Twelve Proposals for 1day Skype Seminars

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01) The Root of Mathematics, Many, dealt with by Block-Numbers, Cup-Counting and Preschool Calculus.

”How old next time?” I asked the child. The answer was four with four fingers shown. But held together two by two created a protest: “That is not four, that is two twos!”. That opened my eyes. Children come to school with two-dimensional block-numbers where all numbers have units. Instead, school teaches cardinality as a one-dimensional line with different number-names; thus disregarding the fact that numbers are two- dimensional blocks all having a unit as shown when writing out fully a total T = 345 = 3 BundleBundles + 4 Bundles + 5 Singles = 3*10^2 + 4*10 + 5*1. So, a number is blocks united (integrated) next-to each other, showing the four ways to unite numbers presented by Algebra, meaning reuniting in Arabic: Power and multiplication and ‘on-top’ and ‘next-to’ addition (integration).
Consequently, mathematics education should develop the two-dimensional block-numbers that children bring to school and allow them to practice counting before adding.

To master Many, we ask ‘how many?’ To answer, we cup-count using a cup for the bundles. So, a number always has some bundles inside and some unbundled outside the cup.
Recounting in the same unit creates overloads or underloads by moving in or out of the cup, T = 5 = 2]1 2s = 1]3 2s = 3]-1 2s. This makes calculation easy: T = 4 x 56 = 4 x 5]6 = 20]24 = 22]4 = 224.
Once counted and recounted, totals may de added. To have 2 3s and 4 5s added on-top as 5s, a unit must be changed, called proportionality. To add them next-to as 8s means adding their areas, called integration; which becomes differentiation when reversed by saying 2 3s + ? 5s = 3 8s’, thus allowing calculus to take place in preschool.

02) 12 Luther-like Theses about how ManyMath can Improve Math Education

1. Digits are icons with as many sticks as they represent.

2. A total T can be ‘cup-counted’ in the normal way or with an overload or underload: T = 5 = 2]1 2s = 1]3 2s = 3]-1 2s.

3. ‘Cup-writing’ makes operations easy: T = 336 /7 = 33]6 /7 = 28]56 /7 = 4]8 = 48.

4. Counting T by bundling, T = (T/B)xB = (5/2)x2 = 2.1 2s, shows a natural number as a decimal number with a unit.

5. Operations are icons showing counting by bundling and stacking. -2 takes away 2. /2 takes away 2s. x2 stacks 2s. +2 adds 2 on-top or next-to.

6. A calculator predicts. Asking T = 4 5s = ? 6s, first (4x5)/6 = 3.some; then (4x5) – (3x6) = 2. So T = 4 5s = 3.2 6s

7. Recounting in tens, calculators leave out the unit and misplace the decimal point: T = 3 7s = 3*7 = 21 = 2.1 tens.

8. Recounting from tens, '? 7s = 3 tens', or 'u*7 = 30 = (30/7)x7', the answer u = 30/7 is found by 'move to opposite side with opposite sign'.

9. Adding totals is ambiguous: OnTop using proportionality, or NextTo using integration?

10. Operations are reversed with reverse operations: With u+3 = 8, u = 8–3; with ux3 = 8, u = 8/3; with u^3 = 8, u = 3√8; with 3^u = 8, u = log3(8); with T1 + u*3 = T2, u = ΔT/3.

11. Double-counting in different units gives 'per-numbers' as 4$/5kg, bridging the two units by recounting: T = 20kg = (20/5)*5kg = (20/5)*4$ = 16$

12. Double-counting in the same unit, per-numbers become fractions as operators, needing a number to become a number, thus adding by their areas as integration.

03) Curing Math Dislike with one Cup and five Sticks

A class is stuck in division and gives up on 234/5. Having heard about '1cup & 5 sticks', the teacher says 'Time out. Next week, no division. Instead we do cup-counting'. Teacher: 'How many sticks?' Class: '5.' Teacher: 'Correct, 5 1s, how many 2s?' Class: '2 2s and 1 left over'. Teacher: 'Correct, we count by bundling. The cup is for bundles, so we put 2 inside the cup and leave 1 outside. With 1 inside, how many outside? And with 3 inside, how many outside?' Class: '1 inside and 3 outside; and 3 inside and 1 lacking outside.' Teacher: 'Correct. A total of 5 sticks can be counted in 3 ways. The normal way with 2 inside and 1 outside. With overload as 1 inside and 3 outside. With underload as 3 inside and less 1 outside.' Class: 'OK'. Teacher. 'Now 37 means 3 inside and 7 unbundled 1s outside. Try recounting 37 with overload and underload. Class: '2 inside and 17 outside; and 4 inside and less 3 outside.'

Teacher: 'Now let us multiply 37 by 2, how much inside and outside?' Class: 6 inside and 14 outside. Or 7 inside and 4 outside. Or 8 inside and less 6 outside.' Teacher: 'Now to divide 74 by 3 we recount 7 inside and 4 outside to 6 inside and 14 outside. Dividing by 3 we get 2 inside and 4 outside; plus 2 leftovers that still must be divided by 3. So 74/3 gives 24 and 2/3.' Class: 'So to divide 234 by 5 we recount 234 as 20 inside and 34 outside. Dividing by 5 we get 4 inside and 6 outside; plus and 4 leftovers that still must be divided by 5. Thus 234/5 gives 46 and 4/5?' Teacher: 'Precisely. Now try multiplication using cup-counting'.

04) DoubleCounting rooting Proportionality - and Fractions and Percentages as PerNumbers

A class is stuck in fractions and percentages and gives up on ¾ = 75%. Having heard about 'per-numbers', the teacher says: Time out. Next week, no fractions, no percentage. Instead we do double-counting. First counting: 42 is how many 7s? The total T = 42 = (42/7)*7 = 6*7 = 6 7s. Then double-counting: Apples double-counted as 3 $ and 4 kg have the per-number 3$ per 4 kg, or 3$/4kg or ¾ $/kg. Asking how many $ for 10kg, we recount 10 in 4s, that many times we have 3$: The total T = 10kg = (10/4)*4kg = (10/4)*5$ = 12.5$. Asking how many kg for 18$, we recount 18 in 5s, that many times we have 4kg: The total T = 18$ = (18/5)*5$ = (18/5)*4kg = 14.4kg. Double-counting in the same unit gives fractions and percentages as 3 per 4, ¾; and 75 per hundred, 75/100 = 75%.

¾ of 200$ means finding 3$ per 4$, so we recount 200 in 4s, that many times we have 3$: The total T = 200$= (200/4)*4$ gives (200/4)*3$ = 150$. 60% of 250$ means finding 60$ per 100$, so we recount 250 in 100s, that many times we have 60$: The total T = 250$= (250/100)*100$ gives (250/100)*60$ = 150$.

To find 120$ in percent of 250$, we introduce a currency # with the per-number 100# per 250$, and then recount 120 in 250s, that many times we have 100#: The total T = 120$ = (120/250)*250$ = (120/250)*100# = 48#. So 120$/250$ = 48#/100# = 48%. To find the end-result of 300$ increasing with 12%, the currency # has the per-number 100# per 300$. 12# increases 100# to 112# that transforms to $ by the per-number. The total T = 112# = (112/100)*100# = (112/100)*300$ = 336$.

05) Algebraic Fractions made easy by Block-Numbers with Units

A class is stuck in algebraic fractions insisting that (2b+4)/2b is 4. Having heard about 'Block-Numbers with units, the teacher says: 'Time out. Next week, no algebraic fractions. Instead we count totals with units.' Teacher, showing six sticks: 'How many sticks?' Class: '6.' Teacher: 'Correct, 6 *1s*, how many *2s*?' Class: '3 *2s'*. Teacher: 'Correct, we count in 2s by taking away 2s, that is by dividing by 2, so T = 6 = (6/2) 2s = 3 *2s* =

3*2. So, factorizing 2b as 2*b, 2b is 2 *bs* or b *2s*. Can 4 be written with a unit?' Class: '4 is 2 *2s'*. Teacher: 'Correct, so 2b and 4 can be written as b 2s and 2 2s totalling b+2 2s or (b+2)*2.' Class: 'OK'. Teacher: 'Now, 6 *2s* divided by 3 *2s* gives 6 divided by 3 or 2. And c *2s* divided by 3 *2s* gives c divided by 3.' Class: 'OK'. Teacher: 'So, b+2 *2s* divided by b *2s* gives b+2 divided by b.' Class: 'OK, and that gives 2?' Teacher: 'Well, division means removing a common unit. So, with b as b *1s* and 2 as 2 *1s* we can remove the *1s*. But b+2 *1s* divided by b *1s* still gives b+2 divided by b, which is the result.' Class: 'OK'. Teacher: 'Now try (3c+9)/6c.' Class: 'We factorize to find a common unit 3: 3c is c *3s*, 9 is 3 *3s*, and 6c is 2x3xc or 2c *3s*. Removing the common unit we get (3c+9)/6c = (c+3)/2c.' Teacher: 'Correct. Now try (b^2c+ bd^3)/bc'. Class: 'We factorize to find a common unit b: b^2c is bxbxc or bc *bs* , bd^3 is d^3 *bs*, and bc is c *bs*. Removing the common unit, we get (b^2c+ bd^3)/bc = (bc + d^3)/c.'

06) Algebra and Geometry, always Together, never Apart

The ancient Greeks used mathematics as a common label for their four knowledge areas, arithmetic, geometry, music and astronomy, seen as many by itself, many in space, many in time and many in space and time. With music and astronomy gone, mathematics was a common label for algebra and geometry until the arrival of the 'New Math' that insisted that geometry must go and that algebra should be defined from above as examples of sets instead of from below as abstractions from examples. Looking at the set of sets not belonging to themselves, Russell showed that set-reference means self-reference as in the classical liar paradox 'this sentence is false' being true if false and vice versa. Still, the new set-based 'meta-matics' entered universities and schools as the only true mathematics; except for the US going 'back to basics', that by separating algebra and geometry crates learning problems that disappear if they are kept together as advocated by Descartes. Thus, in primary school, numbers should be two dimensional LEGO-blocks as 2 3s. And 3x6 should be a block of 3 6s that if recounted in tens must widen its width and shorten its height, so that 3 6s becomes 1.8 tens. And in secondary school bxc should mean b cs; and fractions should be operators needing a number to become a number thus by multiplication becoming areas that are added by integration. Likewise, Euclidean geometry should be introduced in a coordinate system allowing equations to predict the exact position of intersection points of lines in triangles before being constructed with ruler and compasses. And the quadratic equation x^2+bx+c=0 geometrically tells that since x^2+bx = -c, the four parts of a (x+b/2) square reduce to (b/2)^2-c = D, allowing x to be found easily as x = -b/2 ±√D.

07) Calculus in Middle School and High School

A class is stuck in differential calculus and gives up on d/dx(x^2) = 2x. Having heard about 'per-numbers', the teacher says 'Time out. Next week, no differentiation. Instead we go back to middle school and look at per-numbers.' Class: 'Per-numbers, what is that?' Teacher: 'Per-numbers are for example meter per second, dollar per kilo, or dollar per hour. Here is an example: What is the total of 2 kg at 3 $/kg + 4 kg at 5$/kg?'. Class: 'The kg-numbers add to 6, but how do we add per-numbers?' Teacher: 'Can we change $/kg-numbers to $-numbers?'. Class: 'We can multiply 2 and 3 to 6$, and 4 and 5 to 20$ that add up to 26$. But multiplication means adding areas?' Teacher: 'Precisely. Adding per-numbers by their areas is called integral calculus, also called finding the area under the per-number-graph.'

Class: 'But what if the per-number graph is not constant? Then there are too many strips to add!' Teacher: 'We use a trick. Adding 1000 numbers is difficult, but adding 1000 differences is easy since the middle numbers cancel out, so we are left with the difference between the end and the start number.' Class: 'But how can we write area-strips as differences?' Teacher: 'Well, if p is the per-number, then the area-strip with width dx is close to p*dx; but it is also the difference between the end area A2 and the start area A1, so p*dx = A2-A1 = dA, or p = d/dx(A).' Class: 'But that is differentiation?' Teacher: 'Precisely, so if we know that d/dx(x^2) = 2x, then we know that the area under the 2x graph is A2-A1 with A = x^2. So to find a quick way to area-formulas we need to learn to differentiate.' Class: 'OK.'

08) Mathematics, the Grammar of the Number-Language. But why teach Grammar before Language?

Humans have two languages, a word-language and a number-language, assigning words and numbers to things through sentences with a subject and a verb and an object or predicate, 'This is a chair' and '3 chairs have a total of 3x4 legs', abbreviated to 'T = 3x4'.

Both languages have a meta-language, a grammar, that describes the language that describes the world. Thus, the sentence 'this is a chair' leads to a meta-sentence ''is' is a verb'. Likewise, the sentence 'T = 3x4' leads to a meta-sentence ''x' is an operation'.

We master outside phenomena through actions, so learning a word-language means learning actions as how to listen, to read, to write and to speak. Likewise, learning the number-language means learning actions as how to count and to add. We cannot learn how to math, since math is not an action word, it is a label, as is grammar. Thus, mathematics can be seen as the grammar of the number-language.

Since grammar speaks about language, language should be taught and learned before grammar. This is the case with the word-language, but not with the number-language.

Saying 'the number-language is an application of mathematics' implies that then 'of course mathematics must be taught and learned before it can be applied'. However, this corresponds to saying that the word-language is an application of its grammar that therefore must be taught and learned before it can be applied. Which, if implemented, would create widespread illiteracy, as with the present widespread innumeracy resulting from teaching grammar before language in the number-language.

Instead school should follow the word-language and use full sentences 'The total is 3 4s' or 'T = 3x4'. By saying '3x4' only, school removes both the subject and the verb from number-language sentence, thus depriving it of its language nature.

09) Quantitative Literature also has three Genres: Fact and Fiction and Fiddle

Humans communicate in languages: A word language with sentences assigning words to things and actions. And a number language with equations assigning numbers or calculations to things and actions. 'Word stories' come in three genres: Fact, fiction and fiddle. Fact/fiction are stories about factual/fictional things and actions. Fiddle is nonsense like 'This sentence is false'. 'Number stories' are often called mathematical models. They come in the same three genres.

Fact models can be called a 'since-then' models or 'room' models. Fact models quantify quantities and predict predictable quantities: "What is the area of the walls in this room?". Since the model's prediction is what is observed, fact models can be trusted. Algebra's four basic uniting models are fact models: T = a+b, T = axb, T = a^b and T = ∫y dx; as are many models from basic science and economy.

Fiction models can be called 'if-then' models or 'rate' models. Fiction models quantify quantities but predict unpredictable quantities: "My debt is gone in 5 years at this rate!". Fiction models are based upon assumptions and produce fictional numbers to be supplemented with parallel scenarios based upon alternative assumptions. Models from statistics calculating averages assuming variables to be constant are fiction models; as are models from economic theory showing nice demand and supply curves.

Fiddle models can be called 'then-what' models or 'risk' models. Fiddle models quantify qualities that cannot be quantified: "Is the risk of this road high enough to cost a bridge?" Fiddle models should be rejected asking for a word description instead of a number description. Many risk-models are fiddle models: The basic risk model says: Risk = Consequence x Probability. It has meaning in insurance but not when quantifying casualties where it is cheaper to stay in a cemetery than at a hospital.

10) Distance Teacher Education in Mathematics by the CATS method: Count & Add in Time & Space

The MATHeCADEMY.net teaches teachers teach mathematics as 'many-math', a natural science about Many. It is a virus academy saying: To learn mathematics, don't ask the instructor, ask Many. To deal with Many, we Count and Add in Time and Space. The material is question-based.

Primary School. COUNT: How to count Many? How to recount 8 in 3s? How to recount 6kg in $ with 2$ per 4kg? How to count in standard bundles? ADD: How to add stacks concretely? How to add stacks abstractly? TIME: How can counting & adding be reversed? How many 3s plus 2 gives 14? Can all operations be reversed? SPACE: How to count plane and spatial properties of stacks and boxes and round objects?

Secondary School. COUNT: How can we count possibilities? How can we predict unpredictable numbers? ADD: What is a prime number? What is a per-number? How to add per-numbers? TIME: How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable? SPACE: How to predict the position of points and lines? How to use the new calculation technology? Quantitative Literature, what is that? Does it also have the 3 different genres: fact, fiction and fiddle?

PYRAMIDeDUCATION organizes 8 teachers in 2 teams of 4 choosing 3 pairs and 2 instructors by turn.

The instructors instruct the rest of their team. Each pair works together to solve count&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation.

The instructors correct the count&add assignments. In a pair, each teacher corrects the other teacher's routine-assignment. Each pair is the opponent on the essay of another pair.

11) 50 years of Sterile Mathematics Education Research, Why?

PISA scores are still low after 50 years of research. But how can mathematics education research be successful when its three words are not that well defined? Mathematics has meant different things in its 5000 years of history, spanning from a natural science about Many to a self-referring logic.

Within education, two different forms exist at the secondary and tertiary level. In Europe, education serves the nation's need for public servants through multi-year compulsory classes and lines. In North America, education aims at uncovering and developing the individual talent through daily lessons in self-chosen half-year blocks with one-subject teachers.

As to research, academic articles can be at a master level exemplifying existing theories, or at a research level questioning them. Also, conflicting theories create problems as within education where Piaget and Vygotsky contradict each other by saying 'teach as little and as much as possible'.

Consequently, we cannot know what kind of mathematics and what kind of education has been studied, and if research is following traditions or searching for new discoveries. So to answer the question 'How to improve mathematics education research', first we must make the three words well defined by asking: What is meant by mathematics, and by education, and by research? Answers will be provided by the German philosopher Heidegger, asking 'what is 'is'?'

It turns out that, instead of mathematics, schools teaches 'meta-matism' combining 'meta-matics', defining concepts from above as examples of abstractions instead of from below as abstractions from examples; and 'mathe-matism' true inside but seldom outside class, such as adding fractions without units, where 1 red of 2 apples plus 2 red of 3 gives 3 red of 5 and not 7 red of 6 as in the textbook teaching ½+2/3 = 7/6.

So, instead of meta-matism, teach 'many-math' in self-chosen half-year blocks.

12) Difference-Research, a more Successful Research Paradigm?

Despite 50 years of research, many PISA studies show a continuing decline. Maybe, it is time for difference-research searching for hidden differences that make a difference:

1. The tradition teaches cardinality as one-dimensional line-numbers to be added without being counted first. A difference is to teach counting before adding to allow proportionality and integral calculus and solving equations in early childhood: cup-counting in icon-bundles less than ten, recounting in the same and in a different unit, recounting to and from tens, calculator prediction, and finally, forward and reversed on-top and next-to addition.

2. The tradition teaches the counting sequence as natural numbers. A difference is natural numbers with a unit and a decimal point or cup to separate inside bundles from outside singles; allowing a total to be written in three forms: normal, overload and underload: T = 5 = 2.1 2s = 2]1 2s = 1]3 2s = 3]-1 2s.

3. The tradition uses carrying. A difference is to use cup-writing and recounting in the same unit to remove overloads: T = 7x 48 = 7x 4]8 = 28]56 = 33]6 = 336. Likewise with division: T = 336 /7 = 33]6 /7 = 28]56 /7 = 4]8 = 48

4. Traditionally, multiplication is learned by heart. A difference is to combine algebra and geometry by seeing 5x6 as a stack of 5 6s that recounted in tens increases its width and decreases its height to keep the total unchanged.

5. The tradition teaches proportionality abstractly. A difference is to introduce double-counting creating per-number 3$ per 4kg bridging the units by recounting the known number: T = 10kg = (10/4)x4kg = (10/4)x5$ = 12.5$. Double-counting in the same unit transforms per-numbers to fractions and percentages as 3$ per 4$ = ¾; and 75kg per 100kg = 75/100 = 75%.