A TWIN CURRICULUM SINCE CONTEMPORARY MATHEMATICS MAY BLOCK THE ROAD TO ITS EDUCATIONAL GOAL, MASTERY OF MANY

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Mathematics education research still leaves many issues unsolved after half a century. Since it refers primarily to local theory we may ask if grand theory may be helpful. Here philosophy suggests respecting and developing the epistemological mastery of Many children bring to school instead of forcing ontological university mathematics upon them. And sociology warns against the goal displacement created by seeing contemporary institutionalized mathematics as the goal needing eight competences to be learned, instead of aiming at its outside root, mastery of Many, needing only two competences, to count and to unite, described and implemented through a guiding twin curriculum.

POOR PISA PERFORMANCE DESPITE FIFTY YEARS OF RESEARCH

Being highly useful to the outside world, mathematics is a core part of institutionalized education. Consequently, research in mathematics education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 year since 1969. However, despite increased research and funding, the former model country Sweden has seen its PISA result decrease from 2003 to significantly below the OECD average in 2012, causing OECD (2015) to write the report 'Improving Schools in Sweden'. Likewise, math dislike seems to be widespread in high performing countries also. With mathematics and education as social institutions, grand theory may explain this 'irrelevance paradox', the apparent negative correlation between research and performance.

GRAND THEORY

Ancient Greece saw two forms of knowledge, 'sophy'. To the sophists, knowing nature from choice would prevent being patronized by choice presented as nature. To the philosophers, choice was an illusion since the physical is but examples of metaphysical forms only visible to the philosophers educated at Plato's Academy. Christianity eagerly took over metaphysical patronage and changed the academies into monasteries. The sophist skepticism was revived by Brahe and Newton insisting that knowledge about nature comes from laboratory observations, not from library books (Russell, 1945).

Newton's discovery of a non-metaphysical changing will spurred the Enlightenment period: When falling bodies follow their own will, humans can do likewise and replace patronage with democracy. Two republics arose, in the United States and in France. The US still has its first Republic, France its fifth, since its German speaking neighbors tried to overthrow the French Republic again and again.

In North America, the sophist warning against hidden patronization is kept alive in American pragmatism, symbolic interactionism and Grounded theory, the method of natural research resonating with Piaget's principles of natural learning. In France, skepticism towards our four fundamental institutions, words and sentences and cures and schools, is formulated in the poststructuralist thinking of Derrida, Lyotard, Foucault and Bourdieu warning against institutionalized categories, correctness, diagnosed cures, and education; all may hide patronizing choices presented as nature (Lyotard, 1984).

Within philosophy itself, the Enlightenment created existentialism (Marino, 2004) described by Sartre as holding that existence precedes essence, exemplified by the Heidegger warning: In a sentence, trust the subject, it exists; doubt the predicate, it is essence coming from a verdict or gossip.

The Enlightenment also gave birth to sociology. Here Weber was the first to theorize the increasing goal-oriented rationalization that de-enchant the world and create an iron cage if carried to wide. Mills (1959) sees imagination as the core of sociology. Bauman (1990) agrees by saying that sociological thinking "renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now" (p. 16). But he also formulates a warning (p.84): "The ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called goal displacement. (..) The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right".

As to what we say about the world, Foucault (1995) focuses on discourses about humans that, if labeled scientific, establish a truth regime. In the first part of his work he shows how a discourse disciplines itself by only accepting comments to already accepted comments. In the second part he shows how a discourse disciplines its subject also by locking humans up in a predicate prison of abnormalities from which they can only escape by accepting the diagnose and cure offered by the 'pastoral power' of the truth regime. Foucault thus sees a school as a 'pris-pital' mixing the power techniques of a prison and a hospital: the 'pati-mates' must return to their cells daily and accept the diagnose 'un-educated' to be cured by, of course, education as defined by the ruling discourse regime.

Mathematics, stable until the arrrival of SET

In ancient Greece, the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: geometry, arithmetic, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many in space, Many by itself, Many in time and Many in space and time; and together forming the 'quadrivium' recommended by Plato as a general curriculum together with 'trivium' consisting of grammar, logic and rhetoric.

With astronomy and music as independent areas, mathematics became a common label for the two remaining activities, geometry and algebra, both rooted in the physical fact Many through their original meanings, 'to measure earth' in Greek and 'to reunite' in Arabic. And in Europe, Germanic countries taught 'reckoning' in primary school and 'arithmetic' and 'geometry' in the lower secondary school until about 50 years ago when they all were replaced by the 'New Mathematics'.

Here a wish for exactness and unity created a SET-derived 'meta-matics' as a collection of 'well-proven' statements about 'well-defined' concepts, defined top-down as examples from abstractions instead of bottom-up as abstractions from examples. But Russell showed that the self-referential liar paradox 'this sentence is false', being false if true and true if false, reappears in the set sets not belonging to itself, where a set belongs only if it does not: If $M = \{A \mid A \notin A\}$ then $M \in M \Leftrightarrow M \notin M$. The Zermelo–Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating concrete examples from abstract concepts.

SET thus transformed classical grounded 'many-matics' into today's self-referring 'meta-matism', a mixture of meta-matics and 'mathe-matism' true inside but seldom outside a classroom where adding numbers without units as '1 + 2 IS 3' meet counter-examples as e.g. 1week + 2days is 9days.

Mathematics as grand theory

Philosophically, we can ask if Many should be seen ontologically, what it is in itself, or epistemologically, how we perceive and verbalize it. University mathematics holds that many should be treated as cardinality that is linear by its ability to always absorb one more. However, in human number-language, many is a union of blocks coming from counting singles, bundles, bundles of bundles etc., $T = 345 = 3*BB+4*B+5*1$, resonating with what children bring to school, e.g. T = 2 5s.

Likewise, we can ask: in a sentence what is more important, that subject or what we say about it. University mathematics holds that both are important if well-defined and well-proven; and both should be mediated according to Vygotskian psychology. Existentialism holds that existence precedes essence, and Heidegger even warns against predicates as possible gossip. Consequently, learning should come from openly meeting the subject, Many, according to Piagetian psychology.

Sociologically, a Weberian viewpoint would ask if SET is a rationalization of Many gone too far leaving Many de-enchanted and the learners in an iron cage. A Baumanian viewpoint would suggest that, by monopolizing the road to mastery of Many, contemporary university mathematics has created a goal displacement. A Foucaultian viewpoint, seeing education as a pris-pital, would ask, first which structure to choose, European line-organization forcing a return to the same cell after each hour, day and month for several years; or the North American block-organization changing cell each hour, and changing the daily schedule twice a year. Next, that by rejecting the relevance of researching whether the diagnose 'mathematical ignorant' may be cured or not with mathematics, we will be able to see that the road to the goal, mastery of Many, is to develop the existing mastery children bring to school. And, to cure we must be sure the diagnose is not self-referring. So, the word 'mathematics' must go.

Meeting Many, children use block-numbers to count and share

How to master Many can be learned from preschool children. Asked 'How old next time?', a 3year old will say 'Four' and show 4 fingers; but will react strongly to 4 fingers held together 2 by 2, 'That is not 4, that is 2 2s', thus describing what exists, bundles of 2s, and 2 of them.

Children also use block-numbers when talking about Lego bricks as '2 3s' or '3 4s'. When asked 'How many 3s when united?' they typically say '5 3s and 3 extra'; and when asked 'How many 4s?' they may say '5 4s less 2'; and, placing them next-to each other, they typically say '2 7s and 3 extra'.

Children have fun recounting 7 sticks in 2s in various ways, as 1 2s &5, 2 2s &3, 3 2s &1, 4 2s less 1, 1 4s &3, etc. And children don't mind writing a total of 7 using 'bundle-writing' as $T = 7 = 1B5 = 2B3 = 3B1 = 4B\cancel{1}$; or even as 1*BB*3 or 1*BB*1*B*1. Also, children love to count in 3s, 4s, and in hands.

Sharing 9 cakes, 4 children takes one by turn saying they take 1 of each 4. Taking away 4s roots division as counting in 4s. And with 1 left they often say 'let's count it as 4'. Thus 4 preschool children typically share by taking away 4s from 9, and by taking away 1 per 4, and by taking 1 of 4 parts. And they smile when seeing that entering '9/4' allows a calculator to predict the sharing result as 2 1/4; and when seeing that entering '2x5/3' will predict the result of sharing 2 5s between 3 children.

Children thus master sharing, taking parts and splitting into parts before division and counting- and splitting-fractions is taught; which they may like to learn before being forced to add without units.

So why not develop instead of rejecting the core mastery of Many children bring to school?

Proportionality illustrates the variety of mastery and quantitative competence

Proportionality is rooted in the question: "2kg costs 5$, what does 7kg cost; and what does 12$ buy?"

In Europe, the 'Regula de Tri' (rule of three) was used until around 1900: arrange the four numbers with alternating units and the unknown at last. Then from behind, first multiply, the divide.

So first we ask, Q1: '2kg cost 5$, 7kg cost ?$' to get to the answer (7*5/2)$ = 17.5$. Then we ask, Q2: '5$ buys 2kg, 12$ buys ?kg' to get to the answer (12*2)/5$ = 4.8kg.

Then, until the arrival of SET, two methods were common: 'find the unit', and cross multiplication in an equation expressing like proportions or ratios:

Q1: 1kg costs 5/2$, so 7kg cost (5/2)*7 = 17.5$. Q2: 1$ buys 2/5kg, so 12$ buys (2/5)*12 = 4.8kg.

Q1: 2/5 = 7/x, so 2*x = 5*7, x = (5*7)/2 = 17.5. Q2: 2/5 = x/12, so 5*x = 2*12, x = (2*12)/5 = 4.8.

SET chose modeling with linear functions to show the relevance of group theory in abstract algebra: Let us define a linear function $f(x) = c*x$ from the set of kg-numbers to the set of $-numbers, having as domain $DM = \{x \in R \mid x>0\}$. Knowing that $f(2) = 5$ we set up the equation $f(2) = c*2 = 5$ to be solved by multiplying with the inverse element to 2 on both sides and applying the associative law: $c*2 = 5$, $(c*2)*½ = 5*½$, $c*(2*½) = 5/2$, $c*1 = 5/2$, $c = 5/2$. With $f(x) = 5/2*x$, the inverse function is f-1(x) = 2/5*x. So with 7kg, $f(7)$ = 5/2*7 = 17.5$; an with 12$, f-1(12) = 2/5*12 = 4.8kg.

In the future maybe, the recount-formula recounts in the per-number coming from double-counting:

Q1: 7kg = (7/2)*2kg = (7/2)*5$ = 17.5$; Q2: 12$ = (12/5)*5$ = (12/5)*2kg = 4.8kg

A typical contemporary mathematics curriculum

Typically, the core of a curriculum is how to operate on specified and unspecified numbers. Digits are given directly as symbols without letting children discover them as icons with as many strokes or sticks as they represent. Numbers are given as digits respecting a place value system without letting children discover the thrill of bundling, counting both singles and bundles and bundles of bundles. Seldom 0 is included as 01 and 02 in the counting sequence to show the importance of bundling. Never children are told that eleven and twelve comes from the Vikings counting '(ten and) 1 left', '(ten and) 2 left'. Never children are asked to use full number-language sentences, T = 2 5s, including both a subject, a verb and a predicate with a unit. Never children are asked to describe numbers after ten as 1.4 tens with a decimal point and including the unit. Renaming 17 as 2.-3 tens and 24 as 1*B*14 tens is not allowed. Adding without units always precede bundling iconized by division, stacking iconized by multiplication, and removing stacks to look for unbundled singles iconized by subtraction. In short, children never experience the enchantment of counting, recounting and double-counting Many before adding. So, to re-enchant Many will be an overall goal in a twin curriculum in mastery of many through developing the children's existing mastery and quantitative competence.

A QUESTION GUIDED COUNTING CURRICULUM

The question guided re-enchantment curriculum in counting could be named 'Mastering Many by counting, recounting and double-counting'. The design is inspired by Tarp (2018). It accepts that while eight competencies might be needed to learn university mathematics (Niss, 2003), only two are needed to master Many (Tarp, 2002), counting and uniting, motivating a twin curriculum. The corresponding pre-service or in-service teacher education can be found at the MATHeCADEMY.net. Remedial curricula for classes stuck in contemporary mathematics may be found in Tarp (2017).

Q01, icon-making: "The digit 5 seems to be an icon with five sticks. Does this apply to all digits?" Here the learning opportunity is that we may change many ones to one icon with as many sticks or strokes as it represents if written in a less sloppy way. Follow-up activities could be rearranging four dolls as one 4-icon, five 5 cars as one 5-icon, etc.; followed by rearranging sticks on a table or on a paper; and by using a folding ruler to construct the ten digits as icons.

Q02, counting sequences: "How to count fingers?" Here the learning opportunity is that five fingers can also be counted "01, 02, 03, 04, Hand" to include the bundle; and ten fingers as "01, 02, Hand less2, Hand-1, Hand, Hand&1, H&2, 2H-2, 2H-1, 2H". Follow-up activities could be counting things.

Q03, icon-counting: "Can the five fingers be counted in different ways?" Here the learning opportunity is that fingers can be bundle-counted as singles or as pairs (or triplets) allowing both an overload and an underload; and reported in a full number-language sentence with subject, verb and predicate: The total is fives ones, T = 5 1s = 1Bundle3 2s = 2B1 2s = 3B-1 2s = 1BB1, called a bundle- or block-number. The total is shown on a western IKEA abacus in geometry 'space-mode' with the 2 2s on the second and third bar and 1 on the first bar; and in algebra 'time-mode' with 2 on the second bar and 1 on the first bar. Follow-up activities could be counting ten fingers in 3s.

Q04, calculator-prediction: "How can a calculator predict a counting result?" Here the learning opportunity is to see the division sign as an icon for a broom wiping away the bundles: 5/2 means 'from 5, wipe away bundles of 2s'. The calculator answer '2.some' predicts it can be done 2 times. Now the multiplication sign iconizes a lift stacking the bundles into a block. Finally, the subtraction sign iconizes the trace left when dragging away the block to look for unbundled singles. By showing '5-2x2 = 1' the calculator indirectly predicts that a total of 5 1s can be recounted as 2B1 2s. An additional learning opportunity is to write and use the 'recount-formula' $T = (T/B)\text{x}B$ saying 'From T, T/B times B can be taken away.' This formula is also called the proportionality formula occurring all over mathematics and science. Follow-up activities could be counting e.g. 7 in the same way.

Q05, unbundled singles as decimals or fractions: "Where to put the unbundled when using blocks?" Here the learning opportunity is to see that geometrically with blocks, the unbundled can be placed in two ways: next-to the stack as a stack of its own, written as T = 5 = 2.1 2s using a decimal point to separate the bundles from the singles; or on-top as a part of the bundle, written as T = 5 = 2 ½ 2s to show that the unbundled still needs to be counted in 2s. This gives a grounded introduction to decimal numbers and fractions. Follow-up activities could be counting e.g. 7 in the same way.

Q06, prime or folded units: "With fully stacked totals, which units can and cannot be folded?" Here the learning opportunity is to examine the stability of a block that is fully stacked without single leftovers. Writing e.g. T = 2 4s = 2x4, the bundle size 4 is called the unit. Turning over the block, it becomes 4 2s instead, T = 4x2, now with 2 as the unit. Looking at units, 4 can be folded in another unit as 2 2s, whereas 2 cannot be folded since 1 is not a real unit. Consequently, we call 2 a 'prime unit' and 4 a 'folded unit', 4 = 2 2s. Thus, a block of 3 2s cannot be changed by folding, whereas a block of 3 4s can be changed to 6 2s: T = 3 4s = 3 x (2x2) = (3x2) x 2 = 6 2s. Follow-up activities could be examining other fully stacked blocks to make a list of prime units and stable blocks.

Q07, finding units: “What are possible units in *T* = 12?”. Here the learning opportunity is that units come from factorizing in prime units, *T* = 12 = 2*2*3. Follow-up activities could be other examples.

Q08, recounting in another unit: “How to change a unit?” Here the learning opportunity is to observe how the recount-formula changes the unit. Asking e.g. *T* = 3 4s = ? 5s, the recount-formula says *T* = 3 4s = (3x4/5) 5s. Entering 3x4/5, the answer ‘2.some’ shows that a stack of 2 5s can be taken away. Entering 3x4 – 2x5, the answer ‘2’ shows that 3 4s can be recounted in 5s as 2.2 5s. Follow-up activities could be other examples of changing from one icon-unit to another.

Q09, recounting from tens to icons: “How to change unit from tens to icons?” Here the learning opportunity is that asking ‘*T* = 2.4 tens = 24 = ? 8s’ can be formulated as an equation using the letter *u* for the unknown number, *u*x8 = 24. This is easily solved by recounting 24 in 8s as 24 = (24/8)x8 so that the unknown number is *u* = 24/8 attained by moving 8 to the opposite side with the opposite sign. Follow-up activities could be other examples of recounting from tens to icons.

Q10, recounting from icons to tens: “How to change unit from icons to tens?” Here the learning opportunity is that if asking ‘T = 3 7s = ? tens’, the recount-formula cannot be used since the calculator has no ten-button. It is however programmed to give the answer directly by using multiplication alone: *T* = 3 7s = 3x7 = 21 = 2.1 tens, only it leaves out the unit and misplaces the decimal point. An additional learning opportunity using less-numbers geometrically on an abacus, or algebraically with brackets: *T* = 3x7 = 3 x (ten less 3) = 3 x ten less 3x3 = 3ten less 9 = 3ten less (ten less1) = 2ten less less 1 = 2ten1 = 21. Follow-up activities could be other examples of recounting from icons to tens.

Q11, double-counting in two units: “How to double-count in two different units?” Here the learning opportunity is to observe how double-counting in two physical units creates ‘per-numbers’ as e.g. 2$ per 3kg, or 2$/3kg. To answer questions we just recount in the per-number: Asking ‘6$ = ?kg’ we recount 6 in 2s: 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. And vice versa, asking ‘?$ = 12kg’, the answer is 12kg = (12/3)*3kg = (12/3)*2$ = 8$. Follow-up activities could be many other and different examples double-counting in two units since per-numbers and proportionality are core concepts.

Q12, double-counting in the same unit: “How to double-count in the same unit?” Here the learning opportunity is that if double-counted in the same unit, per-numbers take the form of fractions, 3$ per 5$ = 3/5; or percentages, 3 per hundred = 3/100 = 3%. Thus, to find a fraction or a percentage of a total, again we just recount in the per-number. Also, we observe that per-numbers and fractions are not numbers, but operators needing a number to become a number. Follow-up activities could be many other examples of double-counting since fractions and percentages are core concepts.

Q13, recounting the sides in a block. “How to recount the sides in a block halved by its diagonal?” Here the learning opportunity is that in a block with base b and height a and diagonal c, mutual recounting creates the trigonometrical per-numbers: *a* = (*a*/*c*)**c* = sin*A***C*; *b* = (*b*/*c*)**c* = cos*A***c*; *a* = (*a*/*b*)**b* = tan*A***b*. Thus, a rotation becomes a per-number *a*/*b*, or as tan*A* per 1, allowing angles to be found from per-numbers. Follow-up activities could be other blocks e.g. from a folding ruler.

Q14, double-counting in STEM. Here the learning opportunity is that STEM formulas (Science, Technology, Engineering, Mathematics) typically are multiplication formulas with per-numbers coming from double-counting, e.g. kg = (kg/cubic-meter)*cubic-meter = density*cubic-meter; meter = (meter/sec)*sec = velocity*sec; Δ momentum = (Δ momentum/sec) * sec = force * sec; Δ energy = (Δ energy/meter) * meter = force * meter = work. Follow-up activities could be at other examples.

A QUESTION GUIDED UNITING CURRICULUM

The question guided re-enchantment curriculum in adding could be named ‘Mastering Many by uniting and splitting constant and changing unit-numbers and per-numbers’.

A general bundle-formula $T = a*x^2 + b*x + c$ is called a polynomial. It shows the four ways to unite: addition, multiplication, repeated multiplication or power, and block-addition or integration. The tradition teaches addition and multiplication together with their reverse operations subtraction and division in primary school; and power and integration together with their reverse operations factor-finding (root), factor-counting (logarithm) and per-number-finding (differentiation) in secondary school. The formula also includes the formulas for constant change: proportional, linear, exponential, power and accelerated. Including the units, we see there can be only four ways to unite numbers: addition and multiplication unite changing and constant unit-numbers, and integration and power unite changing and constant per-numbers. We might call this beautiful simplicity ‘the algebra square’.

Q21, next-to addition: “With $T1$ = 2 3s and $T2$ = 4 5s, what is $T1+T2$ when added next-to as 8s?” Here the learning opportunity is that next-to addition geometrically means adding by area. Algebraically, the recount-formula predicts the result. If first recounting the totals in tens, then adding, then recounting in 8s, multiplication precedes addition. Next-to addition is called integral calculus. Follow-up activities could be other examples of next-to addition.

Q22, reversed next-to addition: “If $T1$ = 2 3s and $T2$ add next-to as T = 4 7s, what is $T2$?” Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 3s, subtraction precedes division, which is natural as reversed integration, also called differential calculus. Follow-up activities could be other examples of reversed next-to addition.

Q23, on-top addition: “With $T1$ = 2 3s and $T2$ = 4 5s, what is $T1+T2$ when added on-top as 3s; and as 5s?” Here the learning opportunity is that on-top addition means changing units by using the recount-formula. On-top addition thus typically applies proportionality. Follow-up activities could be other examples of on-top addition.

Q24, reversed on-top addition: “If $T1$ = 2 3s and $T2$ as some 5s add to T = 4 5s, what is $T2$?” Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 5s, subtraction precedes division, which again is called differential calculus. Follow-up activities could be other examples of reversed on-top addition.

Q25, adding tens: “With $T1$ = 23 and $T2$ = 48, what is $T1+T2$ when added as tens?” Here the learning opportunity is that addition may create an overload to be removed by recounting in the same unit: $T1+T2 = 23 + 48 = 2B3 + 4B8 = 6B11 = 7B1 = 71$; or $T = 236 + 487 = 2BB3B6 + 4BB8B7 = 6BB11B13 = 6BB12B3 = 7BB2B3 = 723$. Follow-up activities could be other examples of adding tens.

Q26, subtracting tens: “If $T1$ = 23 and $T2$ add to T = 71, what is $T2$?” Here the learning opportunity is that subtraction may create an underload that can be removed by recounting in the same unit: $T2 = 71 – 23 = 7B1 – 2B3 = 5B\text{-}2 = 4B8 = 48$; or $T2 = 956 – 487 = 9BB5B6 – 4BB8B7 = 5BB\text{-}3B\text{-}1 = 4BB7B\text{-}1 = 4BB6B9 = 469$. Follow-up activities could be other examples of subtracting tens.

Q27, multiplying tens: “What is 7 43s recounted in tens?” Here the learning opportunity is that also multiplication may create overloads: $T = 7*43 = 7*4B3 = 28B21 = 30B1 = 301$; or $27*43 = 2B7*4B3 =8BB+6B+28B+21 =8BB34B21 =8BB36B1 = 11BB6B1 = 1161$, solved geometrically by a 2x2 block.

Q28, dividing tens: “What is 348 recounted in 6s?” Here the learning opportunity is that recounting a total with overload often eases division: $T = 348 /6 = 3BB4B8 /6 = 34B8 /6 = 30B48 /6 = 5B8 = 58$. Follow-up activities could be other examples of recounting tens in icons.

Q29, adding per-numbers: “2kg of 3$/kg + 4kg of 5$/kg = 6kg of what?” Here the learning opportunity is that the unit-numbers 2 and 4 add directly whereas the per-numbers 3 and 5 add by areas since they must first be transformed to unit-number by multiplication, creating the areas. Here, the per-numbers are piecewise constant. Asking 2 seconds of 4m/s increasing constantly to 5m/s leads to finding the area in a ‘locally constant’ situation. Follow-up activities could be other examples.

Q30, subtracting per-numbers: “2kg of 3$/kg + 4kg of what = 6kg of 5$/kg?” Here the learning opportunity is that unit-numbers 2 and 4 subtract directly whereas the per-numbers 3 and 5 subtract by areas since they must first be transformed to unit-number by multiplication, creating the areas. In a ‘locally constant’ situation, subtracting per-numbers is called differential calculus.

Q31, finding common units: “Only add with like units, so how to add T = 4ab^2 + 6abc?”. Here the learning opportunity is that possible units comes from factorizing: $T = 2*2*a*b*b + 2*3*a*b*c = 2*b*(2*a*b) + 3*c*(2*a*b) = 2b+3c\ 2ab$s. Follow-up activities could be other examples.

CONCLUSION

A mathematics education curriculum must make a choice. Shall it teach the ontology or the epistemology of Many? Shall it mediate the contemporary university discourse where the set-concept har transformed classical bottom-up ‘many-matics’ into self-referring top-down ‘meta-matism’; or shall it develop the mastery of Many already possessed by children? Shouldn’t classes be allowed to choose between two different curricula, a mediating and a developing? Then we need a twin to the present curriculum, unsuccessfully trying to mediate contemporary university mathematics. So, Luther has a point arguing that reaching a goal is not always helped by institutional patronization.

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