotsky sees education as building ladders from the present theory regime to the learners' learning

zones. Piaget replaces this top-down view with a bottom-up view inspired by American Grounded Theory allowing categories to grow out of concrete experiences and observations. Within Sociology, disagreement about the nature of knowledge began in ancient Greece where the sophists wanted it spread out as enlightenment to enable humans to practice democracy instead of allowing patronizing philosophers to monopolize it. Medieval times saw a controversy between the realists and the nominalists as to whether a name is naming something or a mere sound. In the late Renaissance, a controversy occurred between Hobbes arguing that their destructive nature forces humans to accept patronization, and Locke arguing, like the sophists, that enlightenment enables humans to practice democracy without any physical or metaphysical patronization. As counter-enlightenment, Hegel reinstalled a patronizing Spirit expressing itself through art and through the history of different people. This created the foundation of Europe's line-organized office preparing Bildung schools; and for Marxism and socialism, and for the critical thinking of the Frankfurter School, reviving the ancient sophist-philosopher debate by fiercely debating across the Rhine with the post-structuralism of the French Enlightenment republic. Likewise, the two extreme examples of forced institutionalization in 20th century Europe, both terminated by the low institutionalized American Enlightenment republics, made thinkers as Baumann and Arendt point out that what made termination camps work was the authorized routines of modernity and the banality of evil. Reluctant to follow an order, you can find another job in the private sector, but not in an institution. Here the necessity of keeping a job forces you to carry out both good and evil orders. As an example of a forced institution, this also becomes an issue in Mathematics Education. So I ask: What role do conflicting theories play in Mathematics education and its research?

## REFERENCES

Chomsky, N. & Foucault, M. (2006). *The Chomsky-Foucault Debate on Human Nature*. New York: The New Press.

The Chomsky-Foucault debate on human nature, <https://www.youtube.com/watch?v=3wfNI2L0Gf8&feature=youtu.be>

Paul & Allan debate on postmodern mathe education ([https://www.youtube.com/watch?v=ArKY2y\\_ve\\_U](https://www.youtube.com/watch?v=ArKY2y_ve_U)),

# MIGRANT-MATH: CUPCOUNTING & PRESCHOOL CALCULUS

Allan Tarp

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*Europe receives a continuing migrant flow to benefit from its welfare and educational systems. To benefit from the engineer potential in young migrants allowing them to build up welfare and education in their own country, Europe must rethink its line-organized office directed education dating back to the Napoleon wars; and must replace meaningless top-down MetaMatism with bottom-up ManyMath.*

## BACKGROUND

Increased mathematics education research seems to create a decrease in Nordic PISA results as witnessed by the latest PISA study and the OECD 2015 report ‘Improving Schools in Sweden’. We ask: Can existentialism point to a possible solution?

Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines existentialism as holding that ‘existence precedes essence’ (Marino, 2004 p. 344). Thus a hypothesis can be formulated: Mathematics performance will increase if replacing essence-math with existence-math.

## MATHEMATICS AS A ESSENCE

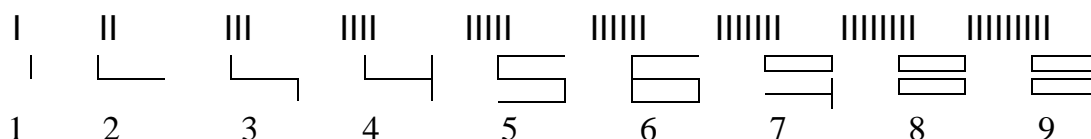
The Pythagoreans labeled their four knowledge areas by a Greek word for knowledge, mathematics. With astronomy and music now as independent areas, today mathematics is a common label for its two remaining activities both rooted in Many: Geometry meaning to measure earth in Greek, and Algebra meaning to reunite numbers in Arabic and replacing Greek Arithmetic (Freudenthal, 1973).

Then the set-concept transformed mathematics to ‘MetaMatics’ defining its concepts by self-reference as examples from internal abstractions instead of as abstractions from external examples. Looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox ‘this sentence it false’ being false if true and true if false: If  $M = \{ A \mid A \notin A \}$  then  $M \in M \Leftrightarrow M \notin M$ .

‘MetaMatism’ means mixing MetaMatics with ‘MatheMatism’ true inside but seldom outside the classroom as e.g. ‘the fraction paradox’ where the textbook insists that  $1/2 + 2/3$  IS  $7/6$  even if the students protest: counting cokes,  $1/2$  of 2 bottles and  $2/3$  of 3 bottles gives  $3/5$  of 5 as cokes and never 7 cokes of 6 bottles.

## MATHEMATICS AS MANY-MATH, A NATURAL SCIENCE ABOUT MANY

A number as  $345 = 3*B^2 + 4*B + 5*1$  shows that to deal with Many, first we iconize then we bundle and stack. Until ten we count in 1s by iconizing, i.e. by rearranging sticks in icons so five ones becomes one five-icon 5 with five sticks, etc.



With icons, a total can be ‘cup-counted’ in icon-bundles so a total T of 7 is bundled in 3s as  $T = 2 \text{ 3s} + 1$  shown with 2 sticks in a bundle-cup and 1 stick outside; reported with ‘cup-writing’,  $T = 2)1 \text{ 3s}$ , then with ‘decimal-writing’ where a decimal point separates the bundles from the singles, and including the unit 3s,  $T = 2.1 \text{ 3s}$ .

A calculator can predict a counting result. A stack of 2 3s is iconized as  $2 \times 3$  showing a lift used 2 times to lift the 3s. Taking away is iconized with ‘/3’ or ‘-3’ showing the broom or the trace when wiping away 3 several times or just once, called division and subtraction. Entering ‘7/3’, we ask the calculator ‘from 7 take away 3s’ and get the answer ‘2.some’. Entering ‘ $7 - 2 \times 3$ ’ we ask ‘from 7 take away 2 3s’ and get the answer 1 leftover. Thus the calculator predicts that  $7 = 2)1 \text{ 3s} = 2.1 \text{ 3s}$ .

Once cup-counted, totals are re-counted, double-counted or added next-to or on-top. To recount in the same unit, changing a bundle to singles creates over- or under-load as when recounting 4 2s as 3.2 2s, or as 5 less 2 2s leading to negative numbers:

$$T = 4 \text{ 2s} = 3.2 \text{ 2s}, \text{ or } T = 4 \text{ 2s} = 5.-2 \text{ 2s}$$

To recount in a different unit means changing unit, called proportionality. Asking ‘3 4s is how many 5s?’ sticks give the result 2.2 5s as predicted by a calculator.

$$T = 3 \text{ 4s} = \text{IIII} \quad \text{IIII} \quad \text{IIII} \rightarrow \text{IIII} \quad \text{IIII} \quad \text{II} \rightarrow 2)2 \text{ 5s} \rightarrow 2.2 \text{ 5s}$$

Recounting in and from tens means resizing number-blocks where the height and the base are inversely proportional as in  $3 \text{ 7s} = 2)1 \text{ tens}$  or  $4 \text{ tens} = 5 \text{ 8s}$ .

Double-counting a physical quantity creates ‘per-numbers’ as  $4\$/5\text{kg}$  allowing 16\$ to be recounted in 4s to bridge to the kg-numbers:  $16\$ = (16/4) \cdot 4\$ = (16/4) \cdot 5\text{kg} = 20\text{kg}$ .

Next-to addition of 2 3s and 4 5s as 3.2 8s means adding areas, called integration. To add on-top the units are made the same by recounting as 1.1 5s and  $4 \text{ 5s} = 5.1 \text{ 5s}$ . Reversed addition is called equations solved by recounting:  $2 \cdot x = 8 = (8/2) \cdot 2$  so  $x = 8/2$ , showing the solving method ‘move to opposite side with opposite sign’.

The root of geometry is a rectangle that halved by a diagonal becomes two right-angled triangles where the sides and the angles are connected by three laws,  $A+B+C = 180$ ,  $a^2+b^2 = c^2$  and  $\tan A = a/b$ . Being filled from the inside by such triangles, a circle with radius  $r$  gets the circumference  $2 \cdot \pi \cdot r$  where  $\pi = n \cdot \tan(180/n)$  for  $n$  large.

## CONCLUSION

There is a fundamental difference between essence- and existence-math, MetaMatism and ManyMath. This means the latter has to be tested outside traditional school in preschool or in special courses for young migrants wanting to become engineers or teachers to help building welfare and education systems in their own country.

## REFERENCE

- Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht-Holland: D. Reidel Publishing Company.
- Marino, G. (2004). *Basic Writings of Existentialism*. New York: Modern Library.

# MigrantMath

- **CupCount & ReCount & DoubleCount**
- **Multiplication** before **Addition**
- **PreSchool Calculus** before **OnTop Addition**

## CupCounting 5 Sticks in 2s

$$\begin{array}{lclclcl}
 5 & = & || & | & | & = & \begin{array}{c} | \\ | \\ | \end{array} & | & | & = & 1)3 & 2s \\
 5 & = & || & || & | & = & \begin{array}{c} || \\ | \\ | \end{array} & | & & = & 2)1 & 2s \\
 5 & = & || & || & | & = & \begin{array}{c} || \\ || \\ | \end{array} & | & & = & 3)-1 & 2s
 \end{array}$$

3 ways to **CupCount**: **Overload**, **Normal**, **Underload**

ReCount 7 in 3s:  $7 = 2)1 \text{ } 3s = 1)4 \text{ } 3s = 3)-2 \text{ } 3s$

NO, **4x7 is not 28**, it is 4 7s = 2)8 = 1)18 = 3)-2 tens

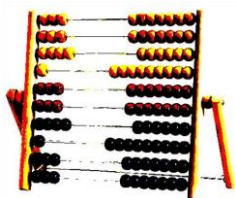
NO, **30/6 is not 30 divided by 6**, it is 3tens recounted in 6s

**CupWriting** tells **InSide Bundles** from **OutSide 1s**

- |                 |                                   |            |
|-----------------|-----------------------------------|------------|
| • $65 + 27$     | $= 6)5 + 2)7 = 8)12 = 9)2 =$      | <b>92</b>  |
| • $65 - 27$     | $= 6)5 - 2)7 = 4)-2 = 3)8 =$      | <b>38</b>  |
| • $7 \times 48$ | $= 7 \times 4)8 = 28)56 = 33)6 =$ | <b>336</b> |
| • $336 / 7$     | $= 33)6 / 7 = 28)56 / 7 = 4)8 =$  | <b>48</b>  |

MatheMatics as **ManyMath** - a Natural Science about Many

**MATHeCADEMY.net**








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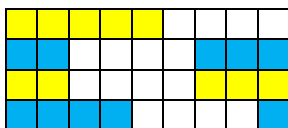
# CupCount <sup>fore you</sup> Add

MatheMatics as **Many**Math, a Natural Science about **MANY**

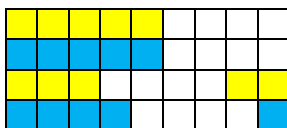
Cure **Math Dislike**: Use Children's own 2D Numbers with Units

<b>Count</b> In <i>Icons</i> In <i>BundleCups</i>	$T = \text{    } = \text{┐} = 4$ $T = 7 = \text{       } = \text{       } = 2)1 \text{ 3s} = 2 \text{ Bundles \& } 1 \text{ 3s}$						
<b>ReCount</b> In same Unit In new Unit	<b>ReBundle</b> to create <b>Overload</b> & <b>Underload</b> $T = 7 = \text{       } = 2)1 \text{ 3s} = 1)4 \text{ 3s} = 3)-2 \text{ 3s}$ $T = 2)1 \text{ 3s} = 1)3 \text{ 4s} = 1)2 \text{ 5s} = 3)1 \text{ 2s} = 1)1)1 \text{ 2s} = 11)1 \text{ 2s}$						
<b>ReCount</b> In Tens From Tens	$3 \text{ 7s} = ? \text{ tens}$ Answer: $3 \times 7 = 21 = 2)1 \text{ tens}$  $? \text{ 7s} = 3 \text{ tens}$ Answer: $(30/7) \times 7 = 4)2 \text{ 7s}$ 						
<b>DoubleCount</b> in <i>PerNumbers</i> in <i>PerFive, 3/5</i> in <i>PerHundred, %</i>	With 4\$ per 5kg, $T = 20\text{kg} = (20/5) \times 5\text{kg} = (20/5) \times 4\$ = 16\$$ $3 \text{ per } 5 \text{ of } 200\$ = ?\$$ . $200\$ = (200/5) \times 5\$ \text{ gives } (200/5) \times 3\$ = 120\$$ $70\% \text{ of } 300\$ = ?\$$ . $300\$ = (300/100) \times 100\$ \text{ gives } (300/100) \times 70\$ = 210\$$						
<b>Calculator</b> Prediction RecountFormula RestackFormula	$T = 2 \text{ 4s} = ? \text{ 5s} = 1)3 \text{ 5s}$ since <div style="border: 1px solid blue; padding: 5px; display: inline-block;"> <math>2 \times 4/5</math>      1.some  <math>2 \times 4 - 1 \times 5</math>      3         </div> $T = (T/B) \times B = T/B \text{ Bs}$ $T = (T-B)+B$						
<b>Add</b> NextTo OnTop	$T = 2 \text{ 3s} + 4 \text{ 5s} = 3)2 \text{ 8s}$  <i>Integration</i>  <i>PreSchool Calculus</i> $T = 2 \text{ 3s} + 4 \text{ 5s} = 1)1 \text{ 5s} + 4 \text{ 5s} = 5)1 \text{ 5s}$  <i>Proportionality</i>						
<b>Reverse Adding</b> Solve Equations Move to Opposite side & sign	<table border="1"> <thead> <tr> <th>OnTop</th><th>NextTo</th></tr> </thead> <tbody> <tr> <td> <math>2 \times ? = 8 = (8/2) \times 2</math>  <math>? = 8/2</math>  <i>Solved by ReCounting</i> </td><td> <math>2 + ? = 8 = (8-2) + 2</math>  <math>? = 8-2</math>  <i>Solved by ReStacking</i> </td></tr> <tr> <td colspan="2"> <math>T = 2 \text{ 3s} + ? \text{ 5s} = 3.2 \text{ 8s}</math>  <math>? = (3.2 \text{ 8s} - 2 \text{ 3s})/5 = \Delta T/5</math>  <i>Solved by Differentiation</i> </td></tr> </tbody> </table>	OnTop	NextTo	$2 \times ? = 8 = (8/2) \times 2$ $? = 8/2$ <i>Solved by ReCounting</i>	$2 + ? = 8 = (8-2) + 2$ $? = 8-2$ <i>Solved by ReStacking</i>	$T = 2 \text{ 3s} + ? \text{ 5s} = 3.2 \text{ 8s}$ $? = (3.2 \text{ 8s} - 2 \text{ 3s})/5 = \Delta T/5$ <i>Solved by Differentiation</i>	
OnTop	NextTo						
$2 \times ? = 8 = (8/2) \times 2$ $? = 8/2$ <i>Solved by ReCounting</i>	$2 + ? = 8 = (8-2) + 2$ $? = 8-2$ <i>Solved by ReStacking</i>						
$T = 2 \text{ 3s} + ? \text{ 5s} = 3.2 \text{ 8s}$ $? = (3.2 \text{ 8s} - 2 \text{ 3s})/5 = \Delta T/5$ <i>Solved by Differentiation</i>							

$T = 7 = 2)1 \text{ 3s}$  on an **Abacus**:



Geometry-mode



Algebra-mode

MrAlTarp

YouTube Videos

Allan.Tarp@MATHeCADEMY.net



MATHeCADEMY.net

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*MatheMatics as **Many**Math*



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Reference ID	Title	Reviewing Process
cfef0eff4c410d62ac3bf4e8464f3a92	A 1YEAR PRE-ENGINEER COURSE FOR YOUNG MIGRANTS, A JOB FOR CRITICAL OR CIVILIZED MATH EDUCATION	<a href="#">Final Result</a> <a href="#">Reviewers Comments</a>
eabba0e06015e477b75e7acc62c9d8f	DEBATE ON HOW TO IMPROVE MATHEMATICS EDUCATION	<a href="#">Final Result</a> <a href="#">Reviewers Comments</a>
88254b84f4667ea1da697e217a377d	ONLINE TEACHER TRAINING FOR CURING MATH DISLIKE: CUP&RE-COUNTING & MULTIPLICATION BEFORE ADDITION	<a href="#">Final Result</a> <a href="#">Reviewers Comments</a>
613d75d931b3a83aa9ce081f49681b1	MIGRANT-MATH: CUPCOUNTING & PRESCHOOL CALCULUS	<a href="#">Final Result</a> <a href="#">Reviewers Comments</a>

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## REVIEW PAPER

Review for submission with Reference ID **cfef0eff4c410d62ac3bf4e8464f3a92**

### Review Details:

**Reviewer ID** 158

**I recommend that the paper/project/poster/symposium be** Rejected

### A. Compatibility with the Aims of MES

**A1. Which of the following does the paper/project/poster/symposium incorporate:**

**A2. Overall assess the compatibility with MES:**

The paper makes an interesting claim that (many) current mathematics curricula draw on a “critical thinking” approach that is “patronizing” in presenting a chosen version of mathematics as if it were natural (“essentialism”) in contrast to a (not yet realised) “civilized thinking” approach in relation to the real (“existentialism”). After an unexciting pathologizing of the state of mathematics education with reference to PISA, the author engages in a very ambitious programme: “How to improve mathematics and its education to better serve the population and migrants?” The paper unfortunately does not make it clear or discuss, however, how the proposed ManyMath would achieve this. While the overall project appears compatible with MES, the answer suggested to the more specific question, “How to design a 1 year pre-

	<p>engineer course for young migrants beginning from scratch?”, is phrased in mathematical terms only, and so remains unrelated the social, political or cultural context.</p>
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	<p>The “theoretical background” offers an interesting comparatively long historical sketch of the development of what is termed in the paper the “critical thinking” approach in education; the arguments are not specific to mathematics education and difficult to follow. A section on mathematics education, highlighting the difference between “critical” and “civilized thinking”, consists mainly of more than a full page direct quote from a previous paper by the author. The work is not situated in relation to any other relevant work in the field of mathematics education.</p>
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	<p>I have reviewed this paper as a theoretical paper. But it does not represent the genre of an academic debate based on some agreement about coherence, logic and warrants of the argument. Also, the narrative contains a couple of inaccuracies (for example that the OECD initiated a report about the Swedish Education System; it was in fact the government who asked the OECD).</p>
<b>B3. Are the discussion and conclusions well founded?</b>	<p>The link between the “theoretical background”, which is concisely summarised in the discussion section, and the impossibility of questioning (the standard version of) rational numbers (in fact, there are many definitions of rational numbers available within the mathematics community as well as in didactic research) and “line-organized education” remains opaque. The conclusion that “education must be organized in flexible half-year blocks aiming at uncovering and developing the talent of the individual learner” has little or no relation to the preceding text. The reading of this paper certainly induces a “scepticism towards ungrounded is-claims” of which I found too many in the text.</p>
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	<p>The paper is composed in two parts – one general cultural-historic, one specific mathematical – each of which is self-contained, with the first operating at a very general level and the second at the level of mathematical detail. A link between these</p>

	parts, however, is not explicitly established but taken for granted, making the paper as a whole incoherent. Also, I had difficulties in understanding the metaphorical introduction in the section entitled “historical background”, probably because of some fixation on logic or coherence regarding the genre of a research paper.
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	Critique of institutions and “is-claims” is needed and welcome, but I have two major concerns with this paper: 1 - It is not situated in the mathematics education research field (for example, not to take the mathematics to be taught for granted is a key feature of the ATD; calculation modes of “ManyMath” are to different extents already proposed and practiced in school mathematics (albeit without the specific formalism used by the author, and probably not in a systematic way), emphasising the counting and re-ordering of objects (such as Dienes blocks) rather than drawing on an internal set-theoretic foundation. 2 - There are no arguments put forward as to why and how a practice drawing on “ManyMath” would solve the ‘problems’ with current institutionalised mathematics education as adumbrated in the introduction.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	More space and effort should be devoted to the question of how and why the ManyMath approach could “improve mathematics and its education to better serve the population and migrants”.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	No; the paper is well written.
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	N.A.
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	N.A.
<b>F. Further Comments</b>	
<b>F1. Comments</b>	Much space is given to (mathematical) details of the suggested approach to perform calculations, while the issue of how and why this approach could “improve mathematics and its education to better serve the

	population and migrants” is not addressed. This ‘unbalance’ could have been avoided as these mathematical elaborations already have been presented elsewhere.
--	---

## File

## Reviewer details

**Reviewer Name:** JE

Review for submission with Reference ID **cfef0eff4c410d62ac3bf4e8464f3a92**

## Review Details:

<b>Reviewer ID</b>	89
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	
<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	I am not sure this paper is ready for MES
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	There is an attempt to draw on considerable literature, the problem is the lack of coherence and clarity I experienced in using this literature. However much of the early literature about reptilian brains etc. is quite irrelevant to what argument might be developed.
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	This does not appear to be an empirical paper
<b>B3. Are the discussion and conclusions well founded?</b>	I certainly could not identify any clear conclusions
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	This is the main problem. There are issues with language - the author appears to be struggling at times to express complex ideas clearly in English.
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	I do not think this paper is clear, or coherent or well structured I am afraid.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	No, I think a lot of work needs to go into it before it is ready for presentation.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	It needs someone to go through it with the author and significantly redraft the paper.
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	n/a
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	n/a
<b>F. Further Comments</b>	
<b>F1. Comments</b>	no

File

Reviewer details

Reviewer Name:

GP

## REVIEW PROJECT

Review for submission with Reference ID **eabba0e0601be477b78e7acc62c59d8f**

<b>Review Details:</b>	
<b>Reviewer ID</b>	148
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	
<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	The topics raised in the symposium proposal are highly compatibly with the aims of MES.
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	Not at all. The author raises a number of questions for discussion in the symposium, questions which have already been addressed in many publications which are not referred to by the proposer. From my point of view, the ignorance of the proposer towards other scholar's contributions and his foregrounding of his personal ideas does not match scientific criteria.
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	Not applicable.
<b>B3. Are the discussion and conclusions well founded?</b>	Not at all. The proposer of the symposium addresses different topics (such as 'mathematics itself' or 'education in general') and then shortly discusses some ideas concerning these concepts, eventually leading to the questioning of the concept as a whole. Firstly, it remains opaque why exactly these thoughts about the concepts are discussed and not others; and secondly, this procedure of questioning the touched concepts could be repeated with every concept without necessarily providing any insights.
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	No. The proposal lacks any argumentative coherence which would explain why certain issues are raised and others not. The form is somewhat unusual; for example the introduction of every paragraph with the phrase "Bo:".
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	Based on the proposal that was submitted, I did not gain the impression that the proposer has a good overview over the research field in which he wants to facilitate a discussion. The absence of references to the work of colleagues lets me wonder in how far the proposer is perceiving and valuing the thoughts of others. Given that the proposer also has a long reputation in establishing rather self-centred forms

	of communication, I do not think that the MES community would benefit from having the proposer offer a symposium on the conference.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	No.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	No.
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	Sessions are not described.
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	The structure of the symposium is not described.
<b>F. Further Comments</b>	
<b>F1. Comments</b>	See attachment for further comments!
<b>File</b>	<a href="#">20161107145418_TarpASymposium.pdf</a>
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	KD

Review for submission with Reference ID **eabba0e0601be477b78e7acc62c59d8f**

<b>Review Details:</b>	
<b>Reviewer ID</b>	119
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	
<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	The aim of the suggested symposium is to debate how mathematics education can be improved, mainly based on the question 'If research cannot improve Math education, then what can?'. Author raises general questions about mathematics, education, mathematics education, the learner, the teacher, the political system, research, and conflicting theories. Although there are issues of relevance for the MES community embedded in text I think a symposium at the MES conference need a clearer focus.
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	The author has no references in the text, but in referencelist there is a book, Chomsky & Foucault (2006) and two Youtube videos showing the Chomsky & Foucault debate and an animated debate between Paul Ernest and the Allan Tarp (script for the debate can be found in Philosophy of Mathematics Education Journal No. 27, April 2013). As far as I understand, the

	references are dealing with the structure of the proposed symposium more than content.
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	Not relevant
<b>B3. Are the discussion and conclusions well founded?</b>	I think the author raises several general issues that are not founded.
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	Since I don't see how the author wants to conduct the symposium I don't see how the different parts of the proposal are linked.
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	As far as I can see the author has not followed the instructions for a symposium proposal since the text does not include "a rationale for its relevance to the conference and a suggested plan for how the symposium will be conducted" (from Call for Papers). Hence, my recommendation is not to accept the symposium proposal.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	I think the proposed symposium would need a clear focus and relate to some research in the field.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	No
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	I cannot see how the author connects his work to previous work in the field and the questions raised in the paper are very general and broad. Since there is no structure of symposium presented it is impossible to see if and how questions of relevance for the conference will be discussed.
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	In my opinion the author does not present a plan for how the symposium will be conducted. The referenced book and the Youtube videos might give a hint about how the author wants to organize the symposium, but the unclear structure of symposium, the general questions, and the lack of focus might mean that discussions will not be productive.
<b>F. Further Comments</b>	
<b>F1. Comments</b>	
<b>File</b>	
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	GH

## REVIEW SYMPOSIUM

<b>Review Details:</b>	
<b>Reviewer ID</b>	206
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	
<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	The aims of the MES in general is not visible in the current paper
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	The author refers to existentialism by Sartre, but it is not clear how the analytic framework embodies the project
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	The methodology is not clearly explained and justified
<b>B3. Are the discussion and conclusions well founded?</b>	The author present some conclusions, however they are not supported by evidence from the study
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	the focus of the paper is not clear
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	no comments
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	no suggestions
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	I can not suggest language edition
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	XXXXXXX
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	XXXXXX
<b>F. Further Comments</b>	
<b>F1. Comments</b>	
<b>File</b>	
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	PM

<b>Review Details:</b>	
<b>Reviewer ID</b>	140
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	



<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	The focus and purpose of the proposal was unclear, and I was unable to discern a connection to the MES community.
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	The author does not situate this work in relation to mathematics education scholarship.
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	The proposal includes what seems to be an advertisement for the author's one-day Skype seminar.
<b>B3. Are the discussion and conclusions well founded?</b>	No.
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	No.
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	This is not a viable proposal.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	No.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	No.
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	Not applicable.
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	Not applicable.
<b>F. Further Comments</b>	
<b>F1. Comments</b>	
<b>File</b>	
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	LG

## REVIEW POSTER

Review for submission with Reference ID **613d75d931b3aa83aa9ced81f49681b1**

<b>Review Details:</b>	
<b>Reviewer ID</b>	158
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	
<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	This poster proposes that “meaningless top-down MetaMatism” must be replaced with “bottom-up ManyMath”. The text is a presentation of how to perform calculations in

	the latter, methods that appear similar to many calculation strategies commonly used in school. There is no elaboration on how or why this would link to the aims of MES.
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	The work is not related to other relevant work in the field; no such references are provided.
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	N.A.
<b>B3. Are the discussion and conclusions well founded?</b>	The “fundamental difference between essence- and existence-math, MetaMatism and ManyMath”, needs to be made more clear and why the latter would solve the ‘math-problems’ (for example in Sweden, as suggested in the introduction).
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	The poster is coherent regarding its mathematical content but not very clear (by arguing with unexplained terminology such as “MetaMatism”).
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	The poster has a focus on mathematical calculations and does not elaborate on the aspects of mathematics education pursued by MES.
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	To be of interest for the MES audience, focus would need to be on why the suggested way of doing mathematics (“ManyMath”) would have social relevance/benefits.
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	No; the poster is well written.
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	N.A.
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	N.A.
<b>F. Further Comments</b>	
<b>F1. Comments</b>	
<b>File</b>	
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	JE
Review for submission with Reference ID <b>613d75d931b3aa83aa9ced81f49681b1</b>	
<b>Review Details:</b>	
<b>Reviewer ID</b>	88
<b>I recommend that the paper/project/poster/symposium be</b>	Rejected
<b>A. Compatibility with the Aims of MES</b>	

<b>A1. Which of the following does the paper/project/poster/symposium incorporate:</b>	
<b>A2. Overall assess the compatibility with MES:</b>	The poster is marginally relevant to the aims of MES
<b>B. Structure of the paper/project/poster/symposium</b>	
<b>B1. Does the author make clear how the work is situated in relation to other relevant literature in the field?</b>	very poorly
<b>B2. Where relevant - is the methodology clearly explained and justified?</b>	the poster suggests teaching approaches on the basis of author's views
<b>B3. Are the discussion and conclusions well founded?</b>	see comment B2
<b>C. Communication and clarity</b>	
<b>C1. Is the paper/project/poster/symposium proposal clear and coherent in both content and form?</b>	content of this poster is presented in a very confusing form
<b>D. General</b>	
<b>D1. Any general comments including reasons for the recommendation.</b>	This poster seems to be a commercial of a particular Educational Services business
<b>D2. Any suggestions for the author regarding the presentation of the paper/project/poster/symposium.</b>	no
<b>D3. Do you think that the paper/project/poster/symposium requires language editing by the program committee? (If you wish, you may edit the paper/project/poster/symposium yourself using the tracking option.)</b>	no
<b>E. Symposium Criteria</b>	
<b>E1. Does the symposium sessions have an overall sense of coherence?</b>	no
<b>E2. Does the structure of the Symposium seem to be realistic and productive?</b>	no
<b>F. Further Comments</b>	
<b>F1. Comments</b>	This poster seems to be a commercial of a particular Educational Services business
<b>File</b>	
<b>Reviewer details</b>	
<b>Reviewer Name:</b>	CD