# A HEIDEGGER VIEW ON HOW TO IMPROVE MATHEMATICS EDUCATION

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## ABSTRACT

*After 50 years of research, mathematics education still has learning problems as witnessed by the PISA studies. So, a suspicion arises: Can we be sure that what has been undertaken is mathematics and education and research? We seek an answer in philosophy by listening to Heidegger that, wanting to establish its meaning, finds two forms of Being: that what is, and how it is. In a Heidegger universe, the core ingredients are I and It and They, where I must neglect the gossip from They to establish an authentic relationship to It. Bracketing mathematics' gossip will allow its root, Many, to open itself and disclose a 'many-matics' as a grounded natural science different in many ways from the traditional self-referring set-based mathe-matics. So to improve its educational sentences, mathematics should bring its subjects to the classroom, but leave its gossip outside.*

## 1. INTRODUCTION

Within education, mathematics is in the front. Consequently, research has grown rapidly for fifty years to solve its many learning problems. The lack of success is shown by PISA studies showing a low level and a continuing decline in many countries. Thus, the former model country Sweden face that 'more than one out of four students not even achieving the baseline level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (OECD 2015, p. 3)

Researchers in mathematics education meet in different fora. In Europe, the Congress of the European Society for Research in Mathematics Education, CERME, meets each second year. At the CERME 10 congress in February 2017 a plenary session asked: What are the solid findings in mathematics education research? (http://cerme10.org/scientific-activities/plenary-sessions/)

By questioning its success, maybe the short answer is: How can mathematics education research be successful when its three words are not that well defined? As to mathematics, it has meant many different things in its almost 5000 years of history spanning from a natural science about the physical fact Many to a self-referring logic.

As to education, two different forms exist. In continental Europe, education serves the nation's need for public servants through multi-year compulsory classes and lines at the secondary and tertiary level. In North America, education aims at uncovering and developing the individual talent through daily lessons in self-chosen half-year blocks together with one-subject teachers at the secondary level, and the tertiary level also has a flexible block organization allowing additional blocks to be taken in the case of unemployment or change of job.

As to research, academic articles can be written at a master level applying or exemplifying existing theories, or at a research level questioning them. Just following ruling theories is especially problematic in the case of conflicting theory as within education where Piaget and Vygotsky contradict each other by saying teach as little and as much as possible respectively.

Consequently, we cannot know what kind of mathematics and what kind of education has been studied, and we cannot know if research is following ruling traditions or searching for new discoveries. So to answer the question ‘How to improve mathematics education research’, first we must try to make the three words well defined by asking: What is meant by Mathematics, what is meant by education and what is meant by research?

Common for all three questions is the word ‘is’, so let us begin by asking ‘what is meant by ‘is’?’.

## 2. WHAT DOES ‘IS’ MEAN

‘To be or not to be’, ‘Cogito, ergo sum’, ‘What is ‘is’?’. Three statements about the nature of being that may or may not have been formulated by Hamlet, Descartes and Heidegger. Still they direct our attention to reflecting and discussing the most used word in sentences, to be.

In his book ‘Being and Time’, ‘Sein und Zeit’ in the original German version, Heidegger writes:

> Do we in our time have an answer to the question of what we really mean by the word ‘being’? Not at all. So it is fitting that we should raise anew the question of the meaning of Being. (..) Our aim in the following treatise is to work out the question of the meaning of Being and to do so concretely. (Heidegger 1962, p. 1)

Going back in time, Heidegger says that the question ‘provided a stimulus for the researches of Plato and Aristotle only to subside from then on as a theme for actual investigation. (p. 2).’ Furthermore, Heidegger says, ‘(..) a dogma has been developed which not only declares the question about the meaning of Being to be superfluous, but sanctions its complete neglect. It is said that Being is the most universal and the emptiest of concepts. As such it resists every attempt at definition (p. 2).’

Heidegger sees this dogma based upon three presuppositions. As to seeing Being as the most universal concept, Heidegger writes ‘In medieval ontology Being is designated as a ‘transcendens’. Aristotle himself knew the unity of this transcendental ‘universal’ as a unity of analogy in contrast to the multiplicity of the highest generic concepts applicable to things (..) So if it is said that Being is the most universal concept, this cannot mean that it is the one which is clearest or that it needs no further discussion. It is rather the darkest of all (p. 3).’

As to seeing the concept of Being is indefinable Heidegger says that ‘Being cannot be derived from higher concepts by definition, nor can it be presented through lower ones (..) We can infer only that Being cannot have the character of an entity (..) The indefinability of Being does not eliminate the question of its meaning (p. 4).’

As to seeing Being as a concept that of all concepts is the one that is self-evident, Heidegger says ‘The very fact that we already live in understanding of Being and that the meaning of Being is still veiled in darkness proves that it is necessary in principle to raise this question again (p. 4).’

Heidegger concludes by saying that

> By Considering these prejudices, however, we have made plain not only that the question of Being lacks an answer, but that the question itself is obscure ad without direction. So if it is to be revived, this means that we must first work out an adequate way of formulating it (p. 4).

To do so, Heidegger says that ‘We must therefore explain briefly what belongs to any question whatsoever, so that from this standpoint the question of Being can be made visible as a very special one with its own distinctive character (p. 5).'

Then Heidegger addresses the nature of a general question aiming at establishing a definition of M by answering the question 'What is M?'. Heidegger assigns to a question the word inquiry and says that 'Every enquiry is a seeking. Every seeking gets guided beforehand by what is sought. Inquiry is a cognizant seeking for an entity both with regard to the fact that it is and with regard to its Being as it is (p. 5).'

Here Heidegger describes the two different uses of being, one that establishes existence, 'M is', and one that establishes 'how M is' to others, since what exists is perceived by humans that begin to categorize it by naming or characterizing or analogizing it, in all three cases using the word 'is'.

Heidegger points to four different uses of the word 'is'. 'Is' can claim a mere existence of M, 'M is'; and 'is' can assign predicates to M, 'M is N', but this can be done in three different ways. 'Is' can point down as a 'naming-is' ('M is for example N or P or Q or …') defining M as a common name for its volume of more concrete examples. 'Is' can point up as a 'judging-is' ('M is an example of N') defining M as member of a more abstract category N. Finally, is can point over as an 'analogizing-is' ('M is like N') portraying M by a metaphor carrying over known aspects from another N.

Heidegger stresses the double meaning of being, 'that M is & how M is' by saying 'Everything we talk about, everything we have in view, everything towards which we comport ourselves in any way, is being; what we are is being and so is how we are. Being lies in the fact that something is and in its Being as it is (p. 6-7).'

To separate that which is from how it is, Heidegger coins the word 'Dasein' by saying 'This entity which each of us is in himself and which includes inquiring as one of the possibilities of its Being, we shall denote by the term "Dasein" (p.7).'

So here Heidegger transforms the 'cogito ergo sum' into 'Ich bin da, und Ich frage' (I exist here and I question). By connecting the word 'da' to existence, Heidegger places existence in time and space since 'da' can mean both there and then. Also, Heidegger sees questioning as the most important ability of Dasein.

Within existentialist thinking, existence and essence are core concepts (Marino 2004). Here Heidegger says

> [Dasein's] Being-what-it-is (essentia) must, so far as we can speak of it at all, be conceived in terms of its Being (existentia). (..) To avoid getting bewildered, we shall always use the Interpretative expression "presence-at-hand" for the term "existentia", while the term "existence", as a designation of Being, will be allotted solely to Dasein. The essence of Dasein lies in its existence. (p. 42)

Here Heidegger reformulates his basic statement 'that M is and how M is' to 'by existing, M has existentia described (by Others) by essentia'; or 'existing, M exists together with presence-at-hand.'

To tell if the essentia of existentia, that is, the characteristics of presence-at-hand, is determined by the Others or by Dasein itself, Heidegger later introduces the concept 'ready-at-hand'

> Equipment can genuinely show itself only in dealings cut to its own measure (hammering with a hammer, for example) (..) In dealings such as this, where something is put to use, our concern subordinates itself to the "in-order-to" which

> is constitutive for the equipment we are employing at the time; the less we just stare at the hammer-Thing, and the more we seize hold of it and use it, the more primordial does our relationship to it become, and the more unveiledly is it encountered as that which it is - as equipment. (..) The kind of Being which equipment possesses - in which it manifests itself in its own right - we call "readiness to-hand". (p. 69)

As to existence, Heidegger talks about authentic an unauthentic existence.

> In each case Dasein is its possibility, and it 'has' this possibility, but not just as a property, as something present-at-hand would. And because Dasein is in each case essentially its own possibility, it can, in its very Being, 'choose' itself and win itself; it can also lose itself and never win itself; or only 'seem' to do so. But only in so far as it is essentially something which can be authentic - that is, something of its own - can it have lost itself and not yet won itself. As modes of Being, authenticity and inauthenticity (these expressions have been chosen terminologically in a strict sense) are both grounded in the fact that any Dasein whatsoever is characterized by mineness. (p. 42-43)

As to the Other, Heidegger talks about a dictatorship.

> We have shown earlier how in the environment which lies closest to us, the public 'environment' already is ready-to-hand and is also a matter of concern. In utilizing public means of transport and in making use of information services such as the newspaper, every Other is like the next. This Being-with-one-another dissolves one's own Dasein completely into the kind of Being of 'the Others', in such a way, indeed, that the Others, as distinguishable and explicit, vanish more and more. In this inconspicuousness and unascertainability, the real dictatorship of the "they" is unfolded. (p. 126)

As to describing the present-at-hand, Heidegger warns against gossip in the form of idle talk, 'Gerede' in German.

> Discourse, which belongs to the essential state of Dasein's Being and has a share in constituting Dasein's disclosedness, has the possibility of becoming idle talk. And when it does so, it serves not so much to keep Being-in-the-world open for us in an articulated understanding, as rather to close it off, and cover up the entities within-the-world. (..) Thus, by its very nature, idle talk is a closing-off, since to go back to the ground of what is talked about is something which it leaves undone. (..) Because of this, idle talk discourages any new inquiry and any disputation, and in a peculiar way suppresses them and holds them back. (p. 169)

## 3. THE HEIDEGGER UNIVERSE

Summing up, from a Heidegger viewpoint the question 'what is 'is'?' leads to two forms of being: that what is; and how it is. Which depends on how They see it: sentenced by a judging-is as an example of an above category, or accepted by a naming-is as a difference among other examples below, or facetted by an analogizing-is as artistically metaphorized by parallel examples.

By his two-fold statement 'that what is; and how it is', Heidegger suggests that an ordinary sentence as 'Peter destroys the apple' is in fact two sentences, on stating existence, 'Peter is', and one stating a judgement 'destroys the apple', that might be gossip since it can be questioned: Is Peter destroying the apple, or preparing it for food, or transforming it in an artistic process, or …?

As to existence statements, the language has seven basic is-statements: I am, you are, he/she is, it is, we are, you are, they are. Heidegger sees three of these as more basic, I am and it is and they are, describing the core of the meaning of being: I exist in a world together with Things and Others.

So, the core of a Heidegger universe is I and It and They. Or, using Heidegger's terms, Dasein is in a world together with Things and They; and to escape unauthenticity, Dasein must constantly question what is present-at-hand to set it free from its prison of ruling They-gossip, so it becomes ready-at-hand, allowing Dasein an authentic existence. Thus, Dasein should be sceptical towards the essence-claims produced by They using judging-is to trap existence in a predicate-prison. Instead, Dasein should ask the judged to open itself to allow alternative authentic terms to arise using naming-is and analogizing-is.

Traditionally, education means teaching learners about the outside world. Here Heidegger sees a learner as a Dasein having as possibility to transform the surrounding presence-at-hand to ready-at-hand; but being hindered by They, teaching presence-at-hand as examples of textbook gossip instead of arranging meetings allowing the transformation to take place.

As to mathematics education, Heidegger sees Dasein in a world with numbers as entities present-at-hand, but caught in essence-claims of idle talk called mathematics. So to establishing an authentic ready-at-hand relationship to them, Dasein must meet them directly and replace the gossip's judgment statements pointing up with naming statements pointing down.

However, numbers come in different forms. Buildings often carry roman numbers, and number plates carry Arabic numbers in two versions, an Eastern and a Western. Apparently, numbers are local gossip about something behind, to be seen in the first three Roman numbers, I and II and III, that is, about different degrees of 'Many'.

So, in the sentence 'here are three apples', three is not in the world by itself, apples are, as well as other units as oranges, chairs, days, hours etc. all having the form of plural to signal the presence of Many. Consequently, what is in the world is Many, and it is Many that Dasein should ask to open itself to establish an authentic relationship free of the restrictions of the gossip called mathematics.

## 4. MEETING MANY

As mammals, humans are equipped with two brains, one for routines and one for feelings. Standing up, we developed a third brain to keep balance and to store sounds assigned to what we grasped with our forelegs, freed to provide the holes in our head with our two basic needs, food for the body and information for the brain. The sounds developed into languages. In fact, we have two languages, a word-language and a number-language.

The word-language assigns words to things through sentences with a subject and a verb and an object or predicate, 'This is a chair'. Observing the existence of many chairs, we ask 'how many totally?' and use the number-language to assign numbers to like things. Again, we use sentences with a subject and a verb and an object or predicate, 'the total is 3 chairs' or, if counting legs, 'the total is 3 fours', abbreviated to 'T = 3 4s' or 'T = 3*4'.

Both languages have a meta-language, a grammar, describing the language, describing the world. Thus, the sentence 'this is a chair' leads to a meta-sentence ''is' is a verb'.

Likewise, the sentence ‘T = 3*4’ leads to a meta-sentence ‘’*’ is an operation’. And since the meta-language speaks about the language, the language should be taught and learned before the meta-language. Which is the case with the word-language, but not with the number-language.

With 2017 as the 500year anniversary for Luther’s 95 theses, we can choose to describe meeting Many in 12 theses.

1. Using a folding ruler we discover that digits are, not symbols as the alphabet, but sloppy writings of icons having in them as many sticks as they represent. (Thus, there are four sticks in the four icon, and five sticks in the five icon, etc. Transforming four ones to one fours allows counting with fours as a unit also.)

2. Using a cup for the bundles we discover that a total can be ‘cup-counted’ in three ways: the normal way or with an overload or with an underload. (Thus, a total of 5 can be counted in 2s as 2 bundles inside the bundle-cup and 1 unbundled single outside, or as 1 inside and 3 outside, or as 3 inside and ‘less 1’ outside; or, if using ‘cup-writing’ to report cup-counting, T = 5 = 2]1 2s = 1]3 2s = 3]-1 2s. Likewise, when counting in tens, T = 37 = 3]7 tens = 2]17 tens = 4]-3 tens. Finally, we discover that also bundles can be bundled, calling for an extra cup for the bundles of bundles: T = 7 = 3]1 2s = 1]1]1 2s. Using a decimal point instead of a bracket to separate the inside bundles from the outside unbundled singles, we discover that a natural number is a decimal number with a unit: T = 3]1 2s = 3.1 2s.)

3. Using recounting a total in the same unit by creating or removing overloads or underloads, we discover that cup-writing offers an alternative way to perform and write down operations. (Thus,

T = 65 + 27 = 6]5 + 2]7 = 8]12 = 9]2 = 92

T = 65 – 27 = 6]5 – 2]7 = 4]-2 = 3]8 = 38

T = 7* 48 = 7* 4]8 = 28]56 = 33]6 = 336

T = 336 /7 = 33]6 /7 = 28]56 /7 = 4]8 = 48)

4. Asking a calculator to predict a counting result, we discover that also operations are icons showing the three tasks involved in counting by bundling and stacking. (Thus, to count 7 in 3s we take away 3 many times iconized by an uphill stoke showing the broom wiping away the 3s. With 7/3 = 2.some, the calculator predicts that 3 can be taken away 2 times. To stack the 2 3s we use multiplication iconizing a lift, 2x3 or 2*3. To look for unbundled singles, we drag away the stack of 2 3s iconized by a horizontal trace: 7 – 2*3 = 1. Thus, by bundling and dragging away the stack, the calculator predicts that 7 = 2]1 3s = 2.1 3s. This prediction holds at a manual counting: I I I I I I I = III III I. Geometrically, placing the unbundled single next-to the stack of 2 3s makes it 0.1 3s, whereas counting it in 3s by placing it on-top of the stack makes it 1/3 3s, so 1/3 3s = 0.1 3s. Likewise when counting in tens, 1/ten tens = 0.1 tens. Using LEGO bricks to illustrate e.g. T = 3 4s, we discover that a block-number contains two numbers, a bundle-number 4 and a counting-number 3. As positive integers, bundle-numbers can be added and multiplied freely, but they can only be subtracted or divided if the result is a positive integer. As arbitrary decimal-numbers, counting-numbers have no restrictions as to operations. Only, to add counting-numbers, their bundle-number must be the same since it is the unit, T = 3*4 = 3 4s.)

5. Wanting to describe the three parts of a counting process, bundling and stacking and dragging away the stack, with unspecified numbers, we discover two formulas. (Thus,

the ‘recount formula’ T = (T/B)*B says that T/B times B can be taken away from T, as e.g. 8 = (8/2)*2 = 4*2 = 4 2s; and the ‘restack formula’ T = (T–B)+B says that T–B is left when B is taken away from T and placed next-to, as e.g. 8 = (8–2)+2 = 6+2. Here we discover the nature of formulas: formulas predict.)

6. Wanting to recount a total in a new unit, we discover that a calculator can predict the result when bundling and stacking and dragging away the stack. (Thus, asking T = 4 5s = ? 6s, the calculator predicts: First (4*5)/6 = 3.some; then (4*5) – (3*6) = 2; and finally T = 4 5s = 3.2 6s. Also, we discover that changing units is officially called proportionality or linearity, a core part of traditional mathematics in middle school and at the first year of university.)

7. Wanting to recount a total in tens, we discover that a calculator predicts the result directly by multiplication; only leaving out the unit and misplacing the decimal point. (Thus, asking T = 3 7s = ? tens, the calculator predicts: T = 21 = 2.1 tens. Geometrically it makes sense that increasing the width of the stack from 7 to ten means decreasing its height from 3 to 2.1 to keep the total unchanged.)

And wanting to recount a total from tens to icons, we discover that this again is an example of recounting to change the unit. (Thus, asking T = 3 tens = ? 7s. the calculator predicts: First 30/7 = 4.some; then 30 – (4*7) = 2; and finally T = 30 = 4.2 7s. Geometrically it again makes sense that decreasing the width means increasing the height to keep the total unchanged.)

8. Using the letter u for an unknown number, we can rewrite recounting from tens as 3 tens = ? 7s, as 30 = u*7 with the answer 30/7 = u, officially called to solve an equation; hereby discovering a natural way to do so: Move a number to the opposite side with the opposite sign. (Thus, the equation 8 = u + 2 describes restacking 8 by removing 2 to be placed next-to; predicted by the restack-formula as 8 = (8–2)+2. So, the equation 8 = u + 2 has the solution is 8–2 = u, again moving a number to the opposite side with the opposite sign.)

9. Once counted, totals can be added, but addition is ambiguous. (Thus, with two totals T1 = 2 3s and T2 = 4 5s, should they be added on-top or next-to each other? To add on-top they must be recounted to get the same unit, e.g. as T1 + T2 = 2 3s + 4 5s = 1.1 5s + 4 5s = 5.1 5s, thus using proportionality. To add next-to, the united total must be recounted in 8s: T1 + T2 = 2 3s + 4 5s = (2*3 + 4*5)/8 * 8 = 3.2 8s. So next-to addition geometrically means to add areas, and algebraically it means to combine multiplication and addition. Officially this is called integration, a core part of traditional mathematics in high school and at the first year of university.)

10. Also we discover that addition and other operations can be reversed. (Thus, in reversed addition, 8 = u+2, we ask: what is the number u that added to 2 gives 8, which is precisely the formal definition of u = 8–2. And in reversed multiplication, 8 = u*2, we ask: what is the number u that multiplied with 2 gives 8, which is precisely the formal definition of u = 8/2. Also we see that the equations u^3 = 20 and 3^u = 20 are the basis for defining the reverse operations root, the factor-finder, and logarithm, the factor-counter, as u = 3√20 and u = log3(20). In all cases we solve the equations by moving to the opposite side with the opposite sign. Reversing next-to addition we ask 2 3s + ? 5s = 3 8s or T1 + ? 5s = T. To get the answer u, from the terminal total T we remove the initial total T1 before we count the rest in 5s: u = (T–T1)/5 = ΔT/5. Combining subtraction and division in this way is called differentiation, the reverse operation to integration combining multiplication and addition.)

11. Observing that many physical quantities are 'double-counted' in two different units, kg and dollar, dollar and hour, meter and second, etc., we discover the existence of 'per-numbers' serving as a bridge between the two units. (Thus, with a bag of apples double-counted as 4$ and 5kg we get the per-number 4$/5kg or 4/5 $/kg. As to 20 kg, we just recount 20 in 5s and get T = 20kg = (20/5)*5kg = (20/5)*4$ = 16$. As to 60$, we just recount 60 in 4s and get T = 60$ = (60/4)*4$ = (60/4)*5kg = 75kg.)

12. Observing that a quantity may be double-counted in the same unit, we discover that per-numbers may take the form of fractions, 3 per 5 = 3/5, or percentages as 3 per hundred = 3/100 = 3%. (Thus, to find 3 per 5 of 20, 3/5 of 20, we just recount 20 in 5s and take that 3 times: 20 = (20/5)*5 = 4 5s, which taken 3 times gives 3*4 = 12, written shortly as 20 counted in 5s taken 3 times, 20/5*3. To find what 3 per 5 is per hundred, 3/5 = ?%, we just recount 100 in 5s, that many times we take 3: 100 = (100/5)*5 = 20 5s, and 3 taken 20 times is 60, written shortly as 3 taken 100-counted-in-5s times, 3*100/5. So 3 per 5 is the same as 60 per 100, or 3/5 = 60%. Also we observe that per-numbers and fractions are not numbers, but operators needing a number to become a number. Adding 3kg at 4$/kg and 5kg at 6$/kg, the unit-numbers 3 and 5 add directly but the per-numbers 4 and 6 add by their areas 3*4 and 5*6 giving the total 8 kg at (3*4+5*6)/8 $/kg. Likewise with adding fractions. Adding by areas means that adding per-numbers and adding fractions become integration as when adding block-numbers next-to each other. So calculus appears at all school levels: at primary, at lower and at upper secondary and at tertiary level.)

## 5. CONCLUSION

To answer the questions 'what is mathematics, education and research' we looked for an answer in a Heidegger universe by allowing the root of mathematics, the physical fact Many, to open itself for us. This disclosed a 'many-matics' with digits as icons containing as many sticks as they represent; and where counting and recounting and double-counting totals come before adding them next-to and on-top, thus creating a natural order for the four basic operations, also being icons present in the counting process: first division draws away bundles then multiplication lift them to a stack that subtraction takes away to look for unbundled singles. This shows that natural numbers are two-dimensional blocks with a counting-number and a bundle-number as a unit, and with a decimal point to separate the bundles from the unbundled. Once counted, blocks can be added where next-to addition means adding areas, also called integration; and where on-top addition means recounting in the same unit to remove or create overloads. And where reversed addition next-to and on-top leads to differentiation and equations. Double-counting in different units leads to per-numbers being added or calculated in calculus, present in primary school as adding blocks, and in middle and high school, as adding piecewise and locally constant per-numbers. Finally, letters and functions are used for unspecified numbers and calculations.

Many-matics differs in many respects from traditional mathematics; that presents digits as symbols and numbers as names for points along a one-dimensional number-line; that neglects counting and recounting and double-counting and next-to addition and goes directly to on-top addition first, then subtraction, then multiplication and in the end division leading on to fractions that by being added without units becomes an example of 'mathe-matism' true inside but seldom outside classrooms: ½ + 2/3 is claimed to be 7/6 in spite of the fact that 1 red of 2 apples plus 2 red of 3 apples total 3 red of 5 apples and certainly not 7 red of 6 apples. Being set-based, definitions use self-referring judging-is statements from above instead of naming-is statements from below, thus defining a concept as ‘meta-matics’, that is, as an example of an abstraction instead of as an abstraction from examples, as it was created historically. Thus a function is defined as an example of a set-relation where first-component identity implies second-component identity, instead of as a placeholder for an unspecified calculation with unspecified numbers. A closer look thus discloses traditional set-based mathematics as ‘meta-matism’, a mixture of meta-matics and mathe-matism.

Meta-matism as ‘2+3 = 5’ adding numbers without units contradicts observations as 2weeks + 3 days = 17 days. And it makes a syntax error in the number-language sentence ‘T = 2+3’ by silencing the subject and the verb. By keeping the gossip part and leaving out the existence part, meta-matism ceases to be a number-language describing the real world. This contradicts the historic origin of mathematics as a common label chosen by the Pythagoreans for their fours knowledge areas: arithmetic, geometry, music and astronomy, seen by the Greeks as knowledge about pure numbers, number in space, number in time, and number in space and time. The four combined in the quadrivium, a general curriculum recommended by Plato. So, with music and astronomy gone, today mathematics should be but a common label for algebra and geometry, both activities rooted in the physical fact Many.

In Greek, geometry means earth measuring, which is done by dividing earth into triangles. In Arabic, algebra means to reunite numbers. Writing out a total T as we say it, T = 345 = 3*ten*ten + 4*ten + 5*1, shows a number as blocks united next-to each other. Also, we see algebra’s four ways to unite numbers: addition, multiplication, repeated multiplication or power, and block-addition also called integration. Which is precisely the core of mathematics: addition and multiplication together with their reversed operations subtraction and division in primary school; and power and integration together with their reversed operations root, logarithm and differentiation in secondary school. Including the units, we see there can only be four ways to unite numbers: addition and multiplication unite variable and constant unit numbers, and integration and power unite variable and constant per-numbers.

As to traditional set-based mathematics, its idea of deriving definitions from the mother concept set leads to meaningless self-reference as in the classical liar paradox ‘This sentence is false’, being true if false and false if true. This was shown by Russell looking at the set of sets not belonging to itself. Here a set belongs to the set if it doesn’t, and does not belong if it does.

To avoid self-reference, Russell created a hierarchical type theory in which fractions could not be numbers if defined by numbers, e.g. as equivalence classes in a set of number-pairs as done by set-based mathematics that consequently invented a new set-theory that by mixing sets and elements also mixes concrete examples and their abstract names, thus mixing concrete apples that can feed humans and the word ‘apple’ that cannot. By mixing things and their names, existence and gossip, set-based mathematics and its meta-matism fill the number-language with both semantic and syntax errors. Still, this language has entered universities worldwide as the only true version of mathematics to be transmitted through education that is improved using research to produce solid findings.

In a Heidegger universe, education means allowing I to meet It directly without They and its patronizing gossip; and to replace judging-is with naming-is when choosing how to label It. Likewise with research seen as a collective education replacing ungrounded categories with grounded ones.

So, maybe the answer to the question about solid findings in mathematics education research is ‘Only one: to improve, mathematics education should ask, not what to do, but what to do differently.’ Maybe research should not study problems but look for hidden differences that make a difference.

However, difference research scarcely exists today since it is rejected at conferences (Tarp 2015) for not applying or extending existing theory that might produce new researchers and feed a growing appliance industry, but being unable to reach its goal, to improve mathematics education.

In short, to be successful, mathematics education research must stop studying the misery coming from teaching meta-matism in compulsory classes. Instead, mathematics must respect its origin as a natural science grounded in Many. And research must search for differences and test if they make a difference, not in compulsory classes, but with daily lessons in self-chosen half-year blocks. Then learning the word-language and the number-language together may not be that difficult, so that all will leave school literate and numerate and use the two languages to discuss how to treat nature and its human population in a civilized way.

Inspired by Heidegger, an existentialist would say: In a sentence, the subject exists, but the sentence about it may be gossip; so stop preaching essence and start teaching existence; or, bring the subject to the classroom and leave the sentence outside.

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