FROM ESSENCE TO EXISTENCE IN MATHEMATICS EDUCATION

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In mathematics and its education, the difference between essence and existence is seldom discussed although central to existentialist thinking. So we can ask: What will an existentialist mathematics education look like? Thus we close the door to the library with today's self-referring mathematics and go outside to rebuild mathematics from its roots, the physical fact Many. Likewise, we can ask if mathematics is learned by exposure to its outside roots or to its inside essence claims.

BACKGROUND

Institutionalized education typically has mathematics as one of its core subjects in primary and secondary school. To evaluate the success OECD arranges PISA studies on a regular basis. Here increasing funding of mathematics education research should improve PISA results. However, the opposite seems to be the case in Scandinavia as witnessed by the latest PISA study and by the OECD report 'Improving Schools in Sweden' (OECD 2015).

As to the content of education, sociology offers understandings of schools and teacher education, psychology of learning, and philosophy of textbooks. Focusing upon existentialist philosophy this paper asks: What will an existentialist mathematics and education look like? The purpose is not to replace one tradition with another but to uncover hidden alternatives to choices presented as nature.

EXISTENTIALISM

The Pythagoreans labeled their four knowledge areas by a Greek word for knowledge, mathematics. With astronomy and music now as independent areas, today mathematics is a common label for the two remaining activities, geometry meaning to measure earth in Greek, and Algebra meaning to reunite numbers in Arabic and replacing Greek arithmetic (Freudenthal 1973).

The Greeks used the word 'sophy' meaning knowledge for men of knowledge, the sophists and the philosophers, disagreeing on the nature of knowledge. Seeing democracy with information and debate and choice as the natural state-form, the sophists emphasized knowing nature from choice to prevent patronization by choices presented as nature. Seeing autocracy patronized by themselves as the natural state-form, the philosophers saw choice as an illusion since to them physical existence was but examples of metaphysical essence only visible to the philosophers educated at the Plato academy having as entrance sign 'Let no one ignorant of geometry enter.'

Today, the sophist skepticism towards false is-claims is carried on by French post-structuralism and by the existentialism of Kierkegaard, Nietzsche, Heidegger and Sartre, defining existentialism as holding that 'existence precedes essence, or (..) that subjectivity must be the starting point' (Marino 2004: 344). In Denmark, a heritage allowed Kierkegaard to publish whatever he wrote. At the end, shortage forced him to shift to flying papers when rebelling against institutionalized Christianity in the form of Christendom. Focusing on the three classical virtues Truth, and Beauty and Goodness,

Kierkegaard left truth to the natural sciences, and argued that to change from a person to a personality the individual should stop admiring beauty created by others and instead realize their own existence through individual choices. Of course, angst is a consequence when fearing to choose the bad instead of the good, and death might follow, but so will forgiveness and resurrection to a new life in real existence, as promised by Christianity in the Holy Communion.

In Germany, Nietzsche saw institutionalized Christendom as the creator of moral is-statements that prevented individuals from realizing their true existence through individual choices and action. To end this serfdom he hoped that someday we will see a '*redeeming* man (..) whose isolation is misunderstood by the people as if it were flight *from* reality – while it is only his absorption, immersion, penetration *into* reality, so that (..) he may bring home the redemption of this reality: its redemption from the curse that the hitherto reigning ideal has laid upon it.' (Marino 2004: 186-187).

Likewise in Germany, Heidegger saw that to avoid traditional essence-claims, is-statements must be replaced by has-statements so that being is characterized by what it has, 'Dasein'. Arendt carried his work further by dividing human activity into labor and work focusing on the private sphere and action focusing on the political sphere thus accepting as the first philosopher political action as a worthy human activity creating institutions that should be treated with care to avoid 'the banality of evil' if turning totalitarian (Arendt 1998).

MATHEMATICS AS ESSENCE

Within mathematics, the existentialist distinction is shown by the function concept, defined by Euler as labeling the existence of calculations combining known and unknown numbers, and today defined as set-relations where first component-identity implies second-component identity thus becoming pure essence through self-reference. The set-concept transformed mathematics to 'meta-matics', a self-referring collection of well-proven statements about well-defined concepts, defined as examples from internal abstractions instead of as abstractions from external examples. Looking at the set of sets not belonging to itself allowed Russell to show that self-reference leads to the classical liar paradox 'this sentence it false' being false if true and true if false:

If $M = \{ A \mid A \notin A) \}$ then $M \in M \Leftrightarrow M \notin M$.

The Zermelo–Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by its inability to separate outside examples from inside abstractions. That institutionalized education ignores this can be seen as an example of 'symbolic violence' used to protect the privileges of today's 'knowledge nobility' (Bourdieu 1977).

Behind colorful illustrations, self-referring metamatics is taught through a gradual presentation of different number types, natural numbers and integers and rational and real numbers, together with the four basic operations, addition and subtraction and multiplication and division, where especially division and letter fractions create learning problems. Equations are introduced as equivalent number names to be changed by identical operations. In pre-calculus polynomial functions are introduced as a basis for calculus presenting differential calculus before integral calculus.

MATHEMATICS AS EXISTENCE

Chosen by the Pythagoreans as a common label, mathematics has no existence itself, only its content has, geometry and algebra, both rooted as natural sciences about the physical fact Many.

The root of geometry is the standard form, a rectangle, that halved by a diagonal becomes two right-angled triangles where the sides and the angles are connected by three laws, A+B+C = 180, a^2+b^2 = c^2 and sinA = a/c. Being filled from the inside by such triangles, a circle with radius r gets the circumference $2\cdot\pi\cdot r$ where $\pi = n\cdot\sin(180/n)$ for n sufficiently large.

Meeting Many we ask 'how many?' Counting and adding gives the answer. We count by bundling and stacking as seen when writing a total T in its full form: $T = 345 = 3\cdot B^2 + 4\cdot B + 5\cdot 1$ where the bundle B typically is ten. This shows the four ways to unite: On-top addition unites variable numbers, multiplication constant numbers, power constant factors, and next-to addition, also called integration, unites variable blocks. As indicated by its name, uniting can be reversed to split a total into parts predicted by the reversed operations, subtraction and division and root & logarithm and differentiation. Likewise, a total can be presented in two forms, an algebraic form using place values to separate the singles from the bundles and the bundle-bundles, and a geometrical form showing three blocks placed next to each other.

Although presented as essence, ten-bundling is a choice. To experience its existence and the root of core mathematics as proportionality and linearity, Many should be bundled in icon-bundles below ten to allow a calculator to predict the result of shifting units: Asking e.g. 'T = 2 3s = ? 4s' the answer is predicted by two formulas, a recount-formula $T = (T/b)\cdot b$ telling that from a total T, bs can be taken away T/b times, and a restack-formula $T = (T-b)+b$ telling that from a total T, b can be taken away and placed next-to. First $T = (2\cdot 3)/4$ gives 1.some. Then $T = 2\cdot 3 - 1\cdot 4$ gives 2. Si the prediction is T = 2 3s = 1 4s & 2 = 1.2 4s. Thus with icon-counting a natural number is a decimal number with a unit where the decimal point separates the singles from the bundled.

With physical units, the need for changing units creates per-numbers as 3$/4kg serving as bridges when recounting $s in 3s or kgs in 4s: $15\$ = (15/3)\cdot 3\$ = (15/3)\cdot 4kg = 20kg$. As per-numbers, fractions are not numbers but operators needing a number to become a number. To add, per-numbers must be multiplied to unit-numbers, thus adding as areas, called integration.

So relinking it to its root, Many, allows today's 'mandarin mathematics' to escape from its present essence-prison. For details, see the 2012 videos from MrAlTarp.

LEARNING AS ESSENCE AND EXISTENCE

Constructivist learning theory contains a European social Vygotskian and a North American radical Piagetian version believing learning taking place through guidance or exposure respectively. The question now is what is to be learned.

Here Vygotsky accepts the ruling essence-claims about the outside fact Many even if self-reference makes them meaningless. Learning is seen as adapting to them and teaching as developing the learner's mind in their direction using outside artefacts as means. Piaget sees learning as a means to adapt to the outside world, and sees teaching as asking guiding questions to outside existence brought inside the classroom to allow learners construct individual schemata to be accommodated through exposure and communication. So to let existence precede essence, Piaget is useful to mediate learning through inside exposure to outside existence. Vygotsky is useful if accepting that outside existence can lead to competing inside essence-claims. However, its lacking skepticism towards the ruling claim involves a high risk for mediating the banality of evil.

INSTITUTIONALIZED EDUCATION AS ESSENCE AND EXISTENCE

Two versions of post-primary education exist, one letting national administration define its essence, the other letting individual talents define its existence. To get Napoleon out of Berlin, a European line-organized office-directed education was created that concentrate teenagers in age-groups and force them to follow the same schedule. To meet the international norm that 95% of an age-group finishes high school, dropout rates are lowered by low passing grades and by strict retention policy.

In the North American republics middle and high schools teachers teach their major subject in their own classroom where they welcome teenagers with recognition: 'Inside, you carry a talent and it is our mutual task to uncover and develop your personal talent through daily lessons in self-chosen half-year blocks. If successful I say 'Good job, you have a talent, you need more'. If not I say 'Good try, you have courage to try uncertainty, now try something else that might be your talent.'

CONCLUSION

An existentialist view replacing essence with existence exposes today's mathematics as pure essence with little existence behind. What has existence is Many waiting to be united by bundling and stacking into a decimal number with a unit presented geometrically or algebraically as a row of blocks or digits. Thus mathematics exists as geometry measuring forms divided into triangles, and as algebra reuniting numbers by four uniting techniques, addition, multiplication, power and integration each with a corresponding reversed splitting technique. So concepts should present themselves as created, not by self-reference as examples from abstractions above, but as abstractions from examples below. And statements should be held true when not falsified. In short, mathematics should be taught and learned as 'many-matics', not as 'metamatism', a mixture of metamatics and 'mathematism' true inside but not outside the classroom as e.g. 'the fraction paradox' where the textbook insists that 1/2 + 2/3 IS 7/6 even if the students protest: counting cokes, 1/2 of 2 bottles and 2/3 of 3 bottles gives 3/5 of 5 as cokes and never 7 cokes of 6 bottles.

As to learning, mediating the ruling essence should be replaced by guided exposure to the roots of mathematics, the physical fact Many, thus replacing Vygotsky with Piaget. And institutionalized education using camps to concentrate teenagers in age-groups obliged to follow forced schedules should be labeled as such allowing mathematics education to avoid the banality of evil. Christianity's Holy Communion offers forgiveness to individuals, not to institutions. Instead institutionalized force should be limited to provide teenagers with daily lessons in self-chosen half-year blocks to uncover and develop their individual talent, as would be the case if the North American Enlightenment republics replaced essence with existence in algebra and geometry.

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