Good & Bad & Evil Math

Tales of

Totals & Numbers & Fractions

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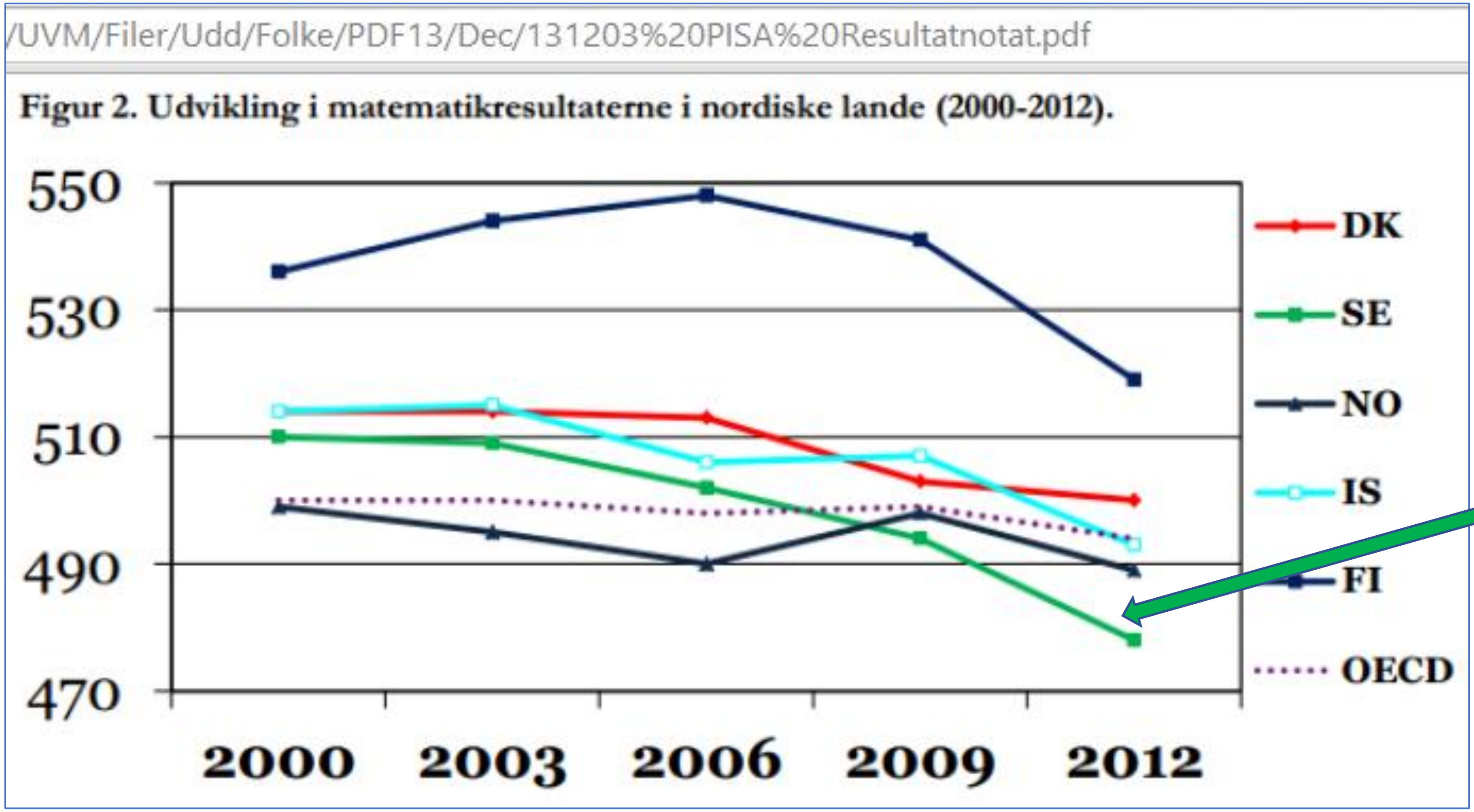
Teaching Teachers to Teach Mathe-Matics as ~~SET~~ MANY-Math

Problem: Poor PISA Performance & Poor Research Results after 50 years



Negative Correlation among Research and Performance

Why?

Is it Really Math we Teach?
Can Math be Different?

Solution in a Nutshell: From BAD to GOOD Math

1) All teach numbers. Don’t. Tell tales about how Totals unite and change
2) All use 1D line-numbers. Don’t. Use 2D block-numbers
3) All begin with addition. Don’t. Begin with counting and division, multiplication and subtraction before adding next-to and on-top
4) All add fractions without units. Don’t. Use units as in integral calculus
5) All include only the predicate (3*5). Don’t. Use full language sentences with a subject, a verb and a predicate (T = 3*5)
6) All call it MatheMatics. Don’t. It is MetaMatism, derived from SET, and falsified by e.g. 2+3 is 17 and not 5 in the case of weeks and days. Real MatheMatics is rooted in MANY.

One Definition of Mathematics

Pythagoras: mathematics, meaning knowledge, is a common label for 4 areas describing Many by itself and in space & time.

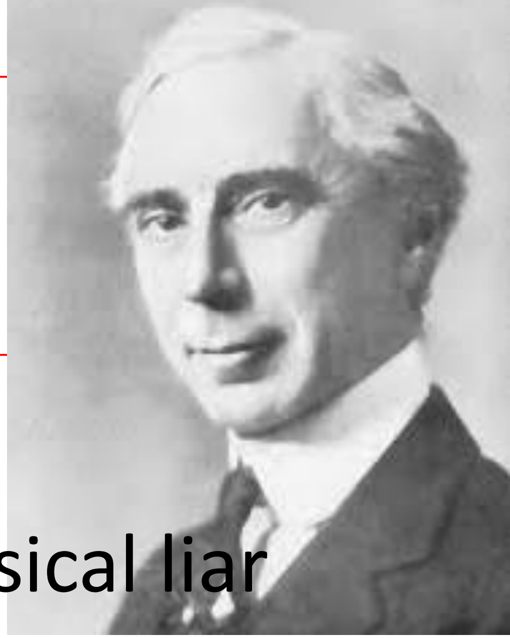
Together they formed the '**quadrivium**' recommended by Plato as a general curriculum after the '**trivium**' consisting of grammar & logic & rhetoric.

Grounded in Many as shown by names:

Geometry means to measure earth in Greek

Algebra means to reunite numbers in Arabic

Another Definition of Mathematics



Around 1900, **SET** made mathematics self-referring.

However, Russell said: Self-reference leads to the classical liar paradox 'this sentence is false', being false if true & opposite.

Let M be the set of sets not belonging to itself, $M = \{A | A \notin A\}$.

Then $M \in M \Leftrightarrow M \notin M$. Forget about sets. Use type theory instead. So, by self-reference, fractions cannot be numbers.

Mathematics: Forget about Russell, he is not a mathematician.

Of course fractions are numbers, they are rational numbers.

Two Different Mathematics

The Ruling Set-based Top-Down Meta-matics from above

- Mathematics exists by itself as a collection of well-proven statements about well-defined concepts
- Concepts are defined from above as **examples from abstractions**
- Mathematics has many applications; and of course it must be taught and learned before it can be applied

a FUNCTION is an example of a set relation where component1-identity implies component2-identity

The Silenced Many-based Bottom-Up Many-matics from below

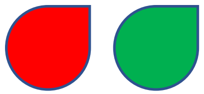
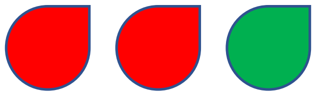
- Many exists all over the outside world, that schools prepare children and teenagers and adults for
- Concepts are defined from below as **abstractions from examples**
- Mathematics has many roots; but teaching it before applied is like teaching a grammar before its language

a FUNCTION is for example 2+x, but not 2+3; i.e. a name for a calculation with an unspecified number

How to Define Good & Bad & Evil Math: Four Questions to Answer (please discuss)

This is true	always	never	sometimes
2 + 3 = 5			
2 x 3 = 6			
$\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$			
$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$			

Four Questions Answered

This is true	always	never	sometimes
$2 + 3 = 5$			X Only with the same unit: 2weeks + 3days = 17days
$2 \times 3 = 6$	X 2x3 is 2 **3s** III III that exist and may be recounted as 6 **1s** I I I I I I		
$\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$			X Depends on the units 1 red of 2 apples + 2 of 3 apples is 3 of 5 apples, and not 7 of 6
$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$			X Only if taken of the same total

Defining Good & Bad & Evil Mathematics

Good mathematics is absolute truths about outside existing things

- T = 2*3 = 6 stating that a total of 2 **3s** can be re-counted as 6 **1s**: III III = I I I I I I
- So good mathematics is tales about how to count and unite and change totals

Bad mathematics ('mathe-matism') is relative truths about outside existing things

- 2+3 = 5, valid with like units, else falsified by e.g. 2weeks + 3days = 17days
- So bad mathematics is tales about numbers without units

Evil mathematics is about what exists only inside classrooms

- 1/2 + 2/3 = 7/6, but 1red of 2 + 2reds of 3 = 3reds of 5, and not 7reds of 6
- So bad mathematics is tales about fractions as numbers.
  Fractions are not numbers, but operators, needing numbers to become numbers.

Today's **BAD** MatheMatics = **MetaMatism** = MetaMatics + MatheMatism

*What is **GOOD** MatheMatics =* **ManyMatics***?*

Difference-Research finds Differences making a Difference, inspired by

Philosopy

- The ancient Greek sophists: To unmask choice masked as nature, find a difference
- In existentialism, Sartre said: EXISTENCE precedes ESSENCE
- Heidegger said: In a sentence, the SUBJECT exists, the PREDICATE is essence that can be different

Sociology (Bauman)

- Sociological imagination *"renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now. "*
- Goal Displacements: *"The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right."*

Psychology

- Don't teach about subjects, bring them to class to allow 'greifen vor begreifen' (Piaget, not Vygotsky)

So let us meet the existing subject **MANY** directly & outside its 'essence-prison' so **MANY** can create its own categories using Grounded Theory

Our Two Language Houses

The **WORD language** assigns words in sentences with a subject, a verb, and a predicate.
The **NUMBER language** assigns numbers instead with a subject, a verb, and a predicate.
Both languages have a meta-language, a grammar, describing the language, describing the world.

The meta-language is about the language, so we should teach and learn language before grammar.
This is the case with the word-language only, since SET-math is a grammar of the number-language.
Mixing language levels creates nonsense: 'The verb smiles' & 'The function increases'.

	WORD language	NUMBER language
Meta-language, grammar	'is' is a verb	'*' is an operation
Language	This is a chair	T = 3*4

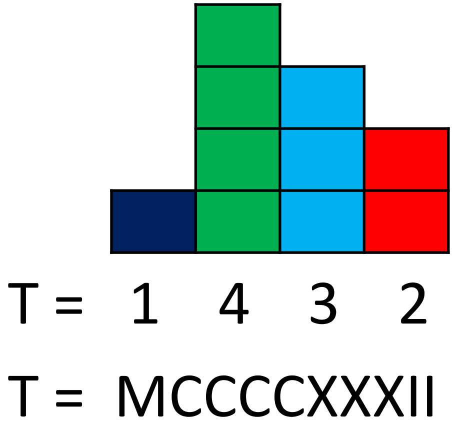
WORLD

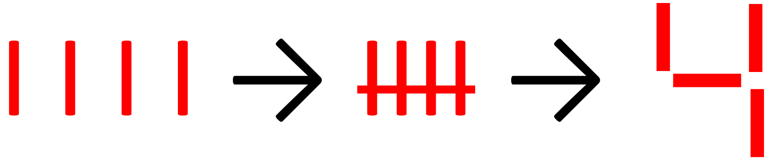
Children see Many as Bundles **<u>with Units</u>**

Asked 'How old next time?', a 3year-old says 4, but reacts when held together 2 by 2:

'<u>That is not 4, that is 2 **2s**</u>'.

Seeing bundles as units, children use 2D LEGO-like **block-numbers**, not 1D **line-numbers**, taught in school, even if 2D Arabic block-numbers replaced 1D Roman line-numbers centuries ago.

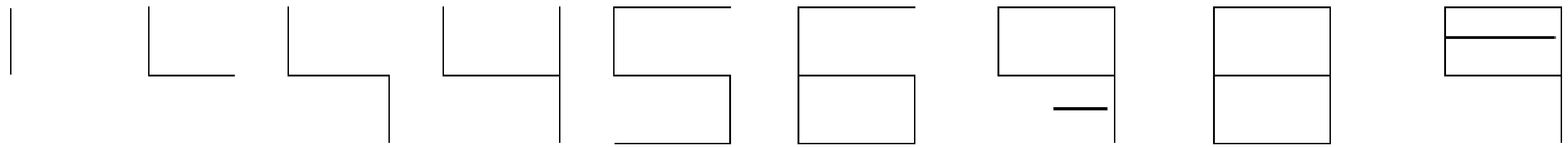


Many as Icons: 

Meeting Many, we ask: “**How Many in Total**?”
To answer, we Math … oops sorry, it’s a label, not an action word.
To answer, first we count, then we add. We name and iconize the degrees of Many until ten, that as 1 bundle has no icon or digit itself.

- Thus there are four sticks in a 4-icon, five in a 5-icon, etc.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII



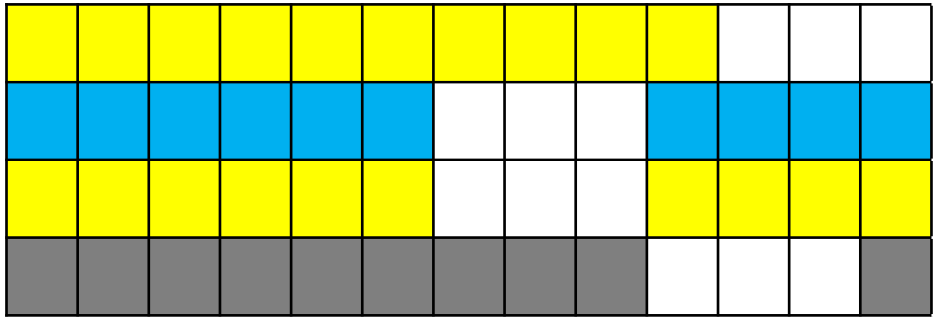
1	2	3	4	5	6	7	8	9

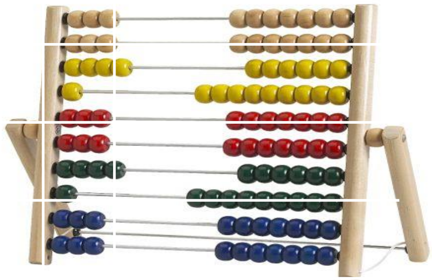
Cup- or BundleCounting in Icons: 9 = ? **4s**

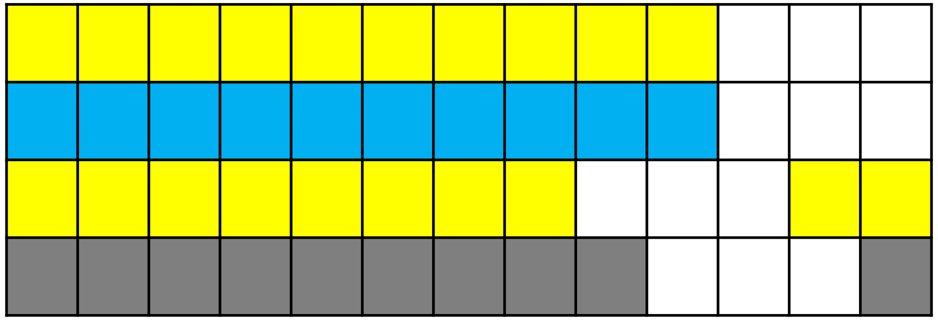
9 = I I I I I I I I I = IIII IIII I = [IIII IIII] I = 2**B**1 **4s** = 2.1 **4s**

To count, we bundle & use a bundle-cup with 1 stick per bundle.
We report with **bundle-writing** or **decimal-writing** where the decimal point separates inside bundles from outside single leftovers.
Shown on a western IKEA **ABACUS,** letting geometry & algebra go together.

Geometry/space mode or **A**lgebra/time mode





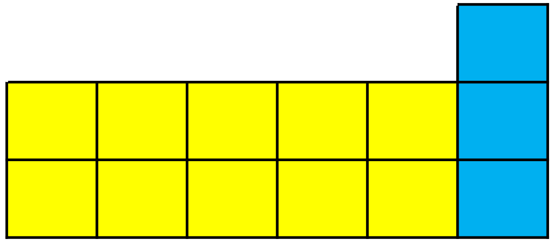


The UnBundled become Decimals or Fractions

0.3 **5s** or 3/5

When counting by bundling and stacking,
the unbundled single leftovers can be placed

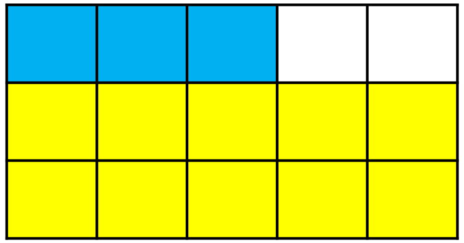
NextTo the stack
counted as a stack of **1s**



T = 2**B**3 **5s** = 2.3 **5s**

A decimal number

OnTop of the stack
counted as a bundle



T = 2 3/5 **5s**

A fraction

Counting Sequences

We may include bundling if saying ‘0**Bundle**3’ or ‘03’ instead of plain ‘3’

- ’0**Bundle**1, 0**B**2, 0**B**3, …, 0**B**8, 0**B**9, 1**B**0, 1**B**1, 1**B**2, … **tens**, or
- ‘01, 02, …, 1**Bundle** less 2, 1**B**-1, 1**B**0, 1**B**1(1left), 1**B**2, … **tens**

Counting fingers gives 1**B**0 **tens**, or

- 2**B**0 **5s** IIII IIII
- 2**B**2 **4s** IIII IIII I I
- 3**B**1 **3s** III III III I or 1**BB**1 **3s** IIIIIIIII I

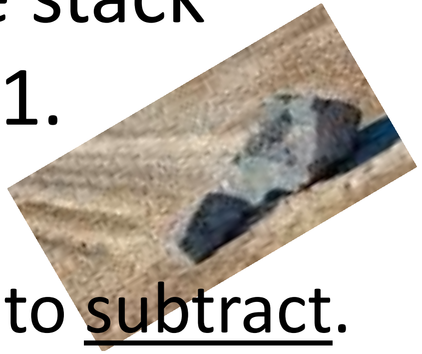
Operations as Icons

- To count 7 in **3s** we take away 3 many times, iconized by an uphill stroke, 7/3, showing the broom wiping away the **3s**.

7/3	**2.some**
7 – 2x3	**1**

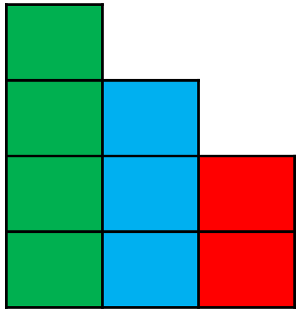
- A calculator predicts: 3 can be taken away 2 times. Stacking the bundles is iconized as a lift, 2x3.

- To look for unbundled singles, we drag away the stack of 2 **3s,** iconized by a horizontal trace: 7 – 2x3 = 1.



Counting creates 3 operations: to <u>divide</u> & to <u>multiply</u> & to <u>subtract</u>.

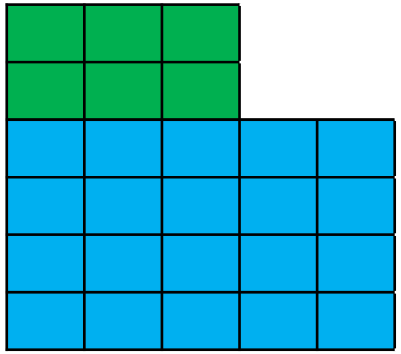
More Operations as Icons



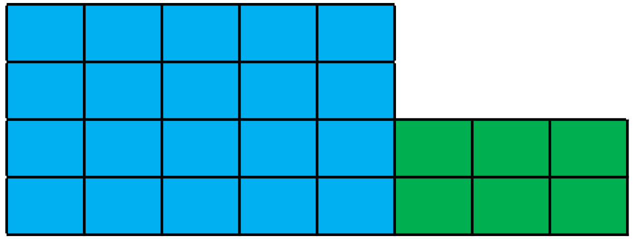
- To bundle bundles also, **power** is iconized as a cap, 5^2, showing the number of times bundles are bundled.
- Counting a Total gives a **BundleFormula,** a polynomial:

T = 432 = 4***BundleBundle** + 3***Bundle** + 2*1 = 4***B**^2 + 3***B**^1 + 2***B**^0

- **Addition** is a cross + showing blocks placed on-top of or next-to each other.



4 **5s** & 2 **3s** added OnTop



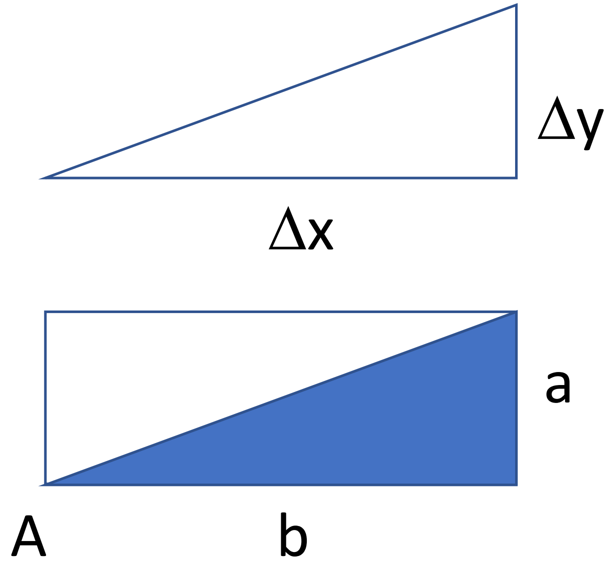
4 **5s** & 2 **3s** added NextTo

The ReCount Formula

7/3	2.some
7 – 2 * 3	1

Predicting T = 7 = 2.1 **3s**, the **ReCount formula T = (T/B)*B** saying 'from T, T/B times, B can be taken away', is all over:

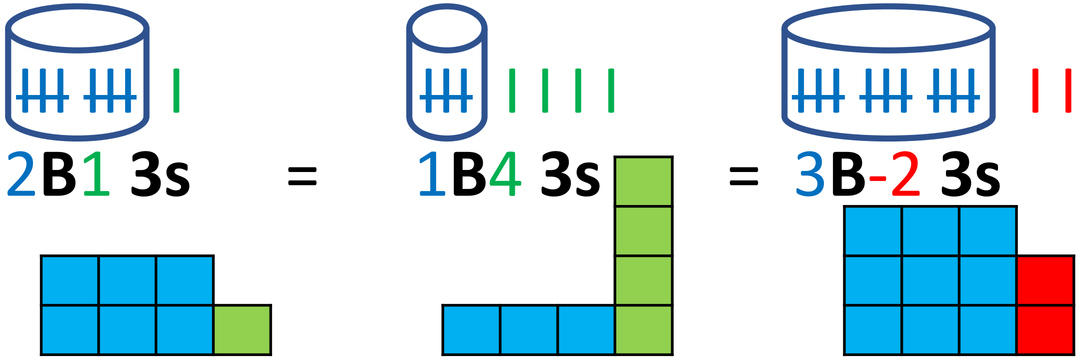
Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second



ReCounting in the Same Unit gives Flexible Totals

A total can be counted in 3 ways: Normal Overload Underload

T = 7 = I I I I I I I =



Or, when counting in tens
T = 37 = 3**B**7 **tens** = 2**B**17 **tens** = 4**B**-3 **tens**

BundleWriting and flexible totals may cure Math Dislike in classes stuck in Division:

☹☹☹ T = 336 /7 = 33**B**6 /7 = 28**B**56 /7 = 4**B**8 = 48 ☺☺☺

Likewise with

Multiplication	T = 7* 48 = 7* 4**B**8 = 28**B**56 = 33**B**6 = 336
Subtraction	T = 53 – 29 = 5**B**3 – 2**B**9 = 3**B**-6 = 2**B**4 = 24
Addition	T = 53 + 29 = 5**B**3 + 2**B**9 = 7**B**12 = 8**B**2 = 82

ReCounting in a Different Unit creates Proportionality & Multiplication & Equations

4*5/6	3.some
4*5 – 3*6	2

ReCounting in different units changes units (**Proportionality**)

- T = 4 **5s** = ? **6s**. A calculator predicts with ReCount-formula: T = 3.2 **6s**

ReCounting from icons to tens gives **Multiplication**

- T = 5 **7s** = ? **tens** = 5*7 = 35 = 3.5 **tens**, predicted by multiplication

So multiplication is a special form of division

ReCounting from tens to icons creates **Equations** solved by recounting

- T = ? **7s** = 42 = (42/7)*7 with the solution ? = 42/7 = 6.

*An equation is solved by moving to **O**pposite **S**ide with **O**pposite **S**ign*

u*7 = 42 = (42/7)*7

u = 42/7 = 6

Solving Equations by ReCounting, we may bracket Group Theory from Abstract Algebra

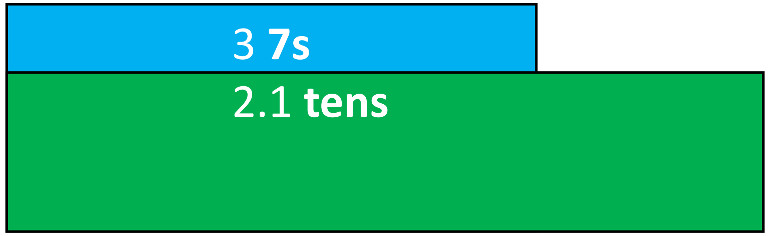
ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: **O**pposite **S**ide & **S**ign

SetMath (Don't test, but do remember bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½as its **inverse element**
⇕	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
⇕	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕	*u* x (2 x (½)) = 4	Applying the **associative** law
⇕	*u* x 1 = 4	Applying the definition of an inverse element
⇕	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

ReCounting Simplifies Multiplication Tables



Geometry: Multiplication means that, recounted in tens, a block increases its width and therefore must decrease its height to keep the total unchanged.

Thus T = 3*7 means 3 **7s** that may be recounted in tens as T = 2.1 **tens** = 21.

Algebra: The full ten-by-ten table can be reduced using that 6 is Bundle less 4, 7 is Bundle less 3, etc. This roots Early Algebra.

T = 2 **6s** = 2*6 = 2*(**B**-4) = 2**B**-8 = 2**B**-(1**B**-2) = 1**B**--2 = 1**B**+2 = 1**B**2 = 12

T = 4 **7s** = 4*7 = 4*(**B**-3) = 4**B** – 1**B**2 = 3**B**-2 = 2**B**8 = 28

T = 8 **7s** = 8*7 = (**B**-2)*(**B**-3) = **BB** – 2**B** – 3**B** + 6 = 10**B** – 2**B** – 3**B** + 6 = 5**B**6 = 56

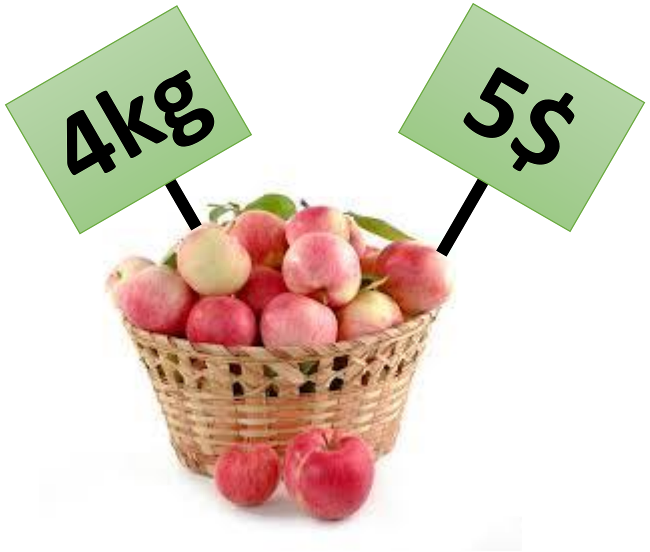
DoubleCounting in 2 Units creates PerNumbers

Apples are double-counted in kg and in $.

With **4kg = 5$** we have the **per-number** 4kg/5$ = 4/5 kg/$

Questions:

12kg = ?$	**20$ = ?kg**
12kg = (12/4)*4kg = (12/4)*5$ = 15$	20$ = (20/5)*5$ = (20/5)*4kg = 16kg



Answer: Recount in the per-number

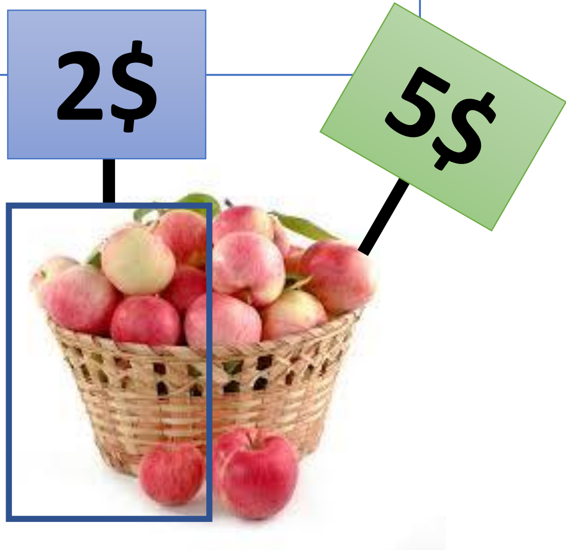
DoubleCounting in the Same Unit creates Fractions

The same unit: 2$ per 5$ = 2$/5$ = 2/5

- *Question: 2/5 = ? per 100; or 2$/5$ is ? per 100$*

Answer: <u>recount</u> 100 in 5s!

100$ = (100/5)*5$ gives (100/5)*2$ = 40$, so 2/5 = 40/100 = 40%



- *Question: 2/5 of 40 = ?; or with units: 2$ per 5$ of 40 $.*

Answer: <u>recount</u> 40 in 5s!

40$ = (40/5)*5$ gives (40/5)*2$ = 16$, so 2/5 of 40 = 16

Trigonometry ReCounts Sides in a HalfBlock

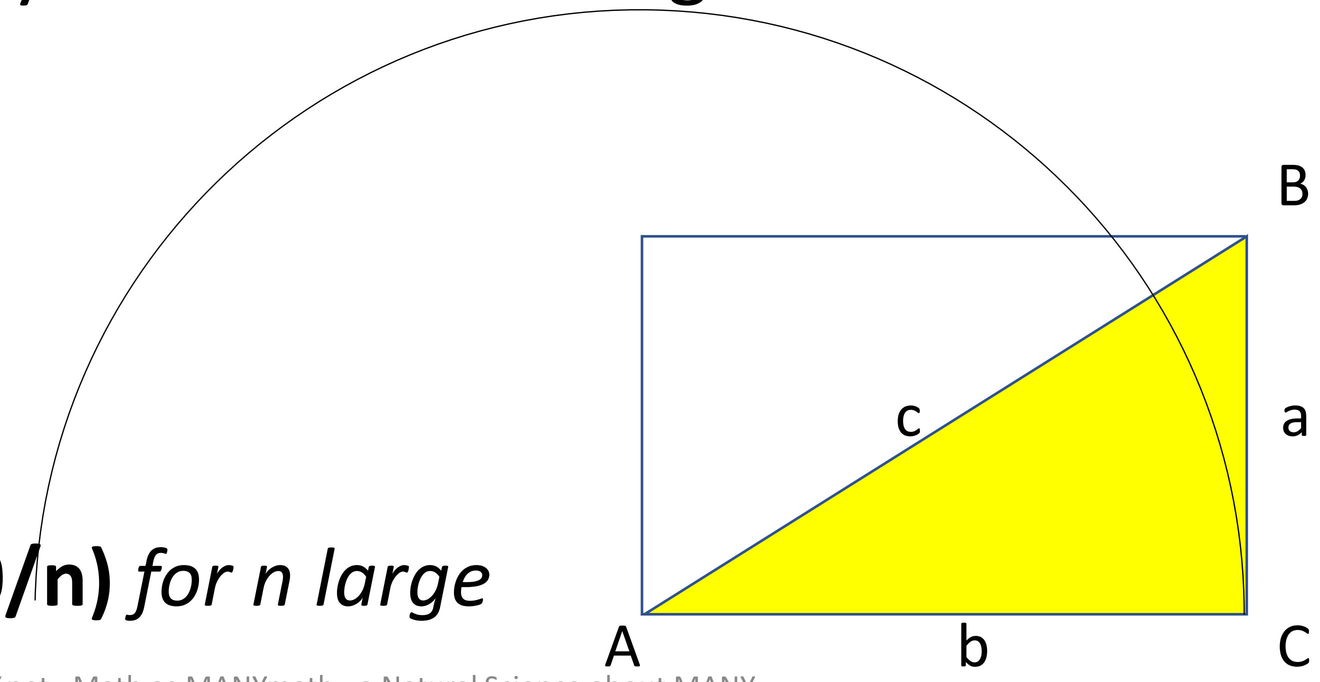
Halved by its diagonal, a block becomes a right angled triangle with three sides: the base b & the height a & the diagonal c, creating trigonometry by mutual recounting.

a = (**a/c**) * c = **sinA** * c

b = (**b/c**) * c = **cosA** * c

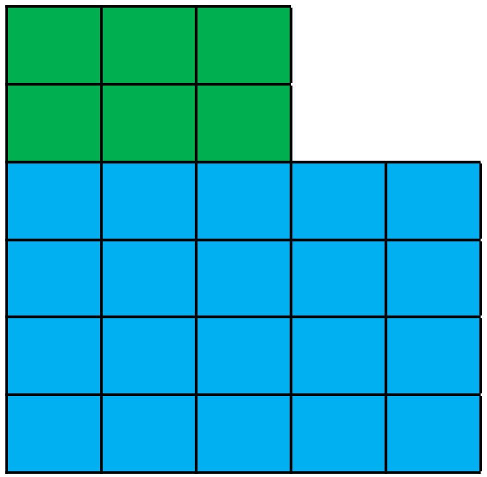
a = (**a/b**) * b = **tanA** * b

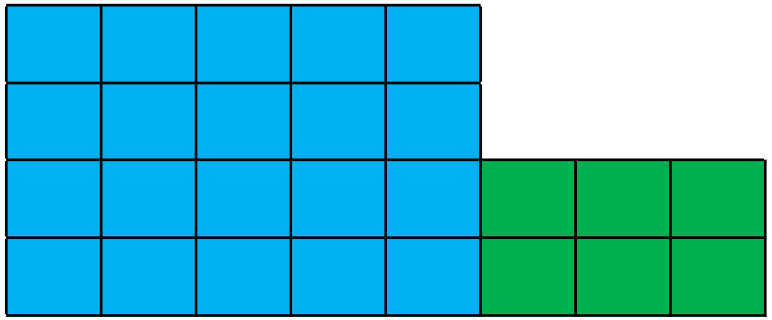
½Circle = **π = n*tan(180/n)** *for n large*



Once Counted & ReCounted, Totals can be Added

OnTop	**NextTo**
4 5s + 2 3s = 4 5s + 1**B**1 5s = 5**B**1 5s	4 5s + 2 3s = 3**B**2 8s
The units are changed to be the same *Change unit = Proportionality*	The areas are added *Adding areas = Integration*





The 3 Numbers in a Total add Differently

From totals as T1 = 2.3 **4s** and T2 = 3.4 **5s** we see that a Total has 3 numbers that add differently:

The bundle-size, the bundle-number, the single-number.

- Bundle-sizes stay unchanged unless the blocks are added next-to each other as in integration
- Bundle-numbers only add with like bundle-sizes.
- Singles always add.

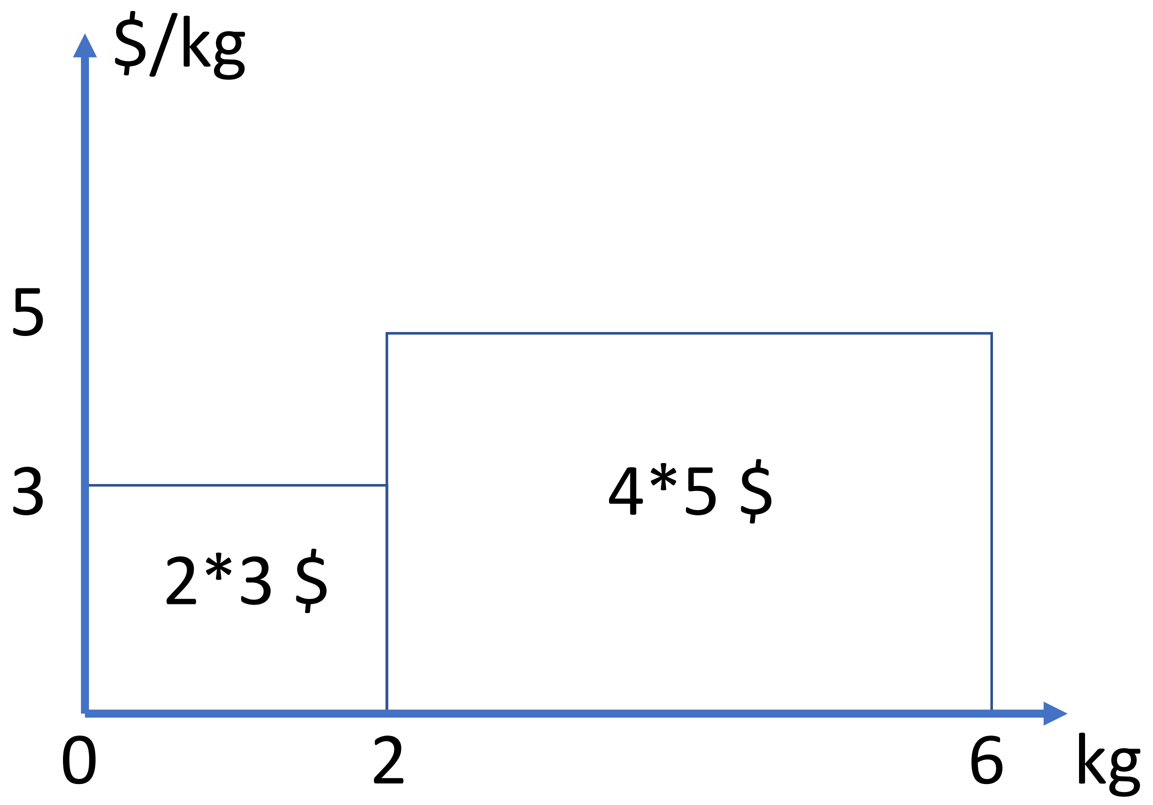
Never add without units: Mars Climate Orbiter, planes?

Adding PerNumbers as Areas (Integration)

2 kg	at	3 $/kg
+ 4 kg	at	5 $/kg
(2+4) kg	at	? $/kg

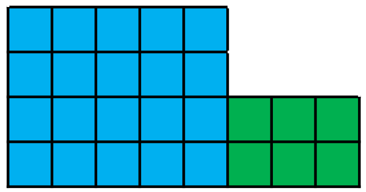
Unit-numbers add on-top.

Per-numbers add next-to as **areas** under the per-number graph.

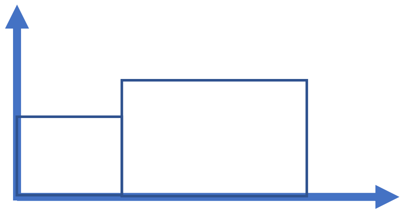


Primary & Middle & High School Calculus

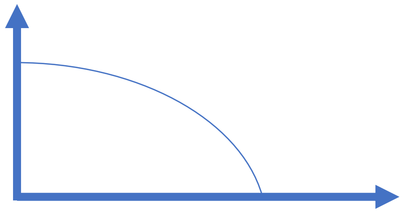
Primary calculus: Next-to addition of block-numbers



Middle calculus: Add piecewise constant per-numbers

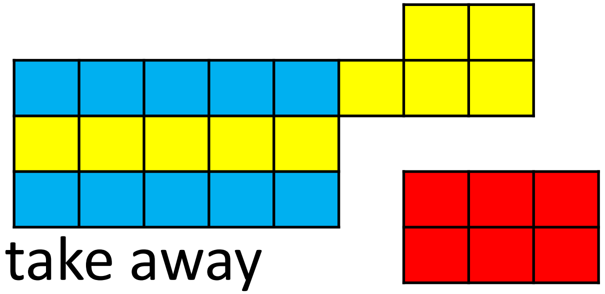


High school calculus: Add locally constant (continuous) per-numbers



Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	**2 3s + ? 5s = 3.2 8s**
? = 8/2	? = 8-2	? = (3.2 8s – 2 3s)/5
Solved by ReCounting	*Solved by ReStacking*	*Solved by differentiation: (T-T1)/5 = ΔT/5*



<u>Hymn to Equations</u>

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Concrete Algebra: 4 ways we Unite, + * ^ ∫ as shown by the Bundle Formula

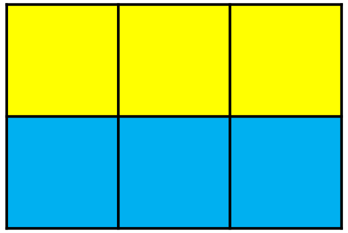
T = 456 = 4*B^2 + 5*B^1 + 6*B^0

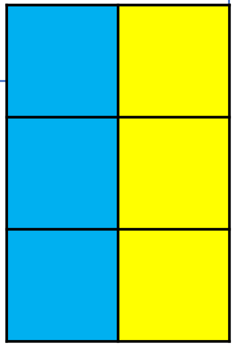
Totals exist as changing or constant **unit-numbers** or **per-numbers**

- Addition & Multiplication unite changing & constant unit-numbers
  - Subtraction & division split into changing & constant unit-numbers
- Integration & Power unite changing & constant per-numbers
  - Differentiation & root/logarithm (factor finder/counter) split into changing & constant per-numbers

Operations **unite** / *split into*	Changing	Constant
Unit-numbers *m, s, $, kg*	**T = a + n** *T – a = n*	**T = a*n** *T/n = a*
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $log_a T = n$, $\sqrt[n]{T} = a$

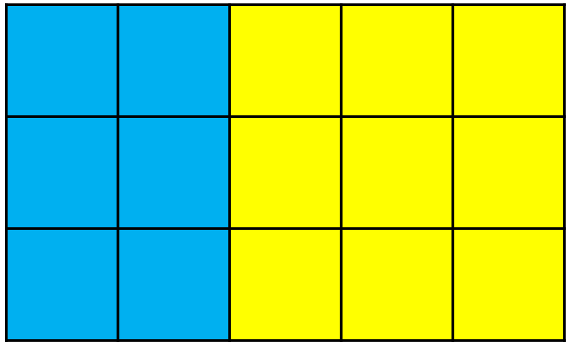
Abstract Algebra: (re)Uniting Units





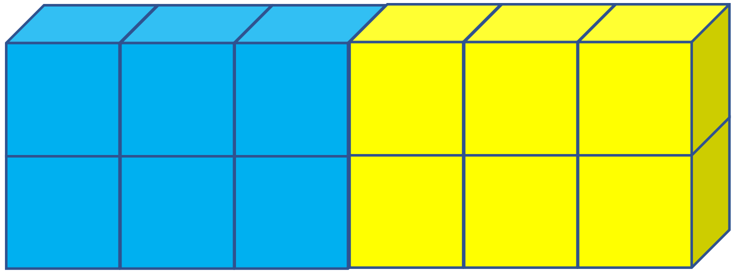
- Turning a block will change the unit
  T = 2 3s = 2*3 -> T = 3 2s = 3*2, so T = 2*3 = 3*2

(The Commutative law)



- A block may be split in two parts
  T = 3 5s = 3 2s + 3 3s or
  T = 3*5 = 3*(2+3) = 3*2 + 3*3

(The Distributive Law)



- A <u>united unit</u> as 6 that can be folded and fully stacked
  - a <u>prime unit</u> as 3 cannot.

T = 2 6s = 2*(2*3) = (2*2)*3

(The Associative law)

$T = 456 = 4*B^2 + 5*B + 6*1$

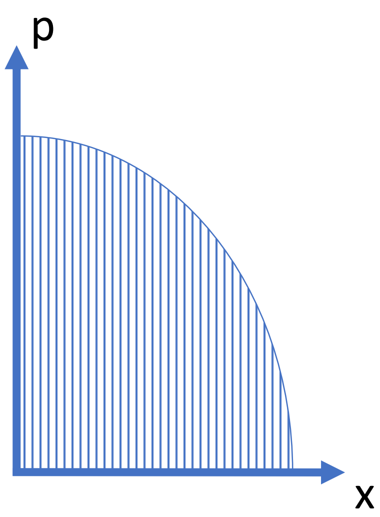
Bundle Formula: 5 ways of Constant Change

The number-formula contains formulas for constant change:

• $T = b*x$	(proportional)	*trade*
• $T = b*x + c$	(linear)	*trends*
• $T = a * x^n$	(elastic)	*science*
• $T = a * n^x$	(exponential)	*economy*
• $T = a*x^2 + b*x + c$	(accelerated)	*physics*

Two forms of NonConstant Change

Adding locally constant per-numbers means finding the area under the per-number graph as a sum of a large number of thin area-strips. But, if written as changes, this reduces to finding one total change since the middle terms cancel out. Writing p*dx = dF, or p = dF/dx motivates differential calculus, also useful to describe non-constant **predictable change**.



Unpredictable change roots statistics to 'post-dict' numbers by a mean and a deviation to be used by probability to pre-dict a confidence interval for numbers we else cannot predict.

Three forms of Constancy

A class is stuck in the **epsilon-delta** definition of continuity and differentiability.   Here a difference is to rename them to ‘local constancy’ and ‘local linearity’. As to constancy:

- y is globally constant c if for all positive numbers **epsilon**, the difference between y and c is less than epsilon.
- y is piecewise constant c if an interval-width **delta** exists such that for all positive numbers **epsilon**, the difference between y and c is less than epsilon, in this interval.
- y is locally constant c if for all positive numbers **epsilon**, an interval-width **delta** exists such that the difference between y and c is less than epsilon, in this interval.

Likewise, the change per-number **Δy/Δx** can be globally, piecewise or locally constant.

If locally constant , it is written as **dy/dx,** and y is called ‘locally linear’.

Quantitative Literature or Modeling comes in 3 Genres also: Fact & Fiction & Fiddle

- Fact models or 'since-then' calculations use numbers and formulas to quantify and to predict predictable quantities as e.g. 'since the base is 4 and the height is 5, then the area of the rectangle is T = 4*5 = 20'. Fact models can be trusted once the numbers and the formulas and the calculation has been checked. Special care must be shown with units to avoid adding meters and inches as in the case of the failure of the 1999 Mars Climate Orbiter.
- Fiction models or 'if-then' calculations use numbers and formulas to quantify and to predict unpredictable quantities as e.g. 'if the unit-price is 4 and we buy 5, then the total cost is T = 4*5 = 20'. Fiction models build upon assumptions that must be complemented with scenarios based upon alternative assumptions before a choice is made.
- Fiddle models or 'what-then' models use numbers and formulas to quantify and to predict unpredictable qualities as e.g. 'since a graveyard is cheaper than a hospital, then a bridge across the highway is too costly.' Fiddle models should be rejected and relegated to a qualitative description.

How Different is the Difference? I
SET Math versus MANY Math

	SET Math	**Many Math**
Goal/ Means	Learn Mathematics / Teach Mathematics	Learn to master Many / Tales of Many as counted, united, changed
Digits	Symbols like letters	Icons with as many sticks as they represent
Numbers	Line-numbers with place-value system Never with units	Block-numbers, stacking singles, bundles, bundle-bundles etc. Always with units
Number-types	Four types: Natural, Integers, Rational, Real	Positive & negative decimal numbers with units
Operations	Mapping from a set-product to the set. Order: Add, subtract, multiply, divide	Counting-icons: bundle /, stack x, remove -, unite on-top & next-to +). Opposite order
Division	8/2 means 8 split in 2	8/2 means 8 split in (counted in) 2s
ReCount PerNumber	Do not exist	Core concepts

How Different is the Difference? II
SET Math versus MANY Math

	SET Math	Many Math
Fractions	Rational numbers without units, and adding without units	Per-numbers, not numbers but operators needing a number to become a number, so added by integration
Equation	Statement about equivalent number-names	A recounting from tens to icons. Reversed operations
Function	A set relation where component1-identity implies comp.2-identity	A number-language sentence about the Total with a subject & a verb & a predicate
Propotio-nality	A linear function	A name for double-counting in two units
Calculus	Differentiation before integration (anti-differentiation)	Integration adds locally constant per-numbers. Integration before differentiation
Geometry	Plane before Coordinate before Trig.	Trigonometry before Coordinate Geometry

Difference-Research, Main Warning: The 3x3 Goal Displacements in Math Education

Primary	Numbers	Could: be icons & predicates in Tales of Many, T = 2 3s = 2*3; show Bundles, T = 47 = 4B7 = 3B17 = 5B-3; T = 456 = 4*BB + 5*B + 6*1 Instead: are changed from predicates to subjects by silencing the real subject, the total. Place-values hide the bundle structure
	Operations	Could: be icons for the counting process as predicted by the RecountFormula T = (T/B)*B, from T pushing Bs away T/B times Instead: hide their icon-nature and their role in counting; are presented in the opposite order (+ - * /) of the natural order (/, *, -, +).
	Addition	Could: wait to after counting & recounting & double-counting have produced unit- and per-numbers; wait to after multiplication Instead: silences counting and next-to addition; silences bundling & uses carry instead of overloads; assumes numbers as ten-based
Middle	Fractions	Could: be per-numbers coming from double-counting in the same unit; be added by areas (integration) Instead: are defined as rational numbers that can be added without units (mathe-matism, true inside, seldom outside classrooms)
	Equations	Could: be introduced in primary as recounting from ten-bundles to icon-bundles; and as reversed on-top and next-to addition Instead: Defined as equivalence relations in a set of number-names to be neutralized by inverse elements using abstract algebra
	Proportionality	Could: be introduced in primary as recounting in another unit when adding on-top; be double-counting producing per-numbers Instead: defined as linear functions, or as multiplicative thinking supporting the claim that fractions and ratios are rational numbers
High	Trigonometry	Could: be introduced in primary as mutual recounting of the sides in a right-angled triangle, seen as a block halved by a diagonal Instead: is postponed till after geometry and coordinate geometry, thus splitting up geometry and algebra.
	Functions	Could: be introduced in primary as formulas, i.e. as the number-language's sentences, T = 2*3, with subject & verb & predicate Instead: are introduced as set-relations where first-component identity implies second-component identity
	Calculus	Could: be introduced in primary as next-to addition; and in middle & high as adding piecewise & locally-constant per-numbers Instead: differential calculus precedes integral calculus, presented as anti-differentiation

ReCounting looks like Dienes MultiBase Blocks

- “Dienes’ name is synonymous with the Multi-base blocks (also known as Dienes blocks) which he invented for the teaching of place value.
- Dienes’ place is unique in the field of mathematics education because of his theories on how mathematical structures can be taught from the early grades onwards using multiple embodiments through manipulatives, games, stories and dance.”

(http://www.zoltandienes.com/about/)

Dienes on Numbers and MultiBase Blocks

"The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. This is why our system of numbering, introduced in the middle ages by Arabs, is called the place value system. My contention has been, that in order to fully understand how the system works, we have to understand the concept of power. (..)

In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..)

Educators today use the "multibase blocks", but most of them only use the base ten, yet they call the set "multibase". These educators miss the point of the material entirely."

(What is a base?, http://www.zoltandienes.com/academic-articles/)

Power & Base from Above, or Bundles from Below

Dienes teaches the 1D place value line-numbers with 2D & 3D blocks to show the importance of the power concept.

- ManyMatics teaches 2D block-numbers with units to show the importance of bundling singles, bundles & bundle-bundles.

Dienes sees numbers as examples of the abstract label **base**

- ManyMatics sees counting as an action with a concrete verb **bundle**

Dienes teaches top-down 'MetaMatics' derived from the concept Set

- ManyMatics teaches a bottom-up natural science about the fact Many; and sees Set as meaningless because of Russell's set-paradox.

base the base
bundle the bundle

Different Education

EU: Line-organized & Office-directed Schools

From secondary school, continental Europe uses **line-organized** education with forced classes and forced schedules, making teenagers stay together in age groups - even if boys are two years behind in mental development.

The classroom belongs to the class. This forces teachers to change room and (in lower secondary school) to teach several subjects outside their training.

Tertiary education is also **line-organized** preparing for offices in the public or private sector. This makes it difficult to change line in the case of unemployment.

This makes reproduction fall to 1.5 child/family, causing the European population to be halved each two generations since per female, $(1.5/2)*(1.5/2) = .75*.75 \approx 0.5$.

Different Education

US: Block-organized & Talent-directed Schools

Alternatively, North America uses **block-organized** education saying to teenagers:

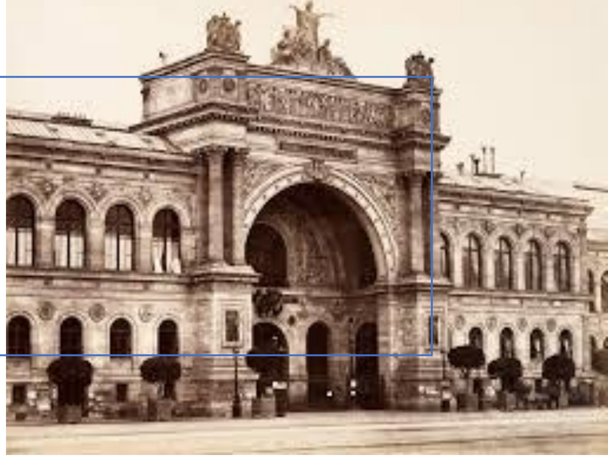
“Welcome, inside you carry a **talent**! Together we will uncover and develop your personal talent through self-chosen daily half-year blocks, academical or practical, together with 1subject teachers. If successful the school will say ‘**good job**, you have a **talent**, you need some more’. If not, the school will say ‘**good try**, you have **courage** to try out the unknown, now try something new’”.

The classroom belongs to the teacher teaching one subject only.

Likewise, college is **block-organized** easy to supplement with additional blocks in the case of unemployment.

At the age of 25, most students have an education, a job and a family with three children, 1 for mother, 1 for father, and 1 for the state to secure reproduction.

Good & Bad Research



- Good research searches for truth about things that exist. It poses a question, and chooses a methodology to transform reliable data into valid statements. Or it uses methodic skepticism to unmask choice masked as nature.
- Bad research is e.g. master level work applying instead of questioning existing research. Or journalism describing something without being guided by a question.
- With these three research genres, peer-review only works inside the same genre.
- All conferences should have a **'salon des refusé'** to foster and boost new paradigms (Kuhn), as it does in art.

More Conflicting Theory in Math Ed Research

Philosophy

- Sophists: Unmask choice masked as nature by finding hidden differences
- Philosophy: All is nature and examples of meta-physical forms only visible to us

Sociology

- Structure: Institutions are good if rational and democratic
- Agent: Goal displacements in institutions lead to 'the banality of evil' (Arendt)

Psychology

- Piaget: Teach little, but allow the learner to meet the **existing** subject directly
- Vygotsky: We need good teaching to mediate institutionalized **essence**

More Enlightenment Sociology in Math Ed Research

Sociology can question institutions by asking: Offering education as a cure for the diagnose ‘uneducated’ is a self-referring irrationality. A power agenda behind?

Thus, inspired by Heidegger’s: ‘In sentences, trust the subject & doubt the predicate’, and wanting to protect its Enlightenment republic, French post-structuralism says:

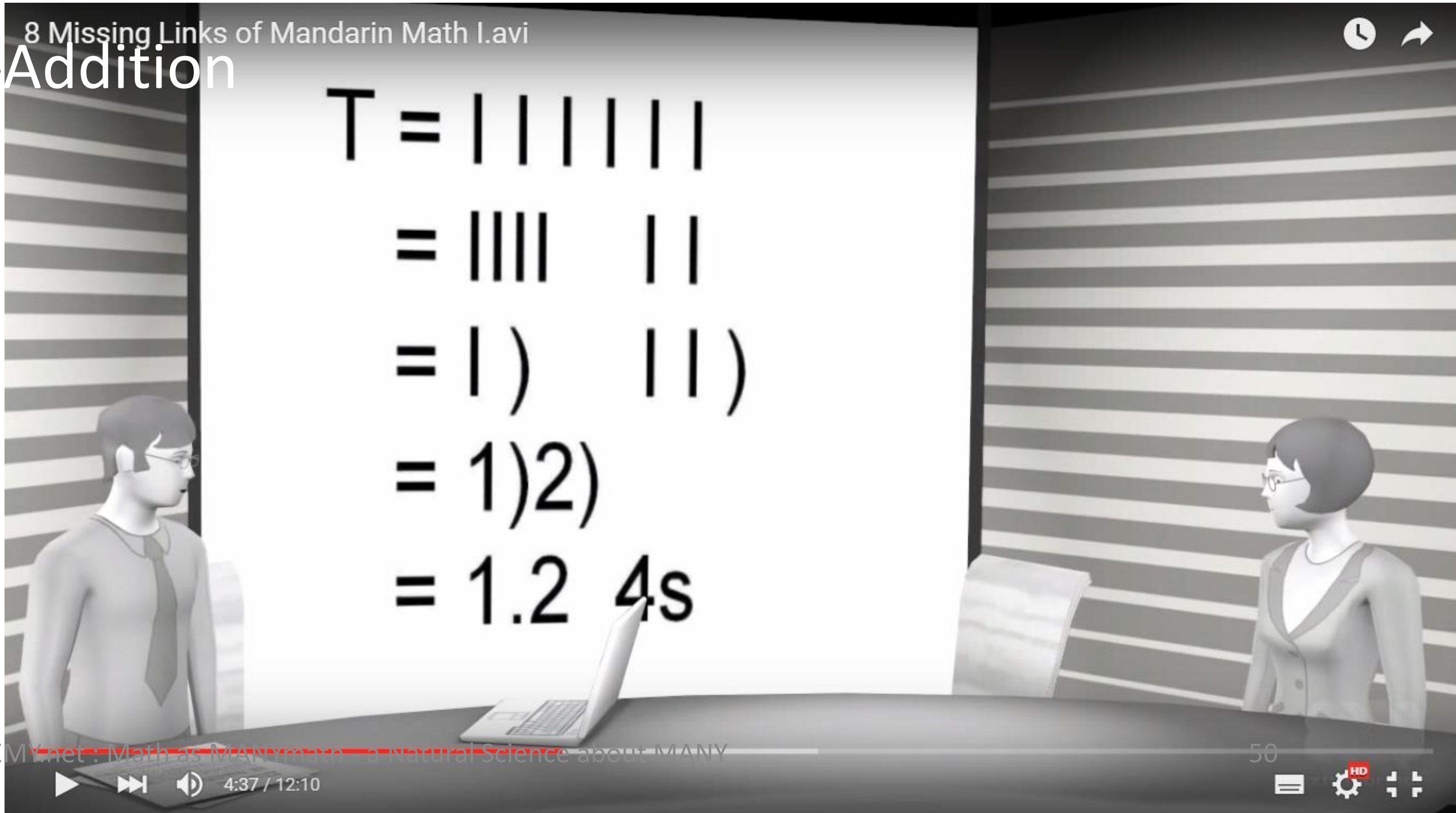
- Derrida: Words can be fake, and install instead of label (DeConstruction)
- Lyotard: Truth can be fake (PostModern skepticism towards meta-narratives)
- Foucault: Diagnoses and discourses can be fake, still allowing curing institutions to expand (a school is really a ‘pris-pital’ mixing power techniques from a prison and a hospital, and with learners as ‘patien-mates’)
- Bourdieu: Education is fake by using symbolic violence (and mathematics especially) to create a new knowledge-nobility

ManyMath is Different
But does it make a Difference? Try it out

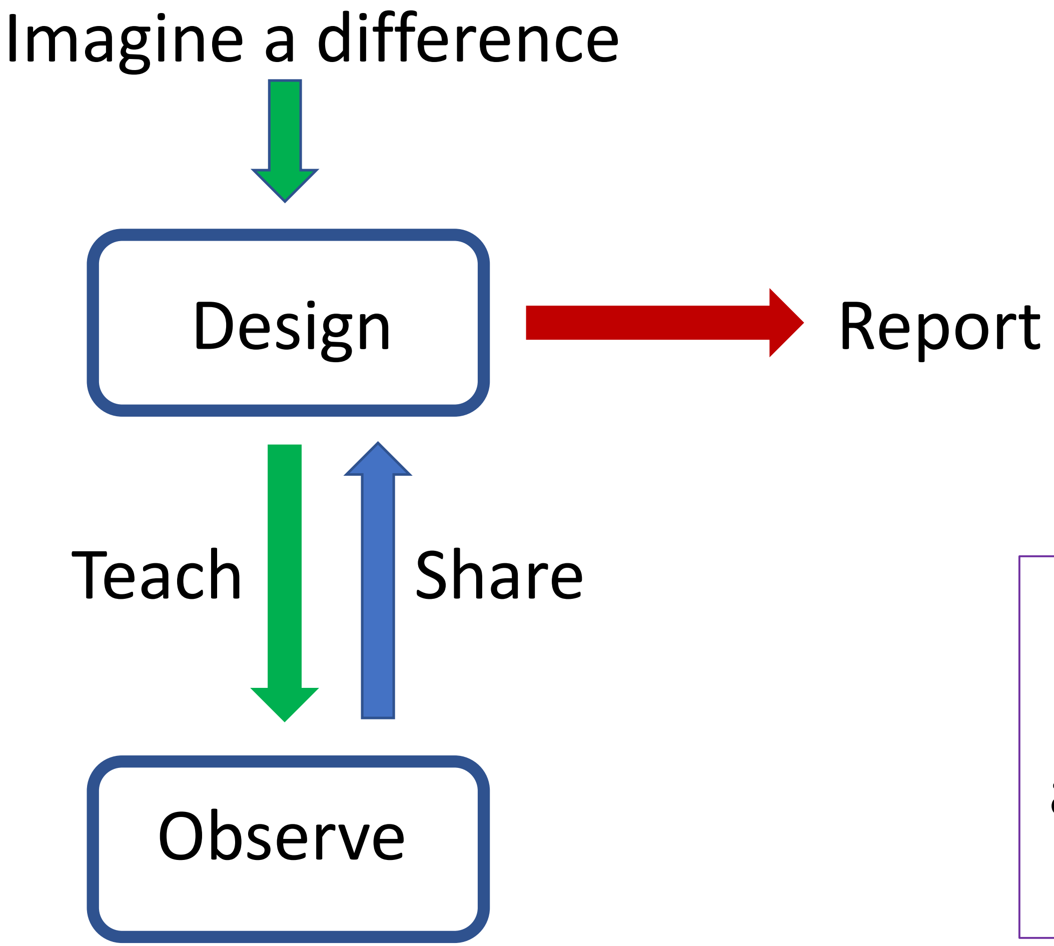
- Watch some YouTube or YouKu videos (MrAlTarp/DrAlTarp)
- Try the **CupCount before you Add** Booklet
- Try a 1day free Skype seminar **How to Cure Math Dislike**
- Try Action Learning and Action Research, e.g. **1Cup & 5Sticks**
- Collect data and Report on 8 **MicroCurricula**, M1-M8
- Try a 1year online InService TeacherTraining at the MATHeCADEMY.net using PYRAMIDeDUCATION to teach teachers to teach MatheMatics as **ManyMatics**, a Natural Science about the root of mathematics, **Many**

Some MrAlTarp YouTube Videos
Screens & Scripts on MATHeCADEMY.net

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



Action Learning & Action Research



Lyotard dissenting Paralogy
Quality indicator:
Ungrounded rejection

Example
Calculators in PreSchool
and Special Needs education
Paper rejected at MADIF10

Numbers as Icons & ReCounting 7 in **5s** & **3s** & **2s**

MatheMatics: Unmask Yourself, Please

- In Greek you mean ‘knowledge’. You were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic. Later only 2 activities remained: Geometry and Algebra
- Then Set transformed you from a natural science about the physical fact Many to a metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism
- So please, unmask your true identity, and tell us how you would like to be presented in education:
- MetaMatism for the few - or ManyMatics for the many.

From Bad & Evil Math
to Good Math:

1) Respect the Child's own 2D Block
2) Count, ReCount & DoubleCount
   before Adding OnTop & NextTo
3) Let Existence precede Essence:
   Think Things

Slides on MATHeCADEMY.net

Details in ***Journal of Mathematics Education***

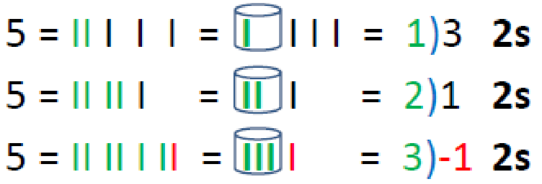
Thank You for Listening

CupCount 'fore you Add Booklet, free to Download

My many Math Tears will not Stay – if I Cup the Stray Away

CupCount 'fore you Add

MathDislike Cured by 1 Cup & 5 Sticks



CupCount 7 in 3s: 7 = 2)1 3s = 1)4 3s = 3)-2 3s

NO, 4x7 is not 28, it is 4 7s = 2)8 = 1)18 = 3)-2 tens
NO, 30/6 is not 30 divided by 6, it is 30 counted in 6s

CupWrite to tell InSide Bundles from OutSide 1s:

- 65 + 27 = 6)5 + 2)7 = 8)12 = 9)2 = 92
- 65 – 27 = 6)5 – 2)7 = 4)-2 = 3)8 = 38
- 7x 48 = 7x 4)8 = 28)56 = 33)6 = 336
- 336 /7 = 33)6 /7 = 28)56 /7 = 4)8 = 48

MatheMatics as **ManyMath**
- a Natural Science about **Many**

Makes Math Potentials Blossom
in Children, Adults & Migrants

Allan.Tarp

MATHeCADEMY.net

Contents

03. CupCounting in Icons

Job		Do	Calculator
9 in 5s	Line Count Bundle Stack Cup Answer	T = I I I I I I I I I 1, 2, 3, 4, B, 1B1, 1B2, 1B3, 1B4 T = IIIII I I I I T = 1)4 5s = 0)9 5s = 2)-1 5s T = 9 = 1.4 5s	9/5 1.some 9 – 1*5 4 9 – 0*5 9 9 – 2*5 -1
9 in 4s	Line Count Bundle Cup Stack Answer	T = I I I I I I I I I 1, 2, 3, B, 1B1, 1B2, 1B3, 2B, 2B1 T = IIII IIII I T = 2)1 4s = 1)5 4s = 3)-3 4s T = 9 = 2.1 4s	9/4 2.some 9 – 2*4 1 9 – 1*4 5 9 – 3*4 -3
9 in 3s	Line Count Bundle Cup Stack Answer		9/ 9 –
8 in 4s	Line Count Bundle Cup Stack Answer		8 8
8 in 3s	Line Count Bundle Cup Stack Answer		8 8

1day free Skype Seminar:
To Cure Math Dislike, CupCount before you Add

Action Learning based on the Child's own 2D NumberLanguage

09-11. Listen and Discuss the PowerPointPresentation

To Cure MathDislike, replace MetaMatism with ManyMath

- **MetaMatism** = MetaMatics + MatheMatism
- **MetaMatics** presents a concept TopDown as an example instead of BottomUp as an abstraction
- **MatheMatism** is true inside but rarely outside classrooms
- **ManyMath**, a natural science about Many mastering Many by BundleCounting & Adding NextTo and OnTop.

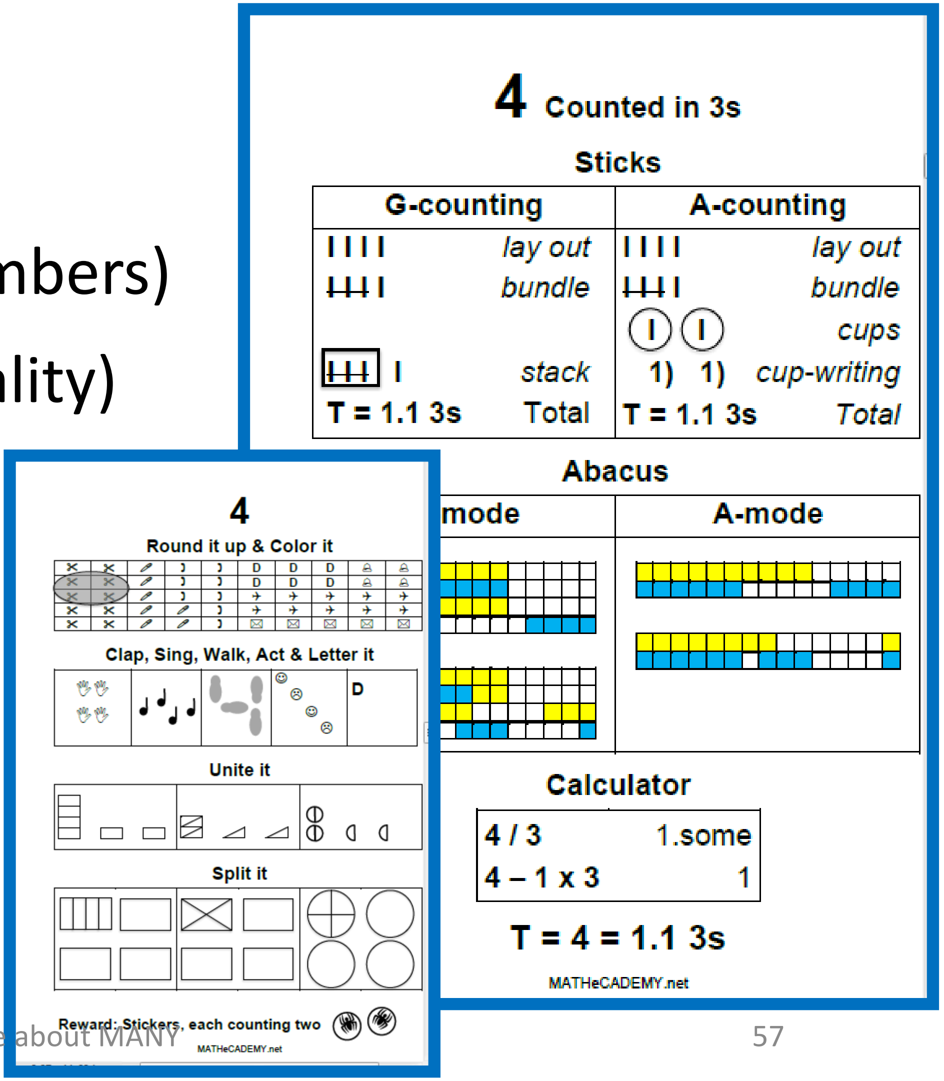
11-13. Skype Conference. Lunch.

13-15. Do: Try out the CupCount before you Add booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in BundleCounting & NextTo Addition.

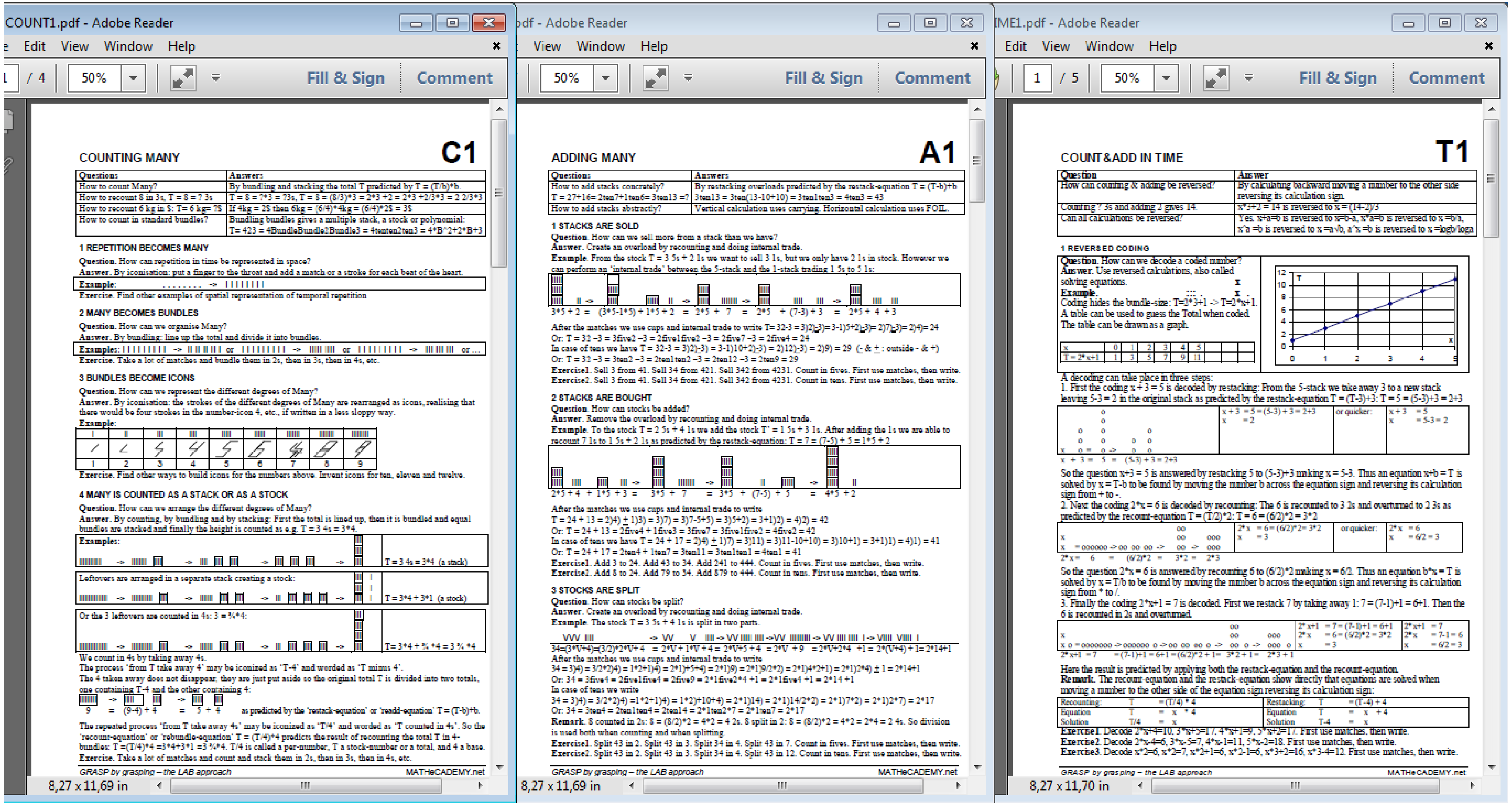
15-16. Coffee. Skype Conference.

8 MicroCurricula for Action Learning & Research

C1. Create Icons
C2. Count in Icons
C3. ReCount in the Same Icon (Negative Numbers)
C4. ReCount in a Different Icon (Proportionality)
A1. Add OnTop (Proportionality)
A2. Add NextTo (Integrate)
A3. Reverse Adding OnTop (Solve Equations)
A4. Reverse Adding NextTo (Differentiate)



Teacher Training in CATS ManyMath
Count & Add in Time & Space



MATHeCADEMY.net

a VIRUSeCADEMY Teaching Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY – the CATS approach: Count & Add in Time & Space

HOME INTRO COUNT & ADD TIME & SPACE DK DISLIKE MIGRANT MATH VIDEOS PAPERS **VARIOUS**

Cure Math Dislike: BundleCounting before Adding

We ACT to deal with the outside world.
We MATH to deal with the natural fact MANY ???
Oops, sorry, MATH is not an action word!
We COUNT & ADD to **Master MANY**.

- **Bundle**Count & **ReC**ount:

T = 7 = I I I I I I I = 卌 卌 I = 2**B**1 **3s** = 2.1 **3s**
T = 2B1 3s = 1**B**4 **3s** = 3**B**-2 **3s** (Overload or Underload)
T = 2B1 **3s** = 1B2 **5s** = 3B1 **2s** = 1BB1B1 **2s**
T = 3×8 = 3 **8s** = 2B6 **9s** = 2B4 **tens**, or the sloppy version 24
T = 336 /7 = 33B6 /7 = 28B56 /7 = 4B8 = 48

Counting gives a decimal number with a unit (a natural number).
Adding **OnTop**, a **T**otal may be **ReC**ounted to shift the unit.
Adding **NextTo**, means **Integration** of areas.

- Add **OnTop** & Add **NextTo**:

T1+T2 = 1B2 **3s** + 4B5 **6s** = 0B5 **6s** + 4B5 **6s** = 5B4 **6s**
T1+T2 = 1B2 **3s** + 4B5 **6s** = 3B7 **9s** or 3B4 **tens** = 34

The **CATS** approach to MATH: **C**ount & **A**dd in **T**ime & **S**pace

Primary school: **C1** & **A1** & **T1** & **S1**
Secondary school: **C2** & **A2** & **T2** & **S2**

FREE teacher education in
MATH as a Natural Science about **MANY**.

RECENT POSTS

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Migrant Math in twenty 1page Sheets
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Heidegger Improving Math Education
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CERME10 with Reviews
Cure Math Dislike, 1day SKYPE Seminar
CupCount & ReCount

Teaches Teachers to Teach
MatheMatics as **ManyMath**,
a Natural Science about **MANY**.
The **CATS** method: To learn Math
Count & **A**dd in **T**ime & **S**pace

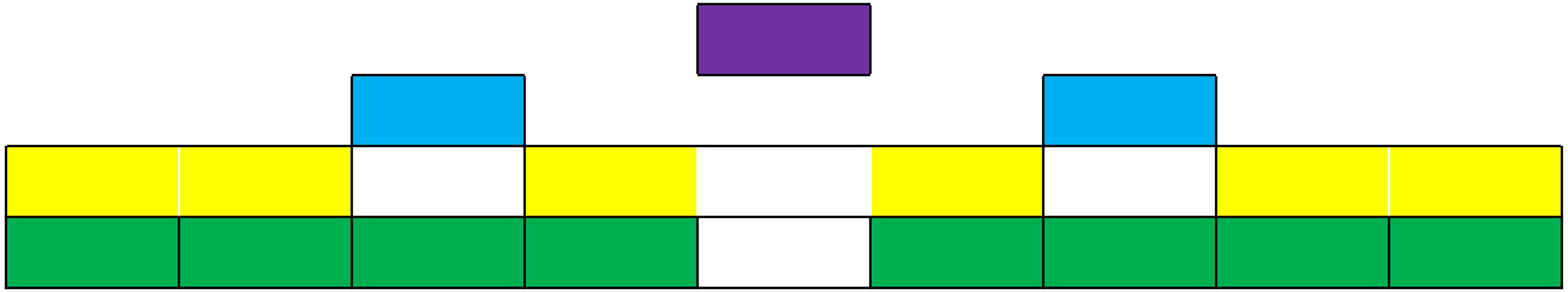
PYRAMIDeDUCATION

To learn MATH: Count&Add MANY
Always ask Many, not the Instructor
MATHeCADEMY.net - a VIRUSeCADEMY

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams



28b. Different Mathematics

Main Parts of a ManyMath Curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage

- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- BundleCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: divide, multiply, subtract, add on-top & next-to

Middle school – integrating algebra and geometry, the content of the label math

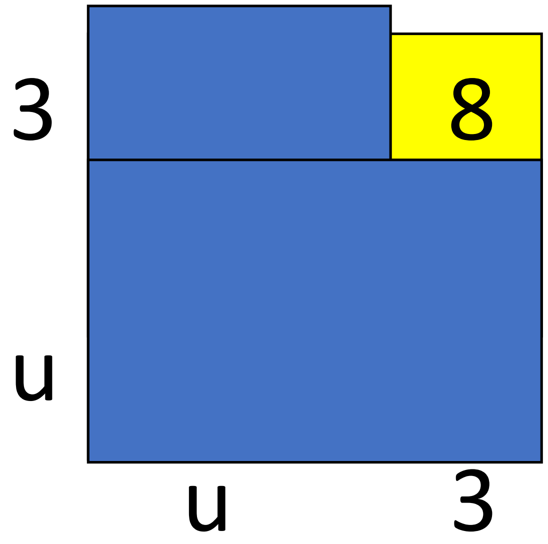
- DoubleCounting produces PerNumbers as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always, so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE

- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

Quadratic Equations with 3 Cards

Solve the quadratic equation **$u^2 + 6u + 8 = 0$**



$(u+3)^2 = u^2 + 6u + 8 + 1$

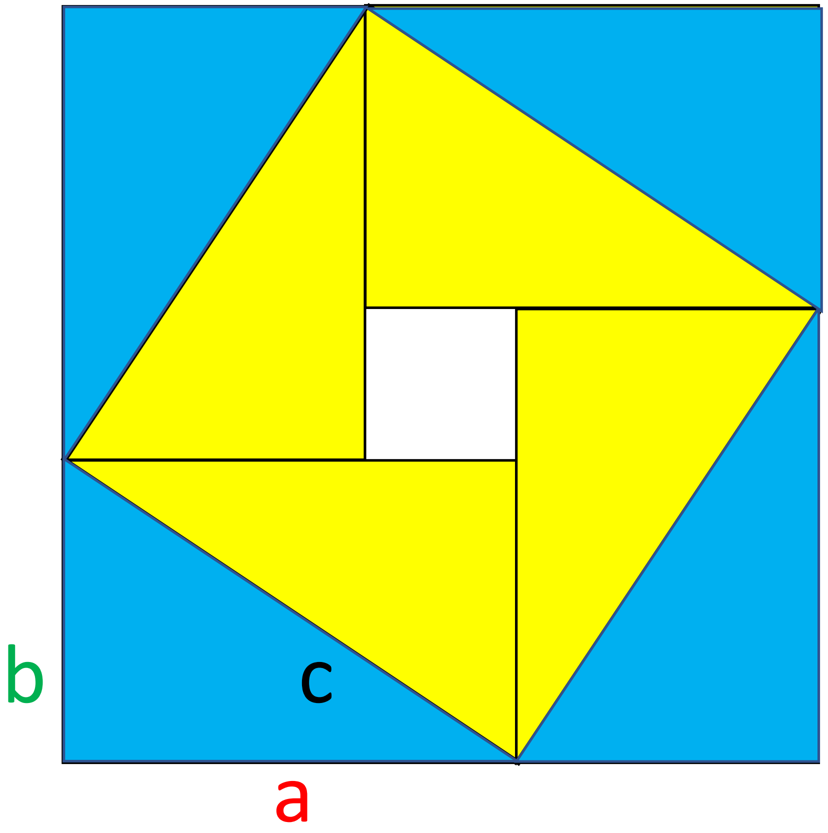
$(u+3)^2 = 0 + 1$

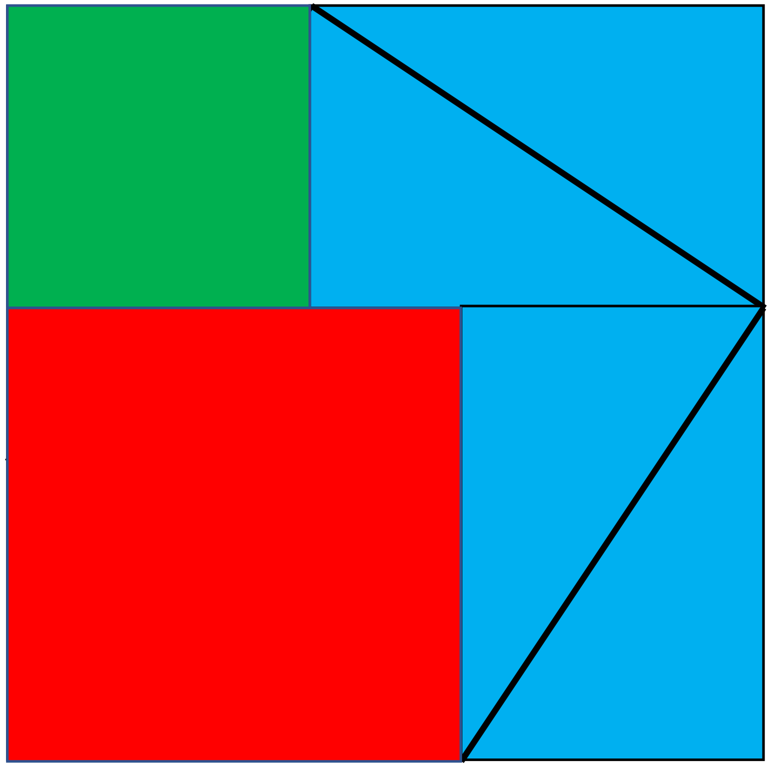
$u+3 = \pm 1$

$u = -3 \pm 1$

Solution: **$u = -4, u = -2$**

Pythagoras shown by 4 Cards with Diagonals





c^2 + 4 ½cards

a^2 + b^2 + 2 cards