# MIGRANT MATH MATERIAL 

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# A 1YEAR PRE-ENGINEER COURSE FOR YOUNG MIGRANTS, A JOB FOR CRITICAL OR CIVILIZED MATH EDUCATION 

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UN population forecasts predict a continuing migrant flow to Europe to benefit from its socialist welfare and educational systems. But a critical question could ask: Is European education ready to benefit from the engineer potential in young migrants allowing them to build up welfare and education in their own country? Is critical socialist thinking able to reform its European line-organized office directed education dating back to the Napoleon wars? A recent OECD report saying that Sweden should urgently reform its school system to improve quality and equity suggests that a solution might instead be provided by the civilized thinking of the North American Enlightenment republics, historically created to receive and integrate migrants through its half-year block-organized talent developing education.

## BACKGROUND AND QUESTION

According to the numbers of hours spend there, education is by far the most extensive public intervention in private life; and with the basic human need for a word- and a number-language for communication, mathematics is one of its core subjects. Consequently, research in math education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 year since 1969. Likewise, funding has increased as witnessed e.g. by the creation of a National Centre for Mathematics Education in Sweden that with its positive attitude to receiving male migrants now beats China with 123 boys/ 100 girls of the $16-17$ years old. However, despite increased research and funding, Sweden has seen its PISA result in mathematics decrease from 509 in 2003 to 478 in 2012, the lowest in the Nordic countries and significantly below the OECD average at 494. This caused OECD to write the report 'Improving Schools in Sweden' describing the Swedish school system as being 'in need of urgent change' (OECD, 2015).

PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (p.3)
In the report OECD writes
Sweden has the highest percentage of students arriving late for school among all OECD countries, especially among socio-economically disadvantaged and immigrant students, and the lack of punctuality has increased between 2003 and 2012. There is also a higher-than-average percentage of students in Sweden who skip classes, in particular among disadvantaged and immigrant students. Arriving late for school and skipping classes are associated highly negatively with mathematics performance in PISA and can have serious
adverse effects on the lives of young people, as they can cut into school learning and also distract other students. (p. 69) The reforms of recent years are important, but evidence suggests they are also somewhat piecemeal, and simply too few, considering the serious situation of the Swedish school system. (p. 55) Sweden faces a serious deterioration in the quality and status of the teaching profession that requires immediate system-wide attention. This can only be accomplished by building capacity for teaching and learning through a long-term human resource strategy for the school sector. (p. 112)

Inspired by the OECD report we can ask: How to improve mathematics and its education to better serve the population and migrants? And more specifically: How to design a lyear pre-engineer course for young migrants beginning from scratch?

Critical and civilized thinking provide two kinds of answers.

## CRITICAL AND CIVILIZED THINKING

As to the content of critical thinking, the Oxford Dictionary of Philosophy writes:
The title is specifically applied to the philosophical approach of the Frankfurt school. This owed its philosophical background to Hegel and to Marx, seeing social and cultural imperfections as defects of rationality, and comparing them with an ideal to which the progress of reason, embodied in pure and undistorting social arrangements, would ideally tend (pp. 88-89)
Civilized thinking mixes existentialism, seeing existence as preceding essence, with the thinking of the two Enlightenment republics, American pragmatism being skeptical towards any philosophical is-claim, and French post-structuralism warning against hidden patronization in choices presented as nature. But to more clearly see the difference between the two we need to go back in history.

## A HISTORICAL BACKGROUND

The distance from its energy source allows water in all three forms: solid, liquid and gas. Thus a continuous flow of incoming high order energy from the sun and outgoing low order waste energy to space during the night allows green cells to store energy to be exploited by grey cells coming in three forms: reptiles, mammals and humans. That by standing up allowed the brain to develop language by remembering sounds given to what the forelegs transformed to hands was grasping. Thus meeting the two fundamental needs shown by the holes in the head: to supply the stomach and the lungs and the brain with food and oxygen and information.
When humans left Africa some went east to the fertile river valleys, some went west to the mountains. Trade took place exchanging eastern silk and pepper with western silver. Its silver mines allowed ancient Greece to develop a culture where men could leave the daily routine work to women and slaves to discuss social matters as 'can adults live together on equal terms or is patronization needed as with children?'
Social theory thus has human interaction as its main focus. As to communication, the most basic interaction, Berne (1964) has developed a transactional analysis describing three different ego-states called Parent and Adult and Child to reflect the social fact
that human interaction can be patronized and non-democratic, or it can be nonpatronized and democratic. In a family the interaction between children and parents will typically be one of patronization. In a society adult interaction typically will be non-patronized, unless the society is a non-democratic autocracy where patronization is carried on into adulthood. In this way Berne describes the main problem in human interaction, the choice between patronization and self-determination or 'Mündigkeit'. The fact that the German word 'Mündigkeit' does not have an English equivalent indicates that social interaction is quite different outside continental Europe and inside where the presence of and resistance against patronization created the label 'Mündigkeit'.
The debate on patronization runs all the way though the history of social theory (Russell, 1945). In ancient Greece the sophists warned against hidden patronization coming from choices presented as nature. Hence to practice the three ingredients of a democracy, information and debate and decision, a population should be enlightened to tell choice from nature. Seeing the physical as examples of metaphysical forms only visible to philosophers from his academy, Plato labelled democratic debate as ignorance. Instead social power should be given to the philosophers who could make wise decisions based upon information coming from insight and knowledge, thus needing neither debate nor democracy. In this way Plato instituted the patronization that Foucault calls 'pastoral power' to be continued first by the Christian church and later by modern universities still using the scholastic research method of only allowing late opponents to already defended texts to be accepted as researchers.

The Greek silver mines lasted about hundred years. Then the Romans took over, financing their empire by silver mines in Spain, eventually captured by the Vandals and by the Arabs. The lack of silver made Europe descend into the dark Middle Age. Here the patronization question reappeared in the controversy on universals between the realists and the nominalists. The realist took the Plato standpoint by renaming his metaphysical forms to universals claimed to have independent existence and to be exemplified in the physical world, and consequently waiting to be discovered by philosophers. In contrast to this the nominalist saw universals as names invented to facilitate human interaction.

Then German silver transported to Italy reopened east-west trade financing the Renaissance, seeing a protestant uprising against the patronization of the Roman Catholic Church resulting in the bloody 30year war from 1618. To avoid the chaos of war, Hobbes in his book 'Leviathan' argued that to protect themselves against their natural egoistic state, humans would have a much better life if accepting the patronization of an autocratic monarch.
Seeing the laboratory as preceding the library, Brahe retrieved data for the motion of planets, which together with Kepler's interpretation allowed Newton to discover that the moon doesn't move among the stars, instead it falls towards the earth as does the apple, both following their own physical will and not the will of a metaphysical patronizor. This inspired Locke to argue against patronization. His chief work, the 'An

Essay Concerning Human Understanding', was highly inspirational in the Enlightenment 1700-century, which resulted in two democracies being installed, one in the US and one in France. American sociology sees human interaction as based upon enlightenment and freed from patronization. Its 'it is true if it works' pragmatism expressed by Peirce and James leads on to symbolic interactionism and to the natural observation rooted research paradigm Grounded Theory resonating with the principles of natural learning expressed by Piaget. In harmony with this, the US enlightenment school, being organized in half-year blocks and aiming at developing the talent of the individual has set the international standard followed worldwide outside Europe.

Inside Europe counter-enlightenment came from Germany where Hegel reinstalled metaphysical patronization in the form of a Spirit expressing itself through the history of the people. Trying to end the French Republic by war resulted in French occupation of Berlin. To get Napoleon out, the king realized that as the French he could no more depend on the blood nobility. So he asked Humboldt to use Hegel to design a lineorganized Bildung education with three goals: Bildung must not enlighten to keep the population from demanding democracy as in France; instead, by imposing upon it a feeling of nationalism, Bildung should transform the population into a people following the will of the Spirit by fighting other people, especially the French. Finally Bildung should use the Sprit expressing itself in romanticism to sort out a knowledge nobility among the people for a central administration (Berglar, 1970).

Opposing Hegel, Nietzsche argued that only by freeing itself from metaphysical philosophical hegemony, western individuals would be able to realize their full potentials. Following Hegel, Marx claimed that until a socialist utopia has been established, a socialist party serving the interest of the working people should patronize people through a dictatorship of the proletariat. Once in power, Hegel-based socialism saw no reason to replace the Hegel-based counter-enlightening line-organized education with the enlightening block- organized education of the American republics. Marxist thinking developed into the critical theory of the Frankfurter school infiltrating the 1968 student revolt to secure that Europe's Bildung education could carry on its Hegel-based patronization.
Today, the sophist warning against unrooted is-claims is carried on by the existentialism of Kierkegaard and Nietzsche and Heidegger and Sartre, defining existentialism as holding that 'existence precedes essence, or (..) that subjectivity must be the starting point' (Marino, 2004: 344); and by French post- structuralism with Derrida and Lyotard and Foucault and Bourdieu showing skepticism towards hidden patronization in our most fundamental institutions: words, correctness, cures and education (Lyotard, 1984), (Tarp, 2004, 2). Foucault thus says:

It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (Chomsky \& Foucault, 2006: 41)

In Germany, Arendt carried his Heidegger's work further by dividing human activity into labor and work focusing on the private sphere and action focusing on the political sphere thus accepting as the first philosopher political action as a worthy human activity creating institutions that should be treated with care to avoid 'the banality of evil' if turning totalitarian by the sheer banality of just following orders (Arendt, 1963). Likewise, Bauman points out that by following authorized routines modernity can create both gas turbines and gas chambers (Baumann, 1989).
As to their meanings, the word 'critical' comes from Greek 'kritike' meaning to pass judgement; and civilized comes from latin 'civis' meaning a free citizen. So civilized thinking means republican thinking always being skeptical towards false is-claims; and critical thinking means passing judgements; but to pass a judgement you must be elected judge by a democratic process, or have insight in the difference between right and wrong as e.g. believing in the Hegel assumption that instead of being free to create their own history, humans are puppets on a string playing out the manuscript of the Spirit. So basically the contradiction between critical and civilized thinking is a replay of the ancient controversy between the Greek philosophers and sophists.

## CRITICAL VERSUS CIVILIZED MATHEMATICS EDUCATION

The difference between critical and civilized mathematics education is seen in a paper describing how to deal with teaching and learning problems in a Brazilian math class (Tarp, 2004, 1)

In Brazil there is a research group, which has focused on issues related to new technologies and mathematics education. This research group has developed software and work with students at different levels and with teachers. A teacher from a nearby school approached the group (..) From the teacher perspective, she had some tough problems to face and she foresaw that the computers would be able to help her. The teacher was teaching a class of 5 th graders, which in her view was really problematic. The kids were older ( 15 years average) than the expected age for this grade: 11. The kids felt humiliated somehow as they were put in a school with kids much younger than them and they had flunked many times, and at several instances they had to repeat all the subjects of a given school year because their 'failure in mathematics'. The students transformed this humiliation into violence in class. The teacher was in fact considering the possibility of just quitting the job since she could not work with those kids in a way she found effective. (..) The teacher was enthusiastic about a software, which deals with rational numbers. (..) both researchers and teacher had the 'intuition' that the computer might have a positive effect in this class and maybe could avoid that the students had to repeat this grade again. (Sec. 2, par. 2-4)
The teacher is supposed to teach rational numbers to a class with a mixture of 11year old students and $15 y e a r$ old repeaters having given up rational numbers and turning to violence. The research group could have asked critical questions as 'is rational numbers defined from below as an abstraction from concrete examples or from above as an example of an abstraction?' and ' why teach addition when it is meaningless to add fractions without units?' Instead the group uncritically assumed 'that the computer might have a positive effect'.

The paper also describes how civilized thinking would work differently:
The research group is working halftime in classrooms and halftime at the university. It focuses on the concerns of typical classrooms as expressed by students, teachers in their stories of complaints. The teacher complains about the violence in the class tempting her to quit the job since she cannot work in a way she finds effective. And the students complain about having to repeat the class because they don't want to learn about fractions, since the teacher by just echoing the textbook is unable to explain to the students, why they shall learn fractions, and what they are useful for. Asked to comment this, the teacher says that mathematics education means education in mathematics, and since rational numbers is part of the mathematics textbook it must be taught and learned. Mathematics is difficult to learn, so the students have to work harder, or be supported by computers. Hence the problems will not disappear before schools can afford computers, or the students decide to become more engaged in mathematics.
Based upon the motto "echo-phrasing is freezing, re-phrasing is freeing" postmodern thinking sees modern institutions frozen in echo-phrasings, that have to be discovered and rephrased. Since the teacher is echoing the textbook, the echoes can be found here. The textbook presents fractions as examples of rational numbers, being example of number sets, being examples of sets. This is the typical way of presentation within modern setbased mathematics explaining concepts as examples of more abstract concepts. This phrasing conflicts with the student demand for explanations relating fractions to their use.
So instead of developing software to supplement, and thus support the existing top-down echo-phrasing of fractions, the group begins to look for alternative bottom-up approaches in journals, other textbooks, other countries, and in other time periods. Also they use their imagination by accessing the silent part of their 'knowledge-iceberg' developed through years of classrooms experience as mathematics educators. Using curriculum architecture they design examples of micro curricula, where fractions emerges from dividing problems, that can be introduced into the ordinary classroom as e.g. games, where students work in pairs throwing dices and splitting the profit, or loss, proportional to their stakes shown by their dice-numbers.

This 'proportional splitting' approach leads to (and thus shows the authenticity and necessity of) fractions, and multiplication of fractions and integers. (Sec. 5, par. 2-5)
So where critical thinking shows no criticism towards the actual mathematics education tradition, civilized thinking asks if this tradition is nature or choice presented as nature and thus hiding alternatives.

## CRITICIZING AND CIVILIZING RATIONAL NUMBERS

In ancient Greece the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas. With astronomy and music as independent knowledge areas, today mathematics is a common label for the two remaining activities, geometry and algebra, (Freudenthal, 1973) both rooted in the physical fact Many through their original meanings, 'to measure earth' in Greek and 'to reunite Many' in Arabic.

Meeting Many we ask 'how many?' Counting and adding gives the answer. We count by bundling and stacking as seen when writing a total T in its block form: $\mathrm{T}=354=$ $3 \cdot B^{\wedge} 2+5 \cdot B+4 \cdot 1$ where the bundle $B$ is ten typically. This illustrates the four ways to unite: On-top addition unites variable numbers, multiplication unites constant numbers, power unites constant factors and per-numbers, and next-to addition, also called integration, unites variable blocks. As indicated by its name, uniting can be reversed to split a total into parts predicted by the reversed operations: subtraction, division, root \& logarithm and differentiation.

| Operations unite/split Totals in | Variable | Constant |
| :--- | :--- | :--- |
| Unit-numbers | $\mathrm{T}=\mathrm{a}+\mathrm{b}$ | $\mathrm{T}=\mathrm{a} \cdot \mathrm{b}$ |
| $\mathrm{m}, \mathrm{s}, \mathrm{kg}, \$$ | $\mathrm{~T}-\mathrm{b}=\mathrm{a}$ | $\mathrm{T} / \mathrm{b}=\mathrm{a}$ |
| Per-numbers | $\mathrm{T}=\int_{\mathrm{a}} \cdot \mathrm{db}$ | $\mathrm{T}=\mathrm{a}^{\wedge} \mathrm{b}$ |
| $\mathrm{m} / \mathrm{s}, \$ / \mathrm{kg}, \$ / 100 \$=\%$ | $\mathrm{dT} / \mathrm{db}=\mathrm{a}$ | $\mathrm{b} \sqrt{ } \mathrm{T}=\mathrm{a}, \log _{\mathrm{a}} \mathrm{T}=\mathrm{b}$ |

Table 1: The four way to unite variable and constant unit- and per-numbers.
Although presented as nature, ten-bundling is a choice. Bundling Many in a 'iconbundles' less than ten means asking e.g. ' $\mathrm{T}=7=$ ? 4 s '. The answer is predicted on a calculator by two formulas, a recount-formula ' $\mathrm{T}=(\mathrm{T} / \mathrm{B}) \cdot \mathrm{B}$ ' telling that from a total $T, T / b$ times $B$ s can be taken away, and a restack-formula ' $T=(T-B)+B$ ' telling that from a total T, T-B is left when B is taken away and placed next-to. First $T=7 / 4$ gives 1.some. Then $T=7-1.4$ leaves 3 . So the prediction is $T=7=14 \mathrm{~s} \& 3=1.34 \mathrm{~s}=1$ $3 / 44 \mathrm{~s}$. Thus with icon-counting, a natural number is a decimal number with a unit where the decimal point separates singles from bundles (Tarp, 2016)
Double-counting physical units creates per-numbers as $3 \$ / 4 \mathrm{~kg}$. With this, units can be changed by recounting $\$ \mathrm{~s}$ in 3 s or kgs in $4 \mathrm{~s}: 15 \$=(15 / 3) \cdot 3 \$=(15 / 3) \cdot 4 \mathrm{~kg}=20 \mathrm{~kg}$. So as per-numbers, fractions are not numbers, but operators, needing a number to become a number. To add, per-numbers must be multiplied to unit-numbers, thus adding as areas, called integration: $1 / 2$ of $4+2 / 3$ of $3=(1 / 2 * 4+2 / 3 * 3)$ of $(4+3)=4$ of 7 .
The root of geometry is the standard form, a rectangle, that halved by a diagonal becomes two right-angled triangles with sides and angles connected by three laws, $\mathrm{A}+\mathrm{B}+\mathrm{C}=180, \mathrm{a}^{\wedge} 2+\mathrm{b}^{\wedge} 2=\mathrm{c}^{\wedge} 2$ and $\tan \mathrm{A}=\mathrm{a} / \mathrm{b}$. Being filled from the inside by triangles, a circle with radius $r$ gets the circumference $2 \cdot \pi \cdot r$ where $\pi=n \cdot \tan (180 / n)$ for $n$ large.
Thus, as a label for algebra and geometry, mathematics is a natural science about the physical fact Many. However, the invention of the concept SET allowed mathematics to become a self-referring collection of 'well-proven' statements about 'well-defined' concepts, i.e. as 'MetaMatics', defined from above as examples from abstractions instead of from below as abstractions from examples. But, by looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false: If $M=\{A \mid A \notin A)\}$ then $\mathrm{M} \in \mathrm{M} \Leftrightarrow \mathrm{M} \notin \mathrm{M}$. The Zermelo-Fraenkel set-theory tries to avoid self-reference
by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract essence. To avoid self-reference Russell introduced a type theory allowing reference only to lower degree types. Consequently fractions cannot be numbers since they refer to numbers in their modern definition: In a set-product of integers, a fraction is an equivalence set created by the equivalence relation $\mathrm{a} / \mathrm{b}{ }^{\sim} \mathrm{c} / \mathrm{d}$ if $\mathrm{a} \cdot \mathrm{d}=\mathrm{b} \cdot \mathrm{c}$.

Thus SET transformed grounded mathematics into a self-referring 'MetaMatism', a mixture of MetaMatics and 'MatheMatism' true inside a classroom but seldom outside where claims as ' $1+2$ IS 3 ' meet counter-examples as e.g. 1 week +2 days is 9 days.
So rational numbers is pure MetaMatism by also being MatheMatism: Inside a classroom, $1 / 2+2 / 3=7 / 6$. Outside 1 coke out of 2 bottles and 2 cokes out of 3 bottles add up to 3 cokes out of 5 bottles, and not 7 cokes out of 6 bottles as taught inside.
Not criticizing rational numbers shows that critical thinking has taboos and that it lacks self-criticism by showing no criticism towards its own un-criticalness.

## 'PRESCHOOL CALCULUS AND MULTIPLICATION BEFORE ADDITION' AS A 1YEAR PRE-ENGINEER MATH COURSE

As a label, mathematics has no content itself, only it ingredients have, algebra and geometry both rooted in the physical fact Many. To deal with Many we count \& add. By counting a total T in bundles, cup-counting creates numbers as blocks of bundles and unbundled occurring in three different ways, normal and overload and underload as in $\mathrm{T}=2] 13 \mathrm{~s}=1] 43 \mathrm{~s}=3]-23 \mathrm{~s}$ when recounted in the same unit. Recounted in a different unit roots proportionality through the recount formula $T=(T / B) \cdot B$ allowing a calculator to predict the result. Recounting in and from tens means resizing blocks where the height and the base are inversely proportional as in $37 \mathrm{~s}=2] 1$ tens or 4 tens $=58 \mathrm{~s}$. Reversed addition is called equations solved by recounting: $2 \cdot x=8=(8 / 2) \cdot 2$ so $x=8 / 2$, showing the solving method 'move to opposite side with opposite sign'. With counting before adding, division and multiplication comes before addition.

Once counted, totals can be added on-top if the units are made the same by proportionality, and next-to as areas also called integration. A composite area always changes with the last block added: change in Area $=$ height* change in base, or $\Delta \mathrm{A}=$ $h \cdot \Delta \mathrm{~b}$ or $\mathrm{h}=\Delta \mathrm{A} / \Delta \mathrm{b}$. So areas can be found by developing $\Delta / \Delta \mathrm{x}$-calculations, also called differentiation in the case of replacing interval changes with local changes: $y^{\prime}=\mathrm{dy} / \mathrm{dx}$ $=\Delta \mathrm{y} / \Delta \mathrm{x}$ for $\Delta \mathrm{x}$ arbitrarily small; as when the per-number is neither globally nor piecewise but locally constant (continuous) (Tarp, 2013).

Finally, double counting a physical quantity in two different units creates pre-numbers or fractions as $2 \$ / 3 \mathrm{~kg}=2 / 3 \$ / \mathrm{kg}$ that must be multiplied to areas before being added.

The difference between a full critical and civilized mathematics education curriculum is illustrated in the appendix.

## DISCUSSION AND CONCLUSION

We asked: wanting to design a lyear pre-engineer course for migrants beginning from scratch, should we use critical and civilized thinking?
Investigating its theoretical background shows that critical thinking is based on Marx, again based on Hegel counter-enlightenment going back to Greek Plato philosophy resonating with the Greek meaning of the word 'kritike', to pass judgement. For Plato, that was precisely what the philosophers were able to do since to them all physical was but examples of metaphysical forms only visible to them. Hegel replaced the forms with a Spirit expressing itself through the history of different people thinking they can decide their future themselves, but in reality just being puppets on a string playing out the masterplan of the Spirit. To Marx, the means to the Spirit's goal, a socialist society, was a proletarian dictatorship with a democracy in the form of a representative pyramid where the top central committee decided the correctness code that justified the judgement passed by critical thinking. Consequently, rational numbers cannot be criticized if part of this code. Likewise, criticizing Hegel-based line-organized office directed education is out of the question. With its lack of self-criticism and dependence on the will of a metaphysical Spirit, critical thinking reminds of a totalitarian religion preaching political correctness instead of teaching enlightenment.
Being skeptical towards ungrounded is-claims, civilized thinking unmasks false nature by uncovering hidden alternatives to choices presented as nature. So categories and correctness are grounded in the outside world; and as means avoiding the banality of evil, its institutions accommodate to resistance from the outside goals they are created to meet. Consequently, mathematics is ManyMath, a natural science accommodating to the physical fact Many; and education must be organized in flexible half-year blocks aiming at uncovering and developing the talent of the individual learner.
So as a 1year pre-engineer course for migrants from scratch we will get to different answers. Uncritically accepting mathematics as meaningless MetaMatism, critical thinking will say it is impossible to learn a pre-engineer background in one year since mathematics is difficult to learn thus taking many hours of hard dedicated work.
Civilized thinking welcomes a course showing that while MetaMatism is difficult, ManyMath is quickly learned: To deal with many, we count and recount and doublecount before performing next-top and on-top addition and reversed addition. First we count in ones to produce icons, then we cup-count in normal, overload and underload form by recounting in the same unit thus realizing that numbers are 2dimensional blocks and not names to the points on a 1dimensional cardinality line as claimed by MetaMatism. Then we recount in a new unit to proportional numbers. Then we recount in and from tens to resize the number blocks. Then we double-count to create pernumbers and fractions. Then to add on-top we must change the unit by proportional recounting; and to add per-numbers we must add next-to as areas where a composite area changes with the last block added. And finally reversed addition leads to solving equations presenting 'opposite side with opposite sign' as a natural method.

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## APPENDIX: A CRITICAL AND A CIVILIZED MATH CURRICULUM

Primary school

| Critical mathematics curriculum | Civilized mathematics curriculum (Tarp, 2016) |
| :--- | :--- |
| 1dim. Number-line with number-names | 2dim. Number-blocks with units |
| No counting, only adding and nextto | Counting before adding, nxto before ontop |
| Addition \& Subtraction before | Multiplication \& Division before |
| Multiplication \& Division | Subtraction \& Addition |
| Multiplication tables to be memorized | Multiplication tables recount to \& from tens |
| No calculator | Calculator from the start as predictors |
| One and two digit numbers | CupCount Many in BundleCups |
| Addition | ReCount Many in same Unit \& in new Unit |
| Subtraction | (Proportionality) |
| Multiplication | ReCount: In Tens \& From Tens |
| Division | (Multiplication \& Division) |
| Simple fractions | Calculator Prediction: RecountFormula |
|  | Addition: NextTo (Integration) \& OnTop |
|  | Reversed addition: Equations |

Middle school

| Fractions are numbers that can be added <br> without units. <br> Letter-fractions must be factorized <br> before added | Fractions are PerNumbers (operators needing a <br> number to become a number) and added by <br> areas (integration) |
| :--- | :--- |
| Negative numbers |  <br> PerFives (fractions) \& PerHundreds ( \%) <br> Fractions <br> Percentages \& Decimals <br> Proportionality |
| working with lettebra go hand in hand when <br> Lormulas; and with lines and forms letter- |  |
| Algebraic fractions | The coordinate system coordinates geometry <br> and algebra so that length can be translated to <br> Solve a linear equation <br> Solve 2 equations w. 2 unknowns |
| $\Delta$-change, and vice versa |  |

High school

| Functions are set-relations | Functions are formulas with two variables |
| :--- | :--- |
| Squares and square roots | Integral Calculus as adding PerNumbers |
| Solve quadratic equations | Change \& Global/Piecewice/Local constancy |
| Linear functions | Root/log as finding/counting change-factors |
| Quadratic functions | Constant change: Proportional, linear, |
| Exponential functions | quadratic, exponential, power |
| Logarithm | Simple and compound interest |
| Differential Calculus |  |
| Integral Calculus | Differential Calculus |
| Statistics \& probability | Unpredictable Change: Statistics \& probability |

# ONLINE TEACHER TRAINING FOR CURING MATH DISLIKE: CUP\&RE-COUNTING \& MULTIPLICATION BEFORE ADDITION 

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Set transformed Mathematics from a mere label for Algebra and Geometry into a selfreferring subject changing the two from example-containers to examples of set, causing massive learning problems as shown by PISA. Re-rooting mathematics in the physical fact Many, the MATeCADEMY.net offers an alternative teacher training.

## BACKGROUND

Despite increased mathematics education research, Swedish PISA results decrease as witnessed by the OECD 2015 report 'Improving Schools in Sweden'. Mathematics seems to be hard, but we could ask: Maybe it is not mathematics that is taught, and maybe there is a hidden mathematics that rooted in the outside world becomes meaningful? And if so, where can teachers learn about it? Existentialist thinking might provide an answer. Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines existentialism as holding that 'existence precedes essence' (Marino, 2004 p. 344). But how does essence-math differ from existence-math?

## A CASE: PETER, STUCK IN DIVISION AND FRACTIONS

Being a mathematics teacher in a class of ordinary students and repeaters flunking division and fractions, Peter is about to give up teaching when he learns about the ' 1 cup \& 5sticks' method to cure mathematics dislike by watching 'CupCount and ReCount before you Add’ (https://www.youtube.com/watch?v=IE5nk2YEQIAxx).
Here 5 sticks are CupCounted in 2 s using a cup for bundles. He sees that a total can be recounted in the same unit in 3 different forms: overload, standard and underload:
$T=5=\| \|\| \|=\underline{\|}\| \|=1] 32 \mathrm{~s}=\underline{I} \underline{\|} \|=2] 12 \mathrm{~s}=\underline{I} \underline{\|} \underline{\|}+=3]-12 \mathrm{~s}$
So counted in bundles, a total has an inside number of bundles and an outside number of singles; and moving a stick out or in creates an over-load or an under-load.
When multiplying, $7 \times 48$ is cup-written as $7 \times 4] 8$ resulting in 28 inside and 56 outside as an overload that can be recounted: $T=7 \times 4] 8=28] 56=33] 6=336$.
And when dividing, $336 / 7$ is cup-written as 33$] 6 / 7$ recounted to 28 inside and 56 outside according to the multiplication table. So 33$] 6 / 7=28] 56 / 7=4] 8=48$.
To try it himself, Peter downloads the 'CupCount \& ReCount Booklet'. He gives a copy to his colleagues and they decide to arrange a free 1day Skype seminar.

In the morning they watch the PowerPoint presentation 'Curing Math Dislike', and discuss six issues: first the problems of modern mathematics, MetaMatism; next the potentials of postmodern mathematics, ManyMath; then the difference between the
two; then a proposal for a ManyMath curriculum in primary and middle and high school; then theoretical aspects; and finally where to learn about ManyMath.

Here MetaMatism is a mixture of MatheMatism, true inside a classroom but rarely outside where ' $2+3=5$ ' is contradicted by 2 weeks +3 days $=17$ days; and MetaMatics, presenting a concept TopDown as an example of an abstraction instead of BottomUp as an abstraction from many examples: A function IS an example of a set-product.

In the afternoon the group works with an extended version of the CupCount \& ReCount Booklet where Peter assists newcomers. At the seminar there are two Skype sessions with an external instructor, one at noon and one in the afternoon.

Bringing ManyMath to his classroom, Peter sees that many difficulties disappear, so he takes a lyear distance learning education at the MATHeCADEMY.net teaching teachers to teach MatheMatics as ManyMath, a natural science about Many. Peter and 7 others experience PYRAMIDeDUCATION where they are organised in 2 teams of 4 teachers choosing 3 pairs and 2 instructors by turn. An external coach assists the instructors instructing the rest of their team. Each pair works together to solve count\&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation. In a pair each teacher corrects the other's routine-assignment. Each pair is the opponent on the essay of another pair. Each teacher pays by coaching a new group of 8 teachers.
At the academy, the 2 x 4 sections are called CATS for primary and secondary school inspired by the fact that to deal with Many, we Count \& Add in Time \& Space.
At the academy, primary school mathematics is learned through educational sentencefree meetings with the sentence subject developing tacit competences and individual sentences coming from abstractions and validations in the laboratory, i.e. through automatic 'grasp-to-grasp' learning.

Secondary school mathematics is learned through educational sentence-loaded tales abstracted from and validated in the laboratory, i.e. through automatic 'gossiplearning': Thank you for telling me something new about something I already knew.

## CONCLUSION

An existentialist distinction between essence and existence shows that what is taught in schools in not mathematics, but a self-referring MetaMatism turning mathematics upside down and containing some statements that do not apply outside. As a common label for Algebra and Geometry meaning reuniting Many and measuring Earth in Arabic and Greek, mathematics should let existence precede essence and become ManyMatics, a natural science about how to divide the earth and its Many products.

## REFERENCE

Marino, G. (2004). Basic Writings of Existentialism. New York: Modern Library.

# Cure MathDislike: CupCount 'fore you Add <br> <br> 1Day Skype Seminar on CupCounting, ReCounting \& CupWriting 

 <br> <br> 1Day Skype Seminar on CupCounting, ReCounting \& CupWriting}

Action Learning on the child's own 2D NumberLanguage as observed when holding 4 fingers together 2 by 2 makes a 3-year-old child say 'No, that is not 4, that is 22 s.'
 demonstrate competencies to actively participate in life'. MATHeCADEMY.net offers UK or DK online Teacher Training based upon Action Learning and Research papers on CupCounting published at the ICME 2004-2012 (mathecademy.net/papers/icme-trilogy). More details on MrAlTarp YouTube videos:


## SUMMARY OF THE 4 PRIMARY AND SECONDARY 4 STUDY UNITS AT THE MATHECADEMY.NET

|  | QUESTIONS | ANSWERS |
| :---: | :---: | :---: |
| C1 COUNT | How to count Many? <br> How to recount 8 in 3 s : $\mathrm{T}=8=$ ? 3 s <br> How to recount 6 kg in $\$: \mathrm{T}=6 \mathrm{~kg}=? \$$ <br> How to count in standard bundles? | By bundling and stacking the total T predicted by $\mathrm{T}=(\mathrm{T} / \mathrm{b}) * \mathrm{~b}$ $\mathrm{T}=8=? * 3=? 3 \mathrm{~s}, \mathrm{~T}=8=(8 / 3) * 3=2) 23 \mathrm{~s}=2.23 \mathrm{~s}=2 * 3+2=22 / 3 * 3$ <br> If $4 \mathrm{~kg}=2 \$$ then $6 \mathrm{~kg}=(6 / 4) * 4 \mathrm{~kg}=(6 / 4) * 2 \$=3 \$$ <br> Bundling bundles gives a multiple stack, a stock or polynomial: $\mathrm{T}=423=4 \text { BundleBundle }+2 \text { Bundle }+3=4 \text { tenten } 2 \operatorname{ten} 3=4 * \mathrm{~B}^{\wedge} 2+2 * \mathrm{~B}+3$ |
| $\begin{gathered} \text { C2 } \\ \text { COUNT } \end{gathered}$ | How can we count possibilities? <br> How can we predict unpredictable numbers? | By using the numbers in Pascal's triangle <br> We 'post-dict' that the average number is 8.2 with the deviation 2.3. <br> We 'pre-dict' that the next number, with $95 \%$ probability, will fall in the confidence interval $8.2 \pm 4.6$ (average $\pm 2 *$ deviation) |
| $\begin{gathered} \text { A1 } \\ \text { ADD } \end{gathered}$ | How to add stacks concretely? $\mathrm{T}=27+16=2 \operatorname{ten} 7+1 \operatorname{ten} 6=3 \operatorname{ten} 13=$ ? How to add stacks abstractly? | By restacking overloads predicted by the restack-equation $\mathrm{T}=(\mathrm{T}-\mathrm{b})+\mathrm{b}$ $\mathrm{T}=27+16=2$ ten $7+1$ ten $6=3$ ten $13=3$ ten 1 ten $3=4$ ten $3=43$ Vertical calculation uses carrying. Horizontal calculation uses FOIL |
| $\begin{gathered} \text { A2 } \\ \text { ADD } \end{gathered}$ | What is a prime number? What is a per-number? How to add per-numbers? | Fold-numbers can be folded: $10=2$ fold5. Prime-numbers cannot: $5=1$ fold5 Per-numbers occur when counting, when pricing and when splitting. The $\$ /$ day-number a is multiplied with the day-number $b$ before added to the total \$number T: T2 $=\mathrm{T} 1+\mathrm{a}^{*} \mathrm{~b}$ |
| T1 TIME | How can counting \& adding be reversed ? Counting? 3s and adding 2 gave 14. Can all calculations be reversed? | By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. $x * 3+2=14$ is reversed to $x=(14-2) / 3$ <br> Yes. $x+a=b$ is reversed to $x=b-a, x * a=b$ is reversed to $x=b / a$, <br> $x^{\wedge} a=b$ is reversed to $x=a \sqrt{ }, a^{\wedge} x=b$ is reversed to $x=\log b / \log a$ |
| $\begin{gathered} \text { T2 } \\ \text { TIME } \end{gathered}$ | How to predict the terminal number when the change is constant? <br> How to predict the terminal number when the change is variable, but predictable? | By using constant change-equations: <br> If $\mathrm{Ko}=30$ and $\Delta \mathrm{K} / \mathrm{n}=\mathrm{a}=2$, then $\mathrm{K} 7=K o+\mathrm{a}$ n $=30+2 * 7=44$ <br> If $\mathrm{Ko}=30$ and $\Delta \mathrm{K} / \mathrm{K}=\mathrm{r}=2 \%$, then $\mathrm{K} 7=\mathrm{Ko}^{*}(1+\mathrm{r})^{\wedge} \mathrm{n}=30^{*} 1.02^{\wedge} 7=34.46$ <br> By solving a variable change-equation: <br> If $\mathrm{Ko}=30$ and $\mathrm{dK} / \mathrm{dx}=\mathrm{K}^{\prime}$, then $\Delta \mathrm{K}=\mathrm{K}-\mathrm{Ko}=\int \mathrm{K}^{\prime} \mathrm{d}_{\mathrm{d}}$ |
| $\begin{gathered} \hline \text { S1 } \\ \text { SPACE } \end{gathered}$ | How to count plane and spatial properties of stacks and boxes and round objects? | By using a ruler, a protractor and a triangular shape. <br> By the 3 Greek Pythagoras', mini, midi \& maxi <br> By the 3 Arabic recount-equations: $\sin A=a / c, \cos A=b / c, \tan A=a / b$ |
| $\begin{gathered} \text { S2 } \\ \text { SPACE } \end{gathered}$ | How to predict the position of points and lines? How to use the new calculation technology? | By using a coordinate-system: If $\operatorname{Po}(\mathrm{x}, \mathrm{y})=(3,4)$ and if $\Delta \mathrm{y} / \Delta \mathrm{x}=2$, then $\mathrm{P} 1(8, y)=$ $\mathrm{P} 1(\mathrm{x}+\Delta \mathrm{x}, \mathrm{y}+\Delta \mathrm{y})=\mathrm{P} 1((8-3)+3,4+2 *(8-3))=(8,14)$ <br> Computers can calculate a set of numbers (vectors) and a set of vectors (matrices) |

# DEBATE ON HOW TO IMPROVE MATHEMATICS EDUCATION 


#### Abstract

Allan Tarp MATHeCADEMY.net, Denmark In this symposium the author invites opponents to debate how to improve mathematics education inspired by the Chomsky-Foucault debate on human nature. The main question is: 'If research cannot improve Math education, then what can?

Bo: Today we discuss Mathematics education and its research. Humans communicate in languages, a word-language and a number-language. In the family, we learn to speak the word language, and we are taught to read and write in institutionalized education, also taking care of the number-language under the name Mathematics, thus emphasizing the three r's: Reading, Writing and Arithmetic. Today governments control education, guided by a growing research community. Still international tests show that the learning of the number language is deteriorating in many countries. This raises the question: If research cannot improve Mathematics education, then what can? I hope our two guests will provide some answers. I hope you will give both a statement and a comment to the other's statement before the floor will comment.


## 1. MATHEMATICS ITSELF

Bo: We begin with Mathematics. The ancient Greeks Pythagoreans used this word as a common label for what we know, which at that time was Arithmetic, Geometry, Astronomy and Music. Later Astronomy and Music left, and Algebra and Statistics came in. So today, Mathematics is a common label for Arithmetic, Algebra, Geometry and Statistics, or is it? And what about the so-called 'New Math' appearing in the 1960 s, is it still around, or has it been replaced by a post New-Math, that might be the same as pre New-Math? In other words, has pre-modern Math replaced modern Math as post-modern Math? So, I would like to ask: 'What is Mathematics, and how is it connected to our number-language?'

## 2. EDUCATION IN GENERAL

Bo: Now let us talk about education in general. On our planet, life takes the form of single black cells, or green or grey cells combined as plants or animals. To survive, plants need minerals, pumped in water from the ground through their leaves by the sun. Animals instead use their heart to pump the blood around, and use the holes in the head to supply the stomach with food and the brain with information. Adapted through genes, reptiles reproduce in high numbers to survive. Feeding their offspring while it adapts to the environment through experiencing, mammals reproduce with a few children per year. Humans only need a few children in their lifetime, since transforming the forelegs to hands and fingers allows humans to grasp the food, and to share information through communication and education by developing a language when associating sounds to what they grasp. Where food must be split in portions, information can be shared. Education takes place in the family and in the workplace; and in institutions with primary, secondary and tertiary education for children, for
teenagers and for the workplace. Continental Europe uses words for education that do not exist in the English language such as Bildung, unterricht, erziehung, didactics, etc. Likewise, Europe still holds on to the line-organized office preparing education that was created by the German autocracy shortly after 1800 to mobilize the population against the French democracy, whereas the North American republics have blockorganized talent developing education from secondary school. As to testing, some countries use centralized test where others use local testing. And some use written tests and others oral tests. So, my next question is 'what is education?'

## 3. MATHEMATICS EDUCATION

Bo: Now let us talk about education in Mathematics, seen as one of the core subjects in schools together with reading and writing. However, there seems to be a difference here. If we deal with the outside world by proper actions, it has meaning to learn how to read and how to write since these are action-words. However, you cannot Math, you can reckon. At the European continent reckoning, called 'Rechnung' in German, was an independent subject until the arrival of the so-called new Mathematics around 1960. When opened up, Mathematics still contains subjects as fraction-reckoning, trianglereckoning, differential-reckoning, probability-reckoning, etc. Today, Europe only offers classes in Mathematics, whereas the North American republics offer classes in algebra and geometry, both being action words meaning to reunite numbers and to measure earth in Arabic and Greek. Therefore, I ask, 'what is Mathematics education?'

## 4. THE LEARNER

Bo: Now let us talk about at the humans involved in Mathematics education: Governments choose curricula, build schools, buy textbooks and hire teachers to help learners learn. We begin with the learners. The tradition sees learning taking place when learners follow external instructions from the teacher in class and from the textbook at home. Then constructivism came along suggesting that instead learning takes place through internal construction. Therefore, I ask 'what is a learner?'

## 5. THE TEACHER

Bo: Now let us talk about the teacher. It seems straightforward to say that the job of a teacher is to teach learners so that learning takes place, checked by written tests. However, continental Europe calls a teacher a 'Lehrer' thus using the same word as for learning. In addition, a Lehrer is supposed to facilitate 'unterrichtung and erziehung and to develop qualifications and competences. In teacher education, the subject didactics, meant to determine the content of Bildung, is unknown outside the continent. And until lately, educating lehrers took place outside the university in special lehrerschools. Thus, being a teacher does not seem to be that well-defined. Therefore, my next question is 'what is a teacher?'

## 6. THE POLITICAL SYSTEM

Bo: Now let us talk about governments. Humans live together in societies with different degrees of patronization. In the debate on patronization, the ancient Greek sophists
argued that humans must be enlightened about the difference between nature and choice to prevent patronization by choices presented as nature. In contrast, the philosophers saw choice as an illusion since physical phenomena are but examples of metaphysical forms only visible to philosophers educated at Plato's Academy who consequently should be accepted as patronizors. Still today, democracies come in two forms with a low and high degree of institutionalized patronization using blockorganized education for individual talent developing or using line-organized education for office preparation. As to exams, some governments prefer them centralized and some prefer them decentralized. As to curricula, the arrival of new Mathematics in the 1960s integrated its subfields under the common label Mathematics. Likewise, constructivism meant a change from lists of concepts to lists of competences. However, these changes came from Mathematics and education itself. So my question is: 'Should governments interfere in Mathematics education?'

## 7. RESEARCH

Bo: Now let us talk about research. Tradition often sees research as a search for laws built upon reliable data and validated by unfalsified predictions. The ancient Greek Pythagoreans found three metaphysical laws obeyed by physical examples. In a triangle, two angles and two sides can vary freely, but the third ones must obey a law. In addition, shortening a string must obey a simple ratio-law to create musical harmony. Their findings inspired Plato to create an academy where knowledge meant explaining physical phenomena as examples of metaphysical forms only visible to philosophers educated at his academy by scholasticism as 'late opponents' defending their comments on an already defended comment against three opponents. However, this method discovered no new metaphysical laws before Newton by discovering the gravitational law brought the priority back to the physical level, thus reinventing natural science using a laboratory to create reliable data and test library predictions. This natural science inspired the 18th century Enlightenment period, which again created counter-enlightenment, so today research outside the natural sciences still uses Plato scholastics. Except for the two Enlightenment republics where American Pragmatism used natural science as an inspiration for its Grounded Theory, and where French post-structuralism has revived the ancient Greek sophist skepticism towards hidden patronization in categories, correctness and institutions that are ungrounded. Using classrooms to gather data and test predictions, Mathematics education research could be a natural science, but it seems to prefer scholastics by researching, not Math education, but the research on Math education instead. To discuss this paradox I therefore ask, 'what is research in general, and within Mathematics education specifically?'

## 8. CONFLICTING THEORIES

Bo: Of course, Mathematics education research builds upon and finds inspiration in external theories. However, some theories are conflicting. Within Psychology, constructivism has a controversy between Vygotsky and Piaget. Vygotsky sees education as building ladders from the present theory regime to the learners' learning
zones. Piaget replaces this top-down view with a bottom-up view inspired by American Grounded Theory allowing categories to grow out of concrete experiences and observations. Within Sociology, disagreement about the nature of knowledge began in ancient Greece where the sophists wanted it spread out as enlightenment to enable humans to practice democracy instead of allowing patronizing philosophers to monopolize it. Medieval times saw a controversy between the realists and the nominalists as to whether a name is naming something or a mere sound. In the late Renaissance, a controversy occurred between Hobbes arguing that their destructive nature forces humans to accept patronization, and Locke arguing, like the sophists, that enlightenment enables humans to practice democracy without any physical or metaphysical patronization. As counter-enlightenment, Hegel reinstalled a patronizing Spirit expressing itself through art and through the history of different people. This created the foundation of Europe's line-organized office preparing Bildung schools; and for Marxism and socialism, and for the critical thinking of the Frankfurter School, reviving the ancient sophist-philosopher debate by fiercely debating across the Rhine with the post-structuralism of the French Enlightenment republic. Likewise, the two extreme examples of forced institutionalization in 20th century Europe, both terminated by the low institutionalized American Enlightenment republics, made thinkers as Baumann and Arendt point out that what made termination camps work was the authorized routines of modernity and the banality of evil. Reluctant to follow an order, you can find another job in the private sector, but not in an institution. Here the necessity of keeping a job forces you to carry out both good and evil orders. As an example of a forced institution, this also becomes an issue in Mathematics Education. So I ask: What role do conflicting theories play in Mathematics education and its research?

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# MIGRANT-MATH: CUPCOUNTING \& PRESCHOOL CALCULUS 

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Europe receives a continuing migrant flow to benefit from its welfare and educational systems. To benefit from the engineer potential in young migrants allowing them to build up welfare and education in their own country, Europe must rethink its lineorganized office directed education dating back to the Napoleon wars; and must replace meaningless top-down MetaMatism with bottom-up ManyMath.

## BACKGROUND

Increased mathematics education research seems to create a decrease in Nordic PISA results as witnessed by the latest PISA study and the OECD 2015 report 'Improving Schools in Sweden'. We ask: Can existentialism point to a possible solution?

Building on the work of Kierkegaard, Nietzsche and Heidegger, Sartre defines existentialism as holding that 'existence precedes essence' (Marino, 2004 p. 344). Thus a hypothesis can be formulated: Mathematics performance will increase if replacing essence-math with existence-math.

## MATHEMATICS AS A ESSENCE

The Pythagoreans labeled their four knowledge areas by a Greek word for knowledge, mathematics. With astronomy and music now as independent areas, today mathematics is a common label for its two remaining activities both rooted in Many: Geometry meaning to measure earth in Greek, and Algebra meaning to reunite numbers in Arabic and replacing Greek Arithmetic (Freudenthal, 1973).

Then the set-concept transformed mathematics to 'MetaMatics' defining its concepts by self-reference as examples from internal abstractions instead of as abstractions from external examples. Looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox 'this sentence it false' being false if true and true if false: If $M=\{A \mid A \notin A)\}$ then $M \in M \Leftrightarrow M \notin M$.
'MetaMatism' means mixing MetaMatics with 'MatheMatism' true inside but seldom outside the classroom as e.g. 'the fraction paradox' where the textbook insists that $1 / 2$ $+2 / 3$ IS $7 / 6$ even if the students protest: counting cokes, $1 / 2$ of 2 bottles and $2 / 3$ of 3 bottles gives $3 / 5$ of 5 as cokes and never 7 cokes of 6 bottles.

## MATHEMATICS AS MANY-MATH, A NATURAL SCIENCE ABOUT MANY

A number as $345=3 * \mathrm{~B}^{\wedge} 2+4 * \mathrm{~B}+5^{*} 1$ shows that to deal with Many, first we iconize then we bundle and stack. Until ten we count in 1 s by iconizing, i.e. by rearranging sticks in icons so five ones becomes one five-icon 5 with five sticks, etc.


With icons, a total can be 'cup-counted' in icon-bundles so a total T of 7 is bundled in 3 s as $\mathrm{T}=23 \mathrm{~s} \& 1$ shown with 2 sticks in a in a bundle-cup and 1 stick outside; reported with 'cup-writing', $\mathrm{T}=2] 13 \mathrm{~s}$, then with 'decimal-writing' where a decimal point separates the bundles from the singles, and including the unit $3 \mathrm{~s}, \mathrm{~T}=2.13 \mathrm{~s}$.
A calculator can predict a counting result. A stack of 23 s is iconized as $2 \times 3$ showing a lift used 2 times to lift the 3 s. Taking away is iconized with ' $/ 3$ ' or ' -3 ' showing the broom or the trace when wiping away 3 several times or just once, called division and subtraction. Entering ' $7 / 3$ ', we ask the calculator 'from 7 take away 3 s' and get the answer ' 2 .some'. Entering ' $7-2 \times 3$ ' we ask 'from 7 take away 23 s ' and get the answer 1 leftover. Thus the calculator predicts that $7=2] 13 \mathrm{~s}=2.13 \mathrm{~s}$.
Once cup-counted, totals are re-counted, double-counted or added next-to or on-top. To recount in the same unit, changing a bundle to singles creates over- or under-load as when recounting 42 s as 3.22 s , or as 5 less 22 s leading to negative numbers:
$\mathrm{T}=42 \mathrm{~s}=3.22 \mathrm{~s}$, or $\mathrm{T}=42 \mathrm{~s}=5 .-22 \mathrm{~s}$
To recount in a different unit means changing unit, called proportionality. Asking '3 4 s is how many 5 s ?' sticks give the result 2.25 s as predicted by a calculator.
$\mathrm{T}=34 \mathrm{~s}=\mathrm{IIII} \mathrm{IIII} \mathrm{IIII} \rightarrow \mathrm{IIIII} \mathrm{IIIII} \mathrm{II} \rightarrow 2 \mathrm{~J} 25 \mathrm{~s} \rightarrow 2.25 \mathrm{~s}$
Recounting in and from tens means resizing number-blocks where the height and the base are inversely proportional as in $37 \mathrm{~s}=2 \mathrm{~J} 1$ tens or 4 tens $=58 \mathrm{~s}$.
Double-counting a physical quantity creates 'per-numbers' as $4 \$ / 5 \mathrm{~kg}$ allowing $16 \$$ to be recounted in 4 s to bridge to the kg-numbers: $16 \$=(16 / 4) * 4 \$=(16 / 4) * 5 \mathrm{~kg}=20 \mathrm{~kg}$.
Next-to addition of 23 s and 45 s as 3.28 s means adding areas, called integration. To add on-top the units are made the same by recounting as 1.15 s and $45 \mathrm{~s}=5.15 \mathrm{~s}$. Reversed addition is called equations solved by recounting: $2 \cdot \mathrm{x}=8=(8 / 2) \cdot 2$ so $\mathrm{x}=$ $8 / 2$, showing the solving method 'move to opposite side with opposite sign'.
The root of geometry is a rectangle that halved by a diagonal becomes two right-angled triangles where the sides and the angles are connected by three laws, $\mathrm{A}+\mathrm{B}+\mathrm{C}=180$, $a^{\wedge} 2+b^{\wedge} 2=c^{\wedge} 2$ and $\tan A=a / b$. Being filled from the inside by such triangles, a circle with radius $r$ gets the circumference $2 \cdot \pi \cdot r$ where $\pi=n \cdot \tan (180 / n)$ for $n$ large.

## CONCLUSION

There is a fundamental difference between essence- and existence-math, MetaMatism and ManyMath. This means the latter has to be tested outside traditional school in preschool or in special courses for young migrants wanting to become engineers or teachers to help building welfare and education systems in their own country.

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## MigrantMath

## - CupCount \& ReCount \& DoubleCount

- Multiplication before Addition
- PreSchool Calculus before OnTop Addition


## CupCounting 5 Sticks in 2 s



3 ways to CupCount: Overload, Normal, Underload
ReCount 7 in 3s:

$$
7=2] 13 s=1] 43 s=3]-23 s
$$

NO, $4 \times 7$ is not 28 , it is $4 \mathbf{7 s}=2$ ] $8=1$ ] $18=3]-2$ tens
NO, $30 / 6$ is not 30 divided by 6 , it is 3tens recounted in 6 s
CupWriting tells InSide Bundles from OutSide 1s

| $\bullet 65+\mathbf{2 7}=6] 5+2] 7=8] 12=9] 2=$ | 92 |  |
| :--- | :--- | ---: |
| $-65-\mathbf{2 7}$ | $=6] 5-2] 7=4]-2=3] 8=$ | 38 |
| - $7 \times 48$ | $=7 \times 4] 8=28] 56=33] 6=$ | 336 |
| $-336 / 7$ | $=33] 6 / 7=28] 56 / 7=4] 8=$ | 48 |

MatheMatics as ManyMath - a Natural Science about Many

## MATHeCADEMY.net

MatheMatics as ManyMath, a Natural Science about MANY
MATHeCADEMY.net Cure Math Dislike: Use Children's own 2D Numbers with Units

| Count <br> In Icons <br> In BundleCups | $\begin{aligned} & \mathrm{T}=\\| \\| \\|=Ч=4 \\ & \mathrm{~T}=7=\\| \\|\\| \\|\\|I=\\|\\| \\| I=2] 1 \mathbf{3 s}=2 \text { Bundles \& } 1 \mathbf{3 s} \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: |
| ReCount In same Unit In new Unit | ReBundle to create Overload \& Underload$\begin{aligned} & T=7=\|\|\|\|\|\| \|=2] 1 \mathbf{3} \mathbf{s}=1] 4 \mathbf{3} \mathbf{s}=3]-2 \mathbf{3 s} \\ & T=2] 1 \mathbf{3} \mathbf{s}=1] 3 \mathbf{4} \mathbf{s}=1] 2 \mathbf{5} \mathbf{s}=3] 1 \mathbf{2 s}=1] 1] 1 \mathbf{2} \mathbf{s}=11] 1 \mathbf{2} \mathbf{s} \end{aligned}$ |  |  |
| ReCount In Tens From Tens | $37 \mathrm{~s}=$ ? tens Answer: $3 \times 7=21=2] 1$ tens <br> ? 7s = 3 tens Answer: $(30 / 7) \times 7=4] 2$ 7s |  |  |
| DoubleCount <br> in PerNumbers <br> in PerFive, 3/5 <br> in PerHundred, \% | With $4 \$$ per $5 \mathrm{~kg}, \mathrm{~T}=20 \mathrm{~kg}=(20 / 5) \times 5 \mathrm{~kg}=(20 / 5) \times 4 \$=16 \$$ <br> 3 per 5 of $200 \$=? \$ .200 \$=(200 / 5) \times 5 \$$ gives $(200 / 5) \times 3 \$=120 \$$ <br> $70 \%$ of $300 \$=? \$ .300 \$=(300 / 100) \times 100 \$$ gives $(300 / 100) \times 70 \$=210 \$$ |  |  |
| Calculator Prediction RecountFormula RestackFormula | $\begin{array}{lrr} T=2 \mathbf{4 s}=? \mathbf{5 s}=1] 3 \mathbf{5} \mathbf{s} \text { since } \\ T=(T / B) \times B=T / B \text { Bs } \\ T=(T-B)+B & 2 \times 4 / 5 & 1 . \text { some } \\ 2 \times 4-1 \times 5 & 3 \\ \hline \end{array}$ |  |  |
| Add NextTo OnTop | $\begin{array}{\|l\|l} \mathrm{T}=23 \mathrm{~s}+45 \mathrm{~s}=312 \mathbf{8 s} \quad \text { Integration } \\ \mathrm{T}=23 \mathrm{~s}+45 \mathrm{~s}=1] 15 \mathrm{~s}+45 \mathrm{~s}=5] 15 \mathrm{~s} \end{array} \quad \begin{array}{\|} \text { Preschool Calculus } \\ \text { Proportionality } \end{array}$ |  |  |
| Reverse Adding | OnTop |  | NextTo |
|  | ? = $8=(8 / 2) \times 2$ | ? $=8=(8-2)+2$ | $\mathrm{T}=23 \mathrm{~s}+$ ? $5 \mathrm{~s}=3.28 \mathrm{~s}$ |
| Move to <br> Opposite side \& sign | $?=8 / 2$ <br> Solved by ReCounting | $?=8-2$ <br> Solved by ReStacking | $?=(3.28 \mathrm{~s}-23 \mathrm{~s}) / 5=\Delta \mathrm{T} / 5$ <br> Solved by Differentiation |

$\mathrm{T}=7=2 \mathrm{l} 1$ 3s on an Abacus:


