The Simplicity of Mathematics reveals a Core Curriculum:

To Master Many
ReCount in Block- & Per-numbers
Postpone or Drop Addition

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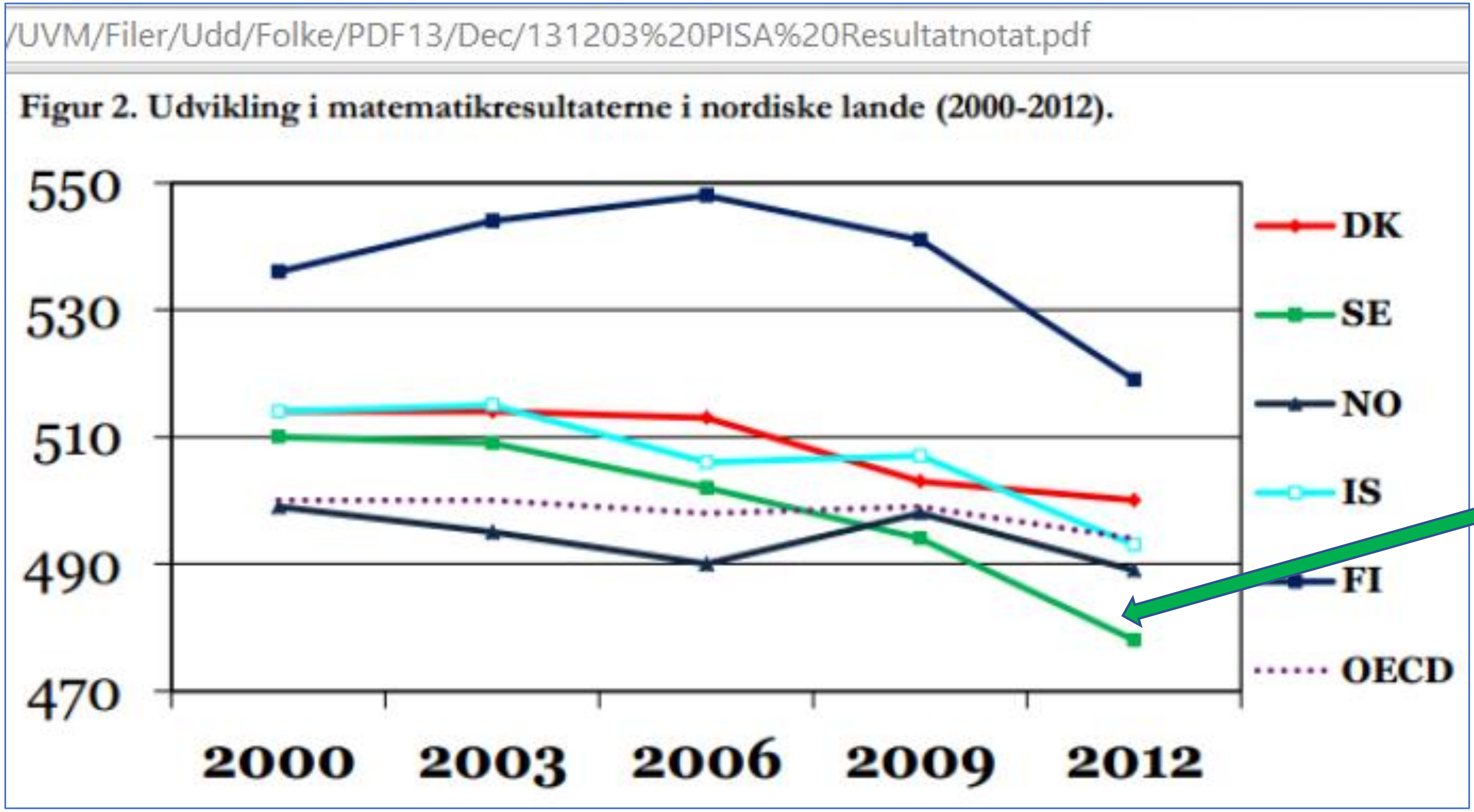
Teaching Teachers to Teach Mathe-Matics as ~~SET~~ MANY-Math

Denmark

Problem: Poor PISA Performance despite 50 years of Math Ed Research



Research Increases Results Decrease, especially in Sweden

Negative Correlation among Research and Performance

Why?

Is it Really Math we Teach?
Can Math be Different?

Solution in a Nutshell
From BAD to GOOD Math

1) All teach numbers. Don't. Tell tales about how Totals unite and change
2) All use 1D line-numbers. Don't. Use 2D block-numbers
3) All begin with addition. Don't. Begin with counting and division, multiplication and subtraction before adding next-to and on-top
4) All add fractions without units. Don't. Use units as in integral calculus
5) All include only the predicate (3*5). Don't. Use full language sentences with a subject, a verb and a predicate (T = 3*5)
6) All call it MatheMatics. Don't. It is MetaMatism, derived from SET, and falsified by e.g. 2+3 is 17 and not 5 in the case of weeks and days. Real MatheMatics is rooted in MANY.

A Call for Curriculum Architects

With many young male migrants in Sweden, a university may write out a competition:

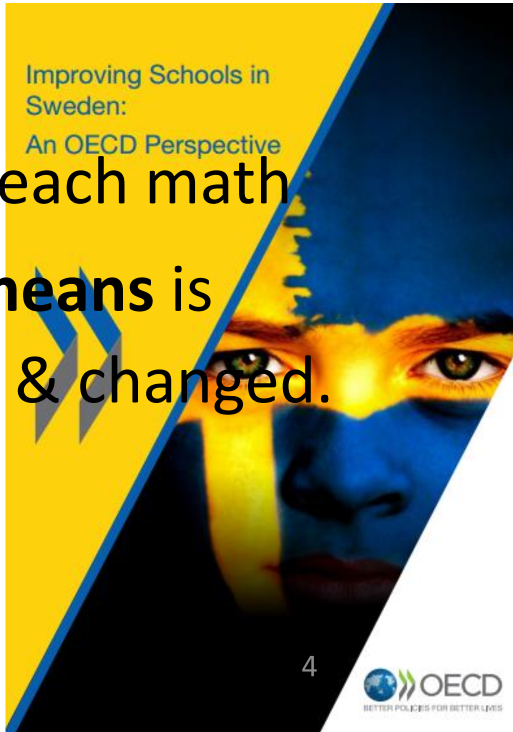
‘Theorize the poor PISA performance; and derive from this a STEM-based core curriculum for young male migrants.’

Didactics: Define one **goal** and several **means**.

The Tradition: The **goal** is to learn mathematics. The **means** is to teach math

A Difference: The **goal** is to master Many in space and time. The **means** is number-language sentences about how Many is counted & added & changed.

Prerequisites: None, start from scratch.



MATHeCDMY

Definitions of MatheMatics

Pythagoras: mathematics, meaning knowledge, is a common label for 4 areas describing Many by itself and in space & time:

- **Geometry** means to <u>measure earth</u> in Greek
- **Algebra** means to <u>reunite</u> numbers in Arabic

Around 1900, **SET** made mathematics self-referring. However, Russell said:

Self-reference leads to the classical liar paradox ‘this sentence is false’, being false if true & opp.

Just look at the set of sets, not belonging to itself. If $M = \{A \mid A \notin A\}$ then $M \in M \Leftrightarrow M \notin M$.

So, forget about sets, and forget about fractions as numbers, by self-reference they cannot be so.

Mathematics: Forget about Russell, he is not a mathematician. Of course fractions are numbers.

Two Different Mathematics

The ruling Set-based Top-Down **Meta-matics**

- Concepts are defined from above as **examples from abstractions**

a FUNCTION is <u>an example of</u> a set relation with component-1 identity implying component-2 identity

The silenced Many-based Bottom-Up **Many-math**

- Concepts are defined from below as **abstractions from examples**

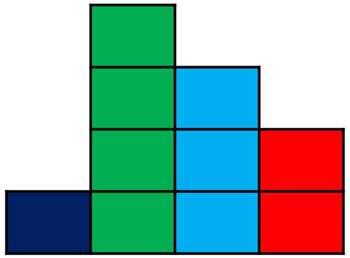
a FUNCTION is <u>for example</u> 2+x, but not 2+3;
i.e. a name for a calculation with an unspecified number

Children see Many as Bundles **with Units**

Asked ‘How old next time?’, a 3year-old says 4, but reacts when held together 2 by 2:

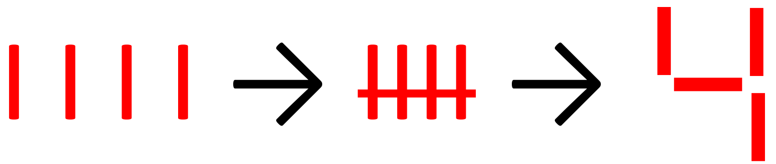
‘That is not 4, that is 2 **2s**’.

Seeing bundles as units, children use 2D LEGO-like **block-numbers**, not 1D **line-numbers**, taught in school, even if 2D Arabic block-numbers replaced 1D Roman line-numbers centuries ago.



T = 1 4 3 2

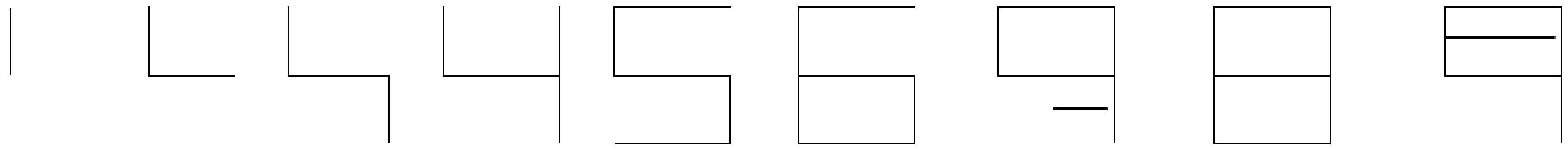
T = MCCCCXXXII

Many as Icons: 

Meeting Many, we ask: “**How Many in Total**?”
To answer, we Math … oops sorry, it’s a label, not an action word.
To answer, first we count, then we add. We name and iconize the degrees of Many until ten, that as 1 bundle has no icon or digit itself.

- Thus there are four sticks in a 4-icon, five in a 5-icon, etc.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9

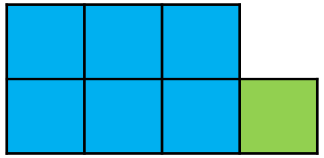


Operations as Icons also

A decimal point
parts inside bundles
from outside singles

We count by bundling & stacking:

T = 7 = I I I I I I I = III III I = III III I = 2B1 **3s** = 2.1 **3s**



- Thus, to count 7 in **3s** we take away 3 many times, iconized by an uphill stroke, 7/3, showing the broom wiping away the **3s**.

7/3	2.some
7 – 2x3	1

- A calculator predicts: 3 can be taken away 2 times. Stacking the bundles is iconized as a lift, 2x3.

- To look for unbundled singles, we drag away the stack of 2 **3s,** iconized by a horizontal trace: 7 – 2x3 = 1.

Counting creates 3 operations: to <u>divide</u> & to <u>multiply</u> & to <u>subtract</u>.

Totals as a Bundle Formula

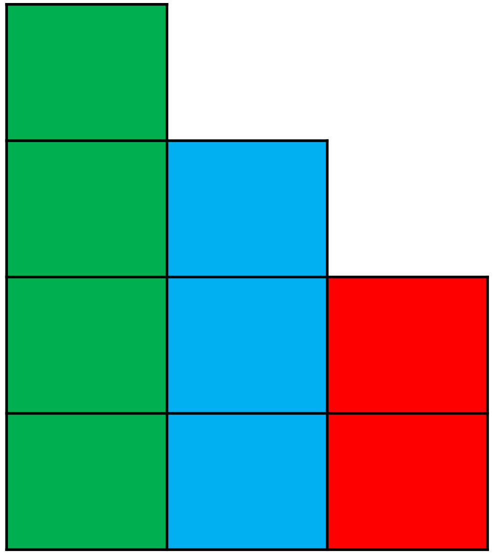
- To bundle bundles also, **power** is iconized as a cap, 5^2, showing the number of times bundles have been bundled.
- **Addition** is a cross + showing blocks juxtaposed next-to or on-top of each other.

Counting gives a Total as a **BundleFormula** called a polynomial.

Here all numbers have units:

T = 432 = 4***BundleBundle** + 3***Bundle** + 2*1

= 4***B^2** + 3***B** **+** 2*1

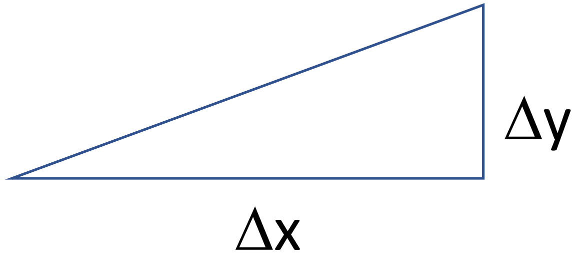


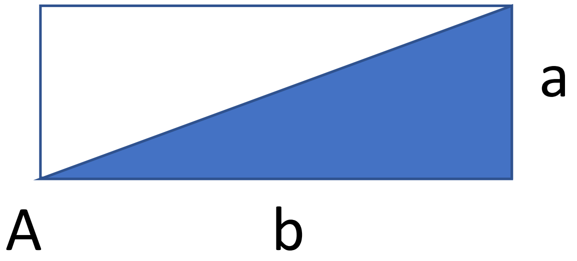
The ReCount Formula

7/3	2.some
7 – 2 * 3	1

Predicting T = 7 = 2.1 **3s**, the **ReCount formula T = (T/B)*B** saying 'from T, T/B times, B can be taken away', is all over:

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second





Trigonometry ReCounts Sides in a HalfBlock

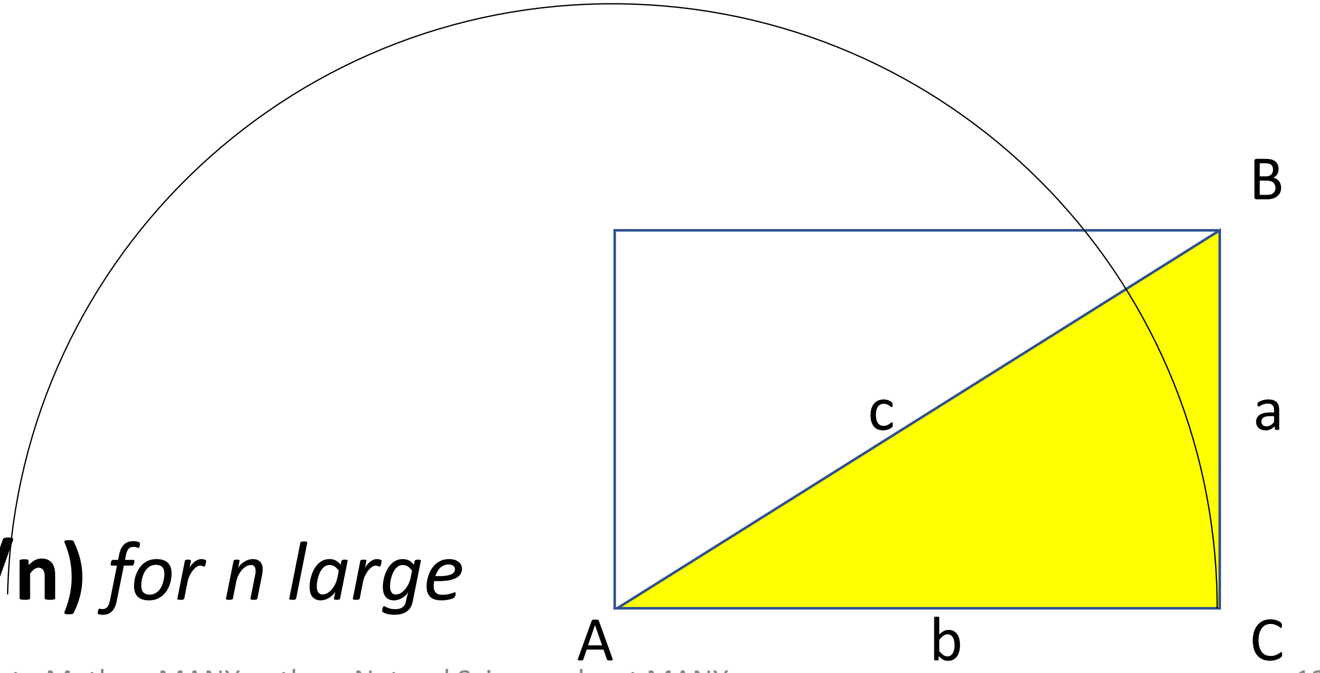
Halved by its diagonal, a block becomes a right angled triangle with three sides: the base b & the height a & the diagonal c, creating trigonometry by mutual recounting.

a = (**a/c**) * c = **sinA** * c

b = (**b/c**) * c = **cosA** * c

a = (**a/b**) * b = **tanA** * b

½Circle = **π = n*tan(180/n)** *for n large*



ReCounting creates Proportionality and Overloads & Underloads

4*5/6	3.some
4*5 – 3*6	2

ReCounting in a **new unit** changes units (**proportionality**)

T = 4 **5s** = ? **6s**. The ReCount-formula predicts T = 3.2 **6s**

ReCounting in the **same unit** creates overloads & underloads

T = 7 = I I I I I I I = ǂǂǂ ǂǂǂ I = ǂǂǂ I I I I = ǂǂǂ ǂǂǂ ǂǂǂ II

T = 7 = 2**B**1 **3s** = 1**B**4 **3s** = 3**B**-2 **3s**

BundleWriting may cure Math Dislike in classes stuck in Division:

T = 336 /7 = 33**B**6 /7 = 28**B**56 /7 = 4**B**8 = 48

Likewise:

Multiplication	T = 7* 48 = 7* 4**B**8 = 28**B**56 = 33**B**6 = 336
Subtraction	T = 53 – 28 = 5**B**3 – 2**B**8 = 3**B**-5 = 2**B**5 = 25
Addition	T = 53 + 28 = 5**B**3 + 2**B**8 = 7**B**11 = 8**B**1 = 81

ReCounting creates Multiplication & Equations

ReCounting <u>from icons to tens</u> is predicted by **Multiplication**

T = 5 **7s** = ? **tens** = 5*7 = 35 = 3.5 **tens**

ReCounting <u>from tens to icons</u> is predicted by **Equations**

T = ? **7s** = 42 = (42/7)*7 recounting 42 in 7s, so ? = 42/7

$$u*7 = 42 = (42/7)*7$$
$$u = 42/7 = 6$$

*An equation is solved by moving to <u>**O**pposite **S**ide with opposite **S**ign</u>*

7 x *u* = 42	Multiplication has 1 as its **neutral element**, and 7 has 1/7 as its **inverse element**
(7 x *u*) x (1/7) = 42x(1/7)	Multiplying 7’s inverse element 1/7 to both [illegible]ber-names
(*u* x 7) x (1/7) = 6	Applying the **commutative law** to u x 7; 6 is the s[illegible]umb[illegible]ne for 42 x 1/7
u x (7 x (1/7)) = 6	Applying the **associative law**
u x 1 = 6	Applying the definition of an inverse element
u = 6	Applying the definition of a neutral element[illegible]*h arrows a test is* [illegible] *needed.*

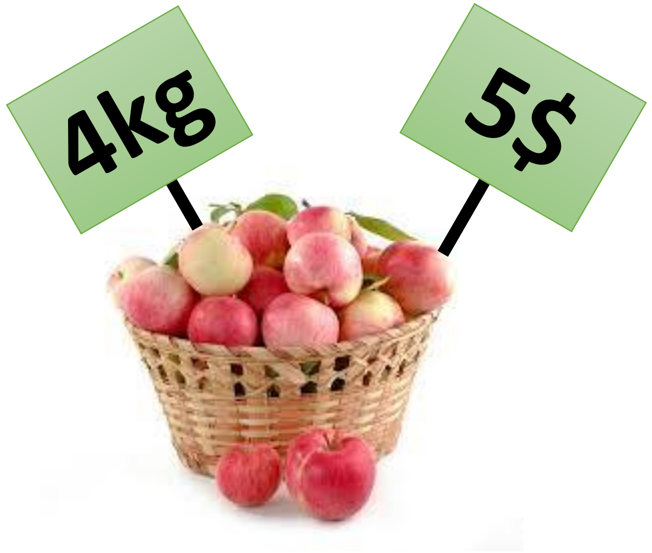
DoubleCounting in 2 units creates PerNumbers

Apples are double-counted in **kg** and in **$**.

With 4**kg = 5$** we have the **PerNumber** 4kg/5$ = 4/5 kg/$

Questions:

12**kg = ?$**	20**$ = ?kg**
12kg = (12/4)*4kg = (12/4)*5$ = 15$	20$ = (20/5)*5$ = (20/5)*4kg = 16kg



Answer: Recount in the per-number

- With like units, per-numbers become fractions: 2$ per 5$ = 2$/5$ = 2/5

The BundleFormula T = 432 = 4*B^2 + 3*B + 2*1 shows the 4 ways, Many Unite (*the Simplicity of Math*)

Many exists as **changing** & **constant block-numbers** & **per-numbers**

- Addition & Multiplication unite changing & constant block-numbers
  Subtraction & Division split into changing & constant block-numbers
- Integration & Power unite changing & constant per-numbers
  Differentiation & Root/Logarithm split into changing & constant per-numbers

Operations **unite** / *split into*	**Changing**	**Constant**
Block-numbers *m, s, $, kg*	T = a + n *T – a = n*	T = a*n *T/n = a*
Per-numbers *m/s, $/kg, m/(100m) = %*	T = ∫ a dn *dT/dn = a*	T = a^n *$\log_a T = n$, $\sqrt[n]{T} = a$*

Theorizing Poor PISA Performance

Poor PISA performance is caused by 4 blind spots:

- Mathematics should respect its nature as a NumberLanguage with 3part sentences (subject-verb-predicate) and a grammar, as in the WordLanguage.
- Seen as a goal in itself, math hides its outside goal, to master Many, so we teach TopDown MetaMatics instead of BottomUp ManyMath
- We use 1D line-numbers instead of 2D block-numbers with 3 numbers: the size of the bundle & the number of bundles & the number of unbundled – and they add differently
- By this complexity, addition OnTop and NextTo should be postponed to after BundleCounting & ReCounting & DoubleCounting in STEM-tasks

1D

+
-
x
/

2D

/
x
-
+

STEM (**S**cience**T**ech**E**ng**M**ath) based Core Curriculum for Migrants

Nature consists of things in motion, combined in momentum = mass*velocity

Things contain **mass** & **molecules** & **eclectric charge**.

Nature is counted in **meter** & **second** & **kilogram** & **mole** & **coulomb**.

Nature is predictable by ReCounting & PerNumbers:

kilogram = (**kilogram/cubic-meter**) * cubic-meter = **density** * cubic-meter

meter = (**meter/second**) * second = **velocity** * second

Δ momentum = (**Δ momentum/second**) * second = **force** * seconds

Δ energy= (**Δ energy/meter**) * meter = **force** * meter = work

*Energy = ½*mass*velocity squared*

PerNumbers/ReCounting in ScienceTechEngMath II

Energy = (**energy/kg**) * kg = **melting/evaporation heat** * kg

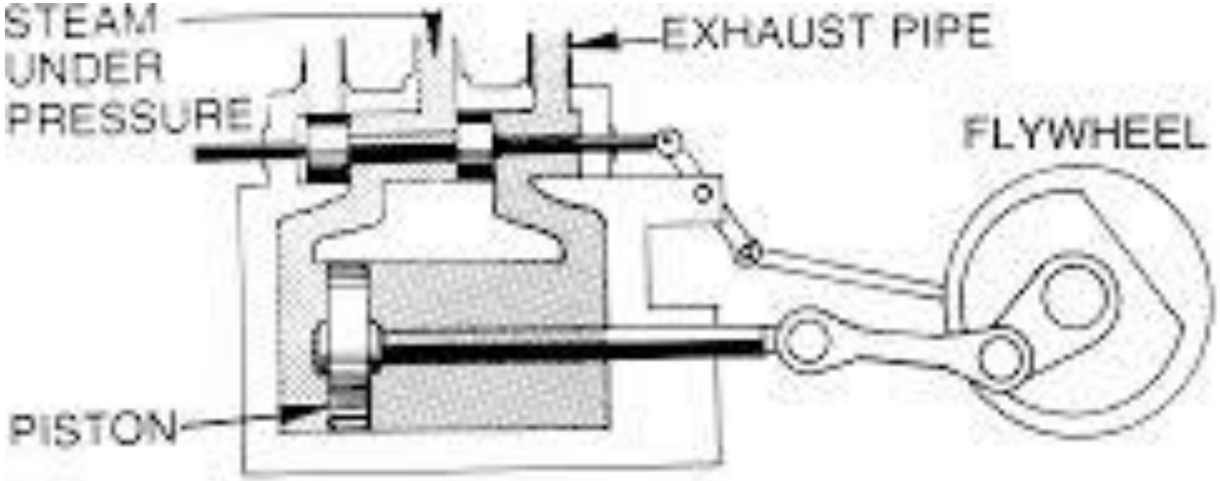
Energy = (**energy/kg/degree**) * kg * degree = **heat** * kg * degree

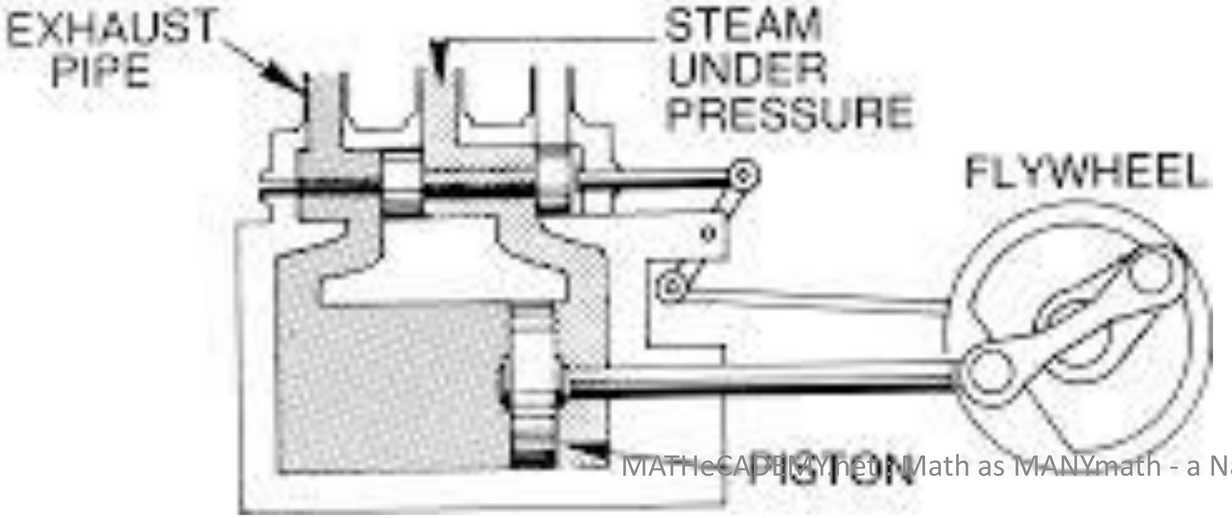
force = (**force/square-meter**) * square-meter = **pressure** * square-meter

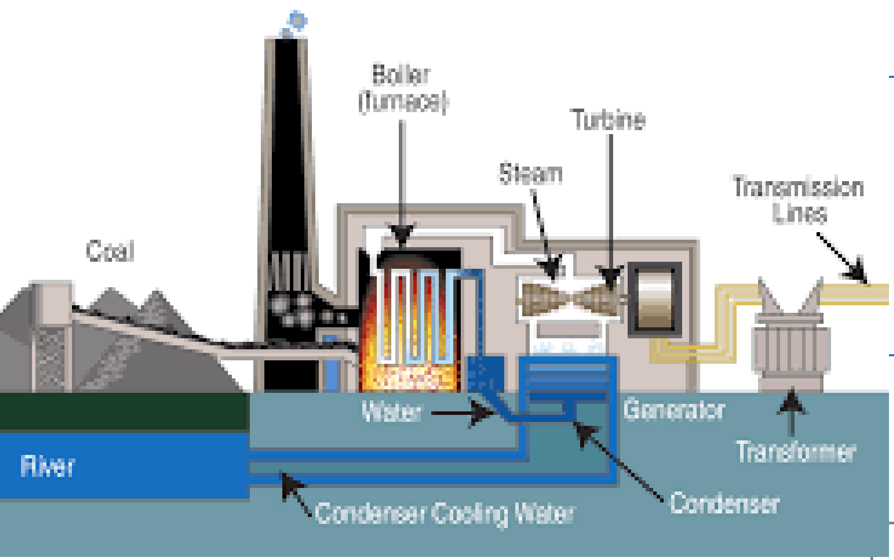
gram = (**gram/mole**) * mole = **molar mass** * mole

mole = (**mole/liter**) * liter = **molarity** * liter

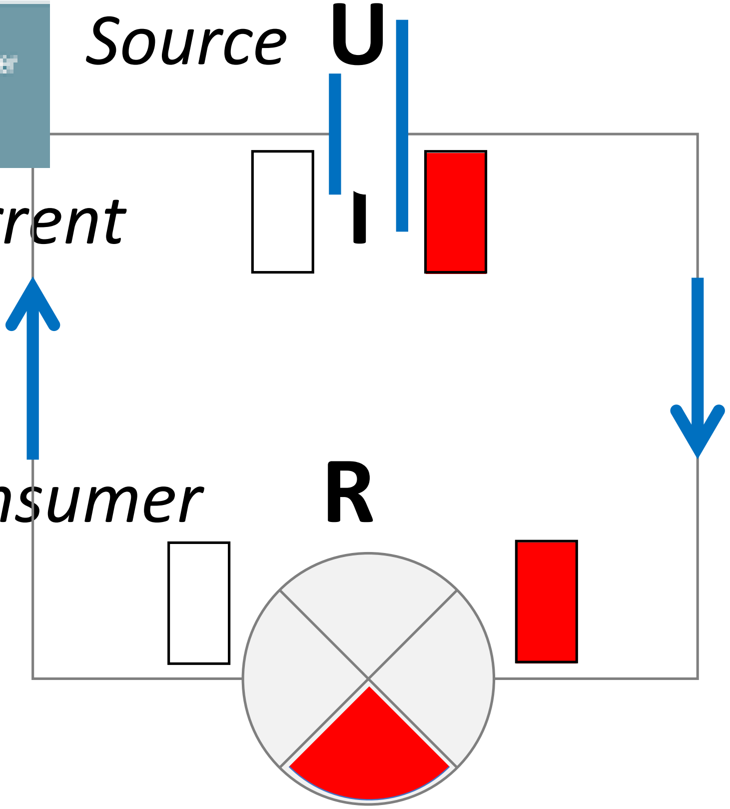
Technology I
Steam at Work: **p*V = n*R*T**







Technology II
Electrons at Work: **P= U*I & U = R*I**



Source **U** — **Volt** = Energy/Coulomb

Current **I** — **Ampere** = Coulomb/second

Consumer **R** — **Resistance** in Ohm

Consumption **P** — **Watt** = Energy/second

Engineering
How many turns up a steep hill?

On a 30 degree hillside, a 10 degree road is to be constructed.
How many turns will there be on a 1 x 1 km hillside?

- We let A and B label the ground corners of the hillside. C labels the point where a road from A meets the edge for the first time, and D is vertically below C on ground level. We want to find the distance BC = u.
- In the triangle BCD, the angle B is 30 degrees, and BD = u*cos(30). With Pythagoras we get u^2 = CD^2 + BD^2 = CD^2 + u^2*cos(30)^2, or CD^2 = u^2(1-cos(30)^2) = u^2*sin(30)^2.
- In the triangle ACD, the angle A is 10 degrees, and AD = AC*cos(10). With Pythagoras we get AC^2 = CD^2 + AD^2 = CD^2 + AC^2*cos(10)^2, or CD^2 = AC^2(1-cos(10)^2) = AC^2*sin(10)^2.
- In the triangle ACB, AB = 1 and BC = u, so with Pythagoras we get AC^2 = 1^2 + u^2, or AC = √(1+u^2).
- Consequently, u^2*sin(30)^2 = AC^2*sin(10)^2, or u = AC*sin(10)/sin(30) = AC*r, or u = √(1+u^2)*r, or u^2 = (1+u^2)*r^2, or u^2*(1-r^2) = r^2, or u^2 = r^2/(1-r^2) = 0.137, giving the distance BC = u = √0.137 = 0.37.

Thus, there will be 2 turns: 370 meter and 740 meter up the hillside.

The Simplicity of Mathematics reveals a Core Curriculum

To Master Many: ReCount in Block- & Per-numbers

Postpone or Drop Addition

Thank You for Listening

Slides & full paper on
MATH**e**CADEMY.net

Details in
Journal of Mathematics Education
vol. 11 #1

DifferenceResearch finds Differences making a Difference, inspired by

- The ancient Greek sophists:

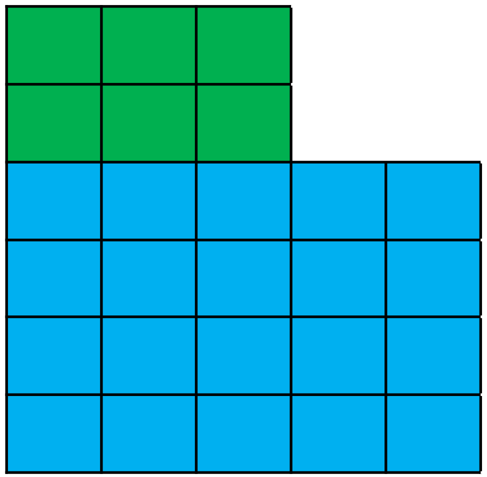
Differences unmask choice masked as nature

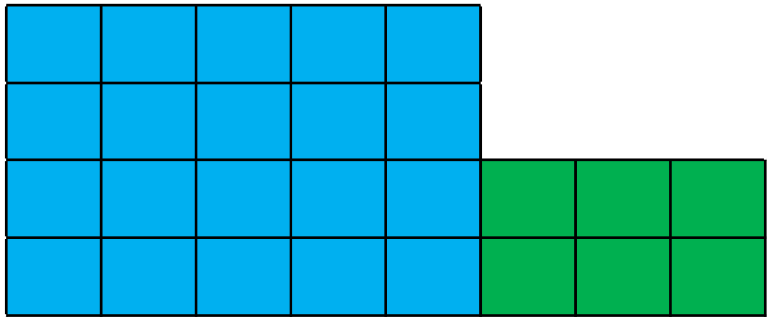
- In existentialism, Sartre: *EXISTENCE precedes ESSENCE.* Heidegger: *In sentences, the SUBJECT exists, but the PREDICATE is essence that often can be different.*

Let’s meet the subject, **MANY**, directly & outside its ‘essence-prison’

Totals Add OnTop & NextTo

OnTop	NextTo
4 5s + 2 3s = 4 5s + 1B1 5s = 5B1 5s	4 5s + 2 3s = 3B2 8s
The units are changed to be the same *Change unit = Proportionality*	The areas are added *Adding areas = Integration*





PerNumbers & Fractions add as **Integral Calculus**

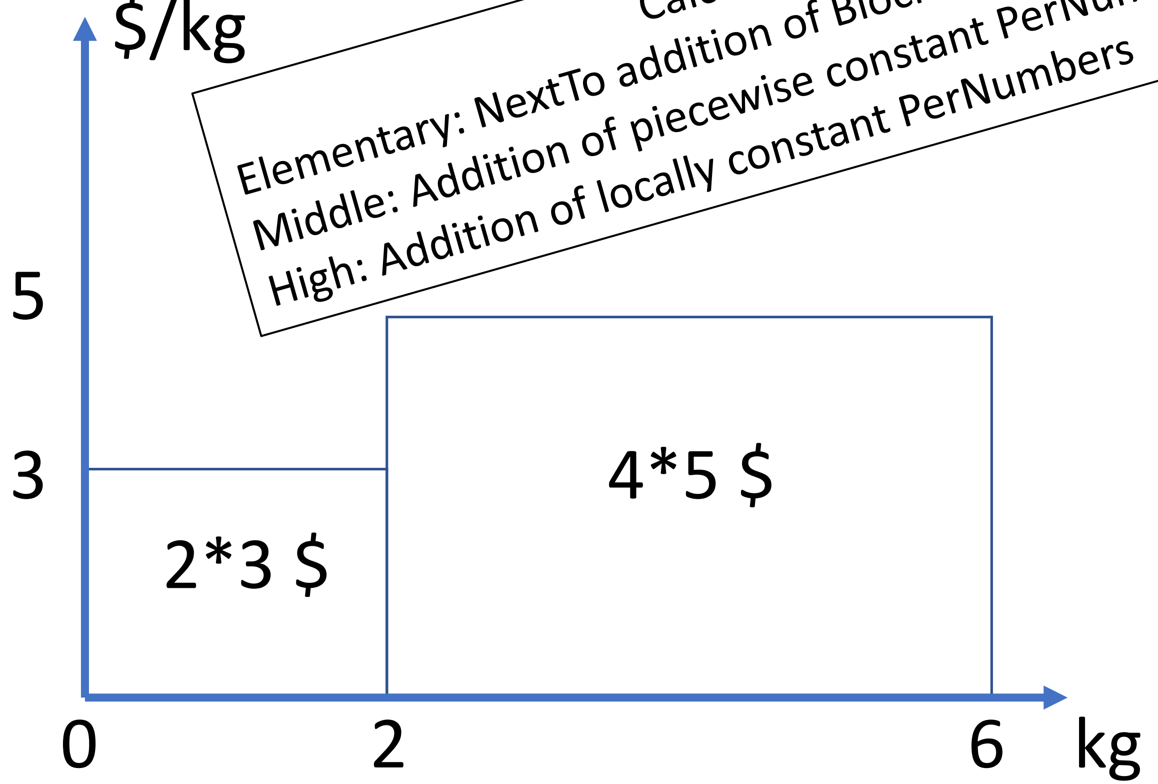
2 kg at 3 $/kg

+ 4 kg at 5 $/kg

(2+4) kg at ? $/kg

Unit-numbers add on-top.

Per-numbers add next-to as **areas** under the per-number graph, i.e. as **integral calculus**



Calculus:
Elementary: NextTo addition of BlockNumbers
Middle: Addition of piecewise constant PerNumbers
High: Addition of locally constant PerNumbers