A NEW CURRICULUM - BUT FOR WHICH OF THE 3X2 KINDS OF MATHEMATICS EDUCATION

An essay on observations and reflections at the ICMI study 24 curriculum conference

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As part of institutionalized education, mathematics needs a curriculum describing goals and means. There are however three kinds of mathematics: pre-, present and post-'setcentric' mathematics; and there are two kinds of education: multi-year lines and half-year blocks. Thus, there are six kinds of mathematics education to choose from before deciding on a specific curriculum; and if changing, shall the curriculum stay within the actual kind or change to a different kind? The absence of federal states from the conference suggests that curricula should change from national multi-year macro-curricula to local half-year micro-curricula; and maybe change to post-setcentric mathematics.

COHERENCE AND RELEVANCE IN THE SCHOOL MATHEMATICS CURRICULUM

The International Commission on Mathematical Instruction, ICMI, has named its 24th study "School mathematics Curriculum Reforms: Challenges, Changes and Opportunities". Its discussion document has 5 themes among which theme B, "Analysing school mathematics curriculum for coherence and relevance" says that "All mathematics curricula set out the goals expected to be achieved in learning through the teaching of mathematics; and embed particular values, which may be explicit or implicit."

So, to analyze we use the verb 'cohere' and the predicate 'relevant' when asking: "to what does this curriculum cohere and to what is it relevant?" As to the meaning of the words 'cohere' and 'relevant' we may ask dictionaries.

The Oxford Dictionaries (en.oxforddictionaries.com) writes that 'to cohere' means 'to form a unified whole' with its origin coming from Latin 'cohaerere', from co- 'together' + haerere 'to stick'; and that 'relevant' means being 'closely connected or appropriate to what is being done or considered.'

We see, that where 'cohere' relates to states, 'relevant' relates to changes or processes taking place.

The Merriam-Webster dictionary (merriam-webster.com) seems to agree upon these meanings. It writes that 'to cohere' means 'to hold together firmly as parts of the same mass'. As to synonyms for cohere, it lists: 'accord, agree, answer, check, chord, coincide, comport, conform, consist, correspond, dovetail, fit, go, harmonize, jibe, rhyme (also rime), sort, square, tally.' And as to antonyms, it lists: 'differ (from), disagree (with).'

In the same dictionary, the word 'relevant' means 'having significant and demonstrable bearing on the matter at hand'. As to synonyms for relevant, it lists: 'applicable, apposite, apropos, germane, material, pertinent, pointed, relative.' And as to antonyms, it lists: 'extraneous, immaterial, impertinent, inapplicable, inapposite, irrelative, irrelevant, pointless.'

If we accept the verb 'apply' as having a meaning close to the predicate 'relevant', we can rephrase the above analysis question using verbs only: "to what does this curriculum cohere and apply?"

Seeing education metaphorically as bridging an individual start level for skills and knowledge to a common end level described by goals and values, we may now give a first definition of an ideal curriculum: “To apply to a learning process as relevant and useable, a curriculum coheres to the start and end levels for skills and knowledge.”

This definition involves obvious choices, and surprising choices also if actualizing the ancient Greek sophist warning against choice masked as nature. The five main curriculum choices are:

- How to make the bridge cohere with the individual start levels in a class?
- How to make the end level cohere to goals and values expressed by the society?
- How to make the end level cohere to goals and values expressed by the learners?
- How to make the bridge cohere to previous and following bridges?
- How to make the bridge (more) passable?

Then specific choices for mathematics education follow these general choices.

GOALS AND VALUES EXPRESSED BY THE SOCIETY

In her plenary address about the ‘OECD 2030 Learning Framework’, Taguma shared a vision:

> The members of the OECD Education 2030 Working Group are committed to helping every learner develop as a whole person, fulfil his or her potential and help shape a shared future built on the well-being of individuals, communities and the planet. (..) And in an era characterised by a new explosion of scientific knowledge and a growing array of complex societal problems, it is appropriate that curricula should continue to evolve, perhaps in radical ways (p. 10).

Talking about learner agency, Taguma said:

> Future-ready students need to exercise agency, in their own education and throughout life. (..) To help enable agency, educators must not only recognise learners’ individuality, (..) Two factors, in particular, help learners enable agency. The first is a personalised learning environment that supports and motivates each student to nurture his or her passions, make connections between different learning experiences and opportunities, and design their own learning projects and processes in collaboration with others. The second is building a solid foundation: literacy and numeracy remain crucial. (p. 11)

By emphasizing learner’s individual potentials, personalised learning environment and own learning projects and processes, Taguma seems to indicate that flexible half-year micro-curricula may cohere better with learners’ future needs than rigid multi-year macro-curricula. As to specifics, numeracy is mentioned as one of the two parts of a solid foundation helping learners enable agency.

DIFFERENT KINDS OF NUMERACY

Numeracy, however, is not that well defined. Oxford Dictionaries and Merriam-Webster agree on saying ‘ability to understand and work with numbers’; whereas the private organization National Numeracy (nationalnumeracy.org.uk) says ‘By numeracy we mean the ability to use mathematics in everyday life’.

The wish to show usage was also part of the Kilpatrick address, describing mathematics as bipolar:

> I want to stress that bipolarity because I think that’s an important quality of the school curriculum and every teacher and every country has to deal with: how much attention do we give to the purer side of mathematics. The New Math thought that it should be entire but that didn’t work really as well as people thought. So how much attention do we give to the pure part of mathematics and how much to the applications and how much do we engage together. Because it turns out if the applications are well-chosen and can be understood by the children then that helps them move toward the purer parts of the field. (p. 20)

After discussing some problems caused by applications in the curriculum, Kilpatrick concludes:

> If we stick with pure mathematics, with no application, what students cannot see, "when will I ever use this?", it's not surprising that they don't go onto take more mathematics. So, I think for self-preservation, mathematicians and mathematics educators should work on the question of: how do we orchestrate the curriculum so that applications play a good role? There is even is even a problem with the word applications, because it implies first you do the mathematics, then you apply it. And actually, it can go the other way. (p. 22)

So, discussing what came first, the hen or the egg, applications or mathematics, makes it problematic to define numeracy as the ability to apply mathematics since it gives mathematics a primacy and a monopoly as a prerequisite for numeracy. At the plenary afterwards discussion, I suggested using the word 're-rooting' instead of 'applying' to indicate that from the beginning, mathematics was rooted in the outside world as shown by the original meanings of geometry and algebra: 'to measure earth' in Greek and 'to reunite' in Arabic.

MATHEMATICS THROUHG HISTORY

In ancient Greece, the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: geometry, arithmetic, music and astronomy, seen by the Greeks as knowledge about Many in space, Many by itself, Many in time, and Many in space and time. Together they formed the 'quadrivium' recommended by Plato as a general curriculum together with 'trivium' consisting of grammar, logic and rhetoric.

With astronomy and music as independent areas, mathematics became a common label for the two remaining activities, geometry and algebra. And in Europe, Germanic countries taught 'reckoning' in primary school and 'arithmetic' and 'geometry' in the lower secondary school until about 50 years ago when they all were replaced by the 'New Mathematics'.

Here a wish for exactness and unity created a 'setcentric' 'meta-matics' as a collection of 'well-proven' statements about 'well-defined' concepts, defined top-down as examples from abstractions instead of bottom-up as abstractions from examples. But Russell showed that the self-referential liar paradox 'this sentence is false', being false if true and true if false, reappears in the set of sets not belonging to itself, where a set belongs only if it does not: If $M = \left\{ A \mid A \notin A \right\}$ then $M \in M \Leftrightarrow M \notin M$. The Zermelo-Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating abstract concepts from concrete examples.

Setcentrism thus changed classical grounded 'many-matics' into a self-referring 'meta-matism', a mixture of meta-matics and 'mathe-matism' true inside but seldom outside a classroom where adding numbers without units as '1 + 2 IS 3' meets counter-examples as e.g. 1week + 2days is 9days.

The introduction of the setcentric New Mathematics created different reactions. Inside the United States it was quickly abandoned with a 'back-to-basics' movement. Outside it was implemented at teacher education, and in schools where it gradually softened. However, it never retook its original form or name, despite, in contrast to 'mathematics', 'reckon' is an action-word better suited to the general aim of education, to teach humans to master the outside world through appropriate actions.

DIFFERENT KINDS OF MATHEMATICS

So, a curriculum must choose between a pre-, a present, and a post-setcentric mathematics as illustrated by an example from McCallum's plenary talk. After noting that "a particularly knotty area in mathematics curriculum is the progression from fractions to ratios to proportional relationships" (p. 4), McCallum asked the audience: "What is the difference between 5/3 and 5÷3".

Pre-setcentric mathematics will say that 5/3 is a number on the number-line reached by taking 5 steps of the length coming from dividing the unit in 3 parts; and that 5÷3 means 5 items shared between 3.

Present setcentric mathematics will say that 5/3 is a rational number defined as an equivalence class in the product set of integers, created by the equivalence relation (*a*,*b*) eq. (*c*,*d*) if cross-multiplication holds, *a*x*d* = *bxc*; and, with 1/3 as the inverse element to 3 under multiplication, 5÷3 should be written as 5x1/3, i.e. the as the solution to the equation 3x*u* = 5, found by applying and thus legitimizing abstract algebra and group theory; thus finally saying goodbye to the Renaissance use of a vertical line to separate addends from subtrahends, and a horizontal line to separate multipliers from divisors.

Post-setcentric mathematics (Tarp, 2018) sees setcentric mathematics as meta-matism hiding the original Greek meaning og mathematics as a science about Many. In this 'Many-math', 5/3 is a per-number coming from double-counting in different units (5$/3kg), becoming a fraction with like units (5$/3$ = 5/3). Here per-numbers and fractions are not numbers but operators needing a number to become a number (5/3 of 3 is 5, 5/3 of 6 is 10); and 5÷3 means 5 counted in 3s occurring in the 'recount-formula' recounting a total *T* in bundles of 3s as *T* = (*T*/3)x3, saying 'from *T*, *T*/3 times, 3 can be taken away'. This gives flexible numbers: *T* = 5 = 1B2 3s = 1.2 3s = 1 2/3 3s = 2B-1 3s = 2.-1 3s, introduced in grade one where bundle-counting and re-counting in another unit precedes adding, and where recounting from tens to icons, *T* = 2.4 tens = ? 6s, leads to the equation *T* = *u*x6 = 24 = (24/6)x6 solved by recounting. In post-setcentric mathematics, per-numbers, fractions, ratios and proportionality melt together since double-counting in two units gives per-numbers as ratios, becoming fractions with like units. And here proportionality means changing units using the recount-formula to recount in the per-number: With 5$/3kg, "how much for 20$?" is found by re-counting 20 in 5s: *T* = 20$ = (20/5)x5$ = (20/5)x3kg = 12 kg. Likewise if asking "how much for 15 kg?"

DIFFERENT KINDS OF EDUCATION

As to education, from secondary school there is a choice between multi-year lines and half-year blocks. At the discussion after the Kilpatrick plenary session I made a comment about these two educational systems, which mas a lady from the United States say I was misinforming since in the states Calculus required a full year block. Together with other comments in the break, this made me realize that internationally there is little awareness of these two different kinds of educational systems. So here is another example of what the Greek sophists warned against, choice masked as nature.

Typically, unitary states have one multi-year curriculum for primary and lower secondary school, followed by parallel multi-year curricula for upper secondary and tertiary education. Whereas, by definition, federal states have parallel curricula, or even half-year curricula from secondary school as in the United States.

At the conference, the almost total absence of federal states as Germany, Canada, the United States and Russia seems to indicate that the problems reside with multi-year national curricula, becoming rigid traditions difficult to change. While federal competition or half-year blocks creates flexibility through an opportunity to try out different curricula.

Moreover, as a social institution involving individual constraint, education calls for sociological perspectives. Seeing the Enlightenment Century as rooting education, it is interesting to study its forms in its two Enlightenment republics, the North American from 1776 and the French from 1789. In North America, education enlightens children about their outside world, and enlighten teenagers about their inside individual talent, uncovered and developed through self-chosen half-year blocks with teachers teaching one subject only in their own classrooms.

To protect its republic against its German speaking neighbors, France created elite schools, criticized today for exerting hidden patronization. Bourdieu thus calls education 'symbolic violence', and Foucault points out that a school is really a 'pris-pital' mixing power techniques from a prison and a hospital, thus raising two ethical issues: On which ethical ground do we force children and teenagers to return to the same room, hour after hour, day after day, week after week, month after month for several years? On which ethical ground do we force children and teenagers to be cured from self-referring diagnoses as e.g., the purpose of mathematics education is to cure mathematics ignorance? Issues, the first Enlightenment republic avoids by offering teenagers self-chosen half-year blocks; and by teaching, not mathematics, but algebra and geometry referring to the outside world by their original meanings.

DIFFERENT KINDS OF COMPETENCES

As to competences, new to many curricula, there are at least three alternatives to choose among. The European Union recommends two basic competences, acquiring and applying, when saying that "Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge."

At the conference two alternatives notions of competences were presented. In his plenary address, Niss recommended a matrix with 8 competences per concept (p. 73). In his paper, Tarp (pp. 317-324) acknowledged that 8 competences may be needed if the goal of mathematics education is to learn present setcentric university mathematics; but if the goal is to learn to master Many with post-setcentric mathematics, then only two competences are needed: counting and adding, rooting a twin curriculum teaching counting, recounting in different units and double-counting before adding.

MAKING THE LEARNING ROAD MORE PASSABLE

Once a curriculum is chosen, the next question is to make its bridge between the start and end levels for skills and knowledge more passable. Here didactics and pedagogy come in; didactics as the captain choosing the way from the start to the end, typically presented as a textbook leaving it to pedagogy, the lieutenants, to take the learners through the different stages.

The didactical choices must answer general questions from grand theory. Thus, philosophy will ask: shall the curriculum follow the existentialist recommendation, that existence precedes essence? And psychology will ask: shall the curriculum follow Vygotsky mediating institutionalized essence, or Piaget arranging learning meetings with what exists in the outside world? And sociology will ask: on which ethical grounds are children and teenagers retained to be cured by institutionalized education?

COLONIZING OR DECOLONIZING CURRICULA

The conference contained two plenary panels, the first with contributors from France, China, The Philippines and Denmark, almost all from the northern hemisphere; the second with contributors from Chile, Australia, Lebanon and South Africa, almost all from the southern hemisphere. Where the first panel talked more about solutions, the second panel talked more about problems.

In the first panel, France and Denmark represented some of the world's most centralized states with war-time educational systems dating back to the Napoleon era, which in France created elite-schools to protect the young republic from the Germans, and in Germany created the Humboldt Bildung schools to end the French occupation by mediating nationalism, and to sort out the population elite for jobs as civil servants in the new central administration; both just replacing the blood-nobility with a knowledge-nobility as noted by Bourdieu. The Bildung system latter spread to most of Europe.

Not surprisingly, both countries see university mathematics as the goal of mathematics education ('mathematics is what mathematicians do'), despite the obvious self-reference avoided by instead formulating the goal as e.g. learning numerical competence, mastery of Many or number-language. Seeing mathematics as the goal, makes mathematics education an example of a goal displacement (Bauman) where a monopoly transforms a means into a goal. A monopoly that makes setcentric mathematics an example of what Habermas and Derrida would call a 'center-periphery colonization', to be decentered and decolonized by deconstruction.

Artigue from France thus advocated an anthropological theory of the didactic, ATD, (p. 43-44), with a 'didactic transposition process' containing four parts: scholarly knowledge (institutions producing and using the knowledge), knowledge to be taught (educational system, 'noosphere'), taught knowledge (classroom), and learned available knowledge (community of study).

> The theory of didactic transposition developed in the early 1980s to overcome the limitation of the prevalent vision at the time, seeing in the development of taught knowledge a simple process of elementarization of scholarly knowledge (Chevallard 1985). Beyond the well-known succession offered by this theory, which goes from the reference knowledge to the knowledge actually taught in classrooms (..), ecological concepts such as those of niche, habitat and trophic chain (Artaud 1997) are also essential in it.

Niss from Denmark described the Danish 'KOM Project' leading to eight mathematical competencies per mathematical topic (pp. 71-72).

> The KOM Project took its point of departure in the need for creating and adopting a general conceptualisation of mathematics that goes across and beyond educational levels and institutions. (..) We therefore decided to base our work on an attempt to define and characterise mathematical competence in an overarching sense that would pertain to and make sense in any mathematical context. Focusing - as a consequence of this approach - first and foremost on the *enactment* of mathematics means attributing, at first, a secondary role to mathematical content. We then came up with the following definition of mathematical competence: Possessing *mathematical competence* – mastering mathematics – is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve actual or potential mathematical challenges of any kind. In order to identify and characterise the fundamental constituents in mathematical competence, we introduced the notion of mathematical competencies: A *mathematical competency* is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve a certain kind of mathematical challenge.

Some of the consequences by being colonized by setcentrism was described in the second panel.

In his paper ‘School Mathematics Reform in South Africa: A Curriculum for All and by All?’ Volmink from South Africa Volmink writes (pp. 106-107):

> At the same time the educational measurement industry both locally and internationally has, with its narrow focus, taken the attention away from the things that matter and has led to a traditional approach of raising the knowledge level. South Africa performs very poorly on the TIMSS study. In the 2015 study South Africa was ranked 38th out of 39 countries at Grade 9 level for mathematics and 47th out of 48 countries for Grade 5 level numeracy. Also in the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), South Africa was placed 9th out of the 15 countries participating in Mathematics and Science – and these are countries which spend less on education and are not as wealthy as we are. South Africa has now developed its own Annual National Assessment (ANA) tests for Grades 3, 6 and 9. In the ANA of 2011 Grade 3 learners scored an average of 35% for literacy and 28% for numeracy while Grade 6 learners averaged 28% for literacy and 30% for numeracy.

After thanking for the opportunity to participate in a cooperative effort on the search of better education for boys, girls and young people around the world, Oteiza from Chile talked about ‘The Gap Factor’ creating social and economic differences. A slide with the distribution of raw scores at PSU mathematics by type of school roughly showed that out of 80 points, the median scores were 40 and 20 for private and public schools respectively. In his paper, Oteiza writes (pp. 81-83):

> Results, in national tests, show that students attending public schools, close to de 85% of school population, are not fulfilling those standards. How does mathematical school curriculum contribute to this gap? How might mathematical curriculum be a factor in the reduction of these differences? (..) There is tremendous and extremely valuable talent diversity. Can we justify the existence of only one curriculum and only one way to evaluate it through standardized tests? (..) There is a fundamental role played by researchers, and research and development centers and institutions. (..) How do the questions that originate in the classroom reach a research center or a graduate program? “*Publish or perish*” has led our researchers to publish in prestigious international journals, but, are the problems and local questions addressed by those publications?”

The Gap Factor is also addressed in a paper by Hoyos from Mexico (pp. 258-259):

> The PISA 2009 had 6 performance levels (from level 1 to level 6). In the global mathematics scale, level 6 is the highest and level 1 is the lowest. (..) It is to notice that, in PISA 2009, 21.8% of Mexican students do not reach level 1, and, in PISA 2015, the percentage of the same level is a little bit higher (25.6%). In other words, the percentage of Mexican students that in PISA 2009 are below level 2 (i.e., attaining the level 1 or zero) was 51%, and this percentage is 57% in PISA 2015, evidencing then an increment of Mexican students in the poor levels of performance. According to the INEE, students at levels 1 or cero are susceptible to experiment serious difficulties in using mathematics and benefiting from new educational opportunities throughout its life. Therefore, the challenges of an adequate educational attention to this population are huge, even more if it is also considered that approximately another fourth of the total Mexican population (33.3 million) are children under 15 years of age, a population in priority of attention”.

As a comment to Volminks remark “Another reason for its lack of efficacy was the sense of scepticism and even distrust about the notion of People's Mathematics as a poor substitute for the “real mathematics”” (p. 104), and inspired by the sociological Centre–Periphery Model for colonizing, by post-colonial studies, and by Habermas’ notion of rationalization and colonization of the lifeworld by the instrumental rationality of bureaucracies, I formulated the following question in the afterwards discussion: “As former colonies you might ask: Has colonizing stopped, or is it still taking place? Is there an outside central mathematics that is still colonizing the mind? What happens to what could be called local math, street math, ethno-math or the child’s own math?”

CONCLUSION AND RECOMMENDATIONS

Designing a curriculum for mathematics education involves several choices. First pre-, present and post-setcentric mathematics together with multi-year lines and half-year blocks constitute 3x2 different kinds of mathematics education. Combined with three different ways of seeing competences, this offers a total of 18 different ways in which to perform mathematics education at each of the three educational levels, primary and secondary and tertiary, which may even be divided into parts.

Once chosen, institutional rigidity may hinder curriculum changes. So, to avoid the ethical issues of forcing cures from self-referring diagnoses upon children and teenagers in need of guidance instead of cures, the absence of participants from federal states might be taken as an advice to replace the national multi-year macro-curriculum with regional half-year micro-curricula. At the same time, adopting the post version of setcentric mathematics will make the curriculum coherent with the mastery of Many that children bring to school, and relevant to learning the quantitative competence and numeracy desired by society.

And, as Derrida says in an essay called 'Ellipsis' in 'Writing and Difference': "Why would one mourn for the centre? Is not the centre, the absence of play and difference, another name for death?"

POSTSCRIPT: MANY-MATH, A POST-SETCENTRIC MATHEMATICS FOR ALL

As post-setcentric mathematics, Many-math, can provide numeracy for all by celebrating the simplicity of mathematics occurring when recounting the ten fingers in bundles of 3s:

T = ten = 1B7 3s = 2B4 3s = 3B1 3s = 4B-2 3s. Or, if seeing 3 bundles of 3s as 1 bundle of bundles,

T = ten = 1BB0B1 3s = 1*B^2 + 0*B + 1 3s, or T = ten = 1BB1B-2 3s = 1*B^2 + 1*B – 2 3s.

This number-formula shows that a number is really a multi-numbering of singles, bundles, bundles of bundles etc. represented geometrically by parallel block-numbers with units. Also, it shows the four ways to unite: on-top addition, multiplication, power and next-to addition, also called integration. Which are precisely the four ways to unite constant and changing unit- and per-numbers numbers into totals as seen by including the units; each with a reverse way to split totals. Thus, addition and multiplication unite changing and constant unit-numbers, and integration and power unite changing and constant per-numbers. We might call this beautiful simplicity 'the Algebra Square', also showing that equations are solved by moving to the opposite side with opposite signs.

Operations **unite**/ *split* Totals in	Changing	Constant
Unit-numbers m, s, kg, $	***T = a + n*** *T – n = a*	***T = a*n*** *T/n = a*
Per-numbers m/s, $/kg, $/100$ = %	***T = ∫a*dn*** *dT/dn = a*	***T = a^n*** *n√T = a logaT = n*

An unbundled single can be placed on-top of the block counted in 3s as T = 1 = 1/3 3s, or next-to the block as a block of its own written as T = 1 = .1 3s Writing T = ten = 3 1/3 3s = 3.1 3s = 4.-2 3s thus introduces fractions and decimals and negative numbers together with counting.

The importance of bundling as the unit is emphasized by counting: 1, 2, 3, 4, 5, 6 or bundle less 4, 7 or B-3, 8 or B-2, 9 or B-1, ten or 1 bundle naught, 1B1, …, 1B5, 2B-4, 2B-3, 2B-2, 2B-1, 2B naught.

This resonates with ‘Viking-counting’: 1, 2, 3, 4, hand, and1, and2, and3, less2, less1, half, 1left, 2left. Here ‘1left’ and ‘2left’ still exist as ‘eleven’ and ‘twelve’, and ‘half’ when saying ‘half-tree’, ‘half-four’ and ‘half-five’ instead of 50, 70 and 90 in Danish, counting in scores; as did Lincoln in his Gettysburg address: “Four scores and seven years ago …”

Counting means wiping away bundles (called division iconized as a broom) to be stacked (called multiplication iconized as a lift) to be removed to find unbundled singles (called subtraction iconized as a horizontal trace). Thus, counting means postponing adding and introducing the operations in the opposite order of the tradition, and with new meanings: 7/3 means 7 counted in 3s, 2x3 means stacking 3s 2 times. Addition has two forms, on-top needing recounting to make the units like, and next-to adding areas, i.e. integral calculus. Reversed they create equations and differential calculus.

The recount-formula, T = (T/B)*B, appears all over mathematics and science as proportionality or linearity formula:

- Change unit, T = (T/B)*B, e.g. T = 8 = (8/2)*2 = 4*2 = 4 2s
- Proportionality, $ = ($/kg)*kg = price*kg
- Trigonometry, a = (a/c)*c = sinA*c, a = (a/b)*b = tanA*b, b = (b/c)*c = cosA*c
- STEM-formulas, meter = (meter/sec)*sec = speed*sec, kg = (kg/m^3)*m^3 = density*m^3
- Coordinate geometry, Δy = (Δy/Δx)*Δx = m*Δx
- Differential calculus, dy = (dy/dx)*dx = y’ * dx

The number-formula also contains the formulas for constant change: $T = b*x$ (proportional), $T = b*x + c$ (linear), $T = a*x^n$ (elastic), $T = a*n^x$ (exponential), $T = a*x^2 + b*x + c$ (accelerated).

If not constant, numbers change: constant change roots pre-calculus, predictable change roots calculus, and unpredictable change roots statistics ‘post-dicting’ what we cannot be ‘pre-dicted’.

THE GENERAL CURRICULUM CHOICES OF POST-SETCENTRIC MATHEMATICS

Making the curriculum bridge cohere with the individual start levels in a class is obtained by always beginning with the number-formula, and with recounting tens in icons less than ten, e.g. T = 4.2 tens = ? 7s, or u*7 = 42 = (42/7)*7, thus solving equations by moving to opposite side with opposite sign. And by always using full number-language sentences with a subject, a verb and a predicate as in the word language, e.g. T = 2*3. This also makes the bridge cohere to previous and following bridges.

Making the end level cohere to goals and values expressed by the society and by the learners is obtained by choosing mastery as the end goal, not of the inside self-referring setcentric construction of contemporary university mathematics, but of the outside universal physical reality, Many.

Making the bridge passable is obtained by choosing Piagetian psychology instead of Vygotskyan.

FLEXIBLE NUMBERS MAKE TEACHERS FOLLOW

Changing a curriculum raises the question: will the teachers follow? Here, seeing the advantage of flexible numbers makes teachers interested in learning more about post-setcentric mathematics:

Typically, division creates problems to students, e.g. 336/7. With flexible numbers a total of 336 can be recounted with an overload as T = 336 = 33B6 = 28B56, so 336/7 = 28B56 /7 = 4B8 = 48; or with an underload as T = 336 = 33B6 = 35B-14, so 336/7 = 35B-14 /7 = 5B-2 = 48.

Flexible numbers ease all operations:

T = 48*7 = 4B8*7 = 28B56 = 33B6 = 336

T = 92 – 28 = 9B2 – 2B8 = 7B-6 = 6B4 = 64

T = 54 + 28 = 5B4 + 2B8 = 7B12 = 8B2 = 82

To learn more about flexible numbers, a group of teachers can go to the MATHeCADEMY.net designed to teach teachers to teach MatheMatics as ManyMatics, a natural science about Many, to watch some of its YouTube videos. Next, the group can try out the "Free 1day Skype Teacher Seminar: Cure Math Dislike by ReCounting" where, in the morning, a power point presentation 'Curing Math Dislike' is watched and discussed locally, and at a Skype conference with an instructor. After lunch the group tries out a 'BundleCount before you Add booklet' to experience proportionality and calculus and solving equations as golden learning opportunities in bundle-counting and re-counting and next-to addition. Then another Skype conference follows after the coffee break.

To learn more, a group of eight teachers can take a one-year in-service distance education course in the CATS approach to mathematics, Count & Add in Time & Space. C1, A1, T1 and S1 is for primary school, and C2, A2, T2 and S2 is for secondary school. For modelling, there is a study unit in quantitative literature. The course is organized as PYRAMIDeDUCATION where the 8 teachers form 2 teams of 4, choosing 3 pairs and 2 instructors by turn. An external coach helps the instructors instructing the rest of their team. Each pair works together to solve count&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e. both assimilation and accommodation. The coach assists the instructors in correcting the count&add assignments. In a pair, each teacher corrects the other's routine-assignment. Each pair is the opponent on the essay of another pair. Each teacher pays for the education by coaching a new group of 8 teachers. The material mediates learning by experimenting with the subject in number-language sentences, i.e. the total T. Thus, the material is self-instructing, saying "When in doubt, ask the subject, not the instructor".

The material for primary and secondary school has a short question-and-answer format. The question could be: "How to count Many? How to recount 8 in 3s? How to count in standard bundles?" The corresponding answers would be: "By bundling and stacking the total T, predicted by T = (T/B)*B. So, T = 8 = (8/3)*3 = 2*3 + 2 = 2*3 + 2/3*3 = 2 2/3*3 = 2.2 3s = 3.-1 3s. Bundling bundles gives multiple blocks, a polynomial: T = 456 = 4BundleBundle + 5Bundle + 6 = 4*B^2 + 5*B + 6*1."

References

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