

Math **Dislike** **CURED**

by 1 **Cup** & 5 Sticks short version

My Many Math Tears will not Stay – if I Cup the Stray Away

CupCOUNT before you **ADD**

$$\begin{array}{lclclcl}
 5 = ||| & = & \text{Cup I} & ||| & = & 1]3 \text{ 2s} \\
 5 = ||| & = & \text{Cup II} & | & = & 2]1 \text{ 2s} \\
 5 = ||| & = & \text{Cup III} & | & = & 3]-1 \text{ 2s}
 \end{array}$$

3 ways to **CupCount**: **Overload**, **Normal**, **Underload**

ReCount 7 in 3s: $7 = 2]1 \text{ 3s} = 1]4 \text{ 3s} = 3]-2 \text{ 3s}$

NO, **4x7 is not 28**, it is 4 **7s** = 2]8 = 1]18 = 3]-2 **tens**

NO, **30/6 is not 30 divided by 6**, it is **3tens** counted in 6s

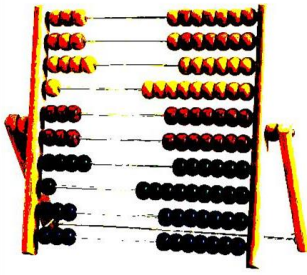
CupWriting tells **InSide Bundles** from **OutSide 1s**

● 65 + 27	=	6]5 + 2]7 = 8]12 = 9]2 =	92
● 65 – 27	=	6]5 – 2]7 = 4]-2 = 3]8 =	38
● 7x 48	=	7x 4]8 = 28]56 = 33]6 =	336
● 336 / 7	=	33]6 / 7 = 28]56 / 7 = 4]8 =	48

MatheMatics as **ManyMath** - a Natural Science about Many
Makes Math Potentials Blossom in Children, Adults & Migrants

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CupCount ^{fore you} **Add**

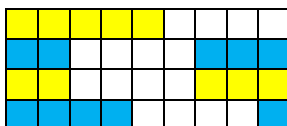
MatheMatics as **ManyMath**
a Natural Science about **MANY**

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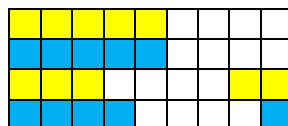
Cure **Math Dislike**: Use Children's own 2D Numbers with Units

Count In <i>Icons</i> In <i>BundleCups</i>	$T = \text{ } = \text{┐} = 4$ $T = 7 = \text{ } = \text{ } = 2]1 \text{ 3s} = 2 \text{ Bundles} \& 1 \text{ 3s}$
ReCount In same Unit In new Unit	ReBundle to create Overload & Underload $T = 7 = \text{ } = 2]1 \text{ 3s} = 1]4 \text{ 3s} = 3]-2 \text{ 3s}$ $T = 2]1 \text{ 3s} = 1]3 \text{ 4s} = 1]2 \text{ 5s} = 3]1 \text{ 2s} = 1]1]1 \text{ 2s} = 11]1 \text{ 2s}$
ReCount In Tens From Tens	$3 \text{ 7s} = ? \text{ tens}$ Answer: $3 \times 7 = 21 = 2]1 \text{ tens}$  $? \text{ 7s} = 3 \text{ tens}$ Answer: $(30/7) \times 7 = 4]2 \text{ 7s}$ 
DoubleCount in <i>PerNumbers</i> in <i>PerFive, 3/5</i> in <i>PerHundred, %</i>	With 4\$ per 5kg, $T = 20\text{kg} = (20/5) \times 5\text{kg} = (20/5) \times 4\$ = 16\$$ 3 per 5 of 200\$ = ?\$. $200\$ = (200/5) \times 5\$ \text{ gives } (200/5) \times 3\$ = 120\$$ 70% of 300\$ = ?\$. $300\$ = (300/100) \times 100\$ \text{ gives } (300/100) \times 70\$ = 210\$$
Calclator Prediction <i>RecountFormula</i>	$T = 2 \text{ 4s} = ? \text{ 5s} = 1]3 \text{ 5s}$ since $2 \times 4/5 = 1.\text{some}$ $T = (T/B) \times B = T/B \text{ Bs}$ $2 \times 4 - 1 \times 5 = 3$
Add NextTo OnTop	$T = 2 \text{ 3s} + 4 \text{ 5s} = 3]2 \text{ 8s}$  <i>Integration</i> $T = 2 \text{ 3s} + 4 \text{ 5s} = 1]1 \text{ 5s} + 4 \text{ 5s} = 5]1 \text{ 5s}$  <i>Proportionality</i>
Multiply, Divide Use <i>CupWriting</i>	$7 \times 463 = 7 \times 4]6]3 = 28]42]21 = 28]44]1 = 32]4]1 = 3241$ $3241 / 7 = 32]4]1 / 7 = 28]44]1 / 7 = 28]42]21 / 7 = 4]6]3 = 463$

$T = 7 = 2]1 \text{ 3s}$ on an **Abacus**:



Geometry-mode



Algebra-mode

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*Teaching Teachers to Teach MatheMatics as **ManyMath***

Piaget: Grasping with Fingers leads to Grasping Mentally

Four as an icon built by four cars, four rhinos, four sticks, a ruler folded in four parts, four beads on an abacus, LEGO blocks, pearls on a pearl board, etc.

Seven sticks cup-counted as 1]2 5s, or as 2]1 3s or as 3]1 2s



The MATHeCADEMY.net stand at the MatematikBiennale in Sweden, 2014

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Preface

"How old will you be next time?" I asked the child. "Four", he answered and showed me four fingers. "Four, you said?" I asked and showed him four fingers held together two by two. "No, that is not four, that is two twos!" the child replied. That opened my eyes. Children come to school with two-dimensional block numbers where all numbers have units. However, the school does not allow the children to count the numbers before being added. Instead the school teaches cardinality as a one-dimensional number line where numbers have different names; thus disregarding the fact that numbers are two dimensional blocks where all numbers have a unit as shown when writing out fully

$$T = 345 = 3 \text{ BundleBundles} + 4 \text{ Bundles} + 5 \text{ Singles} = 3 \cdot 10^2 + 4 \cdot 10 + 5 \cdot 1.$$

This booklet allows schools and parents to choose an education that accepts and develops the 2D number blocks that the children bring to school instead of forcing a 1D number line upon them. Also, the booklet allows the children to practice 'counting before adding' and to include cup-counting and re-counting to different units. The booklet thus is an answer to the question 'How to Save and Develop a Child's Math Potential?'

To master Many we ask 'how many?' To answer, we count by bundling and stacking to get a total T. Once counted, first a total can be recounted in the same unit to create overload or underload, or to create a different unit; next totals can be added NextTo, or OnTop if the units are the same.

Counting a total T of 7 ones in 3s we get the result $T = 7 = 2 \text{ 3s} \ \& \ 1 = 2]1 \text{ 3s}$.

We separate the *inside* bundles from the *outside* unbundled singles by a *cup* becoming a bracket when reporting the result with *cup-writing*: $T = \text{III III} \mid = \text{II}] \mid = 2]1 \text{ 3s}$

Once counted, a total can be *recounted* to create *overload* or *underload*, deficit. To create an overload, we move a stick from the inside to the outside: $T = \text{II}] \mid = \mid] \text{III} \mid = 1]4 \text{ 3s}$.

To create an underload, we borrow foreign sticks to move a bundle from the outside to the inside

$$T = \text{II}] \mid = \text{II}] \mid \text{II} \text{ II} = \text{III}] \text{ II} = 3]-2 \text{ 3s}.$$

Thus a given total can be *recounted* in three ways: normal, with overload and with underload.

$$T = 7 = 2]1 \text{ 3s} = 1]4 \text{ 3s} = 3]-2 \text{ 3s}.$$

A total of 68 can be recounted in four different ways as $T = 68 = 6]8 \text{ tens} = 5]18 \text{ tens} = 7]-2 \text{ tens}$.

Recounting and cup-writing come in handy when we add, subtract, multiply or divide numbers:

Using cup-writing to add 65 and 27 we get an overload outside the bundle cup allowing us to move 10 **1s** from the outside to the inside as 1 **tens**

$$T = 65 + 27 = 6]5 + 2]7 = 8]12 = 9]2 = 92$$

Using cup-writing to subtract 27 from 65 we get an underload outside the bundle cup allowing us to move a bundle of 1 **tens** from the inside to the outside as 10 **1s** to remove the underload.

$$T = 65 - 27 = 6]5 - 2]7 = 4]-2 = 3]8 = 38$$

Alternatively, before subtracting we can create an overload outside by moving 1 **tens** from the inside to the outside as 10 **1s**

$$T = 65 - 27 = 6]5 - 2]7 = 5]15 - 2]7 = 3]8 = 38$$

Using cup-writing to multiply 48 with 7 we get an overload outside the bundle cup allowing us to move 50 **1s** from the outside to the inside as 5 **tens**

$$T = 7 * 48 = 7 * 4]8 = 28]56 = 33]6 = 336$$

Alternatively, before multiplying we can create an underload outside by borrowing 2 **1s**. Later the underload can be removed by moving 2 **tens** outside as 20 **1s**

$$T = 7 * 48 = 7 * 4]8 = 7 * 5]-2 = 35]-14 = 33]6 = 336$$

Using cup-writing to divide 336 with 7 we prefer to have 28 instead of 33 inside the bundle cup, so we create an overload outside by moving 5 bundles outside as 50 **1s**

$$T = 336 = 33]6 = 28]56; \text{ so } T / 7 = 4]8 = 48$$

Alternatively, we can create an underload outside before dividing

$$T = 336 = 33]6 = 35]-14; \text{ so } T / 7 = 5]-2 = 4]8 = 48$$

To divide 338 by 7 we get 2 single leftovers that counted in 7s becomes a fraction $2/7$

$$T = 338 = 33]8 = 28]58 = 28]56 + 2; \text{ so } T / 7 = 4]8 + 2/7 = 48 \text{ } 2/7$$

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Introduction to the Chapters

Chapter 01, From Sticks to Icons, shows how rearranging four sticks creates a 4-icon with as many sticks as it represents; likewise with the other icons until ten having a name but no icon.

Chapter 02, Counting in Icons, shows that when counting by bundling, the bundle-icon is not used. Hence, when counting in tens, ten does not need an icon. A natural counting sequence will report both the bundles and the unbundled: 01, 02, ..., 10, 11; or 0.1, 0.2, ..., 1.0, 1.1 always including the bundle-name as the unit. Each bundle-size has its own counting sequence, but the standard is ten-counting in a sloppy version leaving out the unit and misplacing the decimal point by saying 23 instead of 2.3 tens.

Chapter 03, CupCounting in Icons, shows how a total T can be recounted in icon-bundles. Thus a total of nine things, represented by a line of sticks or beads on an abacus, can be counted in fours by a counting sequence. Also, they can be represented by a stack of bundles placed with one stick per bundle in a bundle cup that can be written as a bracket (cup-writing) and reported as a decimal number with a unit where the decimal point separates the bundles from the unbundled singles, $T = 9 = 2]1\ 4s = 2.1\ 4s$. Alternatively, a calculator can be asked to predict the counting result. Entering '9/4', we ask 'from 9, taking away 4s can be done how many times?' The calculator answers '2.some' so by entering '9 - 2x4' we ask 'from 9, once taking away 2 4s leaves what?' The answer '1' gives the calculator prediction $T = 9 = 2.1\ 4s$. Thus also operations are icons: /4 shows the broom wiping away 4 many times, - 4 shows the trace left when dragging away 4 only once, 2x shows the lifting needed to create a stack of 2 bundles, and +3 shows the juxtaposition of 3 singles left next to a stack of bundles. Moving 1 stick outside the bundle cup creates an overload $T = 1]5\ 4s$; and moving an extra stick in gives an underload, a deficit, $T = 3]-3\ 4s$. Thus by recounting, a total T of nine can be recounted in 4 different ways: $T = \text{nine} = 9\ 1s = 2]1\ 4s = 1]5\ 4s = 3]-3\ 4s$. This comes in handy when totals are added, subtracted, multiplied or divided.

Kapitel 04, CupCounting with dices, shows how a total T can be recounted in icon-bundles where the total is shown on two similar dices and the icon-number is shown on a third dice.

Chapter 05, ReCounting in the same Unit, shows how to recount a total T in the same unit by unbundling a bundle to singles thus creating an overload, or by embezzling extra singles that then has be counted for as a deficit. Thus a total of 2.1 5s can be written with overload as $T = 1]6\ 5s$ or as $T = 1.6\ 5s$, or with embezzlement as $T = 3]-4\ 5s$ or as $T = 3.-4\ 5s$

Chapter 06, ReCounting in a new Unit, shows how once counted in one unit, a total T can be recounted in another unit. Thus a total of 2 9s can be recounted in 6s as in chapter 3, again by lining, counting, bundling, stacking, cup-writing and answering; and again checked by a calculator prediction using two formulas. The ReCount formula $T = (T/B)*B$ saying that 'from T, T/B times Bs can be taken away'; and the ReStack formula $T = (T-B)+B$ saying that 'From T, T-B is left when B is placed next to'. To change a unit is also called **proportionality**.

Chapter 07, ReCounting in BundleBundles, shows how an overload in a bundle-cup can be removed by an extra cup for bundles-of-bundles. Thus counting a total T of 4 8s in 5s gives $T = 6]2\ 5s$. However, with 5 as the bundle-size, 5 bundles can be recounted as 1 bundle-of-bundles of 5s so that $T = 6]2\ 5s = B]1]2\ 5s = 1]1]2\ 5s$ or $T = 6.2\ 5s = 11.2\ 5s$.

Chapter 08, ReCounting in Tens on Squared Paper or an Abacus, shows how easy totals counted in icon-bundles can be recounted in tens since the calculator is programmed to give the answer directly in its sloppy version. Thus to recount 3 8s in tens we enter 3*8 and get the answer 24, so $T = 3\ 8s = 2.4\ \text{tens}$. Recounting icon-numbers in tens systematically will provide the multiplication tables showing individual patterns in a ten by ten square or on an abacus.

Chapter 09, ReCounting from Tens, shows, as in chapter 3, that we can get the answer through a calculator prediction or through lining, rebundling, and cup-writing. Only this time we shorten the line by using Roman numbers as icons. Recounting large numbers from tens, we save time by using a multiplication table. Thus to recount a total T of 253 in 7s we use cup-writing to create overloads guide by the table:
 $T = 253 = 25]3 = 21]43 = 21]42+1 = 3]6\ *7 +1$, so $T = 253 = 36\ 7s + 1$.

Chapter 10, ReCounting BigNumbers in Tens, show how cup-writing may be used to create overloads later to be removed to get the final answer. Thus to recount 7 43s in tens gives a total
 $T = 7\ 43s = 7*43 = 7*4]3 = 28]21 = 30]1 = 301$ as confirmed by a calculator.

Chapter 11, DoubleCounting with PerNumbers, shows that counting a quantity in two different physical units will provide a per-number to be used as a bridge connecting the two units. Thus counting a quantity as 4\$ and as 5 kg gives the per-number 4\$/5kg or 4/5 \$/kg. Asking '8\$ = ? kg', the answer comes from recounting the 8s in 4s to be able to use the per-number as a bridge between the two units: $T = 8\$ = (8/4)*4\$ = (8/4)*5\text{kg} = 10\text{kg}$. Likewise when asking e.g. '?\$ = 12kg'

Chapter 12, DoubleCounting with Fractions and Percentages, shows that fractions and percentages can be treated as per-numbers. Thus asking '3/5 of 200\$' is the same as asking '3 per 5 of 200\$ gives ?'. So we recount the 200 in 5s to get the answer: $T = 200\$ = (200/5)*5\$$ giving $(200/5)*3\$ = 120\$$. And asking '3% of 250\$' is the same as asking '3 per 100 of 250\$'. So we recount the 250 in 100s to get the answer: $T = 250\$ = (250/100)*100\$$ gives $(250/100)*3\$ = 7.5\$$ as confirmed by writing '3/100*250' on a calculator.

Chapter13, ReCounting PerNumbers, Fractions, shows how changing unit transforms pernumbers.

Chapter 14, Adding OnTop, shows that to add two totals T1 and T2 OnTop the units must be the same so recounting may be needed to change a unit. Thus adding 2 3s and 4 5s as 3s, the 4 5s must be recounted as 3s to give a total of 8.2 3s as confirmed by a calculator.

Chapter 15, Reversed Adding OnTop, shows that to reverse OnTop addition, the first total must be taken away before counting the rest in the unit of the second total. Thus asking '2 3s + ? 5s total 5 3s, we take away the 2 3s from the 5 3s before recounting the rest, $T - T1$, in 5s by saying $(T-T1)/5 = \Delta T/5 = 1.4$ 5s as confirmed by a calculator. Subtraction followed by division is called differentiation.

Chapter 16, Adding NextTo, shows that adding two totals T1 and T2 NextTo means adding their areas, also called integration. Thus adding 2 3s and 4 5s NextTo each other as 8s on a ten by ten square or on an abacus gives 3.2 8s as confirmed by a calculator.

Chapter 17, Reversed Adding NextTo, shows that to reverse NextTo addition, the first total must be taken away before counting the rest in the unit of the second total. Thus asking '2 3s + ? 5s total 3 8s, we take away the 2 3s from the 3 8s before recounting the rest, $T - T1$, in 5s by saying $(T-T1)/5 = \Delta T/5 = 3.3$ 5s as confirmed by a calculator. Together, integration and differentiation is called **calculus**.

Chapter 18, Adding Tens, shows that when adding tens, cup-writing can be used to create and remove overloads. Thus adding totals as 27 and 85 creates an overload that can be removed by cup-writing, $T = 27 + 85 = 2]7 + 8]5 = 10]12 = 11]2 = 112$ as confirmed by a calculator.

Chapter 19, Reversed Adding Tens, the number added must be taken away which might result in a deficit calling for a unbundling a bundle, unless this is done first resulting in an overload that allows taking the number away without creating a deficit. Thus asking '? + 27 = 85' or '85 - 27', cup-writing is used to remove the deficit, $85 - 27 = 8]5 - 2]7 = 6] - 2 = 5]8 = 58$; or used to create an overload, $85 - 27 = 8]5 - 2]7 = 7]15 - 2]7 = 5]8 = 58$, both confirmed by a calculator.

Chapter 20, Recounting Solves Equations, shows that equations expressing a reversed calculation can be solved by recounting and restacking. Thus to solve the equation $u*2 = 8$, 8 is recounted in 2s as $8 = (8/2)*2 = 4*2$, so that $u = 4$, checked by a calculator by entering $4*2$. With $u*2 = 8$ solved by $u = 8/2$ we get a shortcut for solving equations: *Move to the opposite side with the opposite sign.*

$u*2 = 8 = (8/2)*2 = 4*2$	Here we recount 8 in 2s as $8 = (8/2)*2 = 4*2$	u = 4
$u+2 = 9 = (9-2)+2 = 7+2$	Here we restack 9 to $9-2+2 = 7+2$	u = 7
$u/3 = 2$	Here we recount 2 in 3s as $2 = (2/3)*3 = 2*3/3 = 6/3$	u = 6
$u-2 = 6$	Here we restack 6 to $6-2+2 = 6+2-2 = 8-2$	u = 8
$2*u+3 = 15$	Here we restack 15 to $15-3+3 = 12+3$, and $2*u = 12 = 12/2*2 = 6*2$	u = 6
$2*u-3 = 15$	Here we restack 15 to $15-3+3 = 15+3-3 = 18-3$, and $2*u = 18 = 18/2*2 = 9*2$	u = 9
$u/2+3 = 15$	Here we restack 15 to $15-3+3 = 12+3$, and $u/2 = 12 = 12/2*2 = 12*2/2 = 24/2$	u = 24
$2/u-3 = 15$	Here we restack 15 to $15-3+3 = 15+3-3 = 18-3$, and $2/u = 18 = 18/2*2 = 18*2/2 = 36/2$	u = 36

01. From Sticks to Icons

Job

Do

[illegible]

1

[illegible]

11

[illegible]

III

[illegible]

||||

[illegible]

|||||

[illegible]

|||||

1

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11

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|||||

III

[illegible]

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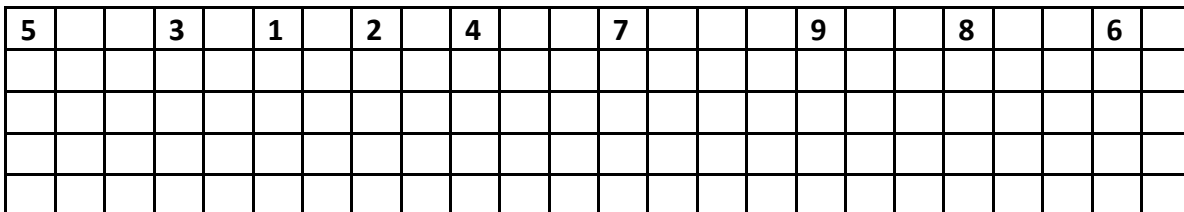
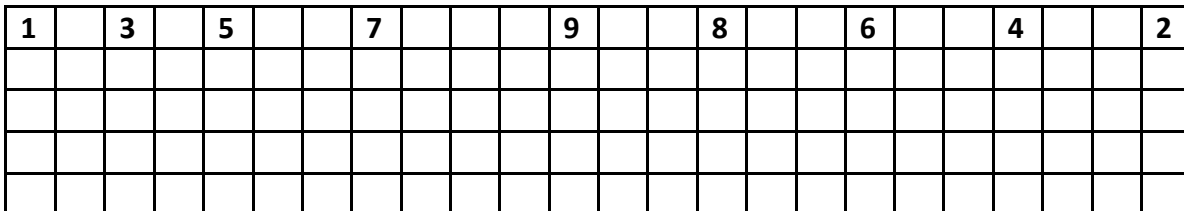
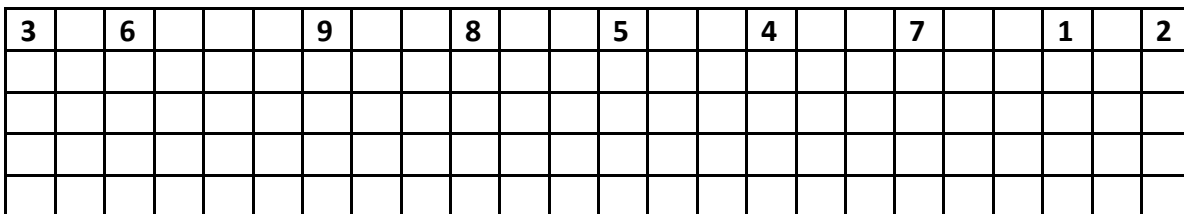
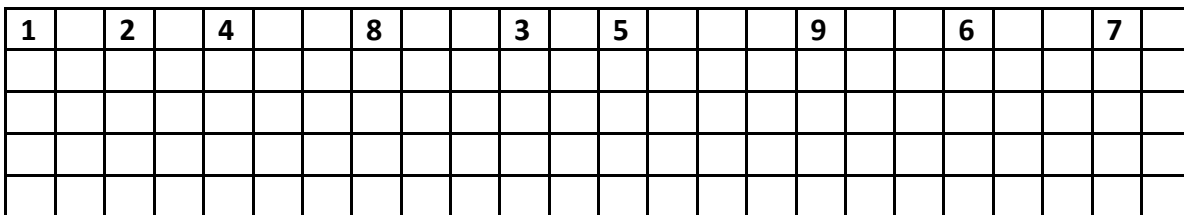
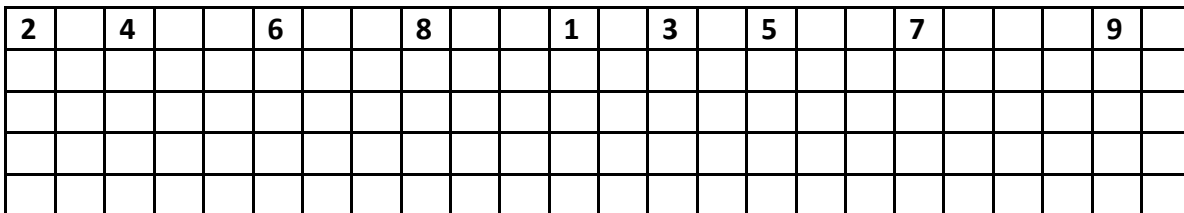
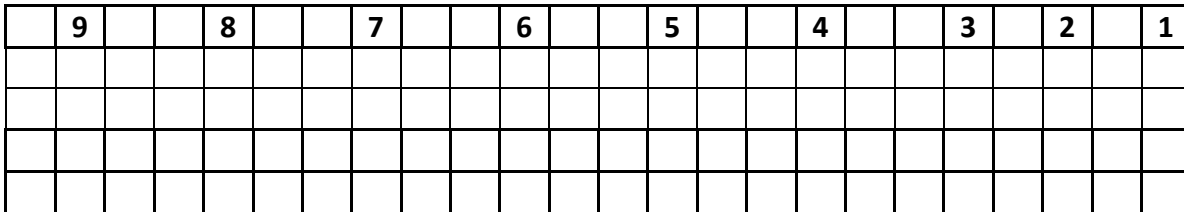
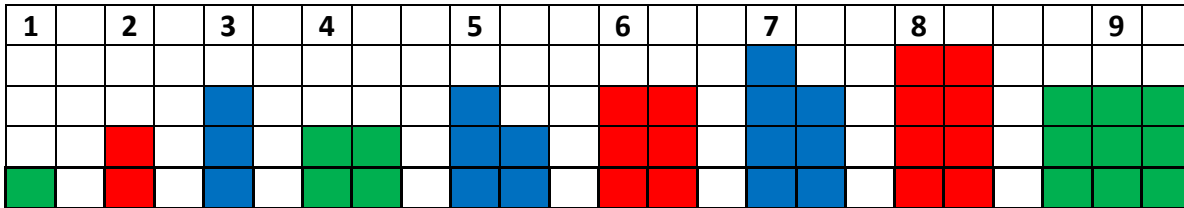
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Count & Color Squares



02. Counting in Icons

ten	1	2	3	4	5	6	7	8	9	10	11	12	13	14
ten	01	02	03	04	05	06	07	08	09	1B	1B1	1B2	1B3	1B4
ten	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.	1.1	1.2	1.3	1.4
9	01	02	03	04	05	06	07	08	1B	1B1	1B2	1B3	1B4	1B5
9	.1	.2	.3	.4	.5	.6	.7	.8	1.	1.1	1.2	1.3	1.4	1.5
8														
8														
7														
7														
6														
6														
5														
5														
4														
4														
3	01	02	1B	1B1	1B2	2B	2B1	2B2	BB	1BB1	1BB2	1BB1B	1BB1B1	1BB1B2
3	.1	.2	1.	1.1	1.2	2.	2.1	2.2	10.	10.1	10.2	11.	11.1	11.2
2														
2														
11	1	2	3	4	5	6	7	8	9	X	1B	1B1	1B2	1B3
11	01													
11	.1													



|| || || || || || || || || || || || ||

ten	02				1B							2B2		
ten														
9														
9														
8														
8														
7														
7														
6														
6														
5														
5														
4														
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3														

















||| ||| ||| ||| ||| ||| ||| ||| ||| ||| ||| ||| ||| ||| |||

ten														
9														
8														
7														
6														
5														
4														
3														
2														

03. CupCounting in Icons

Job		Do	Calculator
9 in 5s	Line	T =	9/5 1.some
	Count	1, 2, 3, 4, B, 1B1, 1B2, 1B3, <u>1B4</u>	9 – 1*5 4
	Bundle	T = 	
	Stack		9 – 0*5 9
	Cup	T = 1]4 5s = 0]9 5s = 2]-1 5s	9 – 2*5 -1
	Answer	<u>T = 9 = 1.4 5s</u>	
9 in 4s	Line	T =	9/4 2.some
	Count	1, 2, 3, B, 1B1, 1B2, 1B3, 2B, <u>2B1</u>	9 – 2*4 1
	Bundle	T = 	
	Cup	T = 2]1 4s = 1]5 4s = 3]-3 4s	9 – 1*4 5
	Stack		9 – 3*4 -3
	Answer	<u>T = 9 = 2.1 4s</u>	
9 in 3s	Line		9/
	Count		9 –
	Bundle		
	Cup		
	Stack		
	Answer		
8 in 4s	Line		8
	Count		8
	Bundle		
	Cup		
	Stack		
	Answer		
8 in 3s	Line		8
	Count		8
	Bundle		
	Cup		
	Stack		
	Answer		

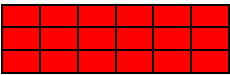
04. CupCounting with Dices

Job		Do	Calculator
   9 i 4s	Line Count Bundle Cup Stack Answer	$T = $ 1, 2, 3, B, 1B1, 1B2, 1B3, 2B, <u>2B1</u> $T = $ $T = 2]1 \quad 4s = 1]5 \quad 4s = 3]-3 \quad 4s$  $T = 9 = 2.1 \quad 4s$	$9/4 \quad 2.\text{some}$ $9 - 2*4 \quad 1$ $9 - 1*4 \quad 5$ $9 - 3*4 \quad -3$
   9 i 4s	Line Count Bundle Cup Stack Answer		$9/$ $9 -$
   9 i 4s	Line Count Bundle Cup Stack Answer		9 9
   9 i 4s	Line Count Bundle Cup Stack Answer		7 7
   9 i 4s	Line Count Bundle Cup Stack Answer		7 7

05. ReCounting in the Same Unit

Job		Do	Cup	Answer
2.1 5s in 5s	Line	T = IIIII IIIII I	2]1	T = 2.1 5s
	UnBundle	T = IIIII I I I I I I	1]6	T = 1.6 5s
	Embezzle	T = IIIII IIIII IIIII	3]-4	T = 3.-4 5s
2.1 4s in 4s	Line			
	UnBundle			
	Embezzle			
2.1 3s in 3s	Line			
	UnBundle			
	Embezzle			
2.1 6s in 6s	Line			
	UnBundle			
	Embezzle			
2.1 7s in 7s	Line			
	UnBundle			
	Embezzle			
3.2 7s in 7s	Line			
	UnBundle			
	Embezzle			
3.2 6s in 6s	Line			
	UnBundle			
	Embezzle			
3.2 5s in 5s	Line			
	UnBundle			
	Embezzle			
3.2 4s in 4s	Line			
	UnBundle			
	Embezzle			
3.2 3s in 3s	Line			
	UnBundle			
	Embezzle			

06. ReCounting in a New Unit

Job		Do	Calculator
2 9s in 6s	Line	T =	
	Count	1, 2, 3, 4, B, 1B1, 1B2, 1B3, 1B4, ..., 3B	
	Bundle	T =	$2*9/6$ 3
	Stack		$2*9 - 3*6$ 0
	Cup	T = 3]	
	Answer	<u>T = 2 9s = 3 6s</u>	
2 9s in 5s	Line		
	Count		
	Bundle		$2*9/$
	Stack		$2*9 -$
	Cup		
	Answer		
2 8s in 6s	Line		
	Count		
	Bundle		$2*8$
	Stack		$2*8$
	Cup		
	Answer		
2 8s in 5s	Line		
	Count		
	Bundle		$2*8$
	Stack		$2*8$
	Cup		
	Answer		
2 7s in 6s	Line		
	Count		
	Bundle		$2*7$
	Stack		$2*7$
	Cup		
	Answer		

07. Recounting in BundleBundles

Job		Do	Calculator
4 8s in 5s	Cup Answer	$T = 4 \text{ 8s} = 6]2 = B1]2 \text{ 5s} = 1]1]2$ <u>$T = 4 \text{ 8s} = 6.2 \text{ 5s} = 11.2 \text{ 5s} = 12.-3 \text{ 5s}$</u>	$4*8/5$ 6.some $4*8 - 6*5$ 2
5 8s in 6s	Cup Answer		
6 9s in 7s	Cup Answer		
9 9s in 8s	Cup Answer		
3 9s in 4s	Cup Answer		
4 5s in 3s	Cup Answer		
6 8s in 5s	Cup Answer		
6 8s in 4s	Cup Answer		
7 8s in 5s	Cup Answer		
7 8s in 4s	Cup Answer		
8 8s in 5s	Cup Answer		
8 8s in 4s	Cup Answer		

08. ReCounting in Tens on Squared Paper or an Abacus

Job		Do	Calculator
7s in tens			$10 * 7 = 70$ $9 * 7 = 63$ $8 * 7 = 56$ $7 * 7 = 49$ $6 * 7 = 42$ $5 * 7 = 35$ $4 * 7 = 28$ $3 * 7 = 21$ $2 * 7 = 14$ $1 * 7 = 7$
8s in tens			$10 * 8 =$ $9 * 8 =$ $8 * 8 =$ $7 * 8 =$ $6 * 8 =$ $5 * 8 =$ $4 * 8 =$ $3 * 8 =$ $2 * 8 =$ $1 * 8 =$
9s in tens			$10 * 9 =$ $9 * 9 =$ $8 * 9 =$ $7 * 9 =$ $6 * 9 =$ $5 * 9 =$ $4 * 9 =$ $3 * 9 =$ $2 * 9 =$ $1 * 9 =$
6s in tens			$10 * 6 =$ $9 * 6 =$ $8 * 6 =$ $7 * 6 =$ $6 * 6 =$ $5 * 6 =$ $4 * 6 =$ $3 * 6 =$ $2 * 6 =$ $1 * 6 =$

09. Recounting From Tens

Job		Do	Calculator
37 in 9s	Line ReBundle Cup Answer	X X X V II 9I 9I 9I V II \rightarrow 9 9 9 X \rightarrow 9 9 9 9 1 $3 \times 7 = 21$ $37 = 36 + 1 = 4 \times 9 + 1$ <u>$T = 37 = 4 \times 9 + 1 = 4.1 \text{ 9s} = 4 \frac{1}{9} \text{ 9s}$</u>	$37/9$ 4.some $37 - 4 \times 9$ 1
37 in 7s	Line ReBundle Cup Answer		
37 in 5s	Line ReBundle Cup Answer		
42 in 7s	Line ReBundle Cup Answer		
42 in 5s	Line ReBundle Cup Answer		
26 in 7s	Line ReBundle Cup Answer		
26 in 5s	Line ReBundle Cup Answer		

10. Recounting Large Numbers in Tens

Job		Do	Calculator
7 43s	Cup Answer	$T = 7 * 4]3 = 28]21 = 30]1 = 301$ <u>$T = 7 \text{ 43s} = 30.1 \text{ tens} = 301$</u>	7*43 301
8 43s	Cup Answer		
9 43s	Cup Answer		
6 43s	Cup Answer		
5 62s	Cup Answer		
4 62s	Cup Answer		
3 62s	Cup Answer		
2 62s	Cup Answer		
27 436s	Cup Answer		
3 436s	Cup Answer		
4 436s	Cup Answer		
5 436s	Cup Answer		
6 436s	Cup Answer		
7 436s	Cup Answer		
8 436s	Cup Answer		

11. DoubleCounting with PerNumbers

Job	Do	Formula
With 4 \$ per 5 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$	$8\$ = (8/4)*4\$ = (8/4)*5\text{kg} = 10\text{kg}$ $12\text{kg} = (12/5)*5\text{kg} = (12/5)*4\$ = 9.6\$$	$\text{Kg} = (\text{kg}/\$)*\$$ $\text{Kg} = (5/4)*8 = 10$ $\$ = (\$/\text{kg})*\text{kg}$ $\$ = (4/5)*12 = 9.6$
With 3 \$ per 5 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 4 \$ per 6 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 4 \$ per 8 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 4 \$ per 5 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 3 \$ per 5 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 4 \$ per 6 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 4 \$ per 8 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 2 \$ per 5 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		
With 2 \$ per 7 kg $8\$ = ?\text{kg}$ $?\$ = 12 \text{ kg}$		

12. DoubleCounting with Fractions and Percentages

Job	Do	Calculator
3 per 5 of 200\$	$200\$ = (200/5) * 5\$$ Giving $(200/5) * 3\$ = 120\$$	$3/5 * 200$ 120
3 per 5 of 400\$		
2 per 5 of 200\$		
1 per 5 of 200\$		
3 per 6 of 240\$		
2 per 6 of 240\$		
5 per 6 of 300\$		
3 per 100 of 250\$ or 3% of 250\$	$250\$ = (250/100) * 100\$$ Giving $(250/100) * 3\$ = 7.5\$$	$3/100 * 250$ 7.5
8 per 100 of 200\$ or 8% of 200\$		
20 per 100 of 200\$ or 20% of 200\$		
3 per 100 of 560\$ or 3% of 560\$		
8 per 100 of 560\$ or 8% of 560\$		
12 per 100 of 560\$ or 12% of 560\$		
20 per 100 of 560\$ or 20% of 560\$		
60 per 100 of 560\$ or 60% of 560\$		

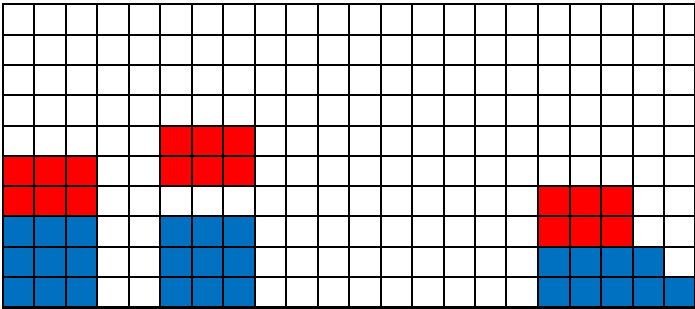
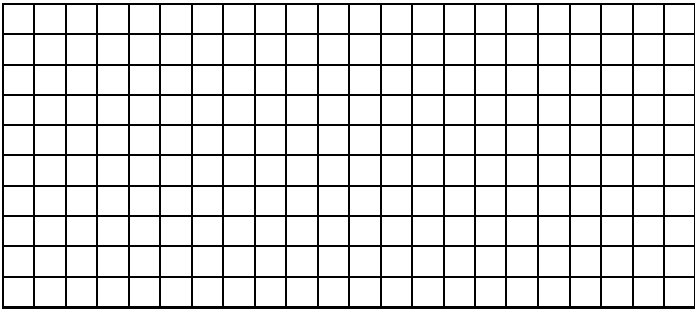
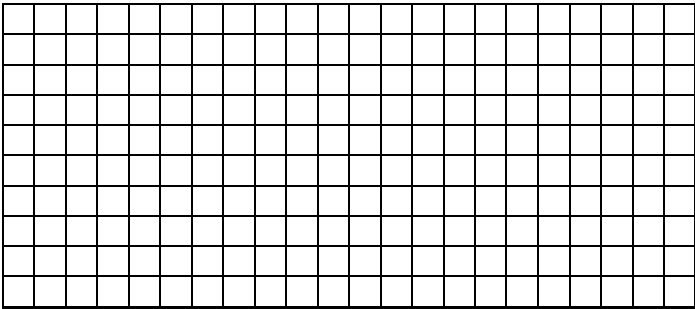
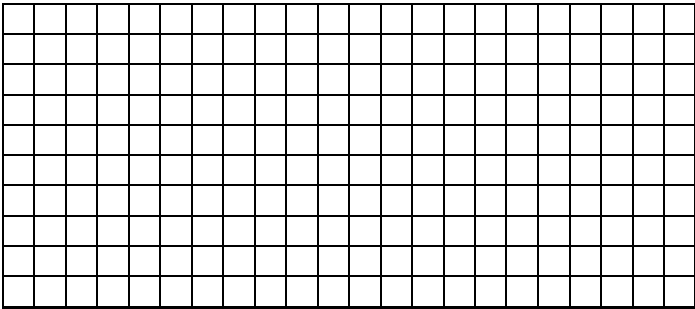
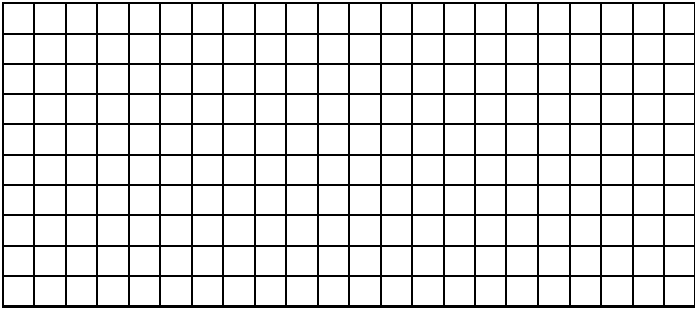
13. ReCounting PerNumbers, Fractions

Job	Do	Do	Calculator	Calculator
2/3 = ?	$2/3 = 2 \text{ 2s} / 3 \text{ 2s} = 4/6$ $2/3 = 2 \text{ 3s} / 3 \text{ 3s} = 6/9$	$2/3 = 2 \text{ 4s} / 3 \text{ 4s} = 8/12$ $2/3 = 2 \text{ 5s} / 3 \text{ 5s} = 10/15$	$2/3 = 0.66..$ $4/6 = 0.66..$	$8/12 = 0.66..$ $10/15 = 0.66..$
1/3 = ?				
1/5 = ?				
2/5 = ?				
3/5 = ?				
4/5 = ?				
4/6 2/6 6/8 2/8	$4/6 = 2 \text{ 2s} / 3 \text{ 2s} = 2/3$ $2/6 = 1 \text{ 2s} / 3 \text{ 2s} = 1/3$	$6/8 = 3 \text{ 2s} / 4 \text{ 2s} = 3/4$ $2/8 = 1 \text{ 2s} / 4 \text{ 2s} = 1/4$	$4/6 = 0.66..$ $2/3 = 0.66..$ $2/6 = 0.33..$ $1/3 = 0.33..$	$6/8 = 0.75$ $3/4 = 0.75$ $2/8 = 0.25$ $1/4 = 0.25$
2/10 4/10 6/10 8/10				
2/12 4/12 6/12 8/12 10/12				
2/14 4/14 6/14 8/14 10/14 12/14				
2/16 4/16 6/16 8/16 10/16 12/16 14/16				

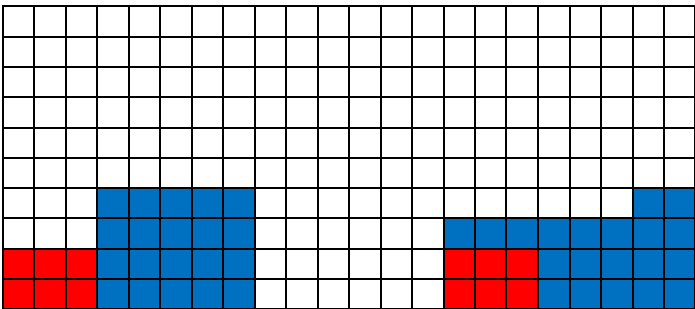
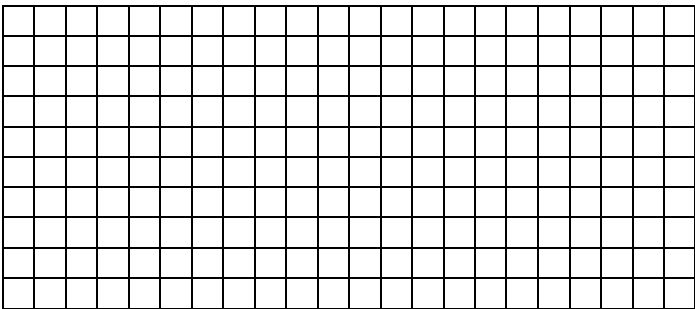
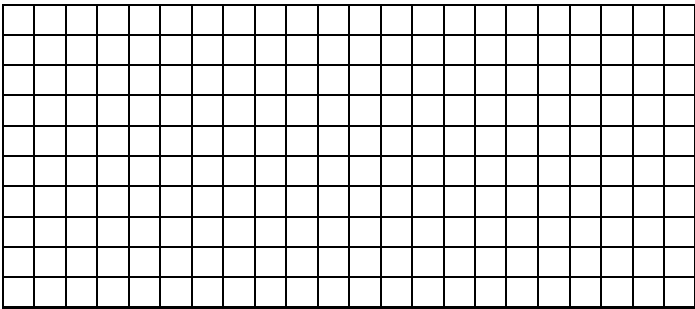
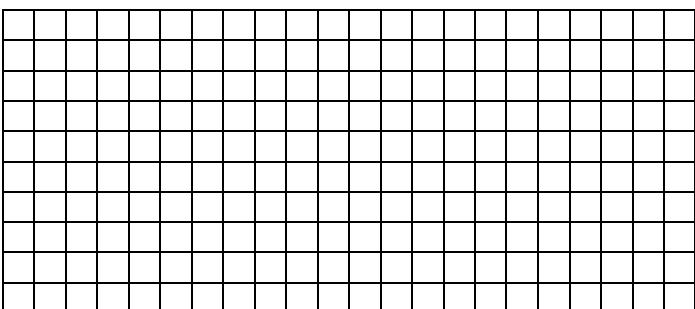
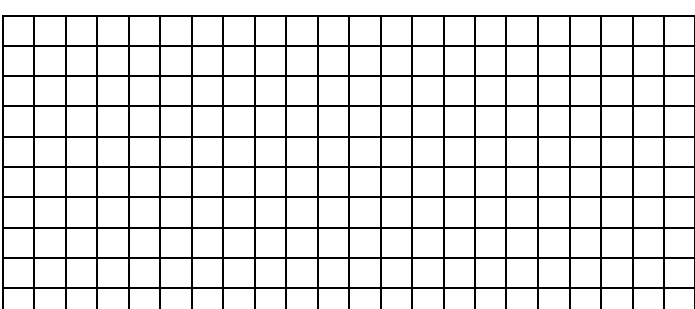
14. Adding OnTop

Job	Do	Calculator
$2\ 3s$ $+$ $4\ 5s$ $=$ $?\ 3s$ $? \ 5s$		$(2*3+4*5)/3$ 8.some $(2*3+4*5) - 8*3$ 2 <u>$2\ 3s + 4\ 5s = 8.2\ 3s$</u> $(2*3+4*5)/5$ 5.some $(2*3+4*5) - 5*5$ 1 <u>$2\ 3s + 4\ 5s = 5.1\ 5s$</u>
$2\ 4s$ $+$ $3\ 5s$ $=$ $? \ 4s$ $? \ 5s$		
$3\ 2s$ $+$ $4\ 6s$ $=$ $? \ 2s$ $? \ 6s$		
$2\ 5s$ $+$ $4\ 3s$ $=$ $? \ 5s$ $? \ 3s$		
$5\ 2s$ $+$ $3\ 4s$ $=$ $? \ 2s$ $? \ 4s$		

15. Reversed Adding OnTop

<p>2 3s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>5 3s</p>		<p>$(5 \cdot 3 - 2 \cdot 3) / 5$ 1.some</p> <p>$(5 \cdot 3 - 2 \cdot 3) - 1 \cdot 5$ 4</p> <p><u>$2 \text{ 3s} + 1.4 \text{ 5s} = 5 \text{ 3s}$</u></p>
<p>2 4s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>5 4s</p>		
<p>2 6s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>4 6s</p>		
<p>2 7s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>6 5s</p>		
<p>2 6s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>4 5s</p>		

16. Adding NextTo

Job	Do	Calculator
$2\ 3s$ $+$ $4\ 5s$ $=$ $?\ 8s$		$(2*3+4*5)/8$ 3.some $(2*3+4*5) - 8*3$ 2 <u>$2\ 3s + 4\ 5s = 3.2\ 8s$</u>
$3\ 2s$ $+$ $4\ 5s$ $=$ $? \ 7s$		
$2\ 3s$ $+$ $4\ 6s$ $=$ $? \ 9s$		
$2\ 4s$ $+$ $4\ 5s$ $=$ $? \ 9s$		
$4\ 3s$ $+$ $2\ 4s$ $=$ $? \ 6s$		

17. Reversed Adding NextTo

<p>2 3s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>3 8s</p>		<p>$(3 \cdot 8 - 2 \cdot 3) / 5$ 3.some</p> <p>$(3 \cdot 8 - 2 \cdot 3) - 3 \cdot 5$ 3</p> <p><u>$2 \text{ 3s} + 3 \cdot 3 \text{ 5s} = 3 \text{ 8s}$</u></p>
<p>2 4s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>3 9s</p>		
<p>2 3s</p> <p>+</p> <p>? 4s</p> <p>=</p> <p>3 7s</p>		
<p>4 3s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>3 8s</p>		
<p>5 2s</p> <p>+</p> <p>? 5s</p> <p>=</p> <p>3 7s</p>		

18. Adding Tens

Job		Do	Calculator
27 + 85	Cup Answer	$T = 2]7 + 8]5 = 10]12 = 11]2 = 112$ <u>$T = 27 + 85 = 11.2 \text{ tens} = 112$</u>	27+85 112
27 + 85	Cup Answer		
33 + 78	Cup Answer		
39 + 71	Cup Answer		
45 + 67	Cup Answer		
58 + 57	Cup Answer		
57 + 49	Cup Answer		
27 + 205	Cup Answer		
33 + 198	Cup Answer		
39 + 191	Cup Answer		
45 + 187	Cup Answer		
58 + 177	Cup Answer		
57 + 169	Cup Answer		
127 + 385	Cup Answer		
433 + 578	Cup Answer		

19. Reversed Adding Tens

Job		Do	Calculator
$27 + ? = 85$ $85 - 27$	Cup Answer	$D = 8]5 - 2]7 = 6]-2 = 5]8 = 58$ $D = 8]5 - 2]7 = 7]15 - 2]7 = 5]8 = 58$ <u>$T = 85 - 27 = 5.8 \text{ tens} = 58$</u>	$85 - 27$ 58
$63 - 17$	Cup Answer		
$55 - 36$	Cup Answer		
$35 - 17$	Cup Answer		
$185 - 27$	Cup Answer		
$235 - 128$	Cup Answer		
$242 - 128$	Cup Answer		
$245 - 167$	Cup Answer		
$312 - 159$	Cup Answer		
$421 - 268$	Cup Answer		

20. ReCounting solves Equations

Do	Equation	Calculator
ReCount Answer	$u*2 = 30 = (30/2)*2 = 15*2$ $u = 15$	15*2 30
ReCount Answer	$u*3 = 15$	
ReCount Answer	$u*4 = 32$	
ReCount Answer	$u*5 = 40$	
ReCount Answer	$u/3 = 12 = (12/3)*3 = 12*3/3 = 36/3$ $u = 36$	36/3 12
ReCount Answer	$u/3 = 10$	
ReCount Answer	$u/4 = 8$	
ReCount Answer	$u/5 = 6$	
ReCount Answer	$u+2 = 30 = (30-2)+2 = 28 + 2$ $u = 28$	28+2 30
ReCount Answer	$u+3 = 24$	
ReCount Answer	$u+4 = 20$	
ReCount Answer	$u+5 = 12$	
ReCount Answer	$u-2 = 30 = (30-2)+2 = 30+2-2 = 32- 2$ $u = 32$	32-2 30
ReCount Answer	$u-3 = 20$	
ReCount Answer	$u-5 = 10$	

ReCount	$2*u+3 = 15 = (15-3)+3 = 12 + 3$		
ReCount	$2*u = 12 = (12/2)*2 = 6*2$	$2*6+3$	15
Answer	$u = 6$		
ReCount	$3*u+4 = 19$		
ReCount			
Answer			
ReCount	$4*u+6 = 38$		
ReCount			
Answer			
ReCount	$2*u-3 = 15 = (15-3)+3 = 15+3-3 = 18 - 3$		
ReCount	$2*u = 18 = (18/2)*2 = 9*2$	$2*9-3$	15
Answer	$u = 9$		
ReCount	$3*u-4 = 8$		
ReCount			
Answer			
ReCount	$4*u-5 = 23$		
ReCount			
Answer			
ReCount	$u/2+3 = 15 = (15-3)+3 = 12 + 3$		
ReCount	$u/2 = 12 = (12/2)*2 = (12*2)/2 = 24/2$	$24/2+3$	15
Answer	$u = 24$		
ReCount	$u/3+4= 12$		
ReCount			
Answer			
ReCount	$u/2-3 = 15 = (15-3)+3 = (15+3)-3 = 18 - 3$		
ReCount	$u/2 = 18 = (18/2)*2 = (18*2)/2 = 36*2$	$36/2-3$	15
Answer	$u = 36$		
ReCount	$u/4-7 = 5$		
ReCount			
Answer			