The Simplicity of Mathematics Designing a STEM-based Core Math Curriculum for Outsiders and Migrants

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Swedish educational shortages challenge traditional mathematics education offered to migrants. Mathematics could be taught in its simplicity instead of as 'meta-matsim', a mixture of 'meta-matics' defining concepts as examples of inside abstractions instead of as abstractions from outside examples; and 'mathe-matism' true inside classrooms but seldom outside as when adding numbers without units. Rebuilt as 'many-matics' from its outside root, Many, mathematics unveils its simplicity to be taught in a STEM context at a 2year course providing a background as pre-teacher or pre-engineer for young male migrants wanting to help rebuilding their original countries.

Decreased PISA Performance Despite Increased Research

Being highly useful to the outside world, mathematics is a core part of institutionalized education. Consequently, research in mathematics education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 year since 1969. Likewise, funding has increased as seen e.g. by the creation of a National Center for Mathematics Education in Sweden. However, despite increased research and funding, the former model country Sweden has seen its PISA result decrease from 2003 to 2012, causing OECD to write the report 'Improving Schools in Sweden' describing its school system as 'in need of urgent change':

> PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life. (OECD, 2015a, p. 3).

Other countries also experience declining PISA results. Since mathematics education is a social institution, social theory might be able to explain 50 years of unsuccessful research in mathematics education.

Social Theory Looking at Mathematics Education

Imagination as the core of sociology is described by Mills (1959); and by Negt (2016) using the term to recommend an alternative exemplary education for outsiders, originally for workers, but today also applicable for migrants.

As to the importance of sociological imagination, Bauman agrees by saying that sociological thinking 'renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now' (p. 16). A wish to uncover unnoticed alternatives motivates a 'difference-research' (Tarp, 2017) asking two questions: 'Can this be different – and will the difference make a difference?' If things work there is no need to ask for differences. But with problems, difference-research might provide a difference making a difference.

Natural sciences use difference-research to keep on searching until finding what cannot be different. Describing matter in space and time by weight, length and time intervals, they all seem to vary. However, including per-numbers will uncover physical constants as the speed of light, the gravitational constant, etc. The formulas of physics are supposed to predict nature's behavior. They cannot be proved as can mathematical formulas, instead they are tested as to falsifiability: Does nature behave different from predicted by the formula? If not, the formula stays valid until falsified.

Social sciences also use difference-research beginning with the ancient Greek controversy between two attitudes towards knowledge, called 'sophy' in Greek. To avoid hidden patronization, the sophists warned: Know the difference between nature and choice to uncover choice presented as nature. To their counterpart, the philosophers, choice was an illusion since the physical was but examples of metaphysical forms only visible to them, educated at the Plato academy. The Christian church transformed the academies into monasteries but kept the idea of a metaphysical patronization by replacing the forms with a Lord deciding world behavior.

Today's democracies implement common social goals through institutions with means decided by parliaments. As to rationality as the base for social organizations, Bauman says:

> **Max Weber,** one of the founders of sociology, saw the proliferation of organizations in contemporary society as a sign of the continuous rationalization of social life. **Rational** action (..) is one in which the *end* to be achieved is clearly spelled out, and the actors concentrate their thoughts and efforts on selecting such *means* to the end as promise to be most effective and economical. (..) the ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called *goal displacement*. (..) The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right. (Bauman, 1990, pp. 79, 84)

As an institution, mathematics education is a public organization with a 'rational action in which the end to be achieved is clearly spelled out', apparently aiming at educating students in mathematics, 'The goal of mathematics education is to teach mathematics'. However, by its self-reference such a goal is meaningless, indicating a goal displacement. So, if mathematics isn't the goal in mathematics education, what is? And, how well defined is mathematics after all?

In ancient Greece, the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: arithmetic, geometry, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many by itself, Many in space, Many in time and Many in space and time. And together forming the 'quadrivium' recommended by Plato as a general curriculum together with 'trivium' consisting of grammar, logic and rhetoric.

With astronomy and music as independent knowledge areas, today mathematics is a common label for the two remaining activities, geometry and algebra, both rooted in the physical fact Many through their original meanings, 'to measure earth' in Greek and 'to reunite' in Arabic. And in Europe, Germanic countries taught counting and reckoning in primary school and arithmetic and geometry in the lower secondary school until about 50 years ago when they all were replaced by the 'New Mathematics'.

Here the invention of the concept SET created a Set-based 'meta-matics' as a collection of 'well-proven' statements about 'well-defined' concepts. However, 'well-defined' meant defining by self-reference, i.e. defining top-down as examples of abstractions instead of bottom-up as abstractions from examples. And by looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false:

If $M = \{A \mid A \notin A)\}$ then $M \in M \Leftrightarrow M \notin M$.

The Zermelo–Fraenkel Set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract concepts. In this way, SET transformed grounded mathematics into today's self-referring 'meta-matism', a mixture of meta-matics and 'mathe-matism' true inside but seldom outside classrooms where adding numbers without units as '1 + 2 IS 3' meet counter-examples as e.g. 1 week + 2 days is 9 days.

So, mathematics has meant many different things during its more than 5000 years of history. But in the end, isn't mathematics just a name for knowledge about shapes and numbers and operations? We all teach 3*8 = 24, isn't that mathematics?

The problem is two-fold. We silence that 3*8 is 3 8s, or 2.6 9s, or 2.4 tens depending on what bundle-size we choose when counting. Also we silence that, which is 3*8, the total. By silencing the subject of the sentence 'The total is 3 8s' we treat the predicate, 3 8s, as if it was the subject, which is a clear indication of a goal displacement.

So, the goal of mathematics education is to learn, not mathematics, but to deal with totals, or, in other words, to master Many. The means are numbers, operations and calculations. However, numbers come in different forms. Buildings often carry roman numbers; and on cars, number-plates carry Arabic numbers in two versions, an Eastern and a Western. And, being sloppy by leaving out the unit and misplacing the decimal point when writing 24 instead of 2.4 tens, might speed up writing but might also slow down learning, together with insisting that addition precedes subtraction and multiplication and division if the opposite order is more natural. Finally, in Lincolns Gettysburg address, 'Four scores and ten years ago' shows that not all count in tens.

To get an answer to the questions 'What is mathematics?' and 'How is mathematics education improved?' we might include philosophy in the form of what Bauman calls 'the second Copernican revolution' of Heidegger asking the question: What is 'is'? (Bauman, 1992, p. ix).

> Inquiry is a cognizant seeking for an entity both with regard to the fact that it is and with regard to its Being as it is. (Heidegger, 1962, p. 5)

Heidegger here describes two uses of 'is'. One claims existence, 'M is', one claims 'how M is' to others, since what exists is perceived by humans wording it by naming it and by characterizing or analogizing it to create 'M is N'-statements.

Thus, there are four different uses of the word 'is'. 'Is' can claim a mere existence of M, 'M is'; and 'is' can assign predicates to M, 'M is N', but this can be done in three different ways. 'Is' can point down as a 'naming-is' ('M is for example N or P or Q or …') defining M as a common name for its volume of more concrete examples. 'Is' can point up as a 'judging-is' ('M is an example of N') defining M as member of a more abstract category N. Finally, 'is' can point over as an 'analogizing-is' ('M is like N') portraying M by a metaphor carrying over known characteristics from another N.

Heidegger sees three of our seven basic is-statements as describing the core of Being: ‘I am’ and ‘it is’ and ‘they are’; or, I exist in a world together with It and with They, with Things and with Others. To have real existence, the ‘I’ (Dasein) must create an authentic relationship to the ‘It’. However, this is made difficult by the ‘dictatorship’ of the ‘They’, shutting the ‘It’ up in a predicate-prison of idle talk, gossip.

> This Being-with-one-another dissolves one’s own Dasein completely into the kind of Being of ‘the Others’, in such a way, indeed, that the Others, as distinguishable and explicit, vanish more and more. In this inconspicuousness and unascertainability, the real dictatorship of the “they” is unfolded. (..) Discourse, which belongs to the essential state of Dasein’s Being and has a share in constituting Dasein’s disclosedness, has the possibility of becoming idle talk. And when it does so, it serves not so much to keep Being-in-the-world open for us in an articulated understanding, as rather to close it off, and cover up the entities within-the-world. (Heidegger, 1962, pp. 126, 169)

In France, Heidegger inspired the poststructuralist thinking of Derrida, Lyotard, Foucault and Bourdieu, pointing out that society forces words upon you to diagnose you so it can offer cures including one you cannot refuse, education, that forces words upon the things around you, thus forcing you into an unauthentic relationship to yourself and your world (Lyotard, 1984. Bourdieu, 1970. Chomsky et al, 2006).

From a Heidegger view a sentence contains two things: a subject that exists, and the rest that might be gossip. So, to discover its true nature hidden by the gossip of traditional mathematics, we need to meet the subject, the total, outside its predicate-prison. We need to allow Many to open itself for us, so that, as curriculum architects, sociological imagination may allow us to construct a core mathematics curriculum based upon exemplary situations of Many in a STEM context, seen as having a positive effect on learners with a non-standard background (Han et al, 2014), aiming at providing a background as pre-teachers or pre-engineers for young male migrants wanting to help rebuilding their original countries.

So, to restore its authenticity, we now return to the original Greek meaning of mathematics as knowledge about Many by itself and in time and space; and use Grounded Theory (Glaser et al, 1967), lifting Piagetian knowledge acquisition (Piaget, 1969) from a personal to a social level, to allow Many create its own categories and properties.

Meeting Many

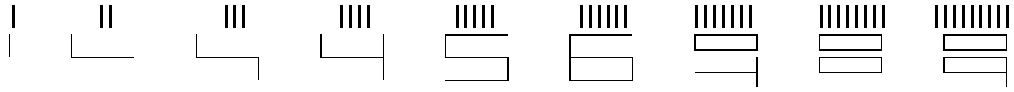
As mammals, humans are equipped with two brains, one for routines and one for feelings. Standing up, we developed a third brain to keep the balance and to store sounds assigned to what we grasped with our forelegs, now freed to provide the holes in our head with our two basic needs, food for the body and information for the brain. The sounds developed into two languages, a word-language and a number-language.

The word-language assigns words to things through sentences with a subject and a verb and an object or predicate, ‘This is a chair’. Observing the existence of many chairs, we ask ‘how many in total?’ and use the number-language to assign numbers to like things. Again, we use sentences with a subject and a verb and an object or predicate, ‘the total is 3 chairs’ or, if counting legs, ‘the total is 3 fours’, abbreviated to ‘T = 3 4s’ or ‘T = 3*4’.

Both languages have a meta-language, a grammar, describing the language, describing the world. Thus, the sentence ‘this is a chair’ leads to a meta-sentence ‘’is’ is a verb’. Likewise, the sentence ‘T = 3*4’ leads to a meta-sentence ‘’*’ is an operation’. And since the meta-language speaks about the language, the language should be taught and learned before the meta-language. Which is the case with the word-language, but not with the number-language.

With 2017 as the 500year anniversary for Luther’s 95 theses, we can choose to describe meeting Many in theses.

01. Using a folding ruler, we discover that digits are, not symbols as the alphabet, but sloppy writings of icons having in them as many sticks as they represent. Thus, there are four sticks in the four icon, and five sticks in the five icon, etc. Counting in 5s, the counting sequence is 1, 2, 3, 4, Bundle, 1-bundle-1, etc. This shows, that the bundle-number does not need an icon. Likewise, when bundling in tens. Instead of ten-1 and ten-2 we use the Viking numbers eleven and twelve meaning 1 left and 2 left in Danish.



1 2 3 4 5 6 7 8 9

02. Transforming four ones to a bundle of 1 4s allows counting with bundles as a unit. Using a cup for the bundles, a total can be ‘cup-counted’ in three ways: the normal way or with an overload or with an underload. Thus, a total of 5 can be counted in 2s as 2 bundles inside the bundle-cup and 1 unbundled single outside, or as 1 inside and 3 outside, or as 3 inside and ‘less 1’ outside; or, if using ‘cup-writing’ to report cup-counting, T = 5 = 2]1 2s = 1]3 2s = 3]-1 2s. Likewise, when counting in tens, T = 37 = 3]7 tens = 2]17 tens = 4]-3 tens. Using a decimal point instead of a bracket to separate the inside bundles from the outside unbundled singles, we discover that a natural number is a decimal number with a unit: T = 3]1 2s = 3.1 2s. Next, we discover that also bundles can be bundled, calling for an extra cup for the bundles of bundles: T = 7 = 3]1 2s = 1]1]1 2s. Or, with tens: T = 234 = 23]4 = 2]3]4.

03. Recounting in the same unit by creating or removing overloads or underloads, cup-writing offers an alternative way to perform and write down operations.

T = 65 + 27 = 6]5 + 2]7 = 8]12 = 9]2 = 92

T = 65 – 27 = 6]5 – 2]7 = 4]-2 = 3]8 = 38

T = 7* 48 = 7* 4]8 = 28]56 = 33]6 = 336

T = 7* 48 = 7* 5]-2 = 35]-14 = 33]6 = 336

T = 336 /7 = 33]6 /7 = 28]56 /7 = 4]8 = 48

T = 338 /7 = 33]8 /7 = 28]58 /7 = 4]8 + 2/7= 48 2/7

04. Asking a calculator to predict a counting result, we discover that also operations are icons showing the three tasks involved when counting by bundling and stacking. Thus, to count 7 in 3s we take away 3 many times iconized by an uphill stoke showing the broom wiping away the 3s. With 7/3 = 2.some, the calculator predicts that 3 can be taken away 2 times. To stack the 2 3s we use multiplication, iconizing a lift, 2x3 or 2*3. To look for unbundled singles, we drag away the stack of 2 3s iconized by a horizontal trace: 7 – 2*3 = 1. Thus, by bundling and dragging away the stack, the calculator

predicts that 7 = 2]1 3s = 2.1 3s. This prediction holds at a manual counting: I I I I I I I = III III I. Geometrically, placing the unbundled single next-to the stack of 2 3s makes it 0.1 3s, whereas counting it in 3s by placing it on-top of the stack makes it 1/3 3s, so 1/3 3s = 0.1 3s. Likewise when counting in tens, 1/ten tens = 0.1 tens. Using LEGO bricks to illustrate T = 3 4s, we discover that a block-number contains two numbers, a bundle-number 4 and a counting-number 3. As positive integers, bundle-numbers can be added and multiplied freely, but they can only be subtracted or divided if the result is a positive integer. As arbitrary decimal-numbers, counting-numbers have no restrictions as to operations. Only, to add counting-numbers, their bundle-number must be the same since it is the unit, T = 3*4 = 3 4s.

05. Wanting to describe the three parts of a counting process, bundling and stacking and dragging away the stack, with unspecified numbers, we discover two formulas. The 'recount formula' T = (T/B)*B says that 'from T, T/B times B can be taken away' as e.g. 8 = (8/2)*2 = 4*2 = 4 2s; and the 'restack formula' T = (T–B)+B says that from T, T–B is left when B is taken away and placed next-to, as e.g. 8 = (8–2)+2 = 6+2. Here we discover the nature of formulas: formulas predict. The recount or proportionality formula turns out to a very basic formula. It turns up in proportionality as $ = ($/kg)*kg when shifting physical units, in trigonometry as a = (a/c)*c = sinA*c when counting sides in diagonals in right-angled triangles, and in calculus as dy = (dy/dx)*dx = y'*dx when counting steepness on a curve.

06. Wanting to recount a total in a new unit, we discover that a calculator can predict the result when bundling and stacking and dragging away the stack. Thus, asking T = 4 5s = ? 6s, the calculator predicts: First (4*5)/6 = 3.some; then (4*5) – (3*6) = 2; and finally T = 4 5s = 3.2 6s. Also we discover that changing units is officially called proportionality or linearity, a core part of traditional mathematics in middle school and at the first year of university.

07. Wanting to recount a total in tens, we discover that a calculator predicts the result directly by multiplication, only leaving out the unit and misplacing the decimal point. Thus, asking T = 3 7s = ? tens, the calculator predicts: T = 21 = 2.1 tens. Geometrically it makes sense that increasing the width of the stack from 7 to ten means decreasing its height from 3 to 2.1 to keep the total unchanged. With 5 as half of ten, and 8 as ten less 2, a 10x10 multiplication table can be reduced to a 3x3 table including the numbers 2, 3 and 4. Thus, 4*8 = 4*(ten less 2) = 4ten less 8 = 32; 5*8 = half of 8ten = 4ten = 40; 7*8 = (ten less 3)*(ten less 2) = tenten, less 3ten, less 2ten, plus 6 = 56.

Wanting to recount a total from tens to icons, we discover this as another example of recounting to change the unit. Thus, asking T = 3 tens = ? 7s, the calculator predicts: First 30/7 = 4.some; then 30 – (4*7) = 2; and finally T = 30 = 4.2 7s. Geometrically it again makes sense that decreasing the width means increasing the height to keep the total unchanged.

08. Using the letter u for an unknown number, we can rewrite the recounting question '? 7s = 3tens' as 'u*7 = 30' with the answer 30/7 = u since 30 = (30/7)*7, officially called to solve an equation. Here we discover a natural way to do so: Move a number to the opposite side with the opposite calculation sign. Thus, the equation 8 = u

+ 2 describes restacking 8 by removing 2 to be placed next-to, predicted by the restack-formula as 8 = (8–2)+2. So, the equation 8 = u + 2 has the solution is 8–2 = u, obtained again by moving a number to the opposite side with the opposite calculation sign.

09. Once counted, totals can be added, but addition is ambiguous. Thus, with two totals T1 = 2 3s and T2 = 4 5s, should they be added on-top or next-to each other? To add on-top they must be recounted to have the same unit, e.g. as T1 + T2 = 2 3s + 4 5s = 1.1 5s + 4 5s = 5.1 5s, thus using proportionality. To add next-to, the united total must be recounted in 8s: T1 + T2 = 2 3s + 4 5s = (2*3 + 4*5)/8 * 8 = 3.2 8s. So next-to addition geometrically means adding areas, and algebraically it means combining multiplication and addition. Officially, this is called integration, a core part of traditional mathematics in high school and at the first year of university.

10. Also we discover that addition and other operations can be reversed. Thus, in reversed addition, 8 = u+2, we ask: what is the number u that added to 2 gives 8, which is precisely the formal definition of u = 8–2. And in reversed multiplication, 8 = u*2, we ask: what is the number u that multiplied with 2 gives 8, which is precisely the formal definition of u = 8/2. Also we see that the equations u^3 = 20 and 3^u = 20 are the basis for defining the reverse operations root, the factor-finder, and logarithm, the factor-counter, as u = 3√20 and u = log3(20). Again we solve the equation by moving to the opposite side with the opposite calculation sign. Reversing next-to addition we ask 2 3s + ? 5s = 3 8s or T1 + ? 5s = T. To get the answer u, from the terminal total T we remove the initial total T1 before we count the rest in 5s: u = (T–T1)/5 = ∆T/5, using ∆ for the difference or change. Letting subtraction precede division is called differentiation, the reverse operation to integration letting multiplication precede addition.

11. Observing that many physical quantities are ‘double-counted’ in two different units, kg and dollar, dollar and hour, meter and second, etc., we discover the existence of ‘per-numbers’ serving as a bridge between the two units. Thus, with a bag of apples double-counted as 4$ and 5kg we get the per-number 4$/5kg or 4/5 $/kg. As to 20 kg, we just recount 20 in 5s and get T = 20kg = (20/5)*5kg = (20/5)*4$ = 16$. As to 60$, we just recount 60 in 4s and get T = 60$ = (60/4)*4$ = (60/4)*5kg = 75kg.

12. Economy is based upon investing money and expecting a return that might be higher or lower than the investment, e.g. 7$ per 5$ or 3$ per 5$. Here when double-counting in the same unit, per-numbers become fractions, 3 per 5 = 3/5, or percentages as 3 per hundred = 3/100 = 3%. Thus, to find 3 per 5 of 20, or 3/5 of 20, as before we just recount 20 in 5s and replace 5 with 3, T = 20 = (20/5)*5 giving (20/5)*3 = 12.

To find what 3 per 5 is per hundred, 3/5 = ?%, we just recount 100 in 5s and replace 5 with 3: T = 100 = (100/5)*5 giving (100/5)*3 = 60. So 3 per 5 is the same as 60 per 100, or 3/5 = 60%. Also we observe that per-numbers and fractions are not numbers, but operators needing a number to become a number. Adding 3kg at 4$/kg and 5kg at 6$/kg, the unit-numbers 3 and 5 add directly, but the per-numbers 4 and 6 add by their areas 3*4 and 5*6 giving the total 8 kg at (3*4+5*6)/8 $/kg. Likewise when adding fractions. Adding by areas means that adding per-numbers and adding fractions become integration as when adding block-numbers next-to each other. So calculus appears at all school levels: at primary, at lower and at upper secondary and at tertiary level.

13. Halved by its diagonal, a rectangle splits into two right-angled triangles. Here the angles are labeled A and B and C at the right angle. The opposite sides are labeled a and b and c.

The height a and the base b can be counted in meters, or in diagonals c creating a sine-formula and a cosine-formula: a = (a/c)*c = sinA*c, and b = (b/c)*c = cosA*c. Likewise, the height can be recounted in bases, creating a tangent-formula: a = (a/b)*b = tanA*b

As to the angles, with a full turn as 360 degrees, the angle between the horizontal and vertical directions is 90 degrees. Consequently, the angles between the diagonal and the vertical and horizontal direction add up to 90 degrees; and the three angles add up to 180 degrees.

An angle A can be counted by a protractor, or found by a formula. Thus, in a right-angled triangle with base 4 and diagonal 5, the angle A is found from the formula cosA = a/c = 4/5 as cos-1(4/5) = 36.9 degrees.

The three sides have outside squares with areas a^2 and b^2 and c^2. Turning a right triangle so that the diagonal is horizontal, a vertical line from the angle C split the square c^2 into two rectangles. The rectangle under the angle A has the area (b*cosA)*c = b*(cosA*c) = b*b = b^2. Likewise, the rectangle under the angle B has the area (a*cosB)*c = a*(cosB*c) = a*a = a^2. Consequently c^2 = a^2 + b^2, called the Pythagoras formula.

This allows finding a square-root geometrically, e.g. x = √24, solving the quadratic equations x^2 = 24 = 4*6, if transformed into a rectangle. On a protractor, the diameter 9.5 cm becomes the base AB, so we have 6units per 9.5cm. Recounting 4 in 6s, we get 4units = (4/6)*6units = (4/6)*9.5 cm = 6.33 cm. A vertical line from this point D intersects the protractor’s half-circle in the point C. Now, with a 4x6 rectangle under BD, BC will be the square-root √24, measured to 4.9, which checks: 4.9^2 = 24.0.

A triangle that is not right-angled transforms into a rectangle by outside right-angled triangles, thus allowing its sides and angles and area to be found indirectly. So, as in right-angled triangles, any triangle has the property that the angles add up to 180 degrees and that the area is half of the height times the base.

Inside a circle with radius 1, the two diagonals of a 4sided square together with the horizontal and vertical diameters through the center form angles of 180/4 degrees. Thus the circumference of the square is 2*(4*sin(180/4)), or 2*(8*sin(180/8)) with 8 sides instead. Consequently, the circumference of a circle with radius 1 is 2*π, where π = n*sin(180/n) for n large.

14. A coordinate system coordinates algebra with geometry where a point is reached by a number of horizontally and vertically steps called the point’s x- and y-coordinates.

Two points A(xo,yo) and B(x,y) with different x- and y-numbers will form a right-angled change-triangle with a horizontal side Δx = x-xo and a vertical side Δy = y-yo and a diagonal distance r from A to B, where by Pythagoras r^2 = Δx^2 + Δy^2. The angle A is found by the formula tanA = Δy/Δx = s, called the slope or gradient for the line from A to B. This gives a formula for a non-vertical line: Δy/Δx = s or Δy = s*Δx, or y-yo = s*(x-xo). Vertical lines have the formula x = xo since all points share the same x-number.

In a coordinate system three points A(x1,y1) and B(x2,y2)and C(x3,y3) not on a line will form a triangle that packs into a rectangle by outside right-angled triangles allowing indirectly to find the angles and the sides and the area of the original triangle.

Different lines exist inside a triangle: Three altitudes measure the height of the triangle depending on which side is chosen as the base; three medians connect an angle with the middle of the opposite side; three angle bisectors bisect the angles; three line bisectors bisect the sides and are turned 90 degrees from the side. Likewise, a triangle has two circles; an outside circle with its center at the intersection point of the line bisectors, and an inside circle with its center at the intersection point of the angle bisectors.

Since Δx and Δy changes place when turning a line 90 degrees, their slopes will be Δy/Δx and -Δx/Δy respectively, so that s1*s2 = -1, called reciprocal with opposite sign.

Geometrical intersection points are predicted algebraically by solving two equations with two unknowns, i.e. by inserting one into the other. Thus with the lines y = 2*x and y = 6-x, inserting the first into the second gives 2*x = 6-x, or 3*x = 6, or x = 2, which inserted in the first gives y = 2*2 = 4, thus predicting the intersection point to be (x,y) = (2,4). The same answer is found on a solver-app; or using software as GeoGebra.

Finding possible intersection points between a circle and a line or between two circles leads to a quadratic equation x^2 + b*x + c = 0, solved by a solver. Or by a formula created by two m-by-x playing cards on top of each other with the bottom left corner at the same place and the top card turned a quarter round clockwise. With k = m-x, this creates 4 areas combining to (x + k)^2 = x^2 + 2*k*x + k^2. With k = b/2 this becomes (x + b/2)^2 = x^2 + b*x + (b/2)^2 + c – c = (b/2)^2 – c since x^2 + b*x + c = 0. Consequently the solution formula is x = -b/2 ±√((b/2)^2 – c).

Finding a tangent to a circle at a point, its slope is the reciprocal with opposite sign of the radius line.

15. A formula predicts a total before counting it. A formula typically contains both specified and unspecified numbers in the form of letters, e.g. T = 5+3*x. A formula containing one unspecified number is called an equation, e.g. 26 = 5+3*x, to be solved by moving to opposite side with opposite calculation sign, (26-5)/3 = x. A formula containing two unspecified numbers is called a function, e.g. T = 5+3*x. An unspecified function containing an unspecified number x is labelled f(x), T = f(x). Thus f(2) is meaningless since 2 is not an unspecified number. Functions are described by a table or a graph in a coordinate system with y = T = f(x), both showing the y-numbers for different x-numbers. Thus, a change in x, Δx, will imply a change in y, Δy, creating a per-number Δy/Δx called the gradient.

16. In a function y = f(x), a small change x often implies a small change in y, thus both remaining ‘almost constant’ or ‘locally constant’, a concept formalized with an ‘epsilon-delta criterium’, distinguishing between three forms of constancy. y is

‘globally constant’ c if for all positive numbers epsilon, the difference between y and c is less than epsilon. And y is ‘piecewise constant’ c if an interval-width delta exists such that for all positive numbers epsilon, the difference between y and c is less than epsilon in this interval. Finally, y is ‘locally constant’ c if for all positive numbers epsilon, an interval-width delta exists such that the difference between y and c is less than epsilon in this interval. Likewise, the change ratio Δy/Δx can be globally, piecewise or locally constant, in the latter case written as dy/dx. Formally, local constancy and linearity is called continuity and differentiability.

17. As to change, a total can change in a predictable or unpredictable way; and predictable change can be constant or non-constant.

Constant change comes in several forms. In linear change, T = b + s*x, s is the constant change in y per change in x, called the slope or the gradient of its graph, a straight line. In exponential change, T = b*(1+r)^x, r is the constant change-percent in y per change in x, called the change rate. In power change, T = b*x^p, p is the constant change-percent in y per change-percent in x, called the elasticity. A saving increases from two sources, a constant $-amount per month, c, and a constant interest rate per month, r. After n months, the saving has reached the level C predicted by the formula C/c = R/r. Here the total interest rate after n months, R, comes from 1+R = (1+r)^n. Splitting the rate r = 100% in t parts, we get the Euler number e = (1+100%/t)^t = (1+1/t)^t if t is large.

Also the change can be constant changing. Thus in T = c + s*x, s might also change constantly as s = c + q*x so that T = b + (c + q*x)*x = b + c*x + q*x^2, called quadratic change, showing graphically as a line with a curvature, a parabola.

If not constant but still predictable, we have a change formula ΔT/Δx = f(x) or dT/dx = f(x) in the case of interval change or local change. Such an equation is called a differential equation which is solved by calculus, adding up all the local changes to a total change being the difference between the end and start number: T2-T1 = Σ ΔT = ∫ dT = ∫f(x)*dx. Thus, with dT/dx = 2*x, T2-T1 = Δ(x^2). Change formula come from observing that in a block, changes Δb and Δh in the base b and the height h impose on the total a change ΔT as the sum of a vertical strip Δb*h and a horizontal strip b*Δh and a corner Δb*Δh that can be neglected for small changes; thus d(b*h) = db*h + b*dh, or counted in T’s: dT/T = db/b + dh/h, or with T’ = dT/dx, T’/T = b’/b + h’/h. Therefore (x^2)’/x^2 = x’/x + x’/x = 2/x, giving (x^2)’ = 2*x since x’ = dx/dx = 1.

18. Unpredictable change can be exemplified by throwing a dice with two results: winning, +1, if showing 4 or above, and losing, 0, if showing 3 or below. Throwing a dice 5 times thus have 6 outcomes, winning from 0 to 5 times. The outcome is called an unpredictable or stochastic or random number or variable. Per definition, random numbers cannot be pre-dicted, instead they can be ‘post-dicted’ using statistics and probability.

Thus the outcome ‘0,0,0,1,1’ can be described by three numbers. The mode is 0 since this number has the highest frequency, 3 per 5, or 3/5. The median is 0 since this is the middle number when aligned in increasing order. The mean u is the fictional number had all numbers been the same: u*5 = 0+0+0+1+1 with the solution u = 2/5 =

0.4. With the outcome ‘0,0,1,1,1’, the mode and median and mean is 1 and 1 and 3/5 = 0.6.

To find the three numbers if the experiment is repeated many times we look at a ‘possibly tree’. The first toss has two results, win or lose, both occurring ½ of the times. Likewise with the following tosses: After two tosses we have three outcomes: 2 wins, 1 win and 0 wins. Here 2 wins and 0 wins occur half of half of the times, i.e. with a probability ¼. 1 win occurs twice, as win-lose or as lose-win, both with a probably of ¼, so the total probability for 1 win is 2*1/4 = ½. Continuing in this way we find that with 5 tosses there are 6 outcomes, winning from 0 to 5 times with the probabilities ½^5 a certain number of times: 1, 5, 10, 10, 5, 1. By calculations we find that the mode is 2 and 3, and that the median and the mean is 2.5, also found by multiplying the number of repetition with the probability for winning.

A spreadsheet random generator can show examples of other outcomes.

19. A sphere may be distorted into a cup. Even if distorted, a rectangle will still divide a sphere into an inside and an outside needing a bridge to be connected. And a sphere with a bridge may be distorted into a cup with a handle or into a donut. Distortion geometry is called topology, useful when setting up networks, thus able to prove that connecting three houses with water, gas and electricity is impossible without a bridge.

20. As qualitative literature, also quantitative literature has three genres, fact and fiction and ‘fiddle’, used when modeling real world situations. Fact is ‘since-then’ calculations using numbers and formulas to quantify and to predict predictable quantities as e.g. ‘since the base is 4 and the height is 5, then the area of the rectangle is T = 4*5 = 20’. Fact models can be trusted once the numbers and the formulas and the calculation has been checked. Special care must be shown with units to avoid adding meters and inches as in the case of the failure of the 1999 Mars-orbiter. Fiction is ‘if-then’ calculations using numbers and formulas to quantify and to predict unpredictable quantities as e.g. ‘if the unit-price is 4 and we buy 5, then the total cost is T = 4*5 = 20’. Fiction models build upon assumptions that must be complemented with scenarios based upon alternative assumptions before a choice is made. Fiddle models is ‘what-then’ models using numbers and formulas to quantify and to predict unpredictable qualities as e.g. ‘since a graveyard is cheaper than a hospital, then a bridge across the highway is too costly.’ Fiddle models should be rejected and relegated to a qualitative description.

Meeting Many in a STEM Context

Having met Many by itself, now we meet Many in time and space in the present culture based upon STEM, described by OECD as follows:

> The New Industrial Revolution affects the workforce in several ways. Ongoing innovation in renewable energy, nanotech, biotechnology, and most of all in information and communication technology will change labour markets worldwide. Especially medium-skilled workers run the risk of being replaced by computers doing their job more efficiently. This trend creates two challenges: employees performing tasks that are easily automated need to find work with tasks bringing other added value. And secondly, it propels people into a global competitive job market. (..) In developed economies, investment in STEM disciplines (science, technology, engineering and mathematics) is increasingly seen as a means to boost innovation and economic growth. The

importance of education in STEM disciplines is recognised in both the US and Europe. (OECD, 2015b)

STEM thus combines basic knowledge about how humans interact with nature to survive and prosper: Mathematics provides formulas predicting nature's physical and chemical behavior, and this knowledge, logos, allows humans to invent procedures, techne, and to engineer artificial hands and muscles and brains, i.e. tools, motors and computers, that combined to robots help transforming nature into human necessities.

A falling ball introduces nature's three main actors, matter and force and motion, similar to the three social actors, humans and will and obedience. As to matter, we observe three balls: the earth, the ball, and molecules in the air. Matter houses two forces, an electro-magnetic force keeping matter together when colliding, and gravity pumping motion in and out of matter when it moves in the same or in the opposite direction of the force. In the end, the ball is lying still on the ground. Is the motion gone? No, motion cannot disappear. Motion transfers through collisions, now present as increased motion in molecules; meaning that the motion has lost its order and can no longer be put to work. In technical terms: as to motion, its energy stays constant but its entropy increases. But, if the disorder increases, how is ordered life possible? Because, in the daytime the sun pumps in high-quality, low-disorder light-energy; and in the nighttime the space sucks out low-quality high-disorder heat-energy; if not, global warming would be the consequence.

Science is about nature itself. How three different Big Bangs, transforming motion into matter and anti-matter and vice versa, fill the universe with motion and matter interacting with forces making matter combine in galaxies, star systems and planets. Some planets have a size and a distance from its sun that allows water to exist in its three forms, solid and gas and liquid, bringing nutrition to green and grey cells, forming communities as plants and animals: reptiles, mammals and humans. Animals have a closed interior water cycle carrying nutrition to the cells and waste from the cells and kept circulating by the heart. Plants have an open exterior water cycle carrying nutrition to the cells and kept circulating by the sun forcing water to evaporate through leaves. Nitrates and carbon-dioxide and water is waste for grey cells, but food for green cells producing proteins and carbon-hydrates and oxygen as food for the grey cells in return.

Technology is about satisfying human needs. First by gathering and hunting, then by using knowledge about matter to create tools as artificial hands making agriculture possible. Later by using knowledge about motion to create motors as artificial muscles, combining with tools to machines making industry possible. And finally using knowledge about information to create computers as artificial brains combining with machines to artificial humans, robots, taking over routine jobs making high-level welfare societies possible.

Engineering is about constructing technology and power plants allowing electrons to supply machines and robots with their basic need for energy and information; and about how to build houses, roads, transportation means, etc.

Mathematics is our number-language allowing us to master Many by calculation sentences, formulas, expressing counting and adding processes. First Many is cup-counted in singles, bundles, bundles of bundles etc. to create a total T that might be recounted in the same or in a new unit or into or from tens; or double-counted in two units to create per-numbers and fractions. Once counted, totals can be added on-top if recounted in the same unit, or next-to by their areas, called integration, which is also how per-numbers and fractions add. Reversed addition is called solving equations. When totals vary, the change can be unpredictable or predictable with a change that might be constant or not. To master plane or spatial shapes, they are divided into right triangles seen as a rectangle halved by its diagonal, and where the base and the height and the diagonal can be recounted pairwise to create the per-numbers sine, cosine and tangent.

So, a core STEM curriculum could be about cycling water. Heating transforms it from solid to liquid to gas, i.e. from ice to water to steam; and cooling does the opposite. Heating an imaginary box of steam makes some molecules leave, so the lighter box is pushed up by gravity until becoming heavy water by cooling, now pulled down by gravity as rain in mountains and through rivers to the sea. On its way down, a dam can transform falling water to electricity. To get to the dam, we must build roads along the hillside.

In the sea, water contains salt. Meeting ice at the poles, water freezes but the salt stays in the water making it so heavy it is pulled down by gravity, elsewhere pushing warm water up thus creating cycles in the ocean pumping warm water to cold regions.

The two water-cycles fueled by the sun and run by gravity leads on to other STEM areas: to the trajectory of a ball pulled down by gravity; to an electrical circuit where electrons transport energy from a source to a consumer; to dissolving matter in water; and to building roads on hillsides.

A short World History

When humans left Africa, some went west to the European mountains, some went east where the fertile valleys in India supplied everything except for silver from the mountains. Consequently, rich trade took place sending pepper and silk west and silver east. European culture flourished around the silver mines, first in Greece then in Spain during the Roman Empire. Then the Vandal conquest of the mines brought the dark middle age to Europe until silver was found in the Harz valley (Tal in German leading to thaler and dollar), transported through Germany to Italy. Here silver financed the Italian Renaissance, going bankrupt when Portugal discovered a sea route to India enabling them to skip the cost of Arab middlemen. Spain looked for a sea route going west and found the West Indies. Here there was neither pepper nor silk but silver in abundance e.g. in the land of silver, Argentine. On their way home, slow Spanish ships were robbed by sailing experts, the Vikings descendants living in England, now forced to take the open sea to India to avoid the Portuguese fortification of Africa's coast.

In India, the English found cotton that they brought to their colonies in North America, but needing labor they started a triangle-trade exchanging US cotton for English weapon for African slaves for US cotton. In the agricultural South, a worker was a cost to be minimized, but in the industrial North a worker was a consumer needed at an industrial market. During the civil war, no cotton came to England that then conquered Africa to bring the plantations to the workers instead. Dividing the world in closed economies kept new industrial states out of the world market that it took two world wars to open for free competition.

Nature Obeys Laws, but from Above or from Below?

In the Lord's Prayer, the Christian Church says: 'Thy will be done, on earth as it is in heaven'. Newton had a different opinion.

As experts in sailing, the Viking descendants in England had no problem stealing Spanish silver on its way across the Atlantic Ocean. But to get to India to exchange it for pepper and silk, the Portuguese fortification of Africa's cost forced them to take the open sea and navigate by the moon. But how does the moon move? The church had one opinion, Newton meant differently.

'We believe, as is obvious for all, that the moon moves among the stars,' said the Church, opposed by Newton saying: 'No, I can prove that the moon falls to the earth as does the apple.' 'We believe that when moving, things follow the unpredictable metaphysical will of the Lord above whose will is done, on earth as it is in heaven,' said the Church, opposed by Newton saying: 'No, I can prove they follow their own physical will, a physical force, that is predictable because it follows a mathematical formula.' 'We believe, as Aristotle told us, that a force upholds a state,' said the Church, opposed by Newton saying: 'No, I can prove that a force changes a state. Multiplied with the time applied, the force's impulse changes the motion's momentum; and multiplied with the distance applied, the force's work changes the motion's energy.' 'We believe, as the Arabs have shown us, that to deal with formulas you just need ordinary algebra,' said the Church, opposed by Newton saying: 'No. I need to develop a new algebra of change which I will call calculus.'

Proving that nature obeys its own will and not that of a patronizor, Newton inspired the Enlightenment century realizing that if enlightened we don't need the double patronization of the physical Lord at the Manor house and the metaphysical Lord above. Citizens only need to inform themselves, debate and vote. Consequently, to enlighten the population, two Enlightenment republics were created, in the US in 1776 and in France in 1789. The US still have their first republic allowing its youth to uncover and develop their personal talent through daily lessons in self-chosen half-year blocks, whereas the Napoleon wars forced France and the rest of continental Europe to copy the Prussians line-organized education forcing teenagers to follow their year-group and its schedule, creating a knowledge nobility (Bourdieu, 1970) for public offices, and unskilled workers, good for yesterday's industrial society, but bad for today's information society where a birth rate at 1.5 child per family will halve the population each 50 years since (1.5/2)^2 = 0.5 approximately.

Counting and DoubleCounting Time, Space, Matter, Force and Energy

Counting time, the unit is seconds. A bundle of 60 seconds is called a minute; a bundle of 60 minutes is called an hour, and a bundle of 24 hours is called a day, of which a bundle of 7 is called a week. A year contains 365 or 366 days, and a month from 28 to 31 days.

Counting space, the international unit is meter, of which a bundle of 1000 is called a kilometer; and if split becomes a bundle of 1000 millimeters, 100 centimeters and 10 decimeters. Counting squares, the unit is 1 square-meter. Counting cubes, the unit is 1 cubic-meter, that is a bundle of 1000 cubic-decimeters, also called liters, that split up as a bundle of 1000 milliliters.

Counting matter, the international unit is gram that splits up into a bundle of 1000 milligrams and that unites in a bundle of 1000 to 1 kilogram, of which a bundle of 1000 is called 1 tons.

Counting force and energy, a force of 9.8 Newton will lift 1 kilogram, that will release an energy of 9.8 Joule when falling 1 meter.

Cutting up a stick in unequal lengths allows the pieces to be double-counted in liters and in kilograms giving a per-number around 0.7 kg/liter, also called the density.

A walk can be double-counted in meters and seconds giving a per-number at e.g. 3 meter/second, called the speed. When running, the speed might be around 10 meter/second. Since an hour is a bundle of 60 bundles of 60 seconds this would be 60*60 meters per hour or 3.6 kilometers per hour, or 3.6 km/h.

A pressure from a force applied to a surface can be double-counted in Newton and in square meters giving a per-number Newton per square-meter, also called Pascal.

Motion can be double-counted in Joules and seconds producing the per-number Joule/second called Watt. To run properly, a bulb needs 60 Watt, a human needs 110 Watt, and a kettle needs 2000 Watt, or 2 kiloWatt. From the Sun the Earth receives 1370 Watt per square meter.

Warming and Boiling water

In a water kettle, a double-counting can take place between the time elapsed and the energy used to warm the water to boiling, and to transform the water to steam.

Heating 1000 gram water 80 degrees in 167 seconds in a 2000 Watt kettle, the per-number will be 2000*167/80 Joule/degree, creating a double per-number 2000*167/80/1000 Joule/degree/gram or 4.18 Joule/degree/gram, called the specific heat of water.

Producing 100 gram steam in 113 seconds, the per-number is 2000*113/100 Joule/gram or 2260 J/g, called the heat of evaporation for water.

Letting Steam Work

A water molecule contains two Hydrogen and one Oxygen atom weighing 2*1+16 units. A collection of a million billion billion molecules is called a mole; a mole of water weighs 18 gram. Since the density of water is roughly 1000 gram/liter, the volume of 1000 moles is 18 liters. Transformed into steam, its volume increases to more than 22.4*1000 liters, or an increase factor of 22,400 liters per 18 liters = 1244 times. The volume should increase accordingly. But, if kept constant, instead the inside pressure will increase.

Inside a cylinder, the ideal gas law, p*V = n*R*T, combines the pressure, p, and the volume, V, with the number of moles, n, and the absolute temperature, T, which adds 273 degrees to the Celsius temperature. R is a constant depending on the units used. The formula expresses different proportionalities: The pressure is direct proportional with the number of moles and the absolute temperature so that doubling one means doubling the other also; and inverse proportional with the volume, so that doubling one means halving the other.

So, with a piston at the top of a cylinder with water, evaporation will make the piston move up, and vice versa down if steam is condensed back into water. This is used in steam engines. In the first generation, water in a cylinder was heated and cooled by turn. In the next generation, a closed cylinder had two holes on each side of an interior moving piston thus decreasing and increasing the pressure by letting steam in and out of the two holes. The leaving steam the is visible on steam locomotives. In the third generation used in power plants, two cylinders, a hot and a cold, connect with two tubes allowing water to circulate inside the cylinders. In the hot cylinder, heating increases the pressure by increasing both the temperature and the number of steam moles; and vice versa in the cold cylinder where cooling decreases the pressure by decreasing both the temperature and the number of steam moles condensed to water, pumped back to the hot cylinder in one of the tubes. In the other tube, the pressure difference makes blowing steam rotate a mill that rotates a magnet over a wire, which makes electrons move and carry electrical power to industries and homes.

An Electrical circuit

To work properly, a 2000 Watt water kettle needs 2000 Joule per second. The socket delivers 220 Volts, a per-number double-counting the number of Joules per charge-unit.

Recounting 2000 in 220 gives (2000/220)*220 = 9.1*220, so we need 9.1 charge-units per second, which is called the electrical current counted in Ampere.

To create this current, the kettle must have a resistance R according to a circuit law Volt = Resistance*Ampere, i.e., 220 = R*9.1, or Resistance = 24.2 Volt/Ampere called Ohm.

Since Watt = Joule per second = (Joule per charge-unit)*(charge-unit per second) we also have a second formula Watt = Volt*Ampere.

Thus, with a 60 Watt and a 120 Watt bulb, the latter needs twice the current, and consequently half the resistance of the former.

Supplied next-to each other from the same source, the combined resistance R must be decreased as shown by reciprocal addition, 1/R = 1/R1 + 1/R2. But supplied after each other, the resistances add directly, R = R1 + R2. Since the current is the same, the Watt-consumption is proportional to the Volt-delivery, again proportional to the resistance. So, the 120 Watt bulb only receives half of the energy of the 60 Watt bulb.

How high up and how far out

A ping-pong ball is send upwards. This allows a double-counting between the distance and the time to the top, 5 meters and 1 second. The gravity decreases the speed when going up and increases it when going down, called the acceleration, a per-number counting the change in speed per second.

To find its initial speed we turn the gun 45 degrees and count the number of vertical and horizontal meters to the top as well as the number of seconds it takes, 2.5 meters and 5 meters and 0,71 seconds. From a folding ruler we see, that now the speed is split into a vertical and a horizontal part, both reducing it with the same factor sin45 = cos45 = 0,707.

The vertical speed decreases to zero, but the horizontal speed stays constant. So we can find the initial speed by the formula: Horizontal distance to the top = horizontal speed * time, or with numbers: 5 = (u*0,707)*0,71, solved as u = 9.92 meter/seconds by moving to the opposite side with opposite calculation sign, or by a solver-app.

The vertical distance is halved, but the vertical speed changes from 9.92 to 9.92*0.707 = 7.01 only. However, the speed squared is halved from 9.92*9.92 = 98.4 to 7.01*7.01 = 49.2.

So horizontally, there is a proportionality between the distance and the speed. Whereas vertically, there is a proportionality between the distance and the speed squared, so that doubling the vertical speed will increase the distance four times.

How many turns on a steep hill

On a 30-degree hillside, a 10 degree road is constructed. How many turns will there be on a 1 km by 1 km hillside?

We let A and B label the ground corners of the hillside. C labels the point where a road from A meets the edge for the first time, and D is vertically below C on ground level. We want to find the distance BC = u.

In the triangle BCD, the angle B is 30 degrees, and BD = u*cos(30). With Pythagoras we get u^2 = CD^2 + BD^2 = CD^2 + u^2*cos(30)^2, or CD^2 = u^2(1-cos(30)^2) = u^2*sin(30)^2.

In the triangle ACD, the angle A is 10 degrees, and AD = AC*cos(10). With Pythagoras we get AC^2 = CD^2 + AD^2 = CD^2 + AC^2*cos(10)^2, or CD^2 = AC^2(1-cos(10)^2) = AC^2*sin(10)^2.

In the triangle ACB, AB = 1 and BC = u, so with Pythagoras we get AC^2 = 1^2 + u^2, or AC = √(1+u^2).

Consequently, u^2*sin(30)^2 = AC^2*sin(10)^2, or u = AC*sin(10)/sin(30) = AC*r, or u = √(1+u^2)*r, or u^2 = (1+u^2)*r^2, or u^2*(1-r^2) = r^2, or u^2 = r^2/(1-r^2) = 0.137, giving the distance BC = u = √0.137 = 0.37.

Thus, there will be 2 turns: 370 meter and 740 meter up the hillside.

Dissolving material in water

In the sea, salt is dissolved in water. The tradition describes the solution as the number of moles per liter. A mole of salt weighs 59 gram, so recounting 100 gram salt in moles we get 100 gram = (100/59)*59gram = (100/59)*1mole = 1.69 mole, that dissolved in 2.5 liter has a strength as 1.69 moles per 2.5 liters or 1.69/2.5 moles/liters, or 0.676 moles/liter.

The Simplicity of Mathematics

Meeting Many, we ask ‘How many in total?’ To answer, we count and add. To count means to use division, multiplication and subtraction to predict unit-numbers as blocks of stacked bundles, but also to recount to change unit, and to double-count to get per-numbers bridging the units, both rooting proportionality.

Adding thus means uniting unit-numbers and per-numbers, but both can be constant or variable, so to predict, we need four uniting operations: addition and multiplication unite variable and constant unit-numbers; and integration and power unite variable and constant per-numbers. As well as four splitting operations: subtraction and division split into variable and constant unit-numbers; and differentiation and root/logarithm split into variable and constant per-numbers. This resonates with the Arabic meaning of algebra, to reunite. And it appears in Arabic numbers written out fully as T = 456 = 4 bundles-of-bundles & 5 bundles & 6 unbundled, showing all four uniting operations, addition and multiplication and power and next-to addition of stacks; and showing that the word-language and the number-language share the same sentence form with a subject and a verb and a predicate or object.

Shapes can split into right-angled triangles, where the sides can be mutually recounted in three per-numbers, sine and cosine and tangent.

So, in principle, mathematics is simple and easy and quick to learn if institutionalized education wants to do so; however, to preserve and expand itself, the institution might want instead to hide the simplicity of mathematics by leaving out the subject and the verb in the number-language sentences; and by avoid counting to hide the block-nature of numbers as stacked bundles in order to impose linear place-value numbers instead; and by reversing the natural order of operations by letting addition precede subtraction, preceding multiplication, preceding division; and by hiding the double nature of addition by silencing next-to addition; and by silencing per-numbers and present fractions as numbers instead of operators needing numbers to become numbers; and by adding fractions without units to hide the true nature of integration as adding per-numbers by their areas; and by postponing trigonometry to after ordinary geometry and coordinate geometry; and by forcing equations to be solved by obeying the commutative and associative laws of abstract algebra; and by hiding that a function is but another name for a number-language sentence; and by forcing differential calculus to precede integral calculus.

Discussion: How does Traditional MatheMatics differ from ManyMatics

But in the end, how different is traditional mathematics from ManyMatics? As their base they have Set and Many, but isn’t that just two different words for the same? Not entirely. Many exists in the world, it is physical, whereas Set exists in a description, it is meta-physical. Thus, traditional mathematics defines its concepts top-down as examples, whereas ManyMatics defines its concepts bottom-up as abstractions. Still, the concepts might be the same, at least when taught? But a comparison uncovers several differences between the Set-derived tradition and its alternative grounded in Many.

The tradition sees the goal of mathematics education as teaching numbers and shapes and operations. In numbers, digits are symbols like letters, ordered according to a place value system, seldom renaming ‘234’ to ‘2tentens 3tens 4’. There are four kinds of numbers: natural and integers and rational and real. The natural numbers are defined by a successor principle making them one dimensional placed along a number line given the name ‘cardinality’. The integers are defined as equivalence classes in a set of ordered number-pairs where (a,b) is equivalent to (c,d) if a+d = b+c. Likewise, the rational numbers are defined by (a,b) being equivalent to (c,d) if a*d = b*c. Finally, the real numbers are defined as limits of number sequences.

The alternative sees the goal of mathematics education as teaching a number-language describing the physical fact Many by full sentences with the total as the subject, e.g. T = 2*3, thus having the same structure as the word-language, both having a language level describing the world, and a meta-language level describing the language. Digits are icons containing as many sticks as they represent if written less sloppy. Numbers occur when counting Many by bundling and stacking produces a block of bundles and unbundled, using cup- or decimal-writing to separate the inside bundles from the outside unbundled. The bundle-number, typically ten, does not need an icon since it is counted as ‘1 bundle’. Thus, a natural number is a decimal number with a unit, illustrated geometrically as a row of blocks containing the unbundled, the bundles, the bundle of bundles etc. Counting includes recounting in the same unit to create overload or underload, as well as recounting in another unit, especially in and from tens. Double-counting in different units gives per-numbers and fractions; however, these are not numbers but operators needing a number to become a number. A diagonal divides a block into two like right-angled triangles where the base and the altitude can be recounted in diagonals or in each other. Real numbers as √2 are calculations with as many decimals as needed, since a single can always be seen as a bundle of parts.

The tradition sees operations in a number set as mappings from a set-product into the set. Addition is the basic operation allowing number sets to be structured with an associative and a commutative and a distributive law as well as a neutral element and inverse elements. Addition is defined as repeating the successor principle, and multiplication is defined as repeated addition. Subtraction and division is defined as adding or multiplying inverse numbers. Standard algorithms for operations are introduced using carrying. Electronical calculators are not allowed when learning the four basic operations. The full ten-by-ten multiplication tables must be learned by heart.

The alternative sees operations as icons describing the counting process. Here division is an uphill stroke showing a broom wiping away the bundles; multiplication is a cross showing a lift stacking the bundles into a block, to be dragged away to look for unbundled singles, shown by a horizontal track called subtraction. Finally, addition is a cross showing that blocks can be juxtaposed next-to or on-top of each other. To add on-top, the blocks must be recounted in the same unit, thus grounding proportionality. Next-to addition means adding areas, thus grounding integration. Reversed adding on-top or next-to grounds equations and differentiation. A calculator is used to predict the result by two formulas, a recount-formula T = (T/B)*B, and a restack-formula T = (T-

B)+B. A multiplication table shows recounting from icons to tens, and is used when recounting from tens to icons introduces equations as reversed calculations. When recounting a total to or from tens, increasing the base means decreasing the altitude, and vice versa. As to multiplication, the commutative law says that the total stays unchanged when turning over a 3 by 4 block to a 4 by 3 block. The associative law says that the total stays unchanged when including or excluding a factor from the unit, T = 2*(3*4) = (2*3)*4. The distributive law says that before adding, recounting must provide a common unit to bracket out, T = 2 3s + 4 5s = 1.1 5s + 4 5s = (1.1 + 4) 5s.

The tradition sees fractions as rational numbers to which the four basic operations can be applied. Thus, fractions can be added without units by finding a common denominator after splitting the numerator and the denominator into prime factors. Fractions are introduced after division, and is followed by ratios and percentages and decimal numbers seen as examples of fractions.

The alternative sees fractions as per-numbers coming from double-counting in the same unit. As per-numbers, fractions are operators needing a number to become a number, thus added by areas, also called integration. Double-counting is introduced before addition. With factors as units, splitting a number in prime factors just means finding all possible units.

After working with number sets, the tradition introduces working with letter sets and polynomial sets to which the four basic operations can be applied once more observing that only like terms can be added, but not mentioning that this is because it means the unit is the same.

The alternative sees letters as units to bracket out during addition or subtraction, and that when multiplied or divided gives a composite unit.

The tradition sees an equation as an open statement expressing equivalence between two number-names containing an unknown variable. The statements are transformed by identical operations aiming at neutralizing the numbers next to the variable by applying the commutative and associative laws.

2*x = 8	an open statement
(2*x)*(1/2) = 8*(1/2)	½, the inverse element of 2, is multiplied to both names
(x*2)*(1/2) = 4	since multiplication is commutative
x*(2*(1/2)) = 4	since multiplication is associative
x*1 = 4	by definition of an inverse element
x = 4	by definition of a neutral element

As to the equation 2 + 3*x = 14, the same procedure as above is carried out twice, first with addition then with multiplication.

The alternative sees an equation as another name for a reversed calculation, to be reversed once more by recounting. Thus in the equation ‘2*x = 8’, recounting some 2s in 1s resulted in 8 1s, which recounted back into 2s gives 2*x = 8 = (8/2)*2, showing that x = 8/2 = 4. And also showing that an equation is solved by moving to the opposite side with opposite calculation sign, the opposite side & sign method.

The equation 2 + 3*x = 14, can be seen in two ways. As reversing a next-to addition of the two blocks, thus solved by differentiation, first removing the initial block and then recounting the rest in 3s: x = (14-2)/3 = 4. Or as a walk that multiplying by 3 and then adding by 2 gives 14,

x *(*3→)* 3*x *(+2→)* 3*x+2 = 14.

Reversing the walk by subtracting 2 and dividing by 3 gives the initial number:

x = 4 = (14-2)/3 *(←/3)* 14-2 *(←-2)* 14

The answer is tested by once more walking forward, 3*4 + 2 = 12 + 2 = 14.

The tradition sees a quadratic equation x^2 + b*x + c = 0 as a pure algebraic problem to be solved, first by factorizing, then by completing the square, and finally by using the solution formula.

The alternative sees solving a quadratic equation as a problem combining algebra and geometry, where a square with the sides x+b/2 creates fives areas, x^2 and b/2*x twice and c and (b^2/4-c) where the first four disappear and leaves (x+b/2)^2 to be the latter, b^2/4-c.

The tradition sees a function as an example of a relation between two sets where first-component identity implies second-component identity. And it gives the name 'linear function' to f(x) = a*x+b even if this is an affine function not satisfying the linear condition f(x+y) = f(x)*f(y), as does the proportionality formula f(x) = a*x.

The alternative sees a function as a name for a formula containing two unspecified numbers or variables, typically x and y. Thus, a function is a fiction showing how the y-numbers depends on the x numbers as shown in a table or by a graph.

The tradition sees proportionality as an example of a function satisfying the linear condition.

The alternative sees proportionality as a name for double-counting in different units creating per-numbers.

The tradition sees geometry to be introduced in the order: plane geometry, coordinate geometry and trigonometry.

The alternative has the opposite order. Trigonometry comes first grounded in the fact that halving a block by its diagonal allows the base and the altitude to be recounted in diagonals or in each other. This also allows a calculator to find pi from a sine formula. Next comes coordinate geometry allowing geometry and algebra to always go hand in hand so that algebraic formula can predict intersection points coming from geometrical constructions.

The tradition has quadratic functions following linear functions, both examples of polynomials.

The alternative sees affine functions as one example of constant change coming in five forms: constant y-change per x-change, constant y-percent-change per x-change, constant y-percent-change per x-percent-change, constant y-change per x-change together with constant y-percent-change per x-change, and finally constantly changing y- change.

The tradition sees logarithm as defined as the integral of the function y = 1/x.

The alternative sees logarithm and root combined both solving power equations. Thus a^x = b gives x = loga(b); and x^a = b gives x = a√b. This shows the logarithm as a factor-counter and the root as a factor-finder.

The tradition sees differential calculus as preceding integral calculus, and the gradient y' = dy/dx is defined algebraically as the limit of $\Delta y/\Delta x$ for Δx approaching 0, and geometrically as the slope of a tangent being the limit position of a secant with approaching intersection points. The limit is defined by an epsilon-delta criterium.

The alternative sees calculus as grounded in adding blocks next-to each other. In primary school calculus occurs when performing next-to addition of 2 3s and 4 5s as 8s. In middle school calculus occurs when adding piecewise constant per-numbers, as 2m at 3m/s plus 4m at 5m/s. In high school calculus occurs when adding locally constant per-numbers, as 5seconds at 3m/s changing constantly to 4m/s. Geometrically, adding blocks means adding areas under a per-number graph. In the case of local constancy this means adding many strips, made easy by writing them as differences since many differences add up to one single difference between the terminal and initial numbers, thus showing the relevance of differential calculus. The epsilon-delta criterium is a straight forward way to formalize the three ways of constancy, globally and piecewise and locally, by saying that constancy means an arbitrarily small difference.

Conclusion

With 50 years of research, mathematics education should have improved significantly. Its lack of success as illustrated by OECD report 'Improving Schools in Sweden' made this paper ask: Applying sociological imagination when meeting Many without having predicates forced upon it by traditional mathematics, can we design a STEM-based core math curriculum aimed at making migrants pre-teachers and pre-engineers in two years? This depends on what we mean by mathematics. And, looking back, mathematics has meant different things through its long history, from a common label for knowledge to today's 'meta-matism' combining 'meta-matics' defining concepts by meaningless self-reference, and 'mathe-matism' adding numbers without units thus lacking outside validity. So, inspired by Heidegger's 'always question sentences, except for its subject' we returned to the original Greek meaning of mathematics: Knowledge about Many by itself and in time and space.

Observing Many by itself allows rebuilding mathematics as a 'many-matics', i.e. as a natural science about the physical fact Many, where counting by bundling leads to block-numbers that recounted in other units leads to proportionality and solving equations; where recounting sides in triangles leads to trigonometry; where double-counting in different units leads to per-numbers and fractions, both adding by their areas, i.e. by integration; where counting precedes addition taking place both on-top and next-to involving proportionality and calculus; where using a calculator to predict the counting result leads to the opposite order of operations: division before multiplication before subtraction before next-to and on-top addition; and where calculus occurs in primary school as next-to addition, and in middle and high school as adding piecewise and locally constant per-numbers; and where integral calculus precedes differential calculus.

With water cycles fueled by the sun and run by gravity as exemplary situations, STEM offers various examples of Many in space and time since science and technology and engineering basically is about double-counting physical phenomena in different units.

The designed STEM-based core math curriculum has been tested in parts with success at the educational level in Danish pre-university classes. It might also be tested on a research level if it becomes known through publishing, i.e., if it will be accepted at the review process. It will offer a sociological imagination absent from traditional research seen by many teachers as useless because of its many references.

Questioning if traditional research is relevant to teachers, Hargreaves argues that

> What would come to an end is the frankly second-rate educational research which does not make a serious contribution to fundamental theory or knowledge; which is irrelevant to practice; which is uncoordinated with any preceding or follow-up research; and which clutters up academic journals that virtually nobody reads (Hargreaves, 1996, p. 7).

Here difference-research tries to be relevant by its very design: A difference must be a difference to something already existing in an educational reality used to collect reliable data and to test the validity of its findings by falsification attempts.

In a Swedish context, obsessive self-referencing has been called the 'irrelevance of the research industry' (Tarp, 2015, p. 31), noted also by Bauman as hindering research from being relevant:

> One of the most formidable obstacles lies in institutional inertia. Well established inside the academic world, sociology has developed a self-reproducing capacity that makes it immune to the criterion of relevance (insured against the consequences of its social irrelevance). Once you have learned the research methods, you can always get your academic degree so long as you stick to them and don't dare to deviate from the paths selected by the examiners (as Abraham Maslow caustically observed, science is a contraption that allows non-creative people to join in creative work). Sociology departments around the world may go on indefinitely awarding learned degrees and teaching jobs, self-reproducing and self-replenishing, just by going through routine motions of self-replication. The harder option, the courage required to put loyalty to human values above other, less risky loyalties, can be, thereby, at least for a foreseeable future, side-stepped or avoided. Or at least marginalized. Two of sociology's great fathers, with particularly sharpened ears for the courage-demanding requirements of their mission, Karl Marx and Georg Simmel, lived their lives outside the walls of the academia. The third, Max Weber, spent most of his academic life on leaves of absence. Were these mere coincidences? (Bauman, 2014, p. 38)

By pointing to institutional inertia as a sociological reason for the lack of research success in mathematics education, Bauman aligns with Foucault saying:

> It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (Chomsky et al., 2006, p. 41)

Bauman and Foucault thus both recommend skepticism towards social institutions where mathematics education and research are two examples. In theory, institutions are socially created as rational means to a common goal, but as Bauman points out, a goal displacement easily makes the institution have itself as the goal instead thus marginalizing or forgetting its original outside goal.

So, if a society as Sweden really wants to improve mathematics education, extra funding might just produce more researchers more eager to follow inside traditions than solving outside problems. Instead funding should force the universities to arrange curriculum architect compositions to allow alternatives to compete as to creativity and effectiveness, thus allowing the universities to rediscover their original outside rational goals and to change its routines accordingly. A situation described in several fairy tales; the Sleeping Beauty hidden behind the thorns of routines becoming rituals until awakened by the kiss of an alternative; and Cinderella making the prince dance, but only found when searching outside the established nobility.

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