50 YEARS OF INEFFECTIVE MATH EDUCATION RESEARCH, WHY? OOPS, WRONG NUMBERS, SORRY

Oral Communication, Allan Tarp, MATHeCADEMY.net

Rejected at the PME42

Many countries face poor PISA results. Does its nature make mathematics so hard to learn despite 50 years of research? We need to read again the two founding fathers.

Freudenthal sees set-based university mathematics as so important to the outside world that it must be taught in schools. Skemp sees a true understanding of mathematics as based upon sets compared as to cardinality by, not counting them, but by establishing a correspondence between them.

Sociology points to a different explanation: maybe mathematics education has a goal displacement where it sees itself as the goal, and its outside root, Many, as a means.

So we may ask: as an alternative to the set-based tradition, is there is a different way to the outside goal of mathematics education, mastery of Many?

By observing the quantitative competences children bring to school, and by using difference-research searching for differences making a difference, we discover an alternative to the present set-based mathematics that was introduced some 50 years ago as ‘New Math’: a ‘many-matics’ seeing mathematics as a natural science about Many.

Here digits are icons with as many sticks as they represent. Also operations are icons where bundle-counting produces two-dimensional block-numbers, ready to be re-counted in the same unit to remove or create overloads or underloads to make operations easier; or in a new unit, later called proportionality; or to and from tens rooting multiplication tables and solving equations.

Here double-counting in two units creates per-numbers, becoming fractions with like units; both being, not numbers, but operators needing numbers to become numbers.

Addition here occurs on-top and next-to rooting proportionality, and integral calculus by adding areas; and here trigonometry precedes plane and coordinate geometry.

So, we need to research what happens if two-dimensional block-numbers replace one-dimensional line-numbers; if the order of operations is reversed; if bundle-counting, re-counting and double-counting precedes adding next-to and on-top; and, if using full sentences about the total in the number-language, as T = 2.1 3s with a subject, a verb and a predicate as in the word-language.

References

Freudenthal, H. (1973). *Mathematics as an Educational Task.* Dordrecht-Holland: D. Reidel Publishing Company.

Skemp, R. R. (1971). *The Psychology of Learning Mathematics.* Middlesex, UK: Penguin Books.

2018. In NNN (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. XX-YY. Umeå, Sweden: PME.

RETHINKING LINE-NUMBER ARITHMETIC AS BLOCK-NUMBER ALGEBRA

Poster Presentation, Allan Tarp, MATHeCADEMY.net

Rejected at the PME42

‘Early Algebra’ recommends further research in seven areas. Apart from rethinking the examples included, this poster addresses area 2 and 4, curricula activity and theorizing numbers and operations. Using Difference-research, searching for differences making a difference, the poster asks: Will a different block-number algebra allow rethinking traditional line-number arithmetic?

As sceptical thinking from the French and American Enlightenment republics, Foucault Concept Archaeology and Existentialism and Grounded Theory is used to look at the roots of Algebra.

In Arabic, Algebra means to reunite. Numbers as T = 345 = 3*B^2+4*B+5*1 show the four ways to unite a total: addition, multiplication, power and integration of juxtaposed blocks. They also show that asking ‘How many in total?’, the answer is expressed as a ‘number-language’ sentence containing, as does a word-language sentence, a subject and a verb and a predicate, thus rooting a formula with an equation sign.

Using grounded theory we observe that, before receiving formal education, preschool children use 2dimensional Bundle-numbers or Block-numbers as T = 2 5s & 1. Re-counting a total into a new unit by asking T = 3 4s = ? 5s, children quickly accept division as an icon for a broom wiping away 5-bundles, and multiplication as an icon for stacking the bundles into a block, and subtraction for the trace left when dragging away the block to look for unbundled leftover singles.

Likewise, children find it natural to formulate the recounting process as ‘from a total T, T divided by the bundle B gives the number of times Bs can be taken away’, shortened to a ‘recount-formula’ T = (T/B)*B. This formula allows using a calculator to predict the result of a re-counting process: Recounting 7 in 3s, we enter 7/3. The answer ‘2.some’ predicts it can be done 2 times. Taking away the stack of 2 3s, the answer ‘7-2x3 = 1’ shows the prediction: 7 can be re-counted as 2 3s & 1.

The recount-formula T = (T/B)*B leads directly to the heart of Algebra by allowing children to use formulas as a natural way to communicate in math education. The commutative, associative and distributive laws follow directly from watching 2- and 3-dimensinal blocks.

And solving equations takes place when re-counting from tens to icons: asking T = 40 = ? 8s the solution is found by re-counting 40 in 8s as 40 = (40/8)*8 = 5*8 = 5 8s.

References

Kieran, C., Pang J. S., Schifter, D. & Ng, S. F. (2016). *Early algebra: research into its nature, its learning, its teaching (ICME 13 topical surveys)*. Hamburg: Springer Open.