Addition-free migrant-math rooted in STEM re-counting formulas

Allan Tarp

MATHeCADEMY.net, Denmark; Allan.Tarp@gmail.com

A curriculum architect is asked to avoid traditional mistakes when designing a curriculum for young migrants that will allow them to quickly become STEM pre-teachers and pre-engineers. Typical multiplication formulas expressing re-counting in different units suggest an addition-free curriculum. To answer the question 'How many in total?' we count and re-count totals in the same or in a different unit, as well as to and from tens; also, we double-count in two units to create per-numbers, becoming fractions with like units. To predict, we use a re-count formula as a core formula in all STEM subjects.

Keywords: STEM, migrant, elementary school mathematics, curriculum, PISA.

Decreased PISA performance despite increased research

Research in mathematics education has grown since the first International Congress on Mathematics Education in 1969. Likewise has funding, see e.g. Swedish Centre for Mathematics Education. Yet, despite extra research and funding, decreasing Swedish PISA result caused OECD to write the report "Improving Schools in Sweden" (2015a) describing its school system as "in need of urgent change" since "more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (p. 3).

To find an unorthodox solution we pretend that a university in southern Sweden, challenged by numerous young male migrants, arranges a curriculum architect competition: "Theorize the low success of 50 years of mathematics education research; and derive from this theory a STEM based core curriculum allowing young migrants to return as STEM pre-teachers and pre-engineers."

Since mathematics education is a social institution, social theory may give a clue to the lacking research success and how to help migrants and how to improve schools in Sweden and elsewhere.

Social theory looking at mathematics education

Imagination as the core of sociology is described by Mills (1959). Bauman (1990) agrees by saying that sociological thinking "renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now" (p. 16).

As to institutions, of which mathematics education is an example, he talks about rational action "in which the end is clearly spelled out, and the actors concentrate their thoughts and efforts on selecting such means to the end as promise to be most effective and economical (p. 79)". He then points out that "The ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called goal displacement (p. 84)."

One such goal displacement is saying that the goal of mathematics education is to learn mathematics since such a goal statement is meaningless by its self-reference. So, if mathematics isn't the goal of mathematics education, what is? And, how well defined is mathematics after all?

In ancient Greece, the Pythagoreans used mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: arithmetic, geometry, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many by itself, Many in space, Many in time and Many in time and space. And together forming the ‘quadrivium’ recommended by Plato as a general curriculum together with ‘trivium’ consisting of grammar, logic and rhetoric.

With astronomy and music as independent knowledge areas, today mathematics should be a common label for the two remaining activities, geometry and algebra, both rooted in the physical fact Many through their original meanings, ‘to measure earth’ in Greek and ‘to reunite’ in Arabic. And in Europe, Germanic countries taught counting and reckoning in primary school and arithmetic and geometry in the lower secondary school until about 50 years ago when they all were replaced by the ‘New Mathematics’. Here the invention of the concept Set created a Set-based ‘meta-matics’, self-referential defining concepts top-down as examples of abstractions instead of bottom-up as abstractions from examples. But, then Russell looked at the set of sets not belonging to itself. Here a set belongs only if it does not: if $M = \{ A \mid A \notin A \}$ then $M \in M \Leftrightarrow M \notin M$. Thus pointing out that self-reference leads to the classical liar paradox ‘this sentence is false’ being false if true and true if false.

In this way, Set changed grounded classical mathe-matics into today’s self-referring ‘meta-matism’, a mixture of meta-matics and ‘mathe-matism’ true inside but seldom outside classrooms where adding numbers without units as ‘2 + 3 IS 5’ meet counter-examples as e.g. 2weeks + 3days is 17 days; in contrast to ‘2*3 = 6’ stating that 2 3s can always be re-counted as 6 1s.

Difference research looks at mathematics education

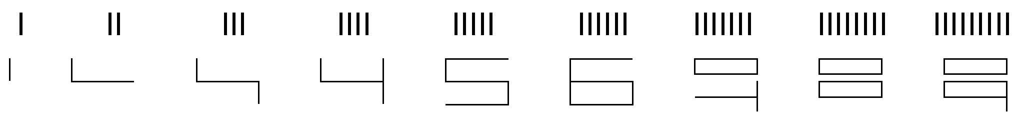
Inspired by the ancient Greek sophists (Russell, 1945), wanting to avoid being patronized by choices presented as nature, ‘Difference-research’ (Tarp, 2017) is searching for hidden differences making a difference. An additional inspiration comes from existentialist philosophy described by Sartre (2007, p. 20) as holding that “Existences precedes essence”. So, to avoid a goal displacement in math education, difference-research asks: How will math look like if grounded in its outside root, Many?

To answer we allow Many to open itself for us, so that, as curriculum architects, sociological imagination may allow us to construct a mathematics core curriculum based upon examples of Many in a STEM context (Lawrenz et al, 2017). So, we now return to the original Greek meaning of mathematics as knowledge about Many by itself and in time and space; and use Grounded Theory (Glaser & Strauss, 1967), lifting Piagetian knowledge acquisition (Piaget, 1969) from a personal to a social level, to allow Many create its own categories and properties.

Meeting Many creates a ‘count-before-adding’ curriculum

Meeting Many, we ask “How many in Total?” To answer, we total by counting to create number-language sentences as e.g. T = 2 3s, containing a subject and a verb and a predicate as in a word-language sentence; and connecting the outside total T with its inside predicate 2 3s (Tarp, 2018b).

Rearranging many 1s into one symbol with as many strokes as it represents (four strokes in the 4-con, five in the 5-icon, etc.) creates icons to be used as units when counting:



1 2 3 4 5 6 7 8 9

Holding 4 fingers together 2 by 2, a 3year-old will say 'That is not 4, that is 2 2s', thus describing what exists, bundles of 2s and 2 of them. This inspires 'bundle-counting', re-counting a total in icon-bundles to be stacked as bundle- or block-numbers, which can be re-counted and double-counted before being processed by on-top and next-to addition, direct or reversed. Thus, a total T of 5 1s is re-counted in 2s as T = 2 2s & 1; described by 'bundle-writing', T = 2B1 2s; or by 'decimal-writing', T = 2.1 2s, where, with a bundle-cup, a decimal point separates the bundles inside from the outside unbundled singles; or by 'deficit-writing', T = 3B-1 2s = 3.-1 2s = 3 bundles less 1 2s.

So, to count a total T we take away bundles B (thus rooting and iconizing division as a broom wiping away the bundles) to be stacked (thus rooting and iconizing multiplication as a lift stacking the bundles into a block) to be moved away to look for unbundled singles (thus rooting and iconizing subtraction as a trace left when dragging the block away). A calculator thus predicts the result by a re-count formula T = (T/B)*B saying that 'from T, T/B times, B can be taken away': entering '5/2' on a calculator gives '2.some', and '5 – 2x2' gives '1', so T = 5 = 2B1 2s. The unbundled can be placed next-to or on-top the stack thus rooting decimals and fractions.

The re-count formula occurs all over. With proportionality: y = c*x; in trigonometry as sine, cosine and tangent: a = (a/c)*c = sinA*c and b = (b/c)*c = cosA*c and a = (a/b)*b = tanA*b; in coordinate geometry as line gradients: Δy = Δy/Δx = c* Δx; and in calculus as the derivative, dy = (dy/dx)*dx = y'*dx. In economics, the re-count formula is a price formula: $ = ($/kg)*kg, $ = ($/day)*day, etc.

Re-counting in the same unit or in a different unit

Once counted, totals can be re-counted in the same unit, or in a different unit. Re-counting in the same unit, changing a bundle to singles allows re-counting a total of 2B1 2s as 1B3 2s with an outside 'overload'; or as 3B-1 2s with an outside 'underload' thus rooting negative numbers. This eases division: 336 = 33B6 = 28B56, so 336/7 = 4B8 = 48; or 336 = 35B-14, so 336/7 = 5B-2 = 48. Re-counting in a different unit means changing unit, also called proportionality. Asking '3 4s is how many 5s?', sticks show that 3 4s becomes 2B2 5s. Entering '3*4/5' we ask a calculator 'from 3 4s we take away 5s'. The answer, '2.some', predicts that the singles come from taking away 2 5s, now asking '3*4 – 2*5'. The answer, '2', predicts that 3 4s can be re-counted in 5s as 2B2 5s or 2.2 5s.

Re-counting to and from tens

Asking '3 4s = ? tens' is called times tables to be learned by heart. Using sticks to de-bundle and re-bundle shows that 3 4s is 1.2 tens. Using the re-count formula is impossible since the calculator has no ten-button. Instead it is programmed to give the answer directly as 3*4 = 12, thus using a short form that leaves out the unit and misplaces the decimal point one place to the right. Re-counting from tens to icons by asking '35 = ? 7s' is called solving an equation x*7 = 35. It is easily solved by re-counting 35 in 7s: x*7 = 35 = (35/7)*7. So x = 35/7, showing that equations are solved by moving to the opposite side with the opposite calculation sign.

Double-counting creates proportionality as per-numbers

Counting a quantity in 2 different physical units gives a 'per-number' as e.g. 2$ per 3kg, or 2$/3kg. To answer the question 'T = 6$ = ?kg', we re-count 6 in the per-number 2s: 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%.

A short curriculum in addition-free mathematics

01. To stress the importance of bundling, the counting sequence can be: 01, 02, …, 09, 10, 11 etc.; or 01, 02, 03, 04, 05, Ten less 4, T-3, T-2, T-1, Ten, Ten and 1, T and 2, etc.

02. Ten fingers can be counted also as 13 7s, 20 5s, 22 4s, 31 3s, 101 3s, 5 2s, and 1010 2s.

03. A Total of five fingers can be re-counted in three ways (standard and with over- and underload): T = 2B1 5s = 1B3 5s = 3B-1 5s = 3 bundles less 1 5s.

04. Multiplication tables can be formulated as re-counting from icon-bundles to tens and use underload counting after 5: T = 4*7 = 4 7s = 4*(ten less 3) = 40 less 12 = 30 less 2 = 28.

05. Dividing by 7 can be formulated as re-counting from tens to 7s and use overload counting: T = 336 /7 = 33B6 /7 = 28B56 /7 = 4B8 = 48.

06. Solving proportional equations as 3*x = 12 can be formulated as re-counting from tens to 3s: 3*x = 12 = (12/3)*3 giving x = 12/3 illustrating the relevance of the 'opposite side & sign' method.

07. Proportional tasks can be done by re-counting in the per-number: With 3$/4kg, 20kg = (20/4)*4kg = (20/4)*3$ = 15$; and 18$ = (18/3)*3$ = (18/3)*4kg = 24kg.

08. Fractions and percentages are per-numbers coming from double-counting in the same unit, 2/3 = 2$/3$. So 2/3 of 60 = 2$/3$ of 60$, where 60$ = (60/3)*3$ then gives (60/3)*2$ = 40$.

09. Trigonometry can precede plane and coordinate geometry to show how, in a box halved by its diagonal, the sides can be mutually re-counted as e.g. a = (a/c)*c = sinA*c, and a = (a/b)*b = tanA*b.

10. Counting by stacking bundles into adjacent blocks leads to the number formula or bundle formula called a polynomial: T = 456 = 4*BundleBundle + 5*Bundle + 6*single = 4*B^2 + 5*B + 6*1. In its general form, the number formula T = a*x^2 + b*x + c contains the different formulas for constant change: T = a*x (proportionality), T = a*x^2 (acceleration), T = a*x^c (elasticity) and T = a*c^x (interest rate); as well as T = a*x+b (linearity, or affinity, strictly).

11. Predictable change roots pre-calculus (if constant) and calculus (if changing). Unpredictable change roots statistics to 'post-dict' numbers by a mean and a deviation to be used by probability to pre-dict a confidence interval for numbers we else cannot pre-dict.

12. Integral calculus can precede differential calculus and include adding both piecewise and locally constant (continuous) per-numbers. Adding 2kg at 3$/kg and 4kg at 5$/kg, the unit-numbers 2 and 3 add directly, but the per-numbers must be multiplied into unit-numbers. So, both per-numbers and fractions must be multiplied by the units before being added as the area under the per-number graph.

Meeting Many in a STEM context

OECD (2015b) says: 'In developed economies, investment in STEM disciplines (science, technology, engineering and mathematics) is increasingly seen as a means to boost innovation and economic growth.' STEM thus combines knowledge about how humans interact with nature to survive and prosper: Mathematical formulas predict nature's behavior, and this knowledge, logos, allows humans to invent procedures, techne, and to engineer artificial hands and muscles and brains, i.e. tools, motors and computers, that combined to robots will help transforming nature into human necessities.

Nature as heavy things in motion

To meet, we must specify space and time in a nature consisting of heavy things at rest or in motion. So, in general, we see that what exists in nature is matter in space and time.

A falling ball introduces nature's three main factors, matter and force and motion, like the three social factors, humans and will and obedience. As to matter, we observe three balls: the earth, the ball, and molecules in the air. Matter houses two forces, an electro-magnetic force keeping matter together when colliding, and gravity pumping motion in and out of matter when it moves in the same or in the opposite direction of the force. In the end, the ball is at rest on the ground having transferred its motion through collisions to molecules in the air; meaning that the motion has lost its order and can no longer be put to work. In technical terms: as to motion, its energy stays constant, but its disorder (entropy) increases. But, if the disorder increases, how is ordered life possible? Because, in the daytime the sun pumps in high-quality, low-disorder light-energy; and in the nighttime the space sucks out low-quality, high-disorder heat-energy; if not, global warming would be the consequence.

So, a core STEM curriculum could be about cycling water. Heating transforms water from solid to liquid to gas, i.e. from ice to water to steam; and cooling does the opposite. Heating an imaginary box of steam makes some molecules leave, so the lighter box is pushed up by gravity until becoming heavy water by cooling, now pulled down by gravity as rain in mountains and through rivers to the sea. On its way down, a dam can transform falling water into electricity.

In the sea, water contains salt. Meeting ice at the poles, water freezes but the salt stays in the water making it so heavy it is pulled down by gravity, elsewhere pushing warm water up thus creating cycles in the ocean pumping warm water to cold regions.

The two water-cycles fueled by the sun and run by gravity leads on to other STEM areas: to dissolving matter in water; to the trajectory of a ball pulled down by gravity; to put steam and electrons to work in a power plant creating an electrical circuit transporting energy from a source to many consumers.

In nature, we count heaviness in kilograms, space in meters and time in seconds. Heavy things in motion have a momentum = mass*velocity, a multiplication formula as most STEM formulas expressing re-counting by per-numbers: kilogram = (kilogram/cubic-meter) * cubic-meter = density * cubic-meter; meter = (meter/second) * second = velocity * second; force = (force/square-meter) * square-meter = pressure * square-meter, where force is the per-number change in momentum per second. Thus, STEM-subjects are swarming with per-numbers: kg/m^3 (density), meter/second (velocity), Joule/second (power), Joule/kg (melting), Newton/m^2 (pressure), etc.

Warming and boiling water

In a water kettle, a double-counting can take place between the time elapsed and the energy used to warm the water to boiling, and to transform the water to steam.

If pumping in 410 kiloJoule will heat 1.4 kg water 70 degrees we get a double per-number 410/70/1.4 Joule/degree/kg or 4.18 kJ/degree/kg, called the specific heat capacity of water. If pumping in 316 kJ will transform 0.14 kg water at 100 degrees to steam at 100 degrees, the per-number is 316/0.14 kJ/kg or 2260 kJ/kg, called the heat of evaporation for water.

Dissolving material in water

In the sea, salt is dissolved in water, described as the per liter number of moles, each containing a million billion billion molecules. A mole of salt weighs 59 gram, so re-counting 100 gram salt in moles we get 100 gram = (100/59)*59 gram = (100/59)*1 mole = 1.69 mole, that dissolved in 2.5 liter has a strength as 1.69 moles per 2.5 liters or 1.69/2.5 moles/liters, or 0.676 moles/liter.

Building batteries with water

At our planet life exists in three forms: black, green and grey cells. Green cells absorb the sun's energy directly; and by using it to replace oxygen with water, they transform burned carbon dioxide to unburned carbohydrate storing the energy for grey cells, releasing the energy by replacing water with oxygen; or for black cells that by removing the oxygen transform carbohydrate into hydrocarbon storing the energy as fossil energy. Atoms combine by sharing electrons. At the oxygen atom the binding force is extra strong releasing energy when burning hydrogen and carbon to produce harmless water H_2O, and carbon dioxide CO_2, producing global warming if not bound in carbohydrate batteries. In the hydrocarbon molecule methane, CH_4, the energy comes from using 4 Os to burn it.

Technology and engineering: letting steam and electrons produce and distribute energy

A water molecule contains two hydrogen and one oxygen atom weighing 2*1+16 units. Thus a mole of water weighs 18 gram. Since the density of water is roughly 1000 gram/liter, the volume of 1000 moles is 18 liters. Transformed into steam, its volume increases to more than 22.4*1000 liters, or an increase factor of 22,400 liters per 18 liters = 1244 times. But, if kept constant, instead the inside pressure will increase as predicted by the ideal gas law, $p*V = n*R*T$, combining the pressure p, and the volume V, with the number of moles n, and the absolute temperature T, which adds 273 degrees to the Celsius temperature. R is a constant depending on the units used. The formula expresses different proportionalities: The pressure is direct proportional with the number of moles and the absolute temperature so that doubling one means doubling the other also; and inverse proportional with the volume, so that doubling one means halving the other.

Thus, with a piston at the top of a cylinder with water, evaporation will make the piston move up, and vice versa down if steam is condensed back into water. This is used in steam engines. In the first generation, water in a cylinder was heated and cooled by turn. In the next generation, a closed cylinder had two holes on each side of an interior moving piston thus increasing and decreasing the pressure by letting steam in and out of the two holes. The leaving steam is visible on e.g. steam locomotives.

Power plants use a third generation of steam engines. Here a hot and a cold cylinder are connected with two tubes allowing water to circulate inside the cylinders. In the hot cylinder, heating increases the pressure by increasing both the temperature and the number of steam moles; and vice versa in the cold cylinder where cooling decreases the pressure by decreasing both the temperature and the number of steam moles condensed to water, pumped back into the hot cylinder in one of the tubes. In the other tube, the pressure difference makes blowing steam rotate a mill that rotates a magnet over a wire, which makes electrons move and carry electrical energy to industries and homes.

An electrical circuit

Energy consumption is given in Watt, a per-number double-counting the number of Joules per second. Thus, a 2000 Watt water kettle needs 2000 Joules per second. The socket delivers 220 Volts, a per-number double-counting the number of Joules per charge-unit. Re-counting 2000 in 220 gives (2000/220)*220 = 9.1*220, so we need 9.1 charge-units per second, which is called the electrical current counted in Ampere. To create this current, the kettle must have a resistance R according to a circuit law Volt = Resistance*Ampere, i.e., 220 = R*9.1, or Resistance = 24.2 Volt/Ampere called Ohm. Since Watt = Joule per second = (Joule per charge-unit)*(charge-unit per second) we also have a second formula, Watt = Volt*Ampere. Thus, with a 60 Watt and a 120 Watt bulb, because of proportionality the latter needs twice the current, and consequently half the resistance of the former.

How high up and how far out

An inclined gun sends a ping-pong ball upwards. This allows a double-counting between the distance and the time to the top, 5 meters and 1 second. The gravity decreases the vertical speed when going up and increases it when going down, called the acceleration, a per-number counting the change in speed per second. To find its initial speed we turn the gun 45 degrees and count the number of vertical and horizontal meters to the top as well as the number of seconds it takes, 2.5 meters and 5 meters and 0,71 seconds. From a folding ruler we see, that now the total speed is split into a vertical and a horizontal part, both reducing the total sped with the same factor sin45 = cos45 = 0,707.

The vertical speed decreases to zero, but the horizontal speed stays constant. So we can find the initial speed u by the formula: Horizontal distance to the top position = horizontal speed * time, or with numbers: 5 = (u*0,707)*0,71, solved as u = 9.92 meter/seconds by moving to the opposite side with opposite calculation sign, or by a solver-app.

Compared with the horizontal, the vertical distance is halved, but the speed changes from 9.92 to 9.92*0.707 = 7.01. However, the speed squared is halved from 9.92*9.92 = 98.4 to 7.01*7.01 = 49.2.

So horizontally, there is a proportionality between the distance and the speed. Whereas vertically, there is a proportionality between the distance and the speed squared, so that doubling the vertical speed will increase the vertical distance four times.

Adding addition to the curriculum

Once counted as block-numbers, totals can be added next-to as areas, thus rooting integral calculus; or on-top after being re-counted in the same unit, thus rooting proportionality. And both next-to and on-top addition can be reversed, thus rooting differential calculus and equations where the question 2 3s + ? 4s = 5 7s leads to differentiation: ? = (5*7 – 2*3)/4 = ΔT/4. Traveling in a coordinate system, distances add directly when parallel; and by their squares when perpendicular.

The number formula T = 456 = 4*B^2 + 5*B + 6*1 shows there are four ways to unite numbers: addition and multiplication add changing and constant unit-numbers; and integration and power unite changing and constant per-numbers. And since any operation can be reversed: subtraction and division split a total into changing and constant unit-numbers; and differentiation and root & logarithm split a total in changing and constant per-numbers (Tarp, 2018b).

Conclusion and recommendation

This paper argues that 50 years of unsuccessful mathematics education research may be caused by a goal displacement seeing mathematics as the goal instead of as an inside means to the outside goal, mastery of Many in time and space. The two views lead to different kinds of mathematics: a set-based top-down ‘meta-matics’ that by its self-reference is indeed hard to teach and learn; and a bottom-up Many-based ‘Many-matics’ simply saying “To master Many, counting and re-counting and double-counting produces constant or changing unit-numbers or per-numbers, uniting by adding or multiplying or powering or integrating.“ A proposal for two separate twin-curricula in counting and adding is found in Tarp (2018a). Thus, the simplicity of mathematics as expressed in a ‘count-before-adding’ curriculum allows replacing line-numbers with block-numbers; and allows learning core mathematics as proportionality, calculus, equations and per-numbers in early childhood. Imbedded in STEM-examples, young migrants learn core STEM subjects at the same time, thus allowing them to become STEM pre-teachers or pre-engineers to help develop or rebuild their own country. The full curriculum can be found in a 27-page paper (Tarp, 2017). Thus, it is possible to solve STEM problems without learning addition, that is not well-defined since blocks can be added both on-top using proportionality to make the units the same, and next-to by areas as integral calculus.

References

Bauman, Z. (1990). *Thinking sociologically.* Oxford, UK: Blackwell.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht-Holland: D. Reidel Publishing Company.

Glaser, B. & Strauss, A. (1967). *The discovery of grounded theory.* New York: Aldine de Gruyter.

Lawrenz, F., Gravemeijer, K., & Stephan, M. (2017). Introduction to this Special Issue. *International Journal of Science and Mathematics Education, Vol. 15, Issue 1 Supplement,* pp. 1-4.

Mills, C. (1959). *The sociological imagination.* Oxford, UK: Oxford University Press.

OECD. (2015a). *Improving schools in Sweden: An OECD Perspective.* Retrieved from: www.oecd.org/edu/school/improving-schools-in-sweden-an-oecd-perspective.htm.

OECD. (2015b). *OECD Forum 2015.* Retrieved from www.oecd.org/forum/oecdyearbook/ we-must-teach-tomorrow-skills-today.htm.

Piaget, J. (1969). *Science of education of the psychology of the child.* New York: Viking Compass.

Russell B. (1945). *A history of western philosophy.* New York: A Touchstone Book.

Sartre, J.P. (2007). *Existentialism is a humanism.* CT. Yale University Press.

Tarp, A. (2017). *Math ed & research 2017.* Retrieved from //mathecademy.net/2017-math-articles/.

Tarp, A. (2018a). *Math ed & research 2018.* Retrieved from //mathecademy.net/2018-math-articles/.

Tarp, A. (2018b). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, March 2018, Vol. 11, No. 1*, pp. 103-117. http://www.educationforatoz.net/images/Allan_Tarp-JME-2018-1.pdf.