COUNTING before ADDING

The Child's Own Twin Curriculum

Count & ReCount & DoubleCount

before Adding NextTo & OnTop

master many with

manymath

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Background: Our two language houses

The **WORD language** assigns words in sentences with The **NUMBER language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. But does mathematics respect teaching language before grammar?

	WORD language	NUMBER language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
		Mathematics
Language	This is a chair	T = 3x4
	Mother tongue	

WORLD

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How well defined is mathematics?

In ancient Greece, Pythagoras used mathematics, meaning knowledge, as a common label for four descriptions of Many by itself & in space & time:

Together they formed the ‘**quadrivium**’ recommended by Plato as a general curriculum after the ‘**trivium**’ consisting of grammar & logic & rhetoric.

Geometry & algebra are both grounded in Many as shown by names:

Geometry means to <u>measure earth</u> in Greek

Algebra means to <u>reunite numbers</u> in Arabic

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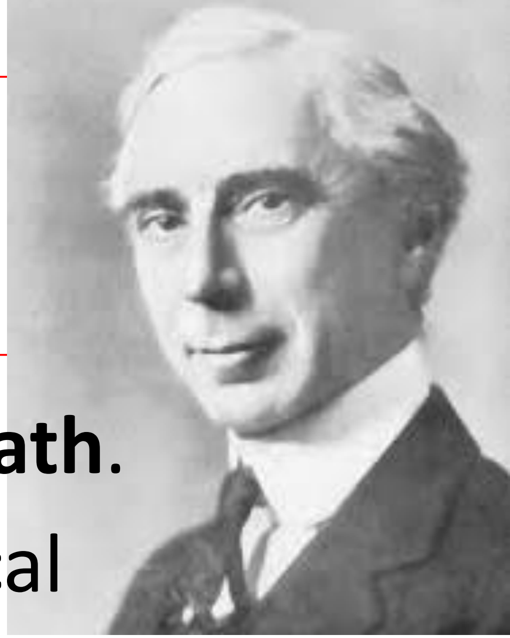
Modern mathematics, MetaMath



Around 1900, **SET** made math a self-referring **MetaMath**.
But Russell saw that self-reference leads to the classical
liar paradox 'this sentence is false', being false if true & opposite:
"Let M be the set of sets not belonging to itself, $M = \{A \mid A \notin A\}$.
Then $M \in M \Leftrightarrow M \notin M$. Forget about sets. Use type theory instead.
So, by self-reference, fractions cannot be numbers."
Mathematics: "Forget about Russell, he is not a mathematician.
We just institutionalize fractions as so-called rational numbers."

Institutions as thorns protecting Sleeping Beauties

- Weber on institutionalization: Rationalized too far, mathematics may become an **iron cage** that **disenchants** its subject.
- Bauman on self-reference: „The ideal model of action subjected to rationality as the supreme criterion contains a danger of so-called **goal displacement**. (..) The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right".
- Arendt: Just following orders may lead to '**the banality of evil**'.
- Sartre on existentialism: „**Existence precedes essence**".

Three curriculum choices to make

What is the goal of mathematics education?

1. To learn institutionalized mathematics, or
2. To learn to master what exists, Many

What is the core means?

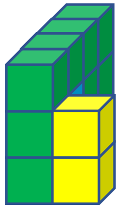
1. To learn about numbers without units, or
2. To learn how to number with units

1, -2, 3/4, √5, π, e

T = 3**B**2 = 3.2 = 4.-2 **4s**

What are numbers?

1. One-dimensional line-numbers without units, or
2. Two-dimensional block-numbers with units



Choosing 1 may have caused 50 years of less successful math education research.

Different curricula: MetaMath or ManyMath

What is the goal of mathematics education?

1. To learn mathematics (self reference pointing up, Vygotsky theory)
2. To learn to Master Many (external reference pointing down, Piaget theory)

What is a core means?

1. To learn about numbers (operations on specified and unspecified numbers)
2. To learn how to number (number-language sentences about counting & adding totals)

What are numbers?

1. 1D line-numbers (integer, natural, rational, real, place value system)
2. 2D bundle-numbers (constant & changing unit-numbers & per-numbers)

Why teach children if they already know?

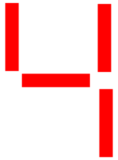
With education curing un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

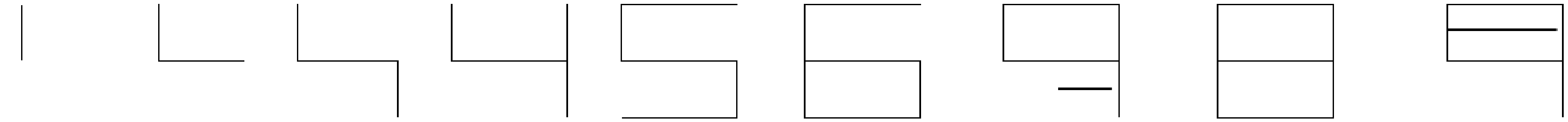
Core Questions:

- What Mastery of Many does the child have already?
- What could be a <u>ChildCenteredCurriculum</u> in Quantitative Competence?

Creating icons: IIII → IIII →  →

Children love making number-icons of cars, dolls, spoons, sticks.
Counting in ones means naming the different degrees of Many.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become units to use when counting in bundles.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII

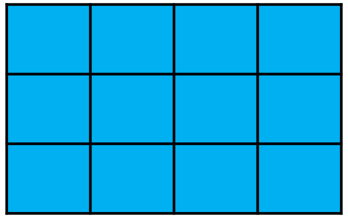


1	2	3	4	5	6	7	8	9

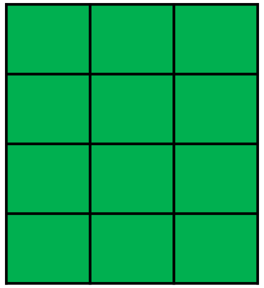
Children see Many as bundles with units

- "How old next time?" A 3year old says "Four" showing 4 fingers: | | | |
- But, the child reacts strongly to 4 fingers held together 2 by 2: || ||
- "That is not four, that is two twos"
- The child describes what exists, and with units: bundles of 2s, and 2 of them
- The block 3 **4s** has two numbers: 3 (the counting-number) and **4** (the unit-number)
- Children also use bundle-numbers with Lego blocks:

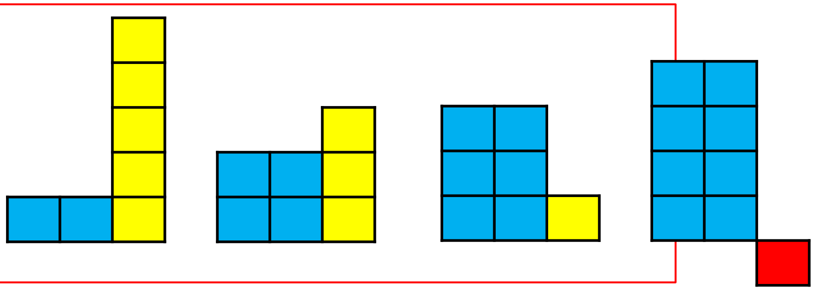
3 **4s**



4 **3s**



To count Many, children bundle



- Children are flexible when re-counting a Total of 7 sticks in **2s**:

I I I I I I I ● ǂ I I I I I ● ǂ ǂ I I I ● ǂ ǂ ǂ I ● ǂ ǂ ǂ ǂ

T = 7 = 1 **2s** &5 = 2 **2s** &3 = 3 **2s** &1 = 4 **2s** less 1

- And children don't mind writing a total of 7 using 'bundle-writing':

T = 7 = 1**B**5 = 2**B**3 = 3**B**1 = 4**B** ~~1~~, or even as

T = 7 = 1**BB**3 = 1**BB**1**B**1 = 2**BB** ~~1~~

- Also, children love to count in **3s**, **4s**, and in **hands**:

Thus, a number is a multi-counting of bundles as units

(…, bundles-of-bundles, bundles, unbundled)

IIII I I

T = 7 = 1 **5s** & 2

T = 7 = 1**B**2 **5s**

Counting bundles gives a number formula

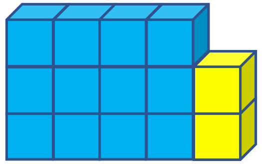
Children have fun when counting bundles, bundles of bundles, etc.:

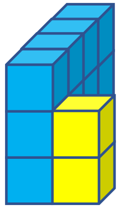
With ten-bundles: 01, 02, …, 09, **Bundle**,

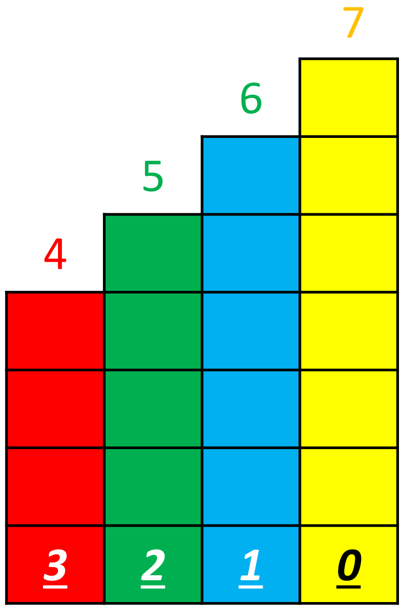
B1, **B**2,…, 9**B**8, 9**B**9, **BundleBundle**,

BB1,…, 2**BB**3**B**4,…, 9**BB**9**B**9, **BundleBundleBundle**,

BBB1,…




With blocks turned to hide the units behind:



B is marked with 1, **BB** with 2, **BBB** with 3, etc., singles with 0.

Later, this is a number formula T = 4567 = 4**BBB**5**BB**6**B**7 = 4x**B**^3 + 5x**B**^2 + 6x**B** + 7

Counting ten fingers & counting in tens

Children have fun when flexibly counting ten fingers in different ways:

- The Roman way: 01, 02, 03, HandLess1, **HAND**, Hand1, H2, H3, 2H-1, 2H, 2H1, 2H2
- The Viking way: 01, 02, 03, 04, HALF, 06, 07, less2, less1, **FULL**, 1left, 2left
- The modern way: 01, 02,…, 09, **ten**, ten1, ten2,…, 9ten8, 9ten9, **tenten**, tenten1,…, 2tenten3ten4,…, 9tenten9ten9, **tententen**, tententen1,…

Division, multiplication & subtraction as icons also

‘From 9 take away **4s**’ we write 9/4

iconizing the sweeping away by a broom, called division.

‘2 times stack **4s**’ we write 2x4

iconizing the stacking up by a lift called multiplication.

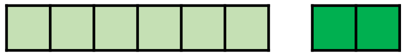
‘From 9 take away 2 **4s**’ to look for un-bundled we write 9 – 2x4

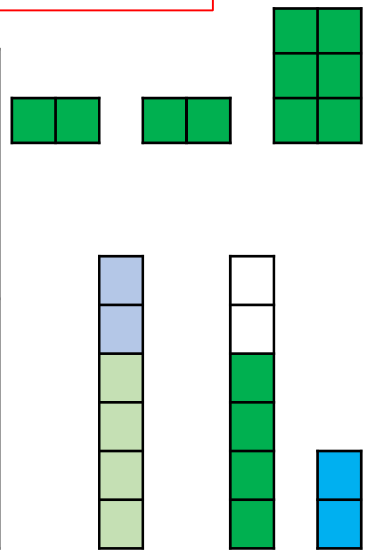
iconizing the dragging away by a trace called subtraction.

So counting includes division and multiplication and subtraction:

Finding the bundles: 9 = 9/4 **4s**. Finding the un-bundled: 9 – 2x4 = 1.

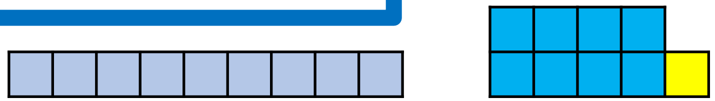
Counting creates two counting formulas

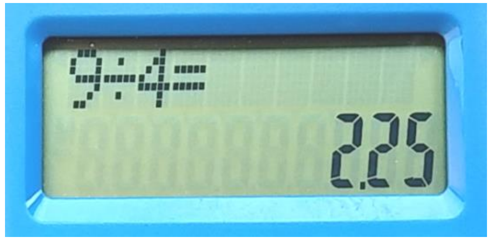
ReCount **T = (T/B) x B**	from a total **T**, **T/B** times,  **Bs** is taken away and stacked on-top
ReStack **T = (T–B) + B**	from a stack **T**, **T–B** is left when **B** is taken away and placed next-to



With formulas, a calculator can **predict** the counting-result 9 = 2**B**1 **4s**

9/4	**2.some**
9 – 2x4	**1**





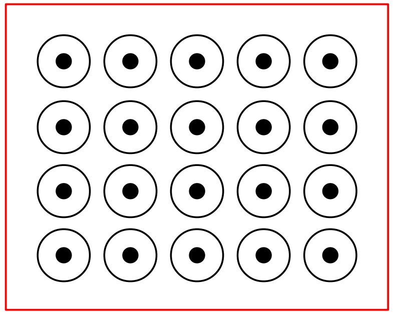
*As sentences of the number language, **formulas predict***

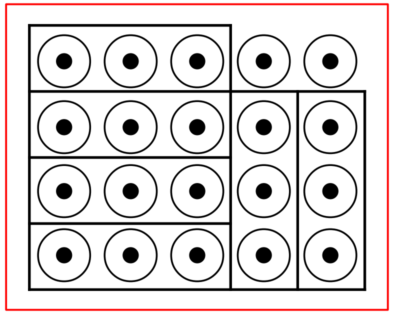
To share Many, children take away bundles predicted by division, multiplication and subtraction

2 preschoolers share 6 cakes by taking away **2s** from 6, thus rooting division as counting in **2s**.

- They smile when seeing that entering ‘6/2’ allows a calculator to predict that they can take cakes 3 times.
- And when seeing that ‘4x5/3’ predicts that 3 children can take cakes 6 times (or 6 cakes 1 time) when sharing 4 rows of 5 cakes.
- And when seeing that ‘4x5-6x3’ predicts that 2 will be left.

6/2	3
4x5/3	6.some
4x5-6x3	2





Question Guided Counting Curriculum

A question guided re-enchanting COUNTING curriculum could be named Mastering Many by counting, re-counting & double-counting.

- The design accepts that while 8 competences might be needed to learn university mathematics, only 2 are needed to Master Many: COUNTING & ADDING, motivating a twin curriculum.
- The corresponding pre-service or in-service question guided teacher education can be found at the MATHeCADEMY.net.
- Remedial micro-curricula for classes stuck in traditional mathematics can be found there also.

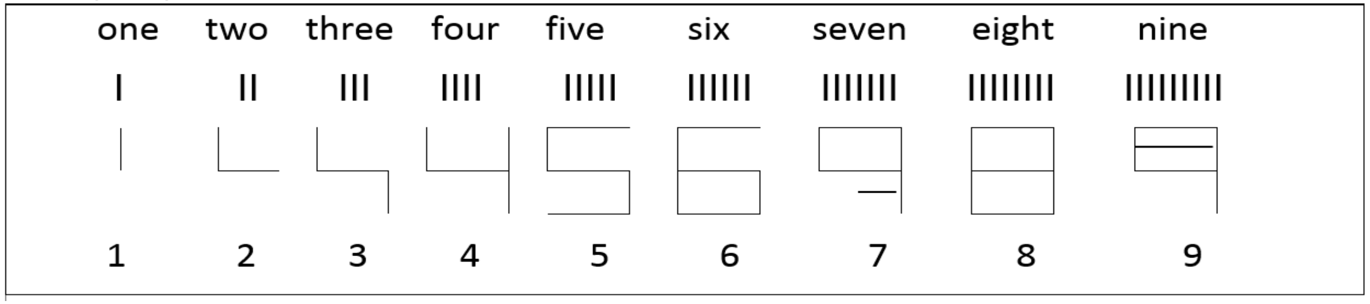
Q01, icon making IIII → IIII → 4

“The digit-icon 4 seems to be have four sticks. Does this apply to all digit-icons?”

We can change many ones to one icon with as many sticks or strokes as it represents, if written in a less sloppy way.

Follow-up activities could be:

- rearranging four dolls as one 4-icon, five cars as one 5-icon, etc.
- rearranging sticks on a table or on a paper
- using a folding ruler to construct the ten digits as icons



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Q02, counting sequences I

"How to count fingers?"

- Using **5s** as the bundle-size, five fingers can be counted as

  "01, 02, 03, 04, **Bundle**"

- And ten fingers can be counted as

  "01, 02, **Bundle** less2, **Bundle** -1, **Bundle**"

  "**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B**".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1**B**3 **7s** = 2**B**2 **4s** = 3**B**1 **3s** = 1**BB**1 **3s**.

Q02, counting sequences II

Counted as 1**B**, the bundle-number needs no icon. So counting a dozen cakes we say:

in	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
4s	01	02	03	**B**	1B1	1B2	1B3	2B	2B1	2B2	2B3	3B
7s	01	02	03	04	05	06	**B**	1B1	1B2	1B3	1B4	1B5
tens	01	02	03	04	05	06	07	08	09	**B**	1B1	1B2

The number names, eleven and twelve come from ‘one left’ and ‘two left’ in Danish, (en / twe levnet), again showing that counting takes place by taking away bundles.

Q03, bundle-counting in icon-units I

"How to count by bundling?"

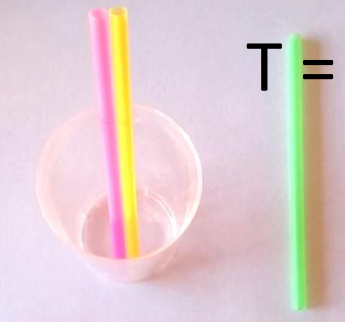
Five fingers can be bundle-counted in pairs or triplets, allowing both an overload and an underload; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

I I I I I • ǁ I I I • ǁ ǁ I • ǁ ǁ ǁ • ǁ ǁ I

T = 5 = 1**Bundle**3 **2s** = 2**B**1 **2s** = 3**B**-1 **2s** = 1**BB**1 **2s**

- Cup- & decimal-writing separates inside bundles from outside singles:

T = 5 = 1]3 **2s** = 2]1 **2s** = 3]-1 **2s** = 1]]0]1 **2s**



T = 5 = 1.3 **2s** = 2.1 **2s** = 3.-1 **2s** = 10.1 **2s**

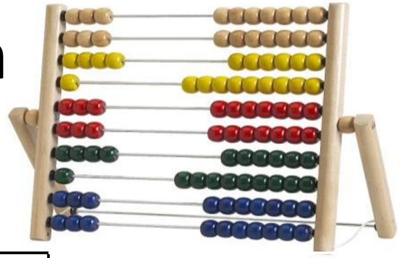
Likewise, if counting in ten-bundles: T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

Q03, bundle-counting in icon-units II

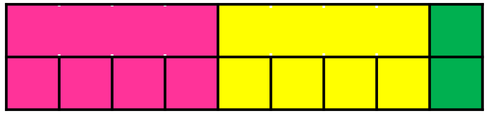
To count 9 in **4s**, we may bundle in a cup with 1 stick per bundle.

9 = I I I I I I I I I = ~~IIII~~ ~~IIII~~ I = **II**]I = 2]1 **4s** = 2**B**1 **4s** = 2.1 **4s**

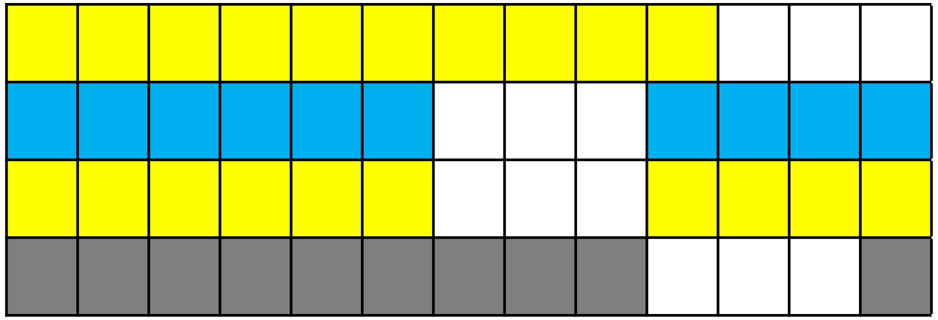
We may report with cup-, bundle- or decimal-writing,
or on a western **ABACUS** in

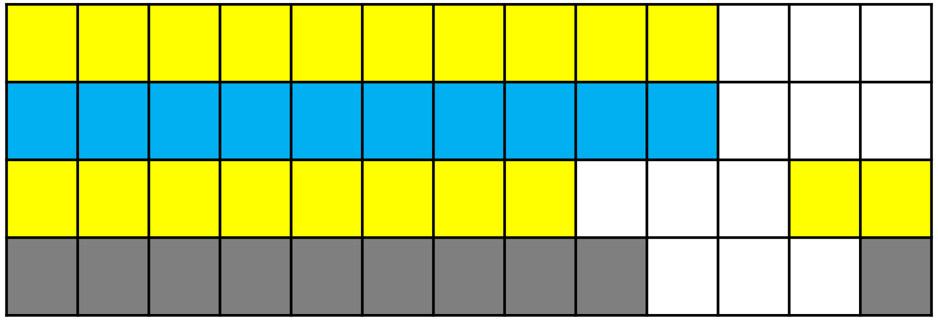


Lego blocks
or CentiCubes

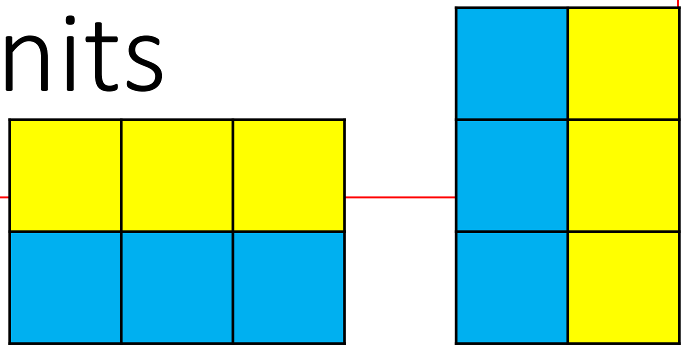


Outside geometry mode or Inside algebra mode

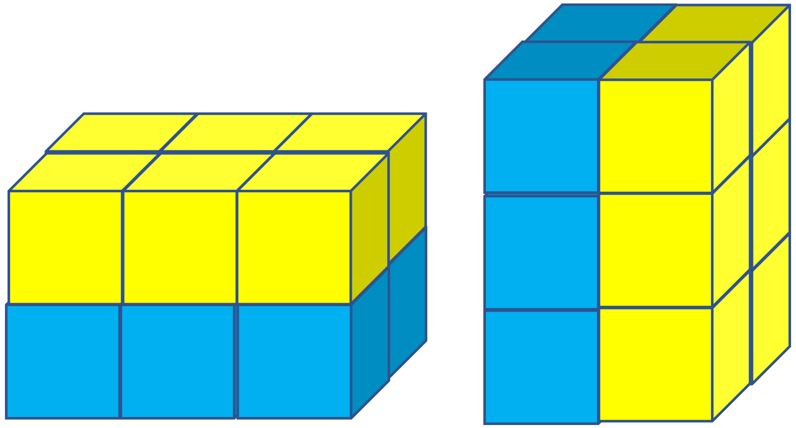




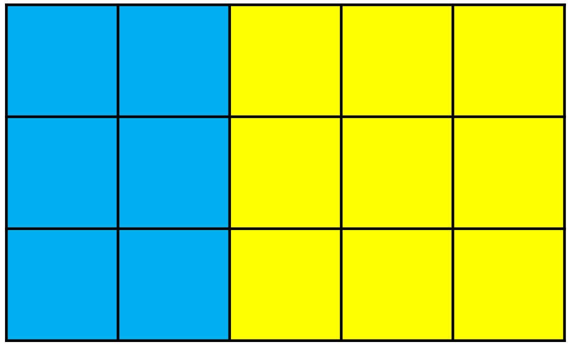
Switching & uniting & splitting units



- Turning a 2D block will change the unit
  T = 2 **3s** = 2x3 → T = 3 **2s** = 3x2,
  So T = 2x3 = 3x2 *(The Commutative law)*



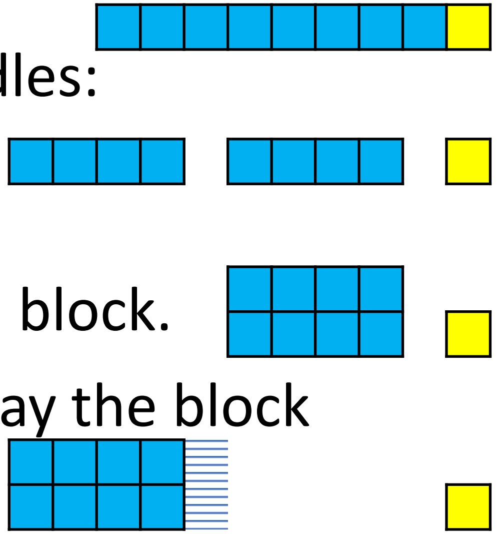
- Turning a 3D block will also change the unit
  So T = 2x(2x3) = (2x2)x3 *(The Associative law)*



- A block may split into two parts
  T = 3 **5s** = 3 **2s** + 3 **3s** or
  So T = 3x5 = 3x(2+3) = 3x2 + 3x3 *(The Distributive law)*

Q04, calculators predict

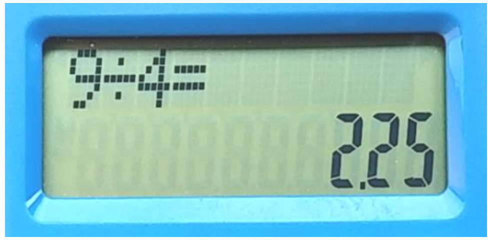
"Can a calculator predict a counting result?"



We may see the division sign as an icon for a broom wiping away bundles:
9/4 means 'from 9, wipe away bundles of 4s'.

- The calculator says '2.some', thus predicting it can be done 2 times.
  Now the multiplication sign iconizes a lift stacking the bundles into a block.
- Finally, the subtraction sign iconizes the trace left when dragging away the block to look for unbundled singles.
- With '9-2x4 = 1' the calculator predicts that 9 can be recounted as 2B1 **4s**.

9/4	**2.some**
9 – 2x4	**1**

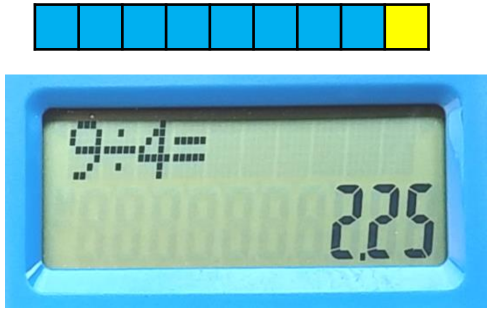


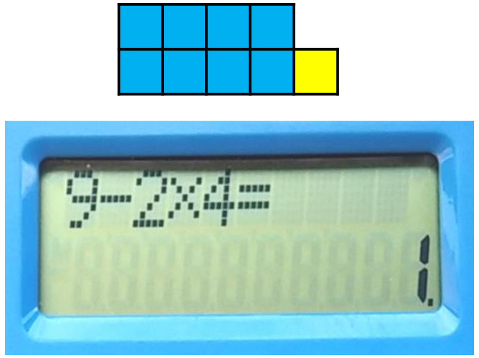
Q04, counting creates 2 counting formulas

ReCount **T = (T/B) x B**	from a total **T**, **T/B** times, **Bs** is taken away and stacked
ReStack **T = (T–B) + B**	from a total **T**, **T–B** is left, when **B** is taken away and placed next-to

As sentences of the number language, **Formulas Predict:**
Predicting that **T** = 9 = 2.1 **4s:**

9/4	**2.some**
9 – 2x4	**1**

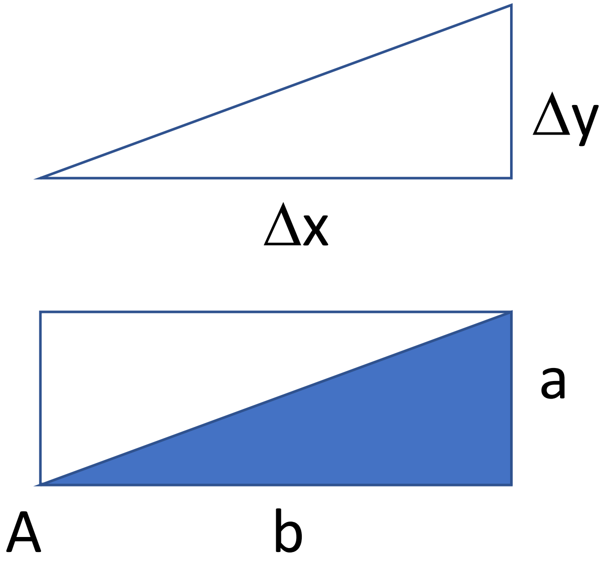




Q04, the recounting formula is a core formula

T = (T/B)*B saying ‘from T, T/B times, Bs can be taken away’, is all over:

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y’** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second

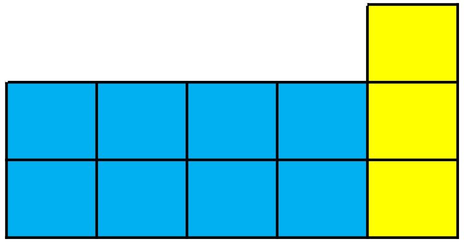


Q05, unbundled as decimals or negatives or fractions
0.3 **4s** or 0.-1 **4s** or 3/4 **4s**

"Where to put the unbundled singles?"

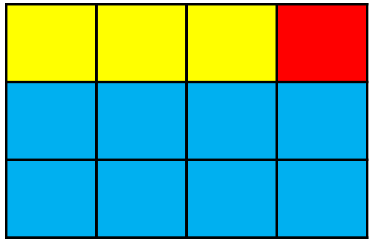
When counting by bundling, the unbundled singles can be placed

NextTo the block	**OnTop** of the block	
counted as a block of **1s**	counted as a bundle	counted in bundles



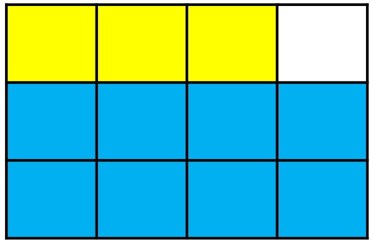
T = 2B3 **4s** = 2.3 **4s**

A decimal number



T = 3B-1 **4s** = 3.-1 **4s**

A negative number



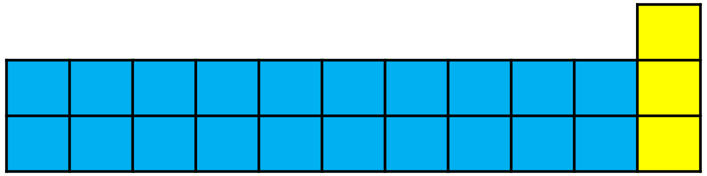
T = 2 3/4 **4s**

A fraction

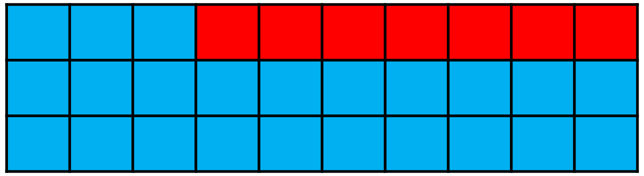
Q05, counting in tens

“Where to put the unbundled singles with tens?”

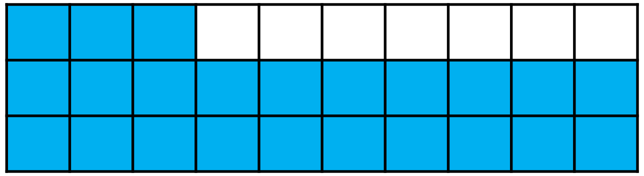
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit, or as



T = 2.3 **tens**



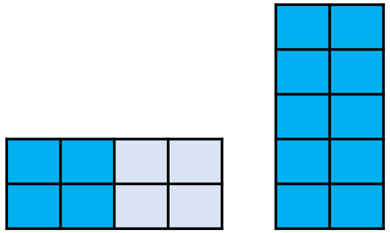
T = 3.-7 **tens**



T = 2 3/10 **tens**

Q06, prime & foldable bundle-units

"When can blocks be folded in like bundles?"



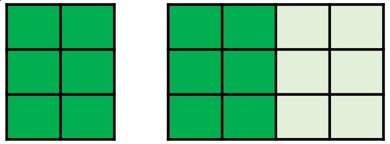
The block T = 2 **4s** = 2 x 4 has 4 as the bundle-unit.

Turning over gives T = 4 **2s** = 4 x 2, now with 2 as the bundle-unit.

4s can be folded in another bundle as 2 **2s**, whereas 2s cannot.

(1 is not a bundle, nor a unit since a bundle-of-bundles stays as 1).

We call 2 a **prime bundle-unit** and 4 a **foldable bundle-unit**, 4 = 2 **2s**.



A block of 3 **2s** cannot be folded.

A block of 3 **4s** can be folded: T = 3 **4s** = 3 x (2 x 2) = (3 x 2) x 2 = 2 **3x2s**.

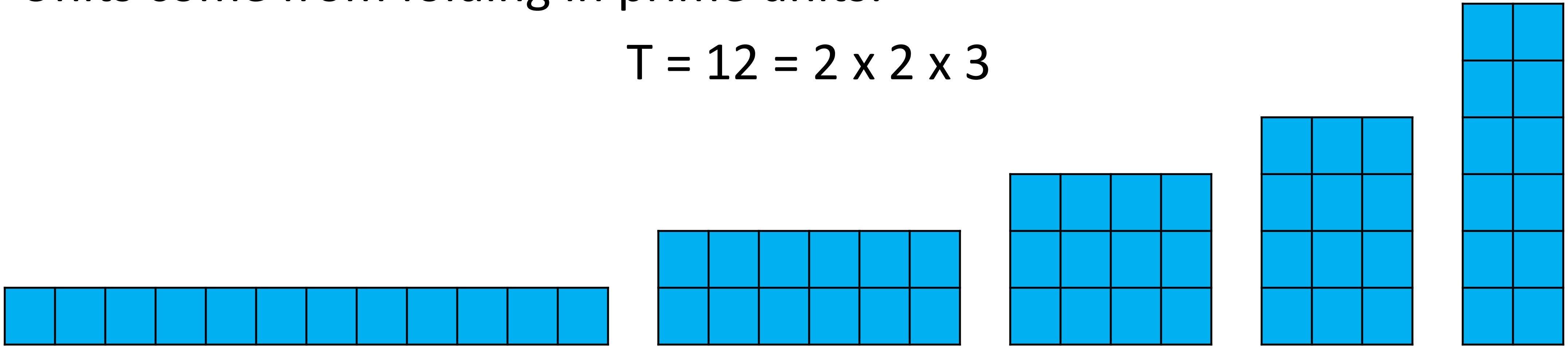
A number is called **even** if it can be written with 2 as the unit, else **odd**.

Q07, finding possible units

"What are possible units in T = 12?"

Units come from folding in prime units:

T = 12 = 2 x 2 x 3



T = 12 **1s** = 2 **2x3s** = 3 **2x2s** = 2x2 **3s** = 2x3 **2s**

Q08, recounting in a different unit

"How to change a unit?"

The recount-formula allows changing the unit.

Asking T = 3 **4s** = ? **5s**, the recount-formula gives T = 3 **4s** = (3x4/5) **5s**.

Entering 3x4/5, the answer '2.some' shows that a block of 2 **5s** can be taken away.

With 3x4–2x5, the answer '2' shows that 3 **4s** can be recounted as 2B2 **5s** or 2.2 **5s**.

3 **4**s = IIII IIII IIII = IIII I III II II = IIIII IIIII II = 2B2 **5**s = 2.2 **5s**

3x4/5	**2.**some
3x4 – 2x5	**2**

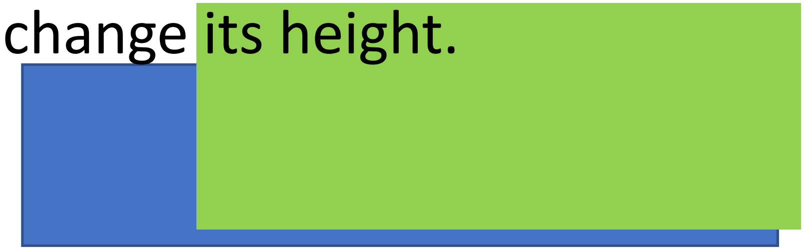
Change Unit = Proportionality

Q09, recounting from tens to icons

"How to change unit from tens to icons?"

Asking 'T = 2.4 **tens** = 24 = ? **8s**', we just recount 24 in **8s**:

T = 24 = (24/8)x8 = 3x8 = 3 **8s.**

Formulated as an **equation** we use ***u*** for the unknown number, ***u***x8 = 24. Recounting 24 in 8s shows that ***u*** is 24/8 attained by moving 8 **to opposite side - with opposite sign**	To keep its size, a block changing its unit must also change its height.  T = 2.4 **tens** = 3 **8s**

u x 8 = 24 = (24/8)x8

u = 24/8 = 3

Q10, recounting from icons to tens (multiplication) 3 **7s** = ? **tens**

"How to change unit from icons to tens?"

Asking 'T = 3 **7s** = ? **tens**', the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone: T = 3 **7s** = 3x7 = 21 = 2.1 **tens**, only it leaves out the unit and the decimal point.

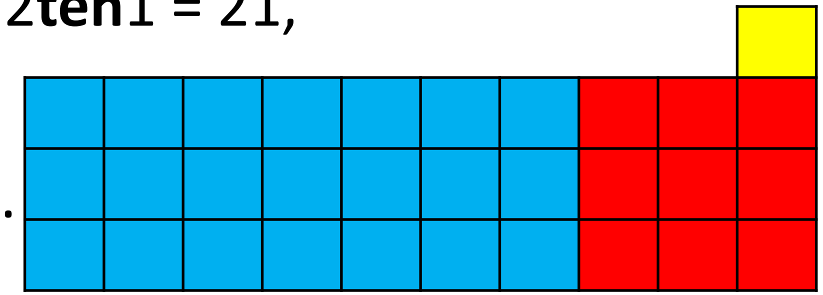
Alternatively, we may use 'less-numbers', so 7 = **ten** less 3

T = 3x7 = 3 x (**ten** less 3) = 3 x **ten** less 3x3 = 3**ten** less 9 = 2**ten**1 = 21,

or with 9 = **ten** less 1:

T = 3**ten** less (**ten** less1) = 2**ten** lessless 1 = 2**ten** & 1 = 21.

showing that 'lessless' cancel out



Recounting large numbers in or from tens: *same size, but new form*

Recounting 6 **47s** in **tens**

Recounting 476 in **7s**

*BundleWriting seprates **INSIDE** bundles from **OUTSIDE** singles*

T = 6 x 47 = 6 x 4**B**7

= 24**B**42

= 28**B**2

= 28.2

tens

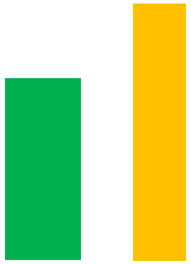
T = 476 = 47.6 **tens**

= 47**B**6

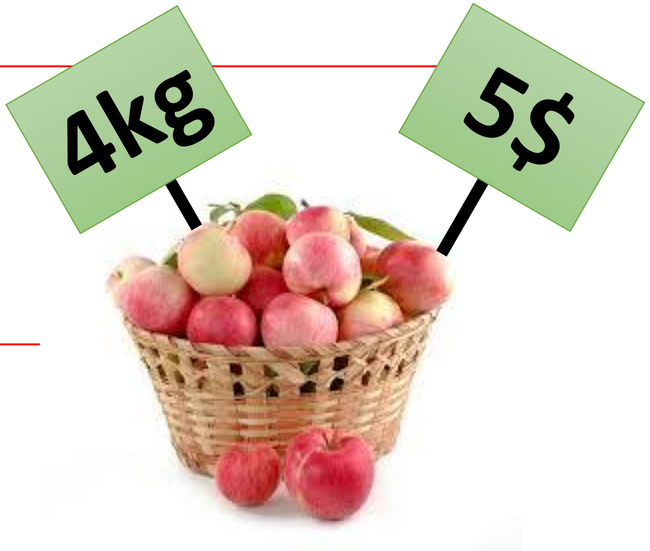
= 42**B**56

= 6x7**B**8x7

= 68 x 7



Q11, double-counting in two units creates bridging PerNumbers & proportionality



"How to double-count in two units?"

DoubleCounting in kg & $, we get **4kg = 5$** or

4kg **per** 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber.

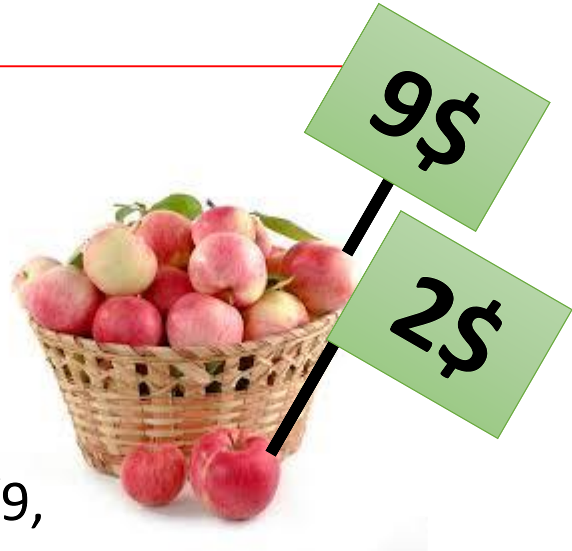
With 4kg bridged to 5$ we answer questions by recounting in the per-number.

Questions:

7kg = ?$	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

Answer: *Recount in the PerNumber (Proportionality)*

Q12, double-counting in the same unit creates fractions



“How to double-count in the same unit?”

Double-counted in the same unit, per-numbers are fractions, 2$ per 9$ = 2/9, or percentages, 2 per 100 = 2/100 = 2%.

To find a fraction or a percentage of a total, again we just recount in the per-number.

- **Taking 3 per 4 = taking ? per 100.** With 3 bridged to 4, we recount 100 in 4s:

100 = (100/4)*4 giving (100/4)*3 = 75, and 75 per 100 = 75%.

- **Taking 3 per 4 of 60 gives ?.** With 3 bridged to 4, we recount 60 in 4s:

60 = (60/4)*4 giving (60/4)*3 = 45.

- **Taking 20 per 100 of 60 gives ?.** With 20 bridged to 100, we recount 60 in 100s:

60 = (60/100)*100 giving (60/100)*20 = 12 .

We observe that per-numbers and fractions are not numbers, but operators needing a number to become a number.

Q12, enlarging or shortening units

“How to enlarge or shorten units in fractions?”

Taking 2/3 of 12 means taking 2 per 3 of 12.

With 2 bridged to 3, we recount 12 in **3s**, 12 = (12/3)*3 = 4*3

So 4 times we can take 2, i.e. 8 of the 12. Thus 2 per 3 = 8 per 12.

This may be used for enlarging or shortening fractions by inserting or removing the same unit above and below the fraction line:

$$\frac{2}{3} = \frac{2\ \mathbf{4s}}{3\ \mathbf{4s}} = \frac{2*4}{3*4} = \frac{8}{12} \quad \bullet \quad \frac{8}{12} = \frac{2*4}{3*4} = \frac{2\ \mathbf{4s}}{3\ \mathbf{4s}} = \frac{2}{3} \quad \bullet \quad \frac{12abc}{8a} = \frac{3*4*a*b}{2*4*a} = \frac{3*b\ \mathbf{4as}}{2\ \mathbf{4as}} = \frac{3b}{2}$$

Q13, recounting the sides in a block

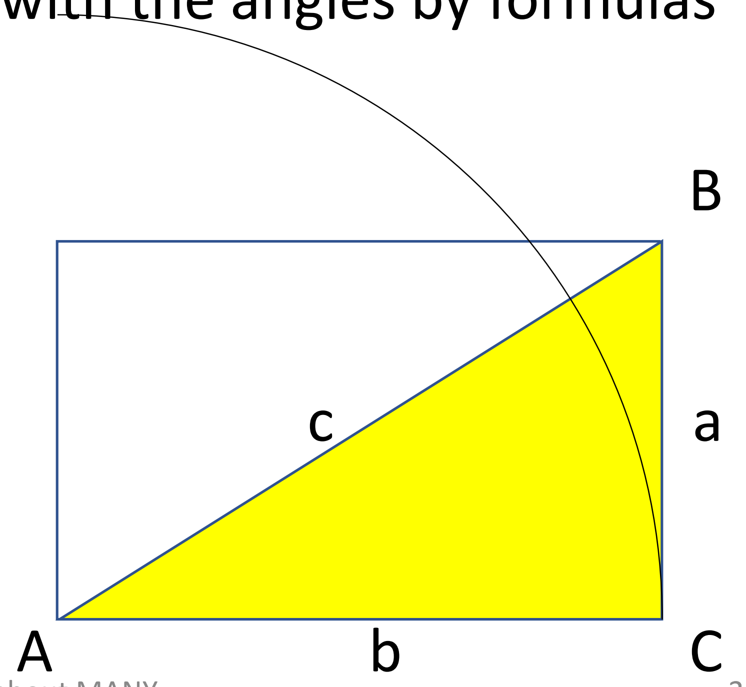
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras formula. And connected with the angles by formulas recounting a side in the other side or in the diagonal:

A+B+C = 180

a*a + b*b = c*c (the Pythagoras formula)

sinA = a/c; cosA = b/c; tanA = a/b = Δy/Δx = gradient

Circle: circum./diam. = π **= n*tan(180/n)** for n large



Q14, double-counting gives per-numbers in STEM multiplication formulas I

STEM (Science, Technology, Engineering, Math) typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

Q14, double-counting gives per-numbers in STEM multiplication formulas II

Extra STEM examples:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec;
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = **(energy/charge)** x **(charge/sec)** or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

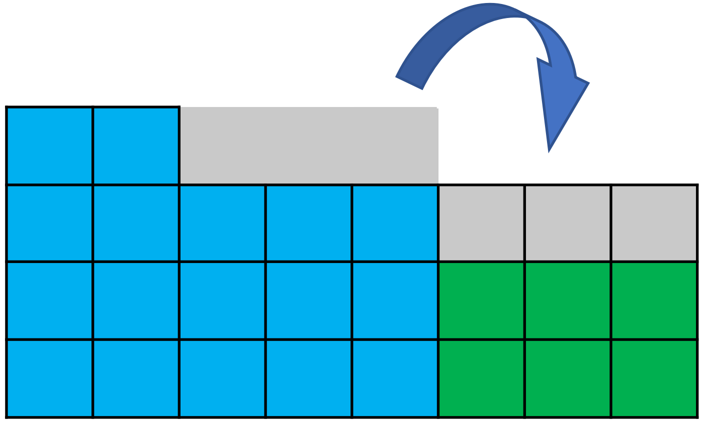
Q15, navigating on a squared paper

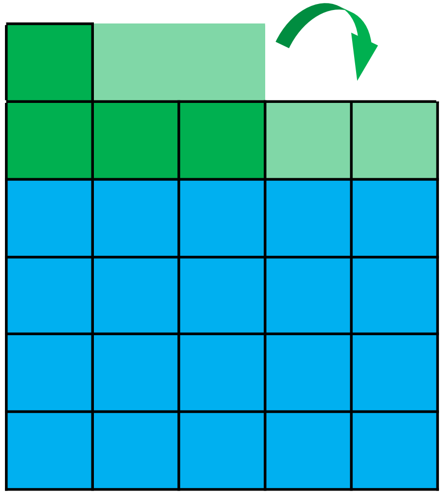
First steps into coordinate geometry, to always keep algebra and geometry together.

"Collect treasures on the rocks "	**"Plan a trip to treasure island"**
Three rocks are placed on a squared paper. The rocks have the values -1, 1, and 2. A journey begins in the midpoint. Two dices tell the out- and up- change, where odd numbers are negative. How many points before reaching the edge? Predict and measure angles on the journey.	Departure point: 3cm out & 2cm up Destination point: 7cm out & 4cm up. Plan a voyage with 1 out per day. How many days before reaching the island? What is your position after 2 days? What is your position after n days? What is the angle traveled?

Counted & recounted, Totals can be added

BUT: **NextTo**	**or**	**OnTop**
4 5s + 2 3s = 3**B**2 8s		4 5s + 2 3s = 4 5s + 1**B**1 5s = 5**B**1 5s
The areas are integrated *Adding areas = Integration*		The units are changed to be the same *Change unit = Proportionality*





Four ways to unite into a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration).
<u>Addition</u> and <u>multiplication</u> add changing and constant unit-numbers.
<u>Integration</u> and <u>power</u> add changing and constant per-numbers.
We might call this beautiful simplicity the 'Algebra Square'.

Operations unite	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n**	**T = a * n**
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn**	**T = a^n**

Five ways to split a Total

The 4 uniting operations (+, *, ^, ∫) each has a reverse splitting operation:
Addition has subtraction (–), and multiplication has division (/).
Power has factor-finding (root, √) and factor-counting (logarithm, log).
Integration has per-number finding (differentiation dT/dn = T‘).

Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / *split into*	**changing**	**constant**
Unit-numbers *m, s, \$, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, \$/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $\log_a T = n$, $\sqrt[n]{T} = a$

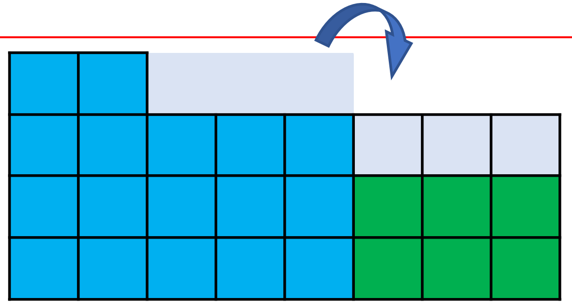
Question Guided Adding Curriculum

A question guided re-enchanting ADDING curriculum could be named '<u>Mastering Many by uniting and splitting constant and changing unit-numbers and per-numbers</u>'.

- A corresponding pre-service and in-service question guided teacher education can be found at the MATHeCADEMY.net.
- Remedial curricula for classes stuck in traditional mathematics can be found there also.

MATHeCADEMY

Q21, next-to addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added next-to as **8s**?"

Outside, next-to addition geometrically means adding areas.
Next-to addition is also called integral calculus.

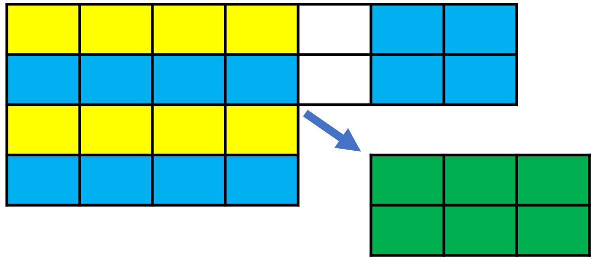
Inside, the recount formula algebraically predicts the result.
Here multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/8) x 8 = 3.2 **8s**

(4x5 + 2x3)/8	**3.some**
(4x5 + 2x3) – 3x8	**2**

Q22, reversed next-to addition



“If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?”

Outside, we remove the initial block T1 and recount the rest in **4s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

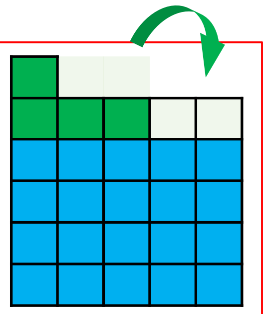
Here subtraction precedes division; which is natural as reversed integration.

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

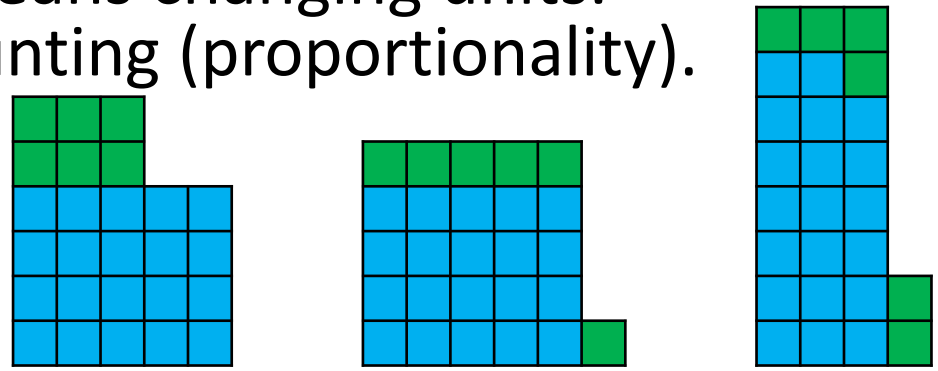
(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

Q23, on-top addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added on-top?"

<u>Outside</u>, on-top addition geometrically means changing units.
On-top addition thus often involves recounting (proportionality).



T = 4 **5s** + 2 **3s** = 4 **5s** + 1.1 **5s** = 5.1 **5s**

T = 4 **5s** + 2 **3s** = 6.2 **3s** + 2 **3s** = 8.2 **3s**

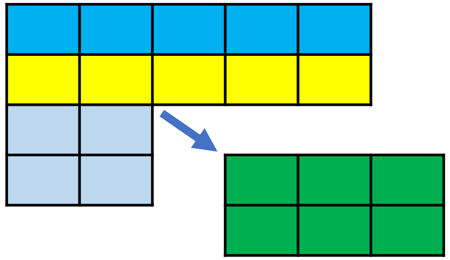
<u>Inside</u>, the recount formula algebraically predicts the result.
Here again, multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/5) x 5 = 5.1 **5s**

(4x5 + 2x3)/5	**5.some**
(4x5 + 2x3) – 5x5	**1**

Q24, reversed on-top addition



"T1 = 2 **3s** and how many **5s** (T2) add on-top as 4 **5s**?"

Outside, we remove the initial block T1 and recount the rest in **5s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed on-top addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

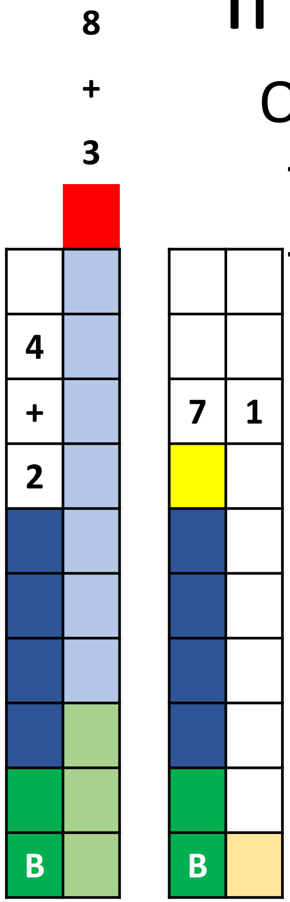
T2 = (T2/**B**) x **B**

= ((4x5 – 2x3)/5) x 5 = 2.4 **5s**

(4x5 – 2x3)/5	**2.some**
(4x5 – 2x3) – 2x5	**4**

Q25, adding tens on-top

"If T1 = 23 and T2 = 48, what is T1+T2 as **tens**?"

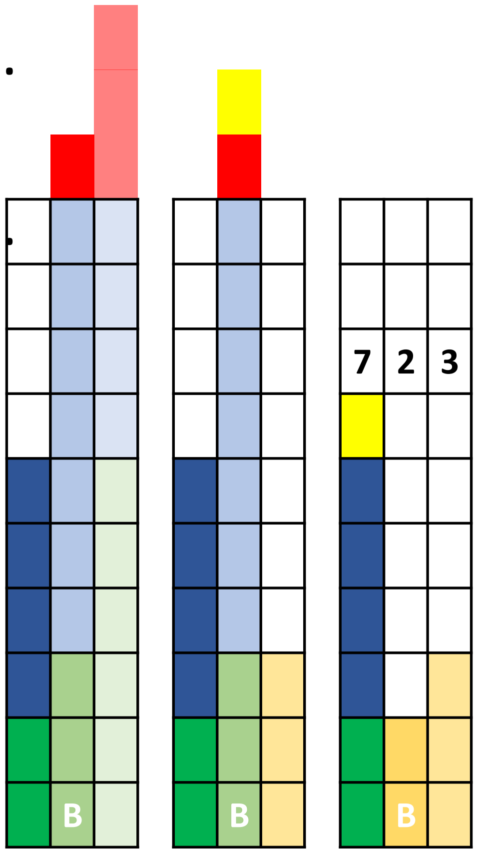


Outside and inside, we recount overloads by changing 1 **tens** to 10 **1s**.

T = 23 + 48 = 2**ten**3 + 4**ten**8 = 6**ten**11 = 6**ten**1**ten**1 = 7**ten**1 =71

T = 236 + 487 = 2**tenten**3**ten**6 + 4**tenten**8**ten**7 = 6**tenten**11**ten**13 = ...

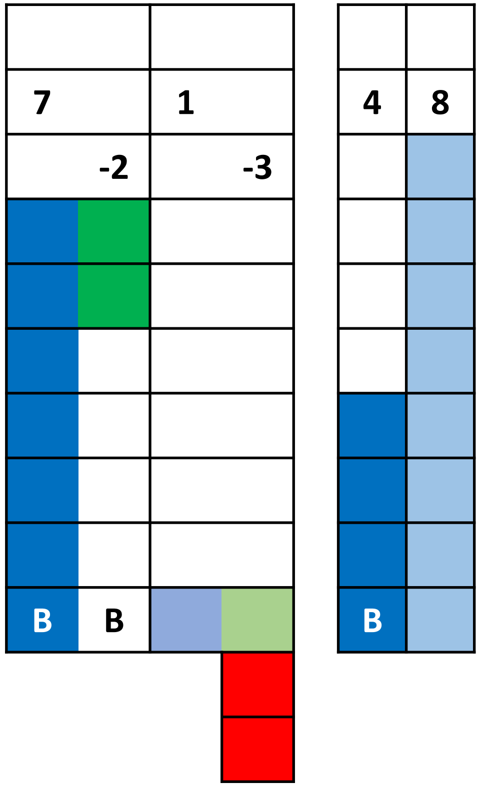
T1+T2 = 23 + 48	T = **2**36 + 487
= 2**B**3 + 4**B**8	= 2**BB**3**B**6 + 4**BB**8**B**7
= 6**B**11	= 6**BB**11**B**13
= 7**B**1	= 6**BB**12**B**3
= 71	= 7**BB**2**B**3
	= 723



Q26, subtracting tens on-top

"If T1 = 23 and T2 add to T = 71, what is T2 as **tens**?"

Outside and inside, we recount underloads by changing 1 **tens** to 10 **1s.**



T = 71 – 23 = 7**ten**1 – 2**ten**3 = 5**ten**-2 = 4**ten**8 = 48

T = 956 – 487 = 9**tenten**5**ten**6 – 4**tenten**8**ten**7 = 5**tenten**-3**ten**-1 = …

T2 = 71 – 23	T2 = 956 – 487
= 7**B**1 – 2**B**3	= 9**BB**5**B**6 – 4**BB**8**B**7
= 5**B**-2	= 5**BB**-3**B**-1
= 4**B**8	= 4**BB**7**B**-1
= 48	= 4**BB**6**B**9
	= 469

Q27, from icons to tens, multiplication

A multiplication table recounts icon-blocks in ten-blocks: T = 7 **3s** = ? **Tens.**

To recount 7 **3s** in **tens** we can use that 7 is **ten** less3, and 3 is 5 less2:

From the 10 **5s** we remove 3 **5s** (/) and 2 **tens** (\).

But then we must add the 3 **2s** that was removed twice.

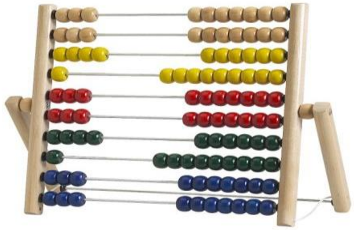
$$T = 7x3 = (ten - 3)x(5 - 2) = tenx5 - 3x5 - tenx2 + 3x2$$

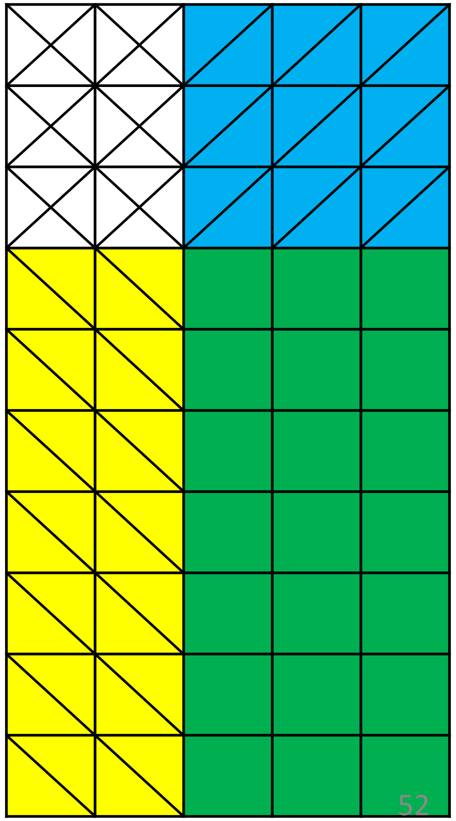
$$= 50 - 15 - 20 + 6 = 21.$$

Shown on a western ten by ten abacus as a 10 by 5 block.

This roots the algebra formula showing that – x – is +

$$\boldsymbol{(a - b) \times (c - d) = a \times c - a \times d - b \times c + b \times d}$$





Q27, multiplication tables

“What is 7 **8s** recounted in **tens**?”

Using underload-numbers after 5, we recount to remove underloads:

T = 7 x 8 = 7 x **B**-2 = 7**B**-14 = 7**B** – 1**B**4 = 6**B**-4 = 5**B**6 = 56	T = 7 x 8 = **B**-3 x **B**-2 = 1**BB** – 3**B** – 2**B** + 6 = 10**B** – 3**B** – 2**B** + 6 = 5**B**6 = 56

	2	**3**	**4**	**5**	**B-4**	**B-3**	**B-2**	**B-1**
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

	2	**3**	**4**	**5**	**B-4**	**B-3**	**B-2**	**B-1**
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
B-4	12	18	24	30	36	42	48	54
B-3	14	21	28	35	42	49	56	63
B-2	16	24	32	40	48	56	64	72
B-1	18	27	36	45	54	63	72	81

Q27, Recounting BundleBundles in tens (squares: ..., 4 **4s** = ? **tens,** 5 **5s** = ? **tens**, ...)

Using the multiplication table, we recount the different bundle-bundles (called squares) in **tens**:

S4 = 4 **4s** = 4x4 = 16

S5 = 5 **5s** = 5x5 = 25, etc.

We see that to get to the next square we add the sides twice, + 1:

5*5 = 4*4 + 2*4 + 1, or with 4 = n:

(n+1)*(n+1) = n*n + 2*n + 1, or

$(n+1)^2 = n^2 + 2*n + 1$

	1	2	3	4	5	6	7	8	9	10
1	1									
2		4								
3			9							
4				16						
5					25					
6						36				
7							49			
8								64		
9									81	
10										100

Q27, recounting from icons to tens (multiplication)

Recount 43 **7s** in **tens**:

T = 43x7 = 301 = 30.1 tens

Horizontally we write 43 as 4**ten**3 or 4**B**3.
Vertically, we write 7.
Multiplying, we get 28**B** and 21.
So, T = 43x7 = 28**B**21 = 30**B**1 = 301.

With underload, 43 is 5**ten**-7 or 5**B**-7.
Vertically, we write 7.
Multiplying, we get 35**B** and -49.
So, T = 43x7 = 35**B**-49 = 30**B**1 = 301.

overload

4**B**	3	43x
?	?	7

4**B**	3	43x
28**B**	21	7
30**B**	1	<u>301</u>

underload

5**B**	-7	43x
?	?	7

5**B**	-7	43x
35**B**	-49	7
30**B**	1	<u>301</u>

Q27, recounting 27 43s in **tens** (multiplication)

Recounting 27 **43s** in **tens**: 27 x 43 = 1161 = 116.1 **tens**

overload

	2B	7	27x43
	?	?	4B
	?	?	3
?BB	?B	?	
	2B	7	27x43
	8BB	28B	4B
	6B	21	3
8BB	34B	21	
8BB	36B	1	
11BB	6B	1	1161

underload

	3B	-3	27x43
	?	?	4B
	?	?	3
?BB	?B	?	
	3B	-3	27x43
	12BB	-12B	4B
	9B	-9	3
12BB	-3B	-9	
12BB	-4B	1	
11BB	6B	1	1161

underload

	3B	-3	27x43
	?	?	5B
	?	?	-7
?BB	?B	?	
	3B	-3	27x43
	15BB	-15B	5B
	-21B	21	-7
15BB	-36B	21	
15BB	-34B	1	
11BB	6B	1	1161

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Q28, recounting from tens to icons (division)

Recount 30.1 **tens** in **7s**: 301/7 = 43

Recount 30.6 **tens** in **7s**: 306/7 = 43 5/7

overload

4**B**	3	43x
28**B**	21	7
30**B**	1	301
?	?	?x
?	?	7
30**B**	1	301

underload

5**B**	-7	43x
35**B**	-49	7
30**B**	1	301
?	?	?x
?	?	7
30**B**	1	301

Multiplying is top-down; division is bottom-up.

Below, we write 301 = 30**B**1. Above we recount 301 as 28**B**21 to count in **7s**. So, T = 301 = 43x7.

Below, we write 306 as 28**B**26 first, then as 28**B**21 + 5 to count in **7s**. So, T = 306 = 43x7 +5.

4**B**	3 +5/7	43 5/7x
28**B**	21+5	7
30**B**	6	306
?	?	?x
?	?	7
30**B**	6	306

Q28, recounting 1161 in **43s** (division)

	2B	7	27x
	8BB	28B	4B
	6B	21	3
8BB	34B	21	
8BB	36B	1	
11BB	6B	1	1161
	? = 2B	?	?x
	8BB	30B	4B
	6B	1	3
?	?	?	
8BB	36B	1	
11BB	6B	1	1161

Overload

Underload

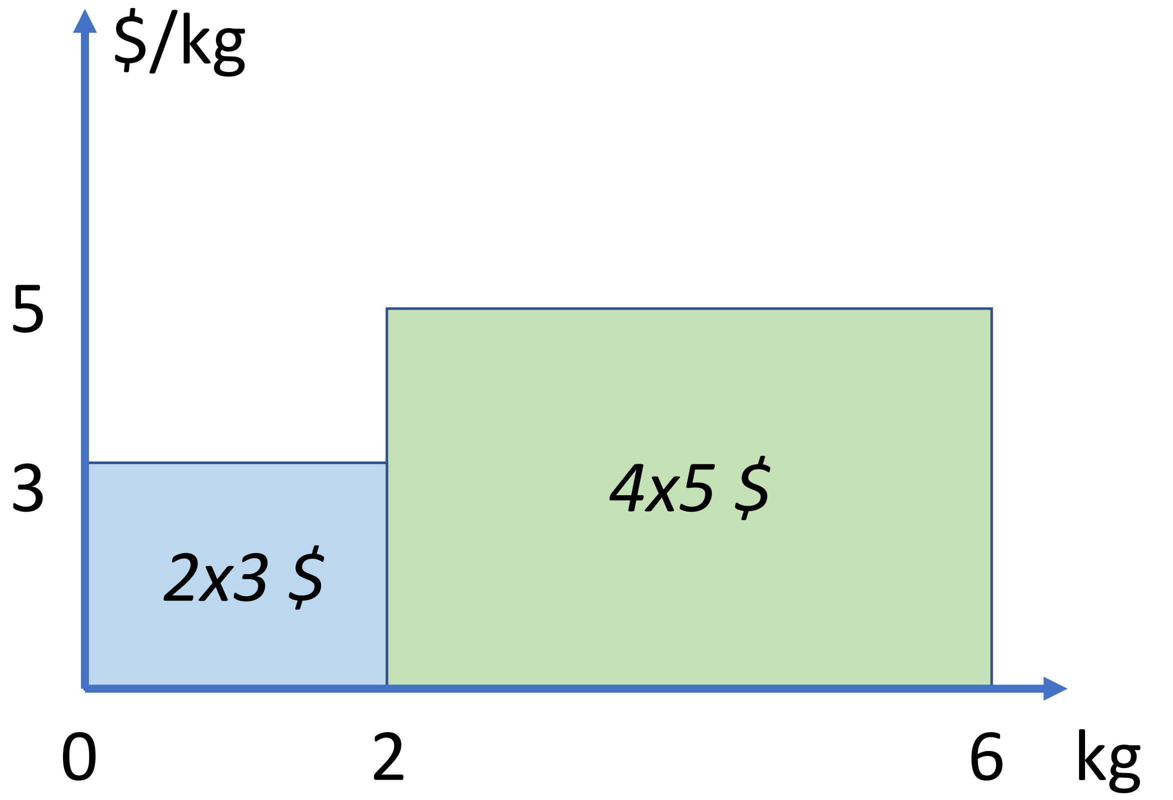
	3B	-3	27x
	8BB	-12B	4B
	9B	-9	3
12BB	-3B	-9	
12BB	-4B	1	
11BB	6B	1	1161
	? = 3B	?	?x
	12BB	-13B	4B
	9B	1	3
?	?	?	
12BB	-4B	1	
11BB	6B	1	1161

Q29, adding PerNumbers as areas (integration)

"2kg at 3$/kg + 4kg at 5$/kg = 6kg at ? $/kg?"

2 kg	at	3 $/kg
+ 4 kg	at	5 $/kg
(2+4) kg	at	? $/kg

- Unit-numbers add on-top.
- Per-numbers add next-to as **areas** under the per-number graph. Here multiplication precedes addition.



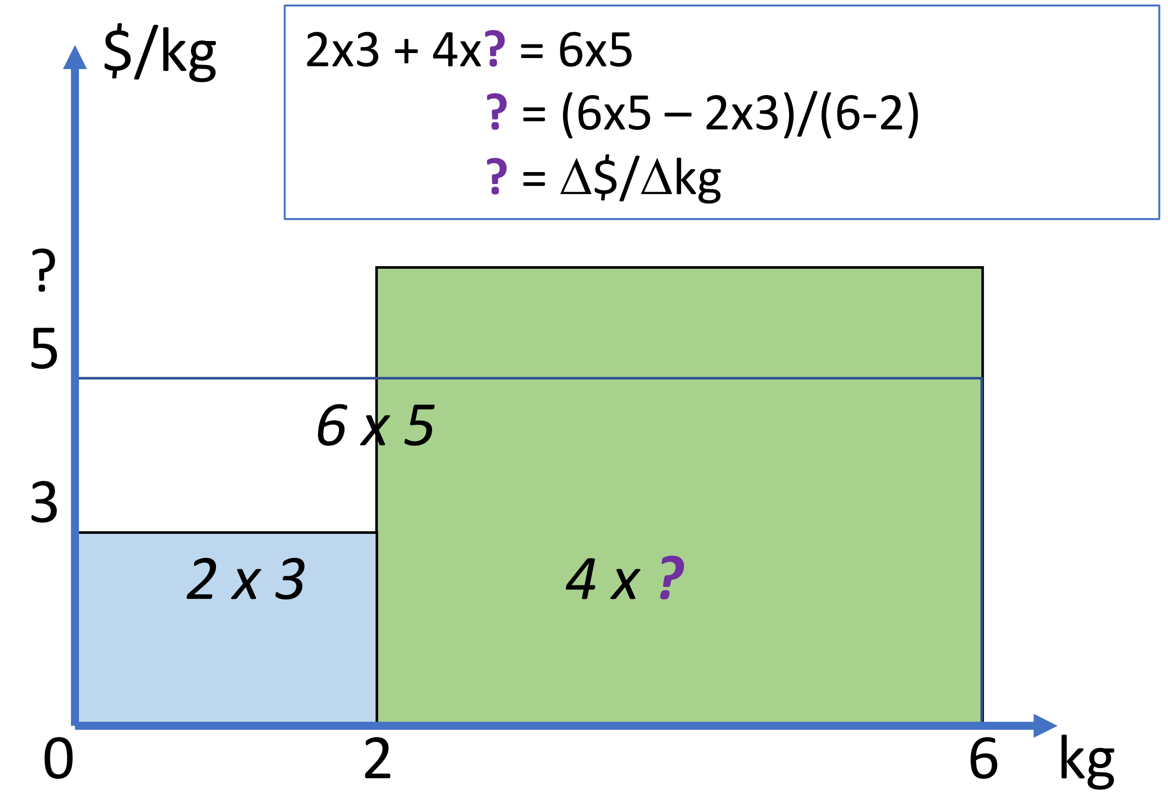
Q30, subtracting PerNumbers (differentiation)

"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

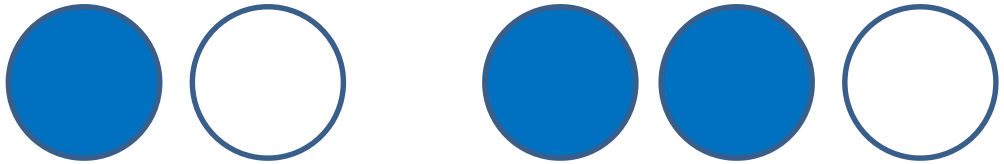
2 kg	at	3 $/kg
+ 4 kg	at	? $/kg
6 kg	at	5 $/kg

Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

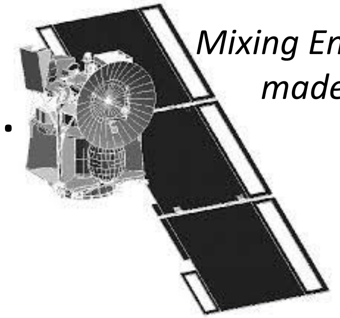
Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) precedes division.



Never add without units, the fraction paradox

The Teacher	The Students
What is 1/2 + 2/3?	Well, 1/2 + 2/3 = (1+2)/(2+3) = 3/5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But 1/2 of 2 cakes + 2/3 of 3 cakes is 1+2 of 2+3 cakes, i.e. 3/5 of 5 cakes! How can it be 7 cakes out of 6 cakes?
Inside this classroom 1/2 + 2/3 **IS** 7/6 !	

Fractions are not numbers, but operators, needing numbers to become numbers.
2+3 IS 5! No, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.
2*3 IS 6! Yes, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.
Adding without units: MatheMatism.



Mixing English and metric units made NASA's Mars Climate Orbiter fail in 1999.

Q31, adding unspecified numbers

"Only add like units, so how to add $T = 4ab^2 + 6abc$?"

Here units come from folding (factoring):

a factor-filter

$T = 4ab^2 + 6abc = T1 + T2$

$= 2 * 2 * a * b * b \quad + \quad 2 * 3 * a * b * c$

$= 2 * b * (2 * a * b) \quad + \quad 3 * c * (2 * a * b)$

$= (2b+3c) * \mathbf{2ab}$

$= 2b+3c\ \mathbf{2ab}_{\mathbf{s}}$

$4ab^2$	2	2	a	b	b
$6abc$	2	3	a	b	c
unit	**2**		$\boldsymbol{a}$	$\boldsymbol{b}$	
T1:		2			b
T2:		3			c

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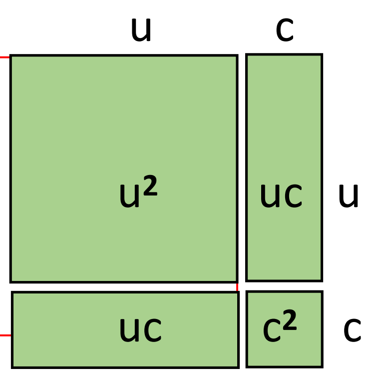
Q31, multiplying unspecified numbers



"How to multiply unspecified two-digit numbers T1 and T2?"

T1*T2 = (2u+3)*(4u-5)

	2u	+3	T1*T2
	?	?	4u
	?	?	-5
?uu	**?u**	?	
	2u	+3	T1*T2
	8uu	+12**u**	4u
	-10**u**	-15	-5
8**u*u**	+2**u**	-15	
8u^2	+2**u**	-15	

T1*T2 = (u+c)*(u-c)

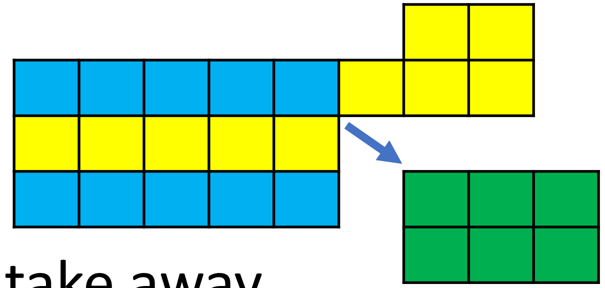
	u	+c	T1*T2
	?	?	u
	?	?	-c
?uu	**?u**	?	
	u	+c	T1*T2
	uu	+cu	u
	-cu	-cc	-c
uu		-cc	
$\mathbf{u^2}$		$\mathbf{-c^2}$	

T1*T2 = (u+c)*(u+c) = $(u+c)^2$

	u	+c	T1*T2
	?	?	u
	?	?	+c
?uu	**?u**	?	
	u	+c	T1*T2
	uu	+cu	u
	+cu	+cc	+c
uu	**+2cu**	+cc	
$\mathbf{u^2}$	**+2cu**	$\mathbf{-c^2}$	

Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 8/2 *Solved by ReCounting*	? = 8-2 *Solved by ReStacking*	? = (3.2 **8s** – 2 **3s**)/5 *Solved by differentiation: (T-T1)/5 = ΔT/5*



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: **O**pposite **S**ide with **O**ppo**S**ite **S**ign

MetaMath (Don't test, but DO remember the bi-implication arrows)

2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
⇕ (2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
⇕ (*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕ *u* x (2 x (½)) = 4	Applying the **associative** law
⇕ *u* x 1 = 4	Applying the definition of an inverse element
⇕ *u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

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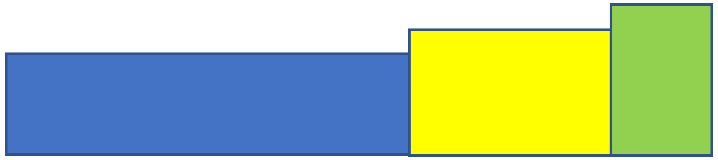
Conclusions

What Mastery of Many does the child have already?

- Children typically see Many as blocks with a number af bundles, and use flexible numbers with units and with over- or underloads

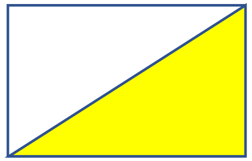
In ManyMath, BLOCKS are fundamental:

- in numbers: 456 = three blocks



- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks



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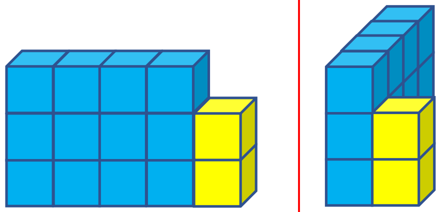
The child's own twin math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving <u>outside</u> geometrical multi-blocks, & (when turned to hide the units behind) <u>inside</u> algebraic bundle-numbers.

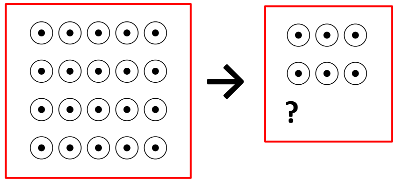
3) Operations are icons, showing the 3 counting steps: Removing bundles & stacking bundles & removing stacks to find the unbundled.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

Question: T = 4 ***5s*** *= ?* ***3s*** ● *Answer: T = 4* ***5s*** *= 6B2* ***3s*** ● *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



Comparing with a traditional math curriculum I

A traditional curriculum: operations on specified and unspecified numbers.

- Digits are given directly as symbols, without letting children discover digits as icons with as many strokes or sticks as they represent.
- Numbers are one-dimensional line-numbers with digits respecting a place value system, without letting children discover the thrill of two-dimensional bundling and stacking counting both <u>singles</u> and <u>bundles</u> and <u>bundles-of-bundles</u> etc., and that includes the unit.
- Seldom, if ever, 0 is included as ‘01, 02, 03’ in the counting sequence to show the importance of bundling.

Comparing with a traditional math curriculum II

- Never children are told that eleven and twelve comes from the Vikings, counting '(ten and) 1 left', '(ten and) 2 left'.
- Never children use full number-language sentences, T = 2 **5s**, including both a subject & a verb & a predicate with a unit.
- Seldom children are asked to describe numbers after ten as 1**B**4 **tens** or 1**ten**4 or 1.4 **tens** with a unit and with a decimal point separating bundles and unbundled singles.
- Seldom 17 is recounted as 2**B**-3 or 2.-3 **tens**. Nor is 24 recounted as 1**B**14 **tens** or 3**B**-6 **tens**.

Comparing with a traditional math curriculum III

- Never it respects the natural order of operations. Instead it turns the order around by giving addition without units priority over subtraction & multiplication & division.
- In short, children never experience the enchantment of counting, re-counting and double-counting Many before being forced to add on-top only, thus neglecting next-to addition.

So, re-enchanting Many is the goal of the twin curriculum in Mastery of Many through respecting and developing the children's existing mastery and quantitative competence.

Proportionality shows the variety of mastery of Many I

Proportionality, **Q1**: “2kg costs 5$, what does 7kg cost”; **Q2**: “What does 12$ buy?”

1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: ‘2kg cost 5$, 7kg cost ?$’. Multiply-then-divide gives the $-number 7x5/2 = 17.5.

Q2: ‘5$ buys 2kg, 12$ buys ?kg’. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

2) Find the unit

Q1: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

3) Cross multiplication

Q1: 2/5 = 7/*u*, so 2**u* = 7*5, *u* = (7*5)/2 = 17.5. **Q2**: 2/5 = *u*/12, so 5**u* = 12*2, *u* = (12*2)/5 = 4.8

4) ‘Re-counting’ in the ‘per-number’ 2kg/5$ coming from ‘double-counting’ the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

Proportionality shows the variety of mastery of Many II

5) Modeling with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive \$-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ ● $(c*2)*½ = 5*½$ ● $c*(2*½) = 5/2$ ● $c*1 = 5/2$ ● $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$\$.
- With 12\$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

Main parts of a ManyMath curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage

- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- BundleCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: divide, multiply, subtract, add on-top & next-to

Middle school – integrating algebra and geometry, the content of the label 'math'

- DoubleCounting produces PerNumbers and fractions as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always, so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE

- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

MATHeCADEMY

Question guided teacher education

MATHeCADEMY.net

Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

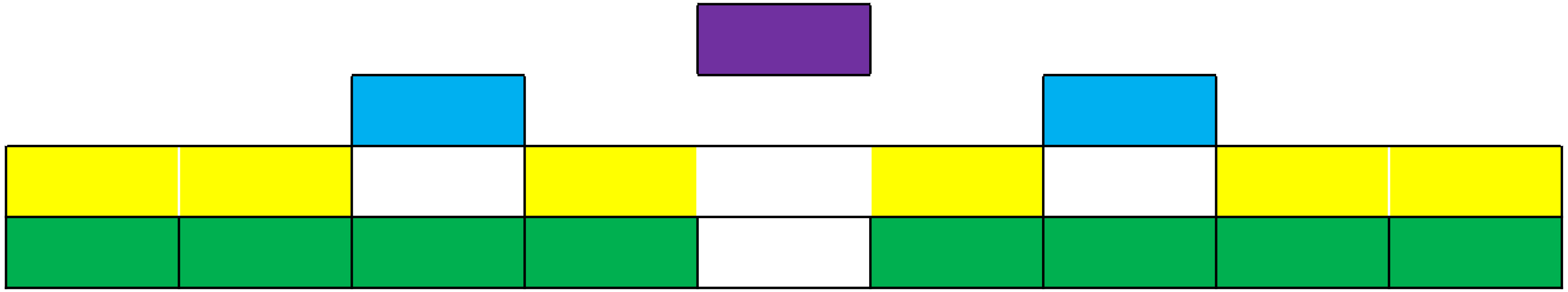
	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

PYRAMIDeDUCATION

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting their Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams



Number Icons

ReCounting 7 in **5s** & **3s** & **2s**

MATHeCADEMY

Theoretical background

Tarp, A. (2018). Mastering Many by counting, recounting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, March 2018, 11*(1), 103-117.

COUNTING before ADDING

The Child's Own Twin Curriculum

Count & ReCount & DoubleCount

before Adding NextTo & OnTop

master many

manymath