THE SIMPLICITY OF MATH REVEALS A CORE CURRICULUM

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Introduction

Research in mathematics education has grown since the first International Congress on Mathematics Education in 1969. Likewise, funding has increased as seen e.g. by the creation of a Swedish centre for Mathematics Education. Yet, despite increased research and funding, decreasing Swedish PISA result caused OECD (2015a) to write the report ‘Improving Schools in Sweden’ describing its school system as ‘in need of urgent change’.

To find an unorthodox solution we pretend that a university in southern Sweden arranges a curriculum architect competition: ‘Theorize the low success of 50 years of mathematics education research, and derive a STEM-based core curriculum from this theory.’

Since mathematics education is a social institution, social theory may give a clue to the lacking success and how to improve schools in Sweden and elsewhere.

Social Theory Looking at Mathematics Education

Imagination as the core of sociology is described by Mills (1959). Bauman (1990) agrees by saying that sociological thinking ‘renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now’ (p. 16).

Mathematics education is an example of ‘rational action (..) in which the end is clearly spelled out, and the actors concentrate their thoughts and efforts on selecting such means to the end as promise to be most effective and economical (p. 79)’. However

> The ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called goal displacement. (..) The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right. (p. 84)

One such goal displacement is saying that the goal of mathematics education is to learn mathematics since, by its self-reference, such a goal statement is meaningless. So, if mathematics isn’t the goal of mathematics education, what is? And, how well defined is mathematics after all?

In ancient Greece, the Pythagoreans used mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: arithmetic, geometry, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many by itself, Many in space, Many in time and Many in time and space. And together forming the ‘quadrivium’ recommended by Plato as a general curriculum together with ‘trivium’ consisting of grammar, logic and rhetoric.

With astronomy and music as independent knowledge areas, today mathematics should be a common label for the two remaining activities, geometry and algebra, both rooted in the physical fact Many through their original meanings, ‘to measure earth’ in Greek and ‘to reunite’ in Arabic. And in Europe, Germanic countries taught counting and reckoning in primary school and arithmetic and geometry in the lower secondary school until about 50 years ago when they all were replaced by the 'New Mathematics'.

Here the invention of the concept SET created a Set-based 'meta-matics' as a collection of 'well-proven' statements about 'well-defined' concepts. However, 'well-defined' meant defining by self-reference, i.e. defining top-down as examples of abstractions instead of bottom-up as abstractions from examples. And by looking at the set of sets not belonging to itself, Russell showed that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false:

If $M = \{ A \mid A \notin A) \}$ then $M \in M \Leftrightarrow M \notin M$.

The Zermelo–Fraenkel Set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract concepts.

In this way, SET transformed grounded mathematics into today's self-referring 'meta-matism', a mixture of meta-matics and 'mathe-matism' true inside but seldom outside classrooms where adding numbers without units as '2 + 3 IS 5' meet counter-examples as e.g. 2weeks + 3days is 17 days; in contrast to '2x3=6' stating that 2 3s can always be re-counted as 6 1s.

Difference Research Looking at Mathematics Education

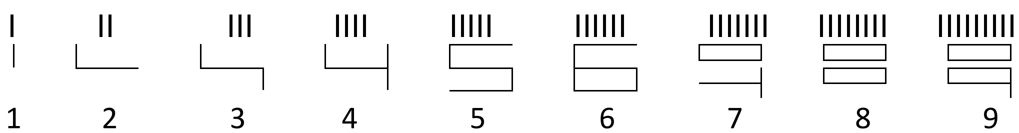
Inspired by the ancient Greek sophists (Russell, 1945), wanting to avoid being patronized by choices presented as nature, 'Difference-research' is searching for hidden differences making a difference. So, to avoid a goal displacement in mathematics education, difference-research asks: How will mathematics look like if grounded in its outside root, Many?

To answer we allow Many to open itself for us, so that, as curriculum architects, sociological imagination may allow us to construct a core mathematics curriculum based upon exemplary situations of Many in a STEM context, seen as having a positive effect on learners with a non-standard background (Han et al, 2014). So, we now return to the original Greek meaning of mathematics as knowledge about Many by itself and in time and space; and use Grounded Theory (Glaser & Strauss, 1967), lifting Piagetian knowledge acquisition (Piaget, 1969) from a personal to a social level, to allow Many create its own categories and properties.

Meeting Many Creates a 'Count-before-Adding' Curriculum

Meeting Many, we ask 'How many in Total?' To answer, we total by counting and adding to create number-language sentences, T = 2 3s, containing a subject and a verb and a predicate as in a word-language sentence.

Rearranging many 1s in 1 icon with as many strokes as it represents (four strokes in the 4-con, five in the 5-icon, etc.) creates icons to be used as units when counting:

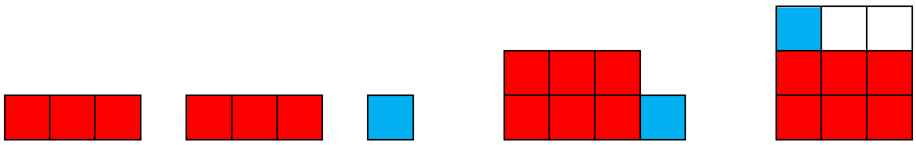


We count in bundles to be stacked as bundle-numbers or block-numbers, which can be re-counted and double-counted and processed by on-top and next-to addition, direct or reversed.

To count a total T we take away bundles B (thus rooting and iconizing division as a broom wiping away the bundles) to be stacked (thus rooting and iconizing multiplication as a lift stacking the bundles into a block) to be moved away to look for unbundled singles (thus rooting and iconizing subtraction as a trace left when dragging the block away). A calculator predicts the result by a re-count formula T = (T/B)*B saying that 'from T, T/B times, B can be taken away':

7/3 gives 2.some, and 7 – 2x3 gives 1, so T = 7 = 2B1 3s.

Placing the singles next-to or on-top of the stack counted as 3s, roots decimals and fractions to describe the singles: T = 7 = 2.1 3s = 2 1/3 3s



T = 7 = 2 3s & 1 = 2B1 3s = 2.1 3s = 2 1/3 3s

A total counted in icons can be re-counted in tens, which roots multiplication tables; or a total counted in tens can be re-counted in icons, T = 42 = ? 7s, which roots equations.

Double-counting in physical units roots proportionality by per-numbers as 3$/4kg bridging the units. Per-numbers become fractions if the units are the same. Since per-numbers and fractions are not numbers but operators needing a number to become a number, they add by their areas, thus rooting integral calculus.

Once counted, totals can be added on-top after being re-counted in the same unit, thus rooting proportionality; or next-to as areas, thus rooting integral calculus. And both on-top and next-to addition can be reversed, thus rooting equations and differential calculus:

2 3s + ? 4s = 5 7s gives differentiation as: ? = (5*7 – 2*3)/4 = ΔT/4

In a rectangle halved by a diagonal, mutual re-counting of the sides creates the per-numbers sine, cosine and tangent. Traveling in a coordinate system, distances add directly when parallel; and by their squares when perpendicular. Re-counting the y-change in the x-change creates change formulas, algebraically predicting geometrical intersection points, thus observing the 'geometry & algebra, always together, never apart' principle.

Predictable change roots pre-calculus (if constant) and calculus (if variable). Unpredictable change roots statistics to 'post-dict' numbers by a mean and a deviation to be used by probability to pre-dict a confidence interval for numbers we else cannot pre-dict.

Alternative Versions of Standard Mathematics

01. To stress the importance of bundling, the counting sequence should be: 01, 02, …, 09, 10,11 etc.

02. The ten fingers should be counted also as 13 7s, 20 5s, 22 4s, 31 3s, 101 3s, 5 2s, and 1010 2s.

03. A Total of five fingers should be re-counted in three ways (standard and with over- and underload): T = 2B1 5s = 1B3 5s = 3B-1 5s = 3 bundles less 1 5s.

04. Multiplication tables should be formulated as re-counting from icon-bundles to tens and use underload counting after 5: T = 4*7 = 4 7s = 4*(ten less 3) = 40 less 12 = 30 less 2 = 28.

05. Dividing by 7 should be formulated as re-counting from tens to 7s and use overload counting: T = 336 /7 = 33B6 /7 = 28B56 /7 = 4B8 = 48

06. Solving proportional equations as 3*x = 12 should be formulated as re-counting from tens to 3s: 3*x = 12 = (12/3)*3 giving x = 12/3 illustrating the relevance of the 'opposite side & sign' method.

07. Proportional tasks should be done by re-counting in the per-number: With 3$/4kg, 20kg = (20/4)*4kg = (20/4)*3$ = 15$; and 18$ = (18/3)*3$ = (18/3)*4kg = 24 kg

08. Fractions and percentages should be seen as per-numbers coming from double-counting in the same unit, 2/3 = 2$/3$. So 2/3 of 60 = 2$/3$ of 60$ = (60/3)*3$ giving (60/3)*2$ = 40$

09. Integral should precede differential calculus and include adding both piecewise and locally constant per-numbers: 2kg at 3$/kg + 4kg at 5$/kg = (2+4)kg at (2*3+4*5)$/(2+4)kg thus showing that per-numbers and fractions are added with their units as the area under the per-number graph.

10. Trigonometry should precede plane and coordinate geometry to show how, in a box halved by its diagonal, the sides can be mutually re-counted as e.g. a = (a/c)*c = sinA*c.

Level & Change Formulas

Re-counting and double-counting leads to the recount-formula T = (T/B)*B occurring all over mathematics: when re-counting or double-counting to change unit in proportional quantities; when re-counting to solve equations; in trigonometry to mutually re-count the sides in a right triangle; and in calculus to mutually re-count the changes as dy = (dy/dx)*dx = y'*dx. In economics, the recount-formula becomes a price-formula: $ = ($/kg)*kg, $ = ($/day)*day, etc.

Counting by stacking bundles into adjacent blocks leads to the number-formula called a polynomial:

T = 456 = 4*BundleBundle + 5*Bundle + 6*single = 4*B^2 + 5*B + 6*1.

In its general form, the number-formula T = a*x^2 + b*x + c contains the different formulas for constant change: T = a*x (proportionality), T = a*x+b (linearity), T = a*x^2 (acceleration), T = a*x^c (elasticity) and T = a*c^x (interest rate).

The number-formula also shows the four ways to unite numbers offered by algebra meaning 'reuniting' in Arabic: addition and multiplication add variable and constant unit-numbers; and integration and power unite variable and constant per-numbers. And since any operation can be reversed: subtraction and division split a total into variable and constant unit-numbers; and differentiation and root & logarithm split a total in variable and constant per-numbers:

Uniting/*splitting into*	Variable	Constant
Unit-numbers m, s, kg, $	T = a + n *T – a = n*	T = a * n *T/n = a*
Per-numbers m/s, $/kg, $/100$ = %	T = ∫ a dn *dT/dn = a*	T = a^n, *loga(T) = n* *n√T = a*

Meeting Many in a STEM Context

Having met Many by itself, we now meet Many in time and space in the present culture based upon STEM, described by OECD (2015b) as follows: ‘In developed economies, investment in STEM disciplines (science, technology, engineering and mathematics) is increasingly seen as a means to boost innovation and economic growth.’

STEM thus combines basic knowledge about how humans interact with nature to survive and prosper: Mathematics provides formulas predicting nature’s physical and chemical behavior, and this knowledge, logos, allows humans to invent procedures, techne, and to engineer artificial hands and muscles and brains, i.e. tools, motors and computers, that combined to robots help transforming nature into human necessities.

A falling ball introduces nature’s three main actors, matter and force and motion, similar to the three social actors, humans and will and obedience. We observe three balls: the earth, the ball, and molecules in the air. Matter houses two forces, an electro-magnetic force keeping matter together when colliding, and gravity pumping motion in and out of matter when it moves in the same or in the opposite direction of the force. In the end, the ball is lying still on the ground. But motion cannot disappear. Motion transfers through collisions, now present as increased motion in molecules; meaning that the motion has lost its order and can no longer be put to work. In the daytime the sun pumps in low-disorder light-energy; and in the nighttime the space sucks out high-disorder heat-energy; if not, global warming would be the consequence.

So, a core STEM curriculum could be about cycling water. Heating transforms water from solid to liquid to gas, i.e. from ice to water to steam; and cooling does the opposite. Heating an imaginary box of steam makes some molecules leave, so the lighter box is pushed up by gravity until becoming heavy water by cooling, now pulled down by gravity as rain in mountains and through rivers to the sea. On its way down, a dam can transform falling water to electricity. To get to the dam, we must build roads along the hillside.

In the sea, water contains salt. Meeting ice at the poles, water freezes but the salt stays in the water making it so heavy it is pulled down by gravity, elsewhere pushing warm water up thus creating cycles in the ocean pumping warm water to cold regions.

The two water-cycles fueled by the sun and run by gravity leads on to other STEM areas: to the trajectory of a ball pulled down by gravity; to an electrical circuit where electrons transport energy from a source to a consumer; to dissolving matter in water; and to building roads on hillsides.

STEM-subjects are swarming with per-numbers: kg/m^3 (density), meter/second (velocity), Joule/second (power), Joule/kg (melting), Newton/m^2 (pressure), $/kg (price), $/hour (wages), etc.

Newton’s Laws of Motion

Observing the consequences of applying a force to a door will illustrate Newton’s three laws combining force and motion. With an open door, it accelerates as long as being pushed. Then it continues with a constant speed until closed, thus illustrating Newton’s second and first law. With the closed, it stays closed whereas the pusher is accelerated backwards, thus illustrating Newton’ third law: what is pushed pushes back. Together they illustrate the second law: with the same force, a high mass means a low acceleration and vice versa.

The core message of Newton's law is that a force will change the motion. Being applied in both space and time, the change depends on the number of meters and seconds the force is applied.. Multiplying the force with the time period, dt, gives the change in the kinetic momentum, F*dt = d(m*v). Including the velocity v = ds/dt, this gives F*ds = d(½*m*v^2), meaning that multiplying the force with the distance gives the change in the kinetic energy.

A Golf Ball

A golf ball is sent away with a vertical and a horizontal velocity at 4 tenmeters per seond and 2 tenmeters per second respectively. On its orbit the ball is subject to the gravitational force changing the vertical speed wiyh approximately 1 tenmeter/second each second (strictly speaking 0.982) and leaving the horisontally speed uncanged. So after 3 seconds the vertical speed is 1 tenmeters per sond ans d horizontal still is 2 tenmeters er second. As to the distances, the horizontal distance is now 3*2 ten meters = 60 meters. As to the vertical distance traveled it is 3*the average spped = 3*1.5= 4.5 en meters = 45 meters. Plotting the positions we get a parabola as a trajectory curcing downwards and haing its maxiumg height af 4 seconds whne all the vertical spped is vanished, thati s ine a hwight 4*2 tenmeters = 80 meter

An Electrical Circuit

To work properly, a 2000Watt water kettle needs 2000Joules per second. The socket delivers 220Volts, a per-number double-counting the number of Joules per charge-unit.

Re-counting 2000 in 220 gives (2000/220)*220 = 9.1*220, so we need 9.1 charge-units per second, which is called the electrical current counted in Ampere.

To create this current, the kettle must have a resistance R according to a circuit law Volt = Resistance*Ampere, i.e., 220 = R*9.1, or Resistance = 24.2Volt/Ampere called Ohm.

Since Watt = Joule per second = (Joule per charge-unit)*(charge-unit per second) we also have a second formula, Watt = Volt*Ampere.

Thus, with a 60Watt and a 120Watt bulb, the latter needs twice the current, and consequently half the resistance of the former.

Supplied next-to each other from the same source, the combined resistance R must be decreased as shown by reciprocal addition, 1/R = 1/R1 + 1/R2. But supplied after each other, the resistances add directly, R = R1 + R2. Since the current is the same, the Watt-consumption is proportional to the Volt-delivery, again proportional to the resistance. So, the 120Watt bulb only receives half of the energy of the 60Watt bulb.

Warming and Boiling Water

In a water kettle, a double-counting can take place between the time and the energy used to warm the water to boiling, and to transform the water to steam.

Heating 1000gram water 80degrees in 167seconds in a 2000Watt kettle, the per-number will be 2000*167/80Joule/degree, creating a double per-number 2000*167/80/1000Joule/degree/gram or 4.18Joule/degree/gram, called the specific heat of water.

Producing 100gram steam in 113seconds, the per-number is 2000*113/100Joule/gram or 2260Joule/g, called the heat of evaporation for water.

Conclusion and Recommendation

This paper argues that the low success of 50 years of mathematics education research may be caused by a goal displacement seeing mathematics as the goal instead of as an inside means to the outside goal, mastery of Many in time and space. The two views offer different kinds of mathematics: a set-based top-down 'meta-matics' that by its self-reference is indeed hard to teach and learn; and a bottom-up Many-based 'Many-matics' simply saying 'To master Many, counting produces constant or variable unit-or per-numbers, uniting by adding or multiplying or powering or integrating.'

Thus, this simplicity of mathematics as expressed in a Count-before-Adding curriculum allows bundle-numbers to replace line-numbers, and to learn core mathematics as proportionality, calculus, equations and per-numbers in early childhood. Imbedded in STEM-examples, young male migrants learn core STEM subjects at the same time, thus allowing them to become STEM-teachers or STEM-engineers to return help develop or rebuild their own country. The full curriculum can be found in a 27-page paper (Tarp, 2017).

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