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ADDITION-FREE STEM-BASED MATH FOR MIGRANTS

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A curriculum architect is asked to avoid traditional mistakes when designing a curriculum for young migrants that will allow them to soon become STEM pre-teachers and pre-engineers. Multiplication formulas expressing recounting in different units suggest an addition-free curriculum. To answer the question 'How many in total?' we count and recount totals by bundling in the same or in a different unit, as well as to and from tens; also, we double-count in two units to create per-numbers, becoming fractions with like units. A recount formula that expresses proportionality when changing units is a core prediction formula in all STEM subjects.

DECREASED PISA PERFORMANCE DESPITE INCREASED RESEARCH

Research in mathematics education has grown since the first International Congress on Mathematics Education in 1969. Likewise has funding, see e.g. the Swedish National Centre for Mathematics Education. Yet, despite extra research and funding, and despite being warned against the possible irrelevance of a growing research industry (Tarp, 2004), decreasing Swedish PISA results caused OECD to write the report "Improving Schools in Sweden" (2015a) describing its school system as "in need of urgent change" since "more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life (p. 3)."

To find an unorthodox solution to poor PISA performance we pretend that a university in southern Sweden, challenged by numerous young male migrants, arranges a curriculum architect competition: "Theorize the low success of 50 years of mathematics education research; and derive from this a STEM based core curriculum allowing young migrants to soon become STEM pre-teachers and pre-engineers."

Since mathematics education is a social institution, social theory may give a clue to the lacking research success and how to improve schools in Sweden and elsewhere.

SOCIAL THEORY LOOKING AT MATHEMATICS EDUCATION

Imagination as the core of sociology is described by Mills (1959). Bauman (1990) agrees by saying that sociological thinking "renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now (p. 16)."

As to institutions, of which mathematics education is an example, Bauman talks about rational action "in which the *end* is clearly spelled out, and the actors concentrate their thoughts and efforts on selecting such *means* to the end as promise to be most effective and economical (p. 79)". He then points out that "The ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called **goal displacement** (p. 84)." Of which one example is saying that the goal of mathematics education is to learn mathematics since such a goal statement is obviously made meaningless by its self-reference.

The link between a goal and its means is also present in existentialist philosophy described by Sartre (2007) as holding that “Existences precedes essence (p. 20)”. Likewise, Arendt (1963) points out that practicing a means blindly without reflecting on its goal might lead to practicing “the banality of evil”. Which makes Bourdieu (1977) says that “All pedagogic action is, objectively, symbolic violence insofar as it is the imposition of a cultural arbitrary by an arbitrary power (p. 5)”. This raises the question if mathematics and education is universal or chosen, more or less arbitrarily.

DIFFERENT KINDS OF EDUCATION

The International Commission on Mathematical Instruction, ICMI, named its 24th study “School mathematics Curriculum Reforms: Challenges, Changes and Opportunities”. At its conference in Tsukuba, Japan, in November 2018 it became clear during plenary discussions that internationally there is little awareness of two different kinds of educational systems practiced from secondary school.

Typically, unitary states have one multi-year curriculum for primary and lower secondary school, followed by parallel multi-year curricula for upper secondary and tertiary education. Whereas, by definition, federal states have parallel curricula, or even half-year curricula from secondary school as in the United States.

Moreover, as a social institution involving monopolizing and individual constraint, education calls for sociological perspectives. Seeing the Enlightenment Century as rooting education, it is interesting to study its forms in its two Enlightenment republics, the North American from 1776 and the French from 1789. In North America, education enlightens children about their outside world, and enlightens teenagers about their inside individual talent, uncovered and developed through self-chosen half-year blocks with teachers teaching only one subject, and in their own classrooms.

To protect its republic against attack from its German speaking neighbors, France created elite schools with multi-year forced classes, called ‘pris-pitals’ by Foucault (1995) pointing out that it mixes power techniques from a prison and a hospital, thus raising two ethical issues: On which ethical ground do we force children and teenagers to return to the same room, hour after hour, day after day, week after week, month after month for several years? On which ethical ground do we force children and teenagers to be cured from self-referring diagnoses as e.g., the purpose of mathematics education is to cure mathematics ignorance? Issues, the first Enlightenment republic avoids by offering teenagers self-chosen half-year blocks; and by teaching, not mathematics, but algebra and geometry referring to the outside world by their original meanings, ‘to measure earth’ in Greek and ‘to reunite’ in Arabic.

DIFFERENT KINDS OF MATHEMATICS

In ancient Greece, the Pythagoreans used mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: arithmetic, geometry, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many by itself, Many in space, Many in time and Many in time and space. And together forming the 'quadrivium' recommended by Plato as a general curriculum together with 'trivium' consisting of grammar, rhetoric and logic (Russell, 1945).

With astronomy and music as independent knowledge areas, today mathematics should be a common label for the two remaining activities, geometry and algebra, both being action-words rooted in the physical fact Many through their original meanings. This resonates with the primary goal of knowledge seeking and education, to be able to master the outside world through proper actions. And in Europe, Germanic countries taught counting and reckoning in primary school and algebra and geometry in the lower secondary school until about 50 years ago when they all were replaced by the setbased 'New Math' even if mathematics is a mere label and not an action-word. But the point was that by being setbased mathematics could become a self-referential 'meta-matics' needing no outside root. Instead it could define concepts top-down as examples of inside abstractions instead of bottom-up as abstractions from outside examples.

Russell objected by pointing to the set of sets not belonging to itself. Here a set belongs only if it does not: if $M = \left\{ A \,\middle|\, A \notin A \right\}$ then $M \in M \Leftrightarrow M \notin M$. In this way Russell shows that self-reference leads to the classical liar paradox 'this sentence is false' being false if true and true if false. Instead Russell proposed a type theory banning self-reference. However, mathematics ignored Russell's paradox and his type theory since it prevented fraction from being numbers by being defined from numbers.

Instead, setbased mathematics changed classical grounded mathe-matics into today's self-referring 'meta-matism', a mixture of meta-matics and 'mathe-matism' true inside but seldom outside classrooms where adding numbers without units as '2 + 3 IS 5' meet counter-examples as e.g. 2weeks + 3days is 17 days; in contrast to '2*3 = 6' stating that 2 3s can always be recounted as 6 1s.

Although spreading around the world, the United States rejected the New Math by going 'back to basics'. So today three kinds of mathematics may be taught: a pre-setbased, a present setbased and a post-setbased version (Tarp, 2017).

THE TRADITION OF MATHEMATICS EDUCATION

The numbers and operations and the equal sign on a calculator suggest that mathematics education should be about the results of operating on numbers, e.g. that 2+3 = 5. This offers a 'natural' curriculum with multidigit numbers obeying a place-value system; and with operations having addition as the base with subtraction as reversed operation, where multiplication is repeated addition with division as reversed operation, and where power is repeated multiplication with the factor-finding root and the factor-counting logarithm as reversed operations.

In some cases, reverse operations create new numbers asking for additional education about the results of operating on these numbers. Subtraction creates negative numbers, where 2 - (-5) = 7. Division creates fractions and decimals and percentages where 1/2

+ 2/3 = 7/6. And root and log create numbers as $\sqrt{2}$ and log 3 where $\sqrt{2}*\sqrt{3} = \sqrt{6}$, and where log 100 = 2. Then halving a block by its diagonal creates a right-angled triangle relating the sides and angles with trigonometrical operations as sine, cosine and tangent where $\sin(60) = \sqrt{3}/2$.

Then calculations with unspecified numbers leads to creating additional education about the results of operating on such numbers, e.g. that $(a+b) * (a-b) = a$^2 - b^2.

In a calculation, changing the input will change the output. Relating the changes creates an operation on the calculation called differentiation, also creating additional education about the results of operating on calculations, e.g. that $(f*g)'/(f*g) = f'/f + g'/g$. And with a reverse operation, integration, again creating additional education about the results of operating on calculations, e.g. that ∫6*x^2dx = 2*x^3.

Having taught inside how to operating on numbers and calculations, its outside use may then be shown as inside-outside applications, or as outside-inside modeling transforming an outside problem into an inside problem transformed back into an outside solution after being solved inside. This introduces quantitative literature, also having three genres as the qualitative: fact, fiction and fiddle (Tarp, 2001).

THEORIZING THE SUCCESS OF MATH EDUCATION RESEARCH

When trying to theorize the low success of 50 years of mathematics education research, the first question must be what we mean by mathematics and education and research.

As to education, who needs it if they already know? So, we must ask: what is it that students do not know and must be educated in? Or in other words: what is the goal of mathematics education? Two answers present themselves, one pointing to on the outside existence rooting mathematics, the other to its inside institutionalized essence.

Giving precedence to inside essence over outside existence the answer is: of course, the goal of mathematics education is to teach mathematics as defined by mathematicians at the universities. Modern societies institutionalize the creation and mediation of knowledge as universities and schools. Here priority should be given to useful knowledge as mathematics; and of course, mathematics must be taught before it can be applied, else there is nothing to apply! However, although very useful, mathematics is at the same time very hard to learn as witnessed again and again by research, carefully and in detail describing students' learning problems. So, 50 years of mathematics education research has not been unsuccessful, on the contrary, it has been extremely successful in proving that, by its very nature, mathematics is indeed difficult. The 'essence precedes existence' stance is typically argued by university scholars as e.g. Bruner (1962), Skemp (1971), Freudenthal (1973), and Niss (1994).

Giving precedence to outside existence over inside essence the answer is: It is correct that research has demonstrated many learning difficulties. However, what has been taught is not an outside rooted mathe-matics, but an inside self-referring meta-matism as defined above. And, until now research has primarily studied the two contemporary versions of mathematics, the pre-setbased and the present setbased version whereas very little if any research has studied the post-setbased mathematics that gives precedence to existence over essence by accepting and developing the mastery of Many in the number-language that children develop before school.

Giving precedence to essence or existence makes a difference to math education.

In its pre-setbased version, mathematics presents digits as symbols, and numbers as a sequence of digits obeying a place value system. Once a counting sequence is established, addition is defined as counting on, after which the other operations are defined from addition. Fractions are seen as numbers.

In its present setbased version, mathematics uses the inside concept set for deriving other concepts. Here numbers describe the cardinality of a set, and an operation is a function from a set product into a set. Again, addition is taught as the first operation.

In its post-setbased version, mathematics presents digits as icons with as many sticks as they represent; and numbers always carry units as part of number-language sentences bridging the outside existence with inside essence, thus connecting outside blocks with inside bundles, T = 2 3s = 2B0 3s. Here operations are icons also, and here counting comes before adding to respect that counting involves taking away bundles by division to be stacked by multiplication, to be pulled away by subtraction to find unbundled ones. And here counting and recounting and double-counting precedes the two forms of addition, on-top and next-to. And here fractions are per-numbers, both being operators needing numbers to become numbers.

Likewise, the core concept 'function' is treated differently. Pre-setbased mathematics sees a function as a calculation containing specified and unspecified numbers. Present setbased mathematics sees a function as a subset of set product where first-component identity implies second-component identity. Post-setbased mathematics sees a function as a number-language sentence T = 2*3 relating an outside existing total with an inside chosen essence.

Choosing an 'inside-outside' view will make mathematics self-referring and difficult by its missing link to its outside roots. Whereas choosing an 'outside-inside' view will allow mathematics develop the language children use to assign numbers to outside things and actions, i.e. a number-language similar to the word-language.

Mathematics as the Grammar of the Number-Language

To communicate we have two languages, a word-language and a number-language. The word-language assigns words to things in sentences with a subject, a verb, and an object or predicate: ”This is a chair”. As does the number-language assigning numbers instead: ”The 3 chairs each have 4 legs”, abbreviated to ”The total is 3 fours”, or ”T = 3 4s” or ”T = 3*4”. Unfortunately, the tradition hides the similarity between word- and number-sentences by leaving out the subject and the verb and just saying ”3*4 = 12”.

Both languages have a meta-language, a grammar, describing the language, describing the world. Thus, the sentence ”This is a chair” leads to a meta-sentence ”The word 'is' is an auxiliary verb”. Likewise, the sentence ”T = 3*4” leads to a meta-sentence ”The operation ‘*’ is commutative”.

Since the meta-language speaks about the language, we should teach and learn the language before the meta-language. This is the case with the word-language only. Instead its self-referring setbased form has turned mathematics into a grammar labeling its outside roots as ‘applications’, used as means to dim the impeding consequences of teaching a grammar before its language.

Before 1970, language was taught as an example of its grammar (Chomsky, 1965). Then a reaction emerged. In his book ‘Explorations in the function of language’ Halliday (1973, p. 7) defines a functional approach to language in the following way:

> A functional approach to language means, first of all, investigating how language is used: trying to find out what are the purposes that language serves for us, and how we are able to achieve these purposes through speaking and listening, reading and writing. But it also means more than this. It means seeking to explain the nature of language in functional terms: seeing whether language itself has been shaped by use, and if so, in what ways - how the form of language has been determined by the functions it has evolved to serve.

Likewise, Widdowson (1978) adopts a “communicative approach to the teaching of language (p. ix)” allowing more students to learn a less correct language to be used for communication about outside things and actions.

Time for a Linguistic Turn in the Number-Language also

Thus, in language teaching a new version of the linguistic turn changed language from being inside grammar-based to being outside world-based. However, this version never made it to the sister-language of the word-language, the number-language.

So, maybe it is time to ask how mathematics will look like if

- instead of being taught as a grammar, it is taught as a number-language communicating about outside things and actions.
- instead of learned before its use, it is learned through its use
- instead of learning about numbers, students learn how to number and enumerate, and how to communicate in full sentences with an outside subject, a linking verb, and an inside predicate as in the word- language.

Maybe the time has come to realize that the two statements ‘2+3 = 5’ and ‘2*3 = 6’ have a different truth status.

The former is a conditional truth depending on the units. But, with 3 as the unit, the latter is an unconditional truth since 2 3s may always be recounted as 6 1s.

In short, maybe it is time to look for a different outside-inside mathematics to replace the present tradition, inside-outside meta-matism? And to ask what kind of math grows from the mastery of Many that children develop through use and before school?

DIFFERENCE RESEARCH LOOKS AT MATHEMATICS EDUCATION

To answer, we let Many open itself for us, so that, as curriculum architects, sociological imagination may allow us to construct a mathematics core curriculum based upon examples of Many in a STEM context (Lawrenz et al, 2017). Using 'Difference-research' (Tarp, 2017) searching for hidden differences making a difference, we now return to the original Greek meaning of mathematics as knowledge about Many by itself and in time and space; and use Grounded Theory (Glaser & Strauss, 1967), lifting Piagetian knowledge acquisition (Piaget, 1969) from a personal to a social level, to allow Many create its own categories and properties.

MEETING MANY CREATES A 'COUNT-BEFORE-ADD' CURRICULUM

Meeting Many, we ask "How many in Total?" To answer, we count by bundling to create a number-language sentence as e.g. T = 2 3s that contains a subject and a verb and a predicate as in a word-language sentence; and that connects the outside total T with its inside predicate 2 3s (Tarp, 2018b). Rearranging many 1s into one symbol with as many sticks or strokes as it represents (four strokes in the 4-con, five in the 5-icon, etc.) creates icons to be used as units when counting by bundling and stacking:

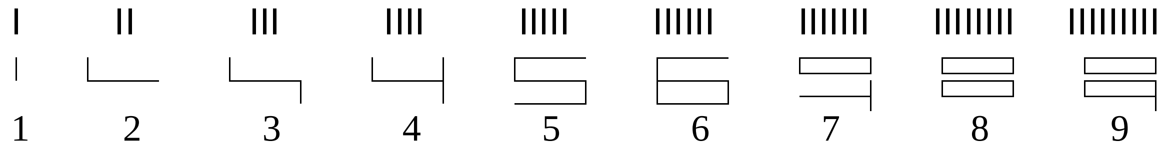

Figure 1: Digits as icons with as many sticks as they represent.

Holding 4 fingers together 2 by 2, a 3year-old will say 'This is not 4, this is 2 2s', thus describing what exists, bundles of 2s and 2 of them. This inspires 'bundle-counting', counting a total in icon-bundles to be stacked as bundle- or block-numbers, which can be recounted and double-counted before being processed by next-to and on-top addition, direct or reversed. Thus, a total T of 5 1s is recounted in 2s as T = 2 2s & 1; described by 'bundle-writing' as T = 2B1 2s; or by 'decimal-writing', T = 2.1 2s, where, with a bundle-cup, a decimal point separates the bundles inside from the outside unbundled singles; or by 'deficit-writing', T = 3B-1 2s = 3.-1 2s = 3 bundles less 1 2s.

To bundle-count a total T we take away bundles B (thus rooting and iconizing division as a broom wiping away the bundles) to be stacked (thus rooting and iconizing multiplication as a lift stacking the bundles into a block) to be moved away to look for unbundled singles (thus rooting and iconizing subtraction as a rope pulling the block away). A calculator thus predicts the result by a 'recount formula' $T = (T/B)*B$ saying that 'from T, T/B times, Bs can be taken away': entering '5/2' on a calculator gives '2.some', and '5 – 2x2' gives '1', so T = 5 = 2B1 2s. The unbundled can be placed next-to the stack as .1 or on-top as ½ counted in 2s, thus rooting decimals and fractions.

The recount formula occurs all over science. With proportionality: $y = c*x$; in trigonometry as sine, cosine and tangent: $a = (a/c)*c = \sin A*c$ and $b = (b/c)*c = \cos A*c$ and $a = (a/b)*b = \tan A*b$; in coordinate geometry as line gradients: $\Delta y = \Delta y/\Delta x = c*\ \Delta x$;

and in calculus as the derivative, dy = (dy/dx)*dx = y'*dx. In economics, the recount formula is a price formula: $ = ($/kg)*kg = price*kg, $ = ($/day)*day = price*day, etc.

Recounting in the Same Unit or in a Different Unit

Once counted, totals can be recounted in the same unit, or in a different unit. Recounting in the same unit, changing a bundle to singles allows recounting a total of 2*B*1 2s as 1*B*3 2s with an outside 'overload'; or as 3*B*-1 2s with an outside 'underload' thus rooting negative numbers. This eases division: 336 = 33*B*6 = 28*B*56, so 336/7 = 4*B*8 = 48; or 336 = 35*B*-14, so 336/7 = 5*B*-2 = 48.

Recounting in a different unit means changing unit, also called proportionality. Asking '3 4s is how many 5s?', sticks show that 3 4s becomes 2*B*2 5s. Entering '3*4/5' we ask a calculator 'from 3 4s we take away 5s'. The answer, '2.some', predicts that the unbundled singles come from taking away 2 5s, now asking '3*4 – 2*5'. The answer, '2', predicts that 3 4s can be recounted in 5s as 2*B*2 5s or 2.2 5s or 2 2/5 5s.

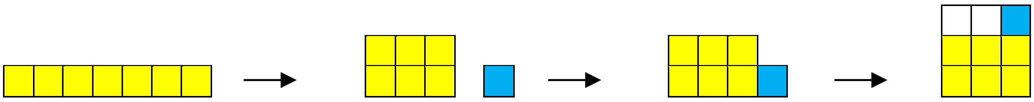

Figure 2: Seven counted as 2 3s & 1 or 2*B*1 3s, and 2.1 3s, and as 2 1/3 3s or 3.-2 3s.

Recounting from Icons to and from Tens

Recounting from icons to tens by asking e.g. '2 7s = ? tens' is eased by using underloads: *T* = 2 7s = 2*7 = 2*(*B*-3) = 20-6 = 14; and T = 6 8s = 6*8 = (*B*-4)*(*B*-2) = *BB* - 4*B* - 2*B* -- 4*2 = 10*B* – 4*B* – 2*B* + 8 = 4*B*8 = 48. This makes sense since widening the base form t7 to ten will shorten the height from 6 to 4.8.

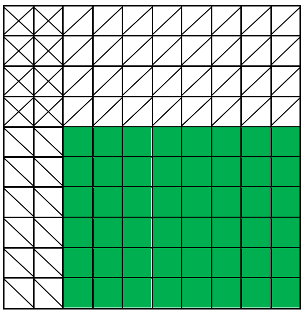

Figure 3: On an abacus 6 8s = 6*8 = (*B*-4)*(*B*-2) = 10*B* - 4*B* - 2*B* + 4 2s = 4*B*8 = 48.

Using the recount formula is impossible since the calculator has no ten-button. Instead it is programmed to give the answer directly as 2*7 = 14, thus using a short form that leaves out the unit and misplaces the decimal point one place to the right.

Recounting from tens to icons by asking '35 = ? 7s' is called an equation *u**7 = 35. It is easily solved by recounting 35 in 7s: *u**7 = 35 = (35/7)*7. So *u* = 35/7, showing that equations are solved by moving to the opposite side with the opposite calculation sign.

Double-Counting Creates Proportionality as Per-Numbers

Counting a quantity in 2 different physical units gives a 'per-number' as e.g. 2$ per 3kg, or 2$/3kg. To answer the question '*T* = 6$ = ?kg', we recount 6 in the per-number

2 and use the per-number to bridge 2$ and 3kg: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg; and vice versa: T = 12kg = (12/3)*3kg = (12/3)*2$ = 8$.

Double-counting in the same unit creates fractions: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%. So 2/3 of 60 = 2$/3$ of 60$, where 60$ = (60/3)*3$ then gives (60/3)*2$ = 40$.

Double-Counting the Sides in a Block Creates Trigonometry

Halving a block by its diagonal allows mutual recounting of the sides, which creates trigonometry to precede plane and coordinate geometry: $a = (a/c)*c = \sin A*c$, and $a = (a/b)*b = \tan A*b$. Filling a circle with blocks shows that $\pi = n*\tan(180/n)$ for n large.

A SHORT CURRICULUM IN ADDITION-FREE MATHEMATICS

00. Playing with '1digit math' (Zybartas et al, 2005): Rearranging 3 cars into one 3-icon, etc. Recounting a total of ten fingers in bundles of e.g. 3s: T = 1Bundle7 = 2B4 = 3B1 = 4Bundle less 2 or 4B-2, and using both fingers and sticks and centi-cubes or LEGO bricks to experience algebra and geometry as always together, never apart. Recounting in a different unit when asking e.g. T = 2 3s = ?4s. Recounting to and from tens when asking e.g. T = 5 6s = ? tens, and T = 4B2 tens = ? 7s. Uniting blocks next-to and on-top when asking e.g. T = 2 3s & 4 5s = ? 8s; and T = 2 3s & 4 5s = ? 3s; and T = 2 3s & 4 5s = ? 5s. Splitting blocks next-to and on-top when asking e.g. T = 2 3s & ? 5s = 3 8s; and T = 2 3s & ? 5s = 7 3s; and T = 2 3s & ? 5s = 4 5s.

01. Until nine, many ones may be rearranged into one icon with as many sticks or strokes as it represents. As one bundle, ten needs no icon. So, a total typically consists of several countings: of unbundled ones, of bundles, of bundles of bundles, etc.

02. Parallel counting sequence stress the importance of bundling: 0Bundle1, 0*B*2, …, 0*B*9, 1*B*0, 1*B*1 etc.; or 0*B*1, 0*B*2, 0*B*3, 0*B*4, 0*B*5 or half Bundle, Bundle less 4, *B*-3, *B*-2, *B*-1, Bundle or 1*B*0, Bundle and 1 or 1*B*1, Bundle and 2 or 1*B*2, etc., thus rooting negative numbers. Here we mention that the Vikings used the words 'eleven' and twelve' as short for 'one-left' and 'two-left'. Using other bundles as units, ten fingers may be counted as 1*B*3 7s, 2*B*0 5s, 2*B*2 4s, 3*B*1 3s, 1*BB*0*B*1 3s, 5*B*0 2s, and 1*BBB*0*BB*1*B*0 2s. A Total of five fingers can be recounted in 2s in three ways, standard or with overload or underload: T = 2*B*1 2s = 1*B*3 2s = 3*B*-1 2s = 3 bundles less 1 2s.

03. Bundle-counting makes operations icons also. First a division-broom pushes away the bundles, then a multiplication-lift creates a stack, to be pulled away by a subtraction-rope to look for unbundles singles separated by the stack by an addition-cross. For prediction, a calculator uses a 'recount formula', $T = (T/B)*B$, saying that 'from T, T/B times, Bs can be taken away'.

04. Recounting in a different unit is called proportionality. Asking '3 4s = ? 5s', we enter '3*4/5' to ask a calculator 'from 3 4s we take away 5s'. The answer '2.some' predicts that the singles come by taking away 2 5s, thus asking '3*4 – 2*5'. The answer '2' predicts that 3 4s can be recounted in 5s as 2*B*2 5s. The unbundled can be placed

next-to the bundles separated by a decimal point, or on-top counted in bundles, thus rooting decimals and fractions, T = 3 4s = 2B2 5s = 2.2 5s = 2 2/5 5s.

05. Recounting from tens to icons by asking '35 = ? 7s' is called an equation u*7 = 35, solved by recounting 35 in 7s: u*7 = 35 = (35/7)*7. So u = 35/7, showing that equations are solved by moving to opposite side with opposite calculation sign. Division is eased by using overloads or underloads: T = 336 = 33B6 = 28B56 = 35B-14, so 336/7 = 4B8 = 5B-2 = 48. As is multiplication: T = 4*78 = 4* 7B8 = 28B32 = 31B2 = 312.

06. Recounting from icons to tens by asking '2 7s = ? tens' is eased by underloads: T = 2*7 = 2*(B-3) = 20-6 = 14; 6*8 = (B-4)*(B-2) = BB-4B-2B--8 = 100 – 60 + 8 = 48.

07. Double-counting a quantity in two units gives a 'per-number' as e.g. 2$ per 3kg, or 2$/3kg. To answer the question 'T = 6$ = ?kg', we recount 6 in the per-number: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions and percent: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%.

08. Trigonometry can precede plane and coordinate geometry to show how, in a block halved by its diagonal, the sides can be mutually recounted as e.g. $a = (a/c)*c = \sin A*c$.

MEETING MANY IN A STEM CONTEXT

OECD (2015b) says: "In developed economies, investment in STEM disciplines (science, technology, engineering and mathematics) is increasingly seen as a means to boost innovation and economic growth." STEM thus combines knowledge about how humans interact with nature to survive and prosper: Mathematical formulas predict nature's behavior, and this knowledge, logos, allows humans to invent procedures, techne, and to engineer artificial hands and muscles and brains, i.e. tools, motors and computers, that combined to robots help transforming nature into human necessities.

Nature as Things in Motion

To meet, we must specify space and time in a nature consisting of things at rest or in motion. So, in general, we see that what exists in nature is matter in space and time.

A falling ball introduces nature's three main ingredients, matter and force and motion, similar to the three social ingredients, humans and will and obedience. As to matter, we observe three balls: the earth, the ball, and molecules in the air. Matter houses two forces, an electro-magnetic force keeping matter together when collisions transfer motion, and gravity pumping motion in and out of matter when it moves in the same or in the opposite direction of the force. In the end, the ball is at rest on the ground having transferred its motion through collisions to molecules in the air; the motion has now lost its order and can no longer be put to work. In technical terms: as to motion, its energy stays constant, but its disorder (entropy) increases. But, if the disorder increases, how is ordered life possible? Because, in the daytime the sun pumps in high-quality, low-disorder light-energy; and in the nighttime the space sucks out low-quality, high-disorder heat-energy; if not, global warming would be the consequence.

So, a core STEM curriculum could be about cycling water. Heating transforms water from solid to liquid to gas, i.e. from ice to water to steam; and cooling does the opposite. Heating an imaginary box of steam makes some molecules leave making gravity push up the lighter box until it becomes heavy water by cooling, now pulled down by gravity as rain in mountains, and through rivers to the sea. On its way down, a dam and magnets can transform moving water into moving electrons, electricity.

In the sea, water contains salt. Meeting ice at the poles, water freezes but the salt stays in the water making it so heavy it is pulled down by gravity, elsewhere pushing warm water up thus creating cycles in the ocean pumping warm water to cold regions.

The two water-cycles fueled by the sun and run by gravity leads on to other STEM areas: to the trajectory of a ball pulled down by gravity; to an electrical circuit where electrons transport energy from a source to a consumer; to dissolving matter in water; and to building roads on hillsides.

In nature, we count matter in kilograms, space in meters and time in seconds. Things in motion have a momentum = mass * velocity, a multiplication formula as most STEM formulas expressing recounting by per-numbers:

- kilogram = (kilogram/cubic-meter) * cubic-meter = density * cubic-meter
- meter = (meter/second) * second = velocity * second
- force = (force/square-meter) * square-meter = pressure * square-meter
- gram = (gram/mole) * mole = molar mass * mole
- mole = (mole/liter) * liter = molarity * liter
- energy = (energy/kg/degree) * kg * degree = heat * kg * degree
- Δ momentum = (Δ momentum/second) * second = force * seconds
- Δ energy= (Δ energy/meter) * meter = force * meter = work
- energy/sec = (energy/charge) * (charge/sec) or Watt = Volt * Amp.

Thus, STEM-subjects swarm with per-numbers: kg/m^3 (density), meter/second (velocity), Joule/second (power), Joule/kg (melting), Newton/m^2 (pressure), etc.

Warming and Boiling Water

In a water kettle, a double-counting can take place between the time elapsed and the energy used to warm the water to boiling, and to transform the water to steam.

If pumping in 410 kiloJoule will heat 1.4 kg water 70 degrees we get a double per-number 410/70/1.4 Joule/degree/kg or 4.18 kJ/degree/kg, called the specific heat capacity of water. If pumping in 316 kJ will transform 0.14 kg water at 100 degrees to steam at 100 degrees, the per-number is 316/0.14 kJ/kg or 2260 kJ/kg, called the heat of evaporation for water.

Dissolving Material in Water

In the sea, salt is dissolved in water, described as the per liter number of moles, each containing a million billion billion molecules. A mole of salt weighs 59 gram, so recounting 100 gram salt in moles we get 100 gram = (100/59)*59 gram = (100/59)*1 mole = 1.69 mole, that dissolved in 2.5 liter has a strength as 1.69 moles per 2.5 liters or 1.69/2.5 mole/liter, or 0.676 mole/liter.

Building Batteries with Water

At our planet life exists in three forms: black, green and grey cells. Green cells absorb the sun's energy directly; and by using it to replace oxygen with water, they transform burned carbon dioxide to unburned carbohydrate storing the energy for grey cells, releasing the energy by replacing water with oxygen; or for black cells that by removing the oxygen transform carbohydrate into hydrocarbon storing the energy as fossil energy. Atoms combine by sharing electrons. At the oxygen atom the binding force is extra strong releasing energy when burning hydrogen and carbon to produce harmless water H2O, and carbon dioxide CO2, producing global warming if not bound in carbohydrate batteries. In the hydrocarbon molecule methane, CH4, the energy comes from using 4 oxygen atoms to burn it.

Technology & Engineering: Steam and Electrons Produce and Distribute Energy

A water molecule contains two hydrogen and one oxygen atom weighing 2*1+16 units making a mole of water weigh 18 gram. Since the density of water is roughly 1 kilogram/liter, the volume of 1000 moles is 18 liters. With about 22.4 liter per mole, its volume increases to about 22.4*1000 liters if transformed into steam, which is an increase factor of 22,400 liters per 18 liters = 1,244 times. But, if kept constant, instead the inside pressure will increase as predicted by the ideal gas law, $p*V = n*R*T$, combining the pressure p, and the volume V, with the number of moles n, and the absolute temperature T, which adds 273 degrees to the Celsius temperature. R is a constant depending on the units used. The formula expresses different proportionalities: The pressure is direct proportional with the number of moles and the absolute temperature so that doubling one means doubling the other also; and inverse proportional with the volume, so that doubling one means halving the other.

Thus, with a piston at the top of a cylinder with water, evaporation will make the piston move up, and vice versa down if steam is condensed back into water. This is used in steam engines. In the first generation, water in a cylinder was heated and cooled by turn. In the next generation, a closed cylinder had two holes on each side of an interior moving piston thus increasing and decreasing the pressure by letting steam in and out of the two holes. The leaving steam is visible on e.g. steam locomotives.

Power plants use a third generation of steam engines. Here a hot and a cold cylinder are connected with two tubes allowing water to circulate inside the cylinders. In the hot cylinder, heating increases the pressure by increasing both the temperature and the number of steam moles; and vice versa in the cold cylinder where cooling decreases the pressure by decreasing both the temperature and the number of steam moles condensed to water, pumped back into the hot cylinder in one of the tubes. In the other tube, the pressure difference makes blowing steam rotate a mill that rotates a magnet over a wire, which makes electrons move and carry electrical energy to consumers.

An Electrical Circuit

Energy consumption is given in Watt, a per-number double-counting the number of Joules per second. Thus, a 2000 Watt water kettle needs 2000 Joules per second. The socket delivers 220 Volts, a per-number double-counting the number of Joules per 'carrier' (charge-unit). Recounting 2000 in 220 gives (2000/220)*220 = 9.1*220, so we need 9.1 carriers per second, which is called the electrical current counted in Ampere, a per-number double-counting the number of carriers per second. To create this current, the kettle must have a resistance R according to a circuit law 'Volt = Resistance*Ampere', i.e., 220 = Resistance*9.1, or Resistance = 24.2 Volt/Ampere called Ohm. Since Watt = Joule per second = (Joule per carrier)*(carrier per second) we also have a second formula, Watt = Volt*Ampere. Thus, with a 60 Watt and a 120 Watt bulb, the latter needs twice the energy and current, and consequently has half the resistance of the former, making the latter receive half the energy if connected in series.

How High Up and How Far Out

A spring sends a ping-pong ball upwards. This allows a double-counting between the distance and the time to the top, e.g. 5 meters and 1 second. The gravity decreases the vertical speed when going up and increases it when going down, called the acceleration, a per-number counting the change in speed per second. To find its initial speed we turn the spring 45 degrees and count the number of vertical and horizontal meters to the top as well as the number of seconds it takes, e.g. 2.5 meters, 5 meters and 0,71 seconds. From a folding ruler we see, that now the total speed is split into a vertical and a horizontal part, both reducing the total speed with the same factor sin45 = cos45 = 0,707. The vertical speed decreases to zero, but the horizontal speed stays constant. So we can find the initial speed *u* by the formula: Horizontal distance to the top position = horizontal speed * time, or with numbers: 5 = (*u**0,707)*0,71, solved as *u* = 9.92 meter/seconds by moving to the opposite side with opposite calculation sign, or by a solver-app. Compared with the horizontal distance, the vertical distance is halved, but the speed changes from 9.92 to 9.92*0.707 = 7.01. However, the speed squared is halved from 9.92*9.92 = 98.4 to 7.01*7.01 = 49.2.

So horizontally, there is a proportionality between the distance and the speed. Whereas vertically, there is a proportionality between the distance and the speed squared, so that doubling the vertical speed will increase the vertical distance four times. Engineering

ADDING ADDITION TO THE CURRICULUM

Once counted as block-numbers, totals can be added next-to as areas, thus rooting integral calculus; or on-top after being recounted in the same unit, thus rooting proportionality. And both next-to and on-top addition can be reversed, thus rooting differential calculus and equations where the question 2 3s + ? 4s = 5 7s leads to differentiation: ? = (5*7 – 2*3)/4 = ΔT/4.

Integral calculus thus precedes differential calculus and include adding both piecewise and locally constant (continuous) per-numbers. Adding 2kg at 3$/kg and 4kg at 5$/kg, the unit-numbers 2 and 3 add directly, but the per-numbers must be multiplied into unit-numbers. So, both per-numbers and fractions must be multiplied by the units before being added as the area under the per-number graph.

Using overloads and underloads eases addition and subtraction: T = 23 + 49 = 2B3 + 4B9 = 6B12 = 7B2 = 72; and T = 56 – 27 = 5B6 – 2B7 = 3B-1 = 2B9 = 29.

Moving in a coordinate system, distances add directly when parallel; and by squares when perpendicular. Re-counting the y-change in the x-change creates a linear change formula $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$, algebraically predicting geometrical intersection points, thus observing a 'geometry & algebra, always together, never apart' principle.

The number-formula T = 456 = 4*B^2 + 5*B + 6*1 shows the four ways to unite numbers offered by algebra meaning 'reuniting' in Arabic: addition and multiplication add changing and constant unit-numbers; and integration and power unite changing and constant per-numbers. And since any operation can be reversed: subtraction and division split a total into changing and constant unit-numbers; and differentiation and root & logarithm split a total in changing and constant per-numbers (Tarp, 2018b):

Uniting/splitting into	Changing	Constant
Unit-numbers m, s, kg, $	$\boldsymbol{T = a + n}$ $T - a = n$	$\boldsymbol{T = a * n}$ $T/n = a$
Per-numbers m/s, $/kg, $/100$ = %	$\boldsymbol{T = \int a\, dn}$ $dT/dn = a$	$\boldsymbol{T = a\wedge n}$ $loga(T) = n$ $n\surd T = a$

Figure 4: An 'Algebra-Square' with the 4 and 5 ways to unite and split totals.

In its general form, the number formula $T = a*x\wedge 2 + b*x$ + c contains the different formulas for constant change: $T = a*x$ (proportionality), $T = a*x\wedge 2$ (acceleration), $T = a*x\wedge c$ (elasticity) and $T = a*c\wedge x$ (interest rate); as well as $T = a*x+b$ (linearity, or affinity, strictly).

As constant/changing, predictable change roots pre-calculus/calculus. Unpredictable change roots statistics to 'post-dict' numbers by a mean and a deviation to be used by probability to pre-dict a confidence interval for numbers we else cannot pre-dict.

Engineering: How Many Turns on a Steep Hill

On a 30-degree hillside, a 10-degree road is constructed. How many turns will there be on a 1 km by 1 km hillside?

We let A and B label the ground corners of the hillside. C labels the point where a road from A meets the edge for the first time, and D is vertically below C on ground level. We want to find the distance $BC = u$.

In the triangle BCD, the angle B is 30 degrees, and $BD = u$*cos(30). With Pythagoras we get u^2 = CD^2 + BD^2 = CD^2 + u^2*cos(30)^2, or CD^2 = u^2(1-cos(30)^2) = u^2*sin(30)^2. In the triangle ACD, the angle A is 10 degrees, and $AD = AC$*cos(10).

With Pythagoras we get $AC^2 = CD^2 + AD^2 = CD^2 + AC^2*\cos(10)^2$, or $CD^2 = AC^2(1-\cos(10)^2) = AC^2*\sin(10)^2$. In the triangle ACB, $AB = 1$ and $BC = u$, so with Pythagoras we get $AC^2 = 1^2 + u^2$, or $AC = \sqrt{(1+u^2)}$.

Consequently, $u^2*\sin(30)^2 = AC^2*\sin(10)^2$, or $u = AC*\sin(10)/\sin(30) = AC*r$, or $u = \sqrt{(1+u^2)}*r$, or $u^2 = (1+u^2)*r^2$, or $u^2*(1-r^2) = r^2$, or $u^2 = r^2/(1-r^2) = 0.137$, giving the distance $BC = u = \sqrt{0.137} = 0.37$.

Thus, there will be 2 turns: 370 meter and 740 meter up the hillside.

CONCLUSION AND RECOMMENDATION

This paper argues that 50 years of unsuccessful mathematics education research may be caused by a goal displacement seeing mathematics as the goal instead of as an inside means to the outside goal, mastery of Many in time and space. The two views lead to different kinds of mathematics: a setbased top-down ‘meta-matics’ that by its self-reference is indeed hard to teach and learn; and a bottom-up Many-based ‘Many-matics’ simply saying “To master Many, counting and recounting and double-counting produces constant or changing unit-numbers or per-numbers, uniting by adding or multiplying or powering or integrating.“ A proposal for two separate twin-curricula in counting and adding is found in Tarp (2018a).

Thus, the simplicity of mathematics as expressed in a ‘count-before-adding’ curriculum allows replacing line-numbers with block-numbers. Imbedded in STEM-examples, young migrants learn core STEM subjects at the same time, thus allowing them to become STEM pre-teachers or pre-engineers to help develop or rebuild their own country. The full curriculum can be found in a 27-page paper (Tarp, 2017).

Thus, it is possible to solve core STEM problems without learning addition, that later should be introduced in both versions since blocks can be added both on-top using proportionality to make the units the same, and next-to by areas as integral calculus.

So, as with another foreign language, why not learn the number-language through its use. And celebrate that core mathematics as proportionality, equations, per-numbers and calculus grow directly from the mastery of Many that children develop through use and before school? Let us see math, not as a goal in itself, but as an inside means to an outside goal that is reached better and by more with quantitative communication.

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BUNDLE-COUNTING PREVENTS & CURES MATH DISLIKE

Allan.Tarp@MATHeCADEMY.net, poster at the 2019 CTRAS

INSIDE-OUTSIDE MATHEMATICS

The numbers and operations and the equal sign on a calculator suggest that mathematics education should be about the results of operating on numbers, e.g. that 2+3 = 5. This offers a 'natural' curriculum with multidigit numbers obeying a place-value system; and with operations having addition as the base with subtraction as reversed operation, where multiplication is repeated addition with division as reversed operation, and where power is repeated multiplication with the factor-finding root and the factor-counting logarithm as reversed operations.

In some cases, reverse operations create new numbers asking for additional education about the results of operating on these numbers. Subtraction creates negative numbers, where 2 - (-5) = 7. Division creates fractions and decimals and percentages where 1/2 + 2/3 = 7/6. And root and log create numbers as $\sqrt{}2$ and log 3 where $\sqrt{}2*\sqrt{}3 = \sqrt{}6$, and where log 100 = 2. Then halving a block by its diagonal creates a right-angled triangle relating the sides and angles with trigonometrical operations as sine, cosine and tangent where sin(60) = $\sqrt{}3/2$, and where π = n*sin(180/n) for n large.

Then calculations with unspecified numbers leads to creating additional education about the results of operating on such numbers, e.g. that (a+b) * (a-b) = a^2 - b^2.

In a calculation, changing the input will change the output. Relating the changes creates an operation on the calculation called differentiation, also creating additional education about the results of operating on calculations, e.g. that (f*g)/(f*g) = f/f + g/g. And with a reverse operation, integration, again creating additional education about the results of operating on calculations, e.g. that ∫ 6*x^2dx = 2*x^3.

Having taught inside how to operating on numbers and calculations, its outside use may then be shown as inside-outside applications, or as outside-inside modeling transforming an outside problem into an inside problem transformed back into an outside solution after being solved inside. This introduces quantitative literature.

OUTSIDE-INSIDE MATHEMATICS

But, as with another foreign language, why not learn the number-language through its use? Is the goal of mathematics education to learn mathematics – or to learn how to master Many? Is math a goal in itself, or an inside means to an outside goal, that may be reached better and by more through quantitative communication? What math grows from the mastery of Many that children develop through use and before school?

01. Meeting Many inspires transforming five ones into one five-icon containing five strokes or sticks. Likewise, with the other digits from one to nine, also containing as many strokes or sticks as they represent if written less sloppy. Icon-building may be

illustrated with a folding ruler. Transforming five ones to one fives allows using five as a unit when counting a total T by bundling and stacking, to be reported in a full number-language sentence with a subject, a verb and a predicate, e.g. T = 2 5s.

02. Icons thus inspires ‘bundle-counting’ and ‘bundle-writing’ where a total T of 5 1s is recounted in 2s as T = 1B3 2s = 2B1 2s = 3B-1 2s, i.e. with or without an overload, or with an underload rooting negative numbers. The unbundled 1 can be placed next to the bundles separated by a decimal point, or on-top of the bundles counted in bundles, thus rooting fractions, T = 5 = 2B1 2s = 2.1 2s = 2 ½ 2s. Recounting in the same unit to create or remove over- or underloads eases operations. Example: T = 336 = 33B6 = 28B56 = 35B-14, so 336/7 = 4B8 = 5B-2 = 48.

03. Bundle-counting makes operations icons also. First a division-broom pushes away the bundles, then a multiplication-lift creates a stack, to be pulled away by a subtraction-rope to look for unbundles singles separated by the stack by an addition-cross. A calculator uses a ‘recount formula‘, $T = (T/B)*B$, to predict that ‘from T, T/B times, Bs can be taken away’. This recount formula occurs all over mathematics and science: when relating proportional quantities as $y = c*x$; in trigonometry as sine and cosine and tangent, e.g. $a = (a/c)*c = \sin A*c$; in coordinate geometry as line gradients, $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$; and in calculus as the derivative, $dy = (dy/dx)*dx = y'*dx$.

04. Recounting in a different unit is called proportionality. Asking ‘3 4s = ? 5s’, sticks say 2B2 5s. Entering ‘3*4/5’ we ask a calculator ‘from 3 4s we take away 5s’. The answer ‘2.some’ predicts that the singles come by taking away 2 5s, thus asking ‘3*4 – 2*5’. The answer ‘2’ predicts that 3 4s can be recounted in 5s as 2B2 5s or 2.2 5s.

05. Recounting from tens to icons by asking ‘35 = ? 7s’ is called an equation $u*7 = 35$. It is easily solved by recounting 35 in 7s: $u*7 = 35 = (35/7)*7$. So $u = 35/7$, showing that equations are solved by moving to opposite side with opposite calculation sign.

06. Recounting to tens by asking ‘2 7s = ? tens’ is eased by using underloads: T = 2*7 = 2*(B-3) = 20-6 = 14; and 6*8 = (B-4)*(B-2) = BB - 4B - 2B -- 8 = 100 – 60 + 8 = 48.

07. Double-counting a quantity in units gives a ‘per-number’ as e.g. 2$ per 3kg, or 2$/3kg. To answer the question ‘T = 6$ = ?kg’, we recount 6 in 2s since the per-number is 2$/3kg: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions and percent: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%.

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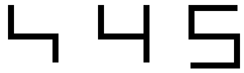
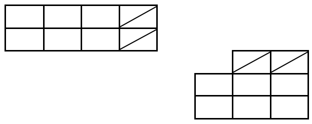
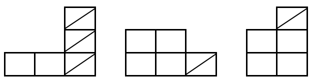
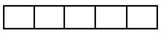
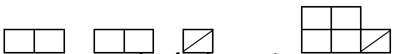
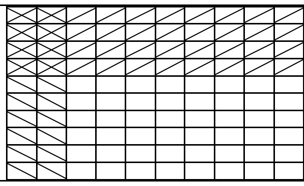
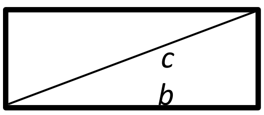
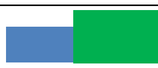
Flexible BundleNumbers

respect & develop **Kids Own Math**

Outside & Inside Math

Digits as ICONS III IIII IIIII		3 4 5
Operations as ICONS	Push Lift Pull	**/ x –**
Count Fingers in 5s using BundleCounting & BundleNumbers	I●●●● II●●● III●● IIII● ~~IIII~~ ~~IIII~~ I●●●●	T = 0*B*1 = 1*B*-4 **5s** T = 0*B*2 = 1*B*-3 **5s** T = 0*B*3 = 1*B*-2 **5s** T = 0*B*4 = 1*B*-1 **5s** T = 1*B*0 = 1*B*0 = 5 T = 1*B*1 = 2*B*-4 **5s**
Unbundled creates Decimals & Fractions & Negative Numbers IIIIIIII ➔ ~~III~~ ~~III~~ II		T = 2*B*2 **3s** = 2.2 **3s** T = 2 2/3 **3s** T = 3*B*-1 3s = 3.-1 **3s** T = 1*BB* 0*B* -1 (T = *p**x^2 + *q**x + *r*)
ReCount in Same Unit creates Flexible Numbers ▌▌▌▌▌III ➔ 53	5: ~~II~~III ~~II~~~~II~~I ~~II~~~~II~~~~II~~ 	T = 1*B*3 *Overload* T = 2*B*1 *Standard* T = 3*B*-1 *Underload* T = 53 = 5*B*3 = 4*B*13 = 6*B*-7 **tens**
Flexible BundleNumbers ease Operations	65 + 27 = ? = 65 – 27 = ? = 7* 48 = ? = 336 /7 = ? =	6*B*5 + 2*B*7 = 8*B*12 = 9*B*2 = 92 6*B*5 – 2*B*7 = 4*B*-2 = 3*B*8 = 38 7* 4*B*8 = 28*B*56 = 33*B*6 = 336 33*B*6 /7 = 28*B*56 /7 = 4*B*8 = 48
ReCount in New Unit  5 = ? **2s** *ReCount-Formula:*	 T = (T/*B*) * *B*	T = 5 = (5/2)*2 = ? = 2*B*1 **2s** 5/2 2.some 5 - 2*2 1
ReCount: Tens to Icons ▌▌▌IIIIII = ? **7s**	3*B*5 **tens** = *u**7	*u**7 = 35 = (35/7)*7 so *u* = 35/7
ReCount: Icons to Tens 6 **8s** = ? **tens** 		T = 6 **8s** = 6*8 = (*B*-4)*(*B*-2) = ***BB*** - 4***B*** - 2***B*** - - 8 = 10***B*** - 6***B*** + 8 = 4*B*8 = 4.8 **tens** = 48
DoubleCount gives PerNumbers	2$ per 3kg = 2$/3kg	T = 6$ = (6/2) * 2$ = (6/2) * 3kg = 9kg
Like Units: Fractions 5% of 40	5$/100$ of 40$	T = 40$ = (40/100)*100$ gives (40/100)*5$ = 2$
DoubleCount a Block halved by its Diagonal		*a* = (*a*/*c*)**c* = sin *A***c* *a* = (*a*/*b*)**b* = tan *A***b* π= *n**tan(180/*n*) *for n large* *c***c* = *a***a* + *b***b*

Allan.Tarp@MATHeCADEMY.net MrAlTarp YouTube:

Add NextTo	T = 2 **3s** + 4 **5s** = 3*B*2 **8s**  *Integration*
OnTop	T = 2 **3s** + 4 **5s** = 1*B*1 **5s** + 4 **5s** = 5*B*1 **5s**  *Proportionality*
MatheMatism	ADDING WITHOUT UNITS Digits or Fractions or Per-numbers

Flexible BundleNumbers respect & develop **Kids Own Math**

01. Meeting Many inspires transforming five ones into one five-icon containing five strokes or sticks. Likewise, with the other digits from one to nine, also containing as many strokes or sticks as they represent if written less sloppy. Icon-building may be illustrated with a folding ruler. Transforming five ones to one fives allows using five as a unit when counting a total T by bundling and stacking, to be reported in a full number-language sentence with a subject, a verb and a predicate, e.g. T = 2 5s.

02. Icons thus inspires 'bundle-counting' and 'bundle-writing' where a total T of 5 1s is recounted in 2s as T = 1*B*3 2s = 2*B*1 2s = 3*B*-1 2s, i.e. with or without an overload, or with an underload rooting negative numbers. The unbundled 1 can be placed next to the bundles separated by a decimal point, or on-top of the bundles counted in bundles, thus rooting fractions, T = 5 = 2*B*1 2s = 2.1 2s = 2 ½ 2s. Recounting in the same unit to create or remove over- or underloads eases operations. Example: T = 336 = 33*B*6 = 28*B*56 = 35*B*-14, so 336/7 = 4*B*8 = 5*B*-2 = 48.

03. Bundle-counting makes operations icons also. First a division-broom pushes away the bundles, then a multiplication-lift creates a stack, to be pulled away by a subtraction-rope to look for unbundles singles separated by the stack by an addition-cross. A calculator uses a 'recount formula', $T = (T/B)*B$, to predict that 'from T, T/B times, Bs can be taken away'. This recount formula occurs all over mathematics and science: when relating proportional quantities as $y = c*x$; in trigonometry as sine and cosine and tangent, e.g. $a = (a/c)*c = \sin A*c$; in coordinate geometry as line gradients, $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$; and in calculus as the derivative, $\mathrm{d}y = (\mathrm{d}y/\mathrm{d}x)*\mathrm{d}x = y'*\mathrm{d}x$.

04. Recounting in a different unit is called proportionality. Asking '3 4s = ? 5s', sticks say 2*B*2 5s. Entering '3*4/5' we ask a calculator 'from 3 4s we take away 5s'. The answer '2.some' predicts that the singles come by taking away 2 5s, thus asking '3*4 – 2*5'. The answer '2' predicts that 3 4s can be recounted in 5s as 2*B*2 5s or 2.2 5s.

05. Recounting from tens to icons by asking '35 = ? 7s' is called an equation $u*7 = 35$. It is easily solved by recounting 35 in 7s: $u*7 = 35 = (35/7)*7$. So $u = 35/7$, showing that equations are solved by moving to opposite side with opposite calculation sign.

06. Recounting to tens by asking '2 7s = ? tens' is eased by using underloads: $T = 2*7 = 2*(B-3) = 20-6 = 14$; and $6*8 = (B-4)*(B-2) = BB - 4B - 2B -- 8 = 100 – 60 + 8 = 48$.

07. Double-counting a quantity in units gives a 'per-number' as e.g. 2$ per 3kg, or 2$/3kg. To answer the question 'T = 6$ = ?kg', we recount 6 in 2s since the per-number is 2$/3kg: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions and percent: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%.

08. Next-to addition geometrically means adding by areas, so multiplication precedes addition. Next-to addition is also called integral calculus, or differential if reversed.

09. On-top addition means using the recount-formula to get like units. Changing units is also called proportionality, or solving equations if reversed.

References

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education*, *11*(1), 103-117.

WORKSHOP IN ADDITION-FREE STEM-BASED MATH

Allan Tarp@MATHeCADEMY.net, workshop at the 2019 CTRAS

NATURE AS HEAVY THINGS IN MOTION IN TIME AND SPACE

A falling ball introduces nature's three main ingredients, matter and force and motion, similar to the three social ingredients, humans and will and obedience. As to matter, we observe three balls: the earth, the ball, and molecules in the air. Matter houses two forces, an electro-magnetic force keeping matter together when collisions transfer motion, and gravity pumping motion in and out of matter when it moves in the same or in the opposite direction of the force. In the end, the ball is at rest on the ground having transferred its motion through collisions to molecules in the air; the motion has now lost its order and can no longer be put to work. In technical terms: as to motion, its energy stays constant, but its disorder (entropy) increases. But, if the disorder increases, how is ordered life possible? Because, in the daytime the sun pumps in high-quality, low-disorder light-energy; and in the nighttime the space sucks out low-quality, high-disorder heat-energy; if not, global warming would be the consequence.

So, a core STEM curriculum could be about cycling water. Heating transforms water from solid to liquid to gas, i.e. from ice to water to steam; and cooling does the opposite. Heating an imaginary box of steam makes some molecules leave making gravity push up the lighter box until it becomes heavy water by cooling, now pulled down by gravity as rain in mountains, and through rivers to the sea. On its way down, a dam and magnets can transform moving water into moving electrons, electricity.

Matter and force and motion all represent different degrees of Many, thus calling for a science about Many. This is how mathematics arose in ancient Greece, so it should respect its root as a natural science by letting multiplication precede addition since the basic science formulas are multiplication formulas expressing 'per-numbers' coming from double-counting: kg = (kg/cubic-meter)*cubic-meter = density*cubic-meter; force = (force/square-meter)*sq.-meter = pressure*sq.-meter; meter = (meter/sec)*sec = velocity*sec; energy = (energy/sec)*sec = Watt*sec; energy = (energy/kg)*kg = heat*kg; Δ momentum = (Δ momentum/sec)*sec = force*sec = impulse; Δ energy = (Δ energy/meter)*meter = force*meter = work; gram = (gram/mole)*mole = molar mass*mole; energy/sec = (energy/charge)*(charge/sec) or Watt = Volt*Amp.

Counting in Icon-Bundles Allows Recounting in the Same and in a Different Unit

Meeting many, we observe that five ones may be recounted as one five-icon. Likewise, with the other digits; thus being, not symbols, but icons with as many strokes or sticks as they represent. 'Bundle-counting' in icon-bundles allows 'bundle-writing' where a total T of 5 1s is recounted in 2s as T = 1B3 2s = 2B1 2s = 3B-1 2s, i.e. with or without an overload, or with an underload rooting negative numbers. The unbundled 1 can be placed next to the bundles separated by a decimal point, or on-top of the bundles counted in bundles, thus rooting fractions, T = 5 = 2B1 2s = 2.1 2s = 2 ½ 2s.

Recounting in the same unit to create or remove over- or underloads eases operations. Example: T = 336 = 33B6 = 28B56 = 35B-14, so 336/7 = 4B8 = 5B-2 = 48.

Bundle-counting makes operations icons also. First a division-broom pushes away the bundles, then a multiplication-lift creates a stack, to be pulled away by a subtraction-rope to look for unbundles singles separated by the stack by an addition-cross.

This creates a 'recount formula', $T = (T/B)*B$, saying that 'from T, T/B times, Bs can be taken away'. This formula predicts the result of recounting in another unit, called proportionality: Asking '3 4s is how many 5s?', sticks show that 3 4s becomes 2B2 5s. Entering '3*4/5' we ask a calculator 'from 3 4s we take away 5s'. The answer '2.some' predicts that the unbundled singles come by taking away 2 5s, thus asking '3*4 – 2*5'. The answer '2' predicts that 3 4s recount in 5s as 2B2 5s or 2.2 5s or 2 2/5 5s.

This recount formula occurs all over mathematics and science: when relating proportional quantities as $y = c*x$; in trigonometry as sine and cosine and tangent, e.g. $a = (a/c)*c = \sin A*c$; in coordinate geometry as line gradients, $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$; and in calculus as the derivative, $dy = (dy/dx)*dx = y'*dx$.

Recounting to and from Tens

Times tables ask '2 7s = ? tens', eased by using underloads: T = 2*7 = 2*(B-3) = 20-6 = 14; and 6*8 = (B-4)*(B-2) = BB - 4B - 2B -- 8 = 100 – 60 + 8 = 48. Using the recount formula is impossible since the calculator has no ten-button. Instead it is programmed to give the answer as 3*4 = 12, using a short form that leaves out the unit and misplaces the decimal point one place to the right, strangely enough called a 'natural' number.

Recounting from tens to icons by asking '35 = ? 7s' is called an equation $u*7 = 35$. It is easily solved by recounting 35 in 7s: $u*7 = 35 = (35/7)*7$. So $u = 35/7$, showing that equations are solved by moving to the opposite side with the opposite calculation sign.

Double-counting Creates Proportionality as Per-Numbers

Counting a quantity in 2 different physical units gives a 'per-number' as e.g. 2\$ per 3kg, or 2\$/3kg. To answer the question 'T = 6\$ = ?kg', we recount 6 in 2s since the per-number is 2\$/3kg: T = 6\$ = (6/2)*2\$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions and percent: 2\$/3\$ = 2/3, and 2\$/100\$ = 2/100 = 2%.

References

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

WORKSHOP EXERCISES IN ADDITION-FREE STEM-BASED MATH

STEM: Mathematics as one of the natural Sciences, applied in Technology and Engineering

Allan.Tarp@MATHeCADEMY.net, June 2019

Using 'Outside-Inside Math' allows outside science problems to be solved by inside math formulas.

Outside degrees of Many create inside number-icons with the number of strokes they represent. Outside counting-operations, occurring when bundles are pushed away, lifted and pulled away to find unbundled ones, create the operation-icons division, /, and multiplication, x, and subtraction -.

Once bundle-counted, recounting in different units (called proportionality) create a 'recount-formula', $T = (T/B)*B$, saying that 'from T, T/B times, Bs can be taken away'; occurring all over math and science: when relating proportional quantities as $y = c*x$; in trigonometry as sine and cosine and tangent, e.g. $a = (a/c)*c = \sin A*c$; in coordinate geometry as line gradients, $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$; in calculus as the derivative, $dy = (dy/dx)*dx = y'*dx$; in science as speed: $\Delta s = (\Delta s/\Delta t)*\Delta t = v*\Delta t$.

Asking '3 **4s** is how many **5s**?', outside sticks show that 3 **4s** becomes 2*B*2 **5s**: IIII IIII IIII -> VV II.

To predict inside, we enter '3*4/5' to ask a calculator 'from 3 **4s** we take away **5s'**. The answer '2.some' predicts that the unbundled ones come by taking away 2 **5s**. Now, asking '3*4 – 2*5' gives '2'. So, 3 **4s** = 2*B*2 **5s = 2.2 5s**.

3*4/5	2.some
3*4 – 2*5	2

Recounting a quantity in two different physical units gives a 'per-number' as e.g. 2m per 3sec, or 2m/3sec. To answer the question '*T* = 6m = ?sec', we recount 6 in **2s** since the per-number is 2m/3sec: *T* = 6m = (6/2)*2m = (6/2)*3sec = 9sec. Double-counting in the same unit creates fractions and %: 2\$/3\$ = 2/3, and 2\$/100\$ = 2/100 = 2%. 5% of 40 = ?; T = 40 = (40/100)*100 gives (40/100)*5 = 2.

kg = (kg/cubic-meter)*cubic-meter = density*cub.-meter	Δ momentum = (Δ mom./sec)*sec = force*sec = impulse
force = (force/square-meter)*sq.-meter = press.*sq.-meter	Δ energy = (Δ energy/meter)*meter = force*meter = work
meter = (meter/sec)*sec = velocity*sec	gram = (gram/mole)*mole = molar mass*mole
energy = (energy/sec)*sec = Watt*sec	energy/sec = (energy/charge)*(charge/sec), or
energy = (energy/kg)*kg = heat*kg	Watt = Volt*Amp.

Science multiplication formulas expressing 'per-numbers' coming from double-counting

Five Ways to Solve Proportionality Questions

Inside recounting solves outside questions as "If 2m need 5sec, then 7m need ?sec; and 12sec gives ?m"

- Europe used the 'Regula de Tri' (rule of three) until around 1900: arrange the four numbers with alternating units and the unknown at last. Now, from behind, first multiply, then divide. So first we ask, Q1: '2m takes 5s, 7m takes ?s' to get to the answer (7*5/2)s = 17.5s. Then we ask, Q2: '5s gives 2m, 12s gives ?m' to get to the answer (12*2)/5s = 4.8m.

Then, two new methods appeared, 'find the unit', and cross multiplication in an equation expressing like proportions or ratios:

- Q1: 1m takes 5/2s, so 7m takes 7*(5/2) = 17.5s. Q2: 1s gives 2/5m, so 12s gives 12*(2/5) = 4.8m.
- Q1: 2/5 = 7/*x*, so 2**x* = 7*5, *x* = (7*5)/2 = 17.5. Q2: 2/5 = *x*/12, so 5**x* = 12*2, *x* = (12*2)/5 = 4.8.
- Alternatively, we may recount in the 'per-number' 2m/5s coming from 'double-counting' the total *T*. Q1: *T* = 7m = (7/2)*2m = (7/2)*5s = 17.5s; Q2: *T* = 12s = (12/5)*5s = (12/5)*2m = 4.8m.
- SET introduced modeling with linear functions to show the relevance of abstract algebra's group theory: Let us define a linear function $f(x) = c*x$ from the set of m-numbers to the set of s-numbers, having as domain $DM = \{x \in R \mid x>0\}$. Knowing that $f(2) = 5$, we set up the equation $f(2) = c*2 = 5$ to be solved by multiplying with the inverse element to 2 on both sides and applying the associative law: $c*2 = 5$, $(c*2)*½ = 5*½$, $c*(2*½) = 5/2$, $c*1 = 5/2$, $c = 5/2$. With $f(x) = 5/2*x$, the inverse function is $f\text{-}1(x) = 2/5*x$. So with 7m, $f(7) = 5/2*7 = 17.5$s; and with 12s, $f\text{-}1(12) = 2/5*12 = 4.8$m.

Three different kinds of mathematics answering the question: What is a function?

pre-setcentric:	*a function is a calculation with specified and unspecified numbers.*
present setcentric:	*a function is a subset of a set-product where component identity transfers.*
post-setcentric:	*a function is a number-language sentence with a subject, a verb and a predicate.*

EXERCICES

E01. With sticks, transform many OUTSIDE ones into one INSIDE many-icon with as many strokes as it represents.

E02. Name fingers as 5s using BundleCounting & BundleNumbers: 0**B**1 = 1**B**-4, 0**B**2 = 1**B**-3, …**5s**

E03. Count 5 fingers in 2s using flexible bundle-numbers: T = 5 = 1**B**3 = 2**B**1 = 3**B**-1 **2s** (overload, standard, underload)

E04. Recount ten fingers in **4s**, **3s** and **2s**: T = ten = 1**B**6 = 2**B**2 = 3**B**-2 **4s**; T = 3**B**1 = 1**BB**1 = 1**BB** 0**B** 1 = 10.1 **3s**; T = 5**B**0 = 4**B**2 = 2**BB** 1**B** 0 = 1**BBB** 0**BB** 1**B** 0 **2s**. ReCount 7 fingers in **3s**: T = 7 = 2B1= 1BB-2 = 2.1 = 3.-2 = 2 1/3.

E05. Write traditional numbers as flexible BundleNumbers: T = 53 = 5**B**3 = 4**B**13 = 6**B**-7 tens

E06. Flexible BundleNumbers ease Operations	65 + 27 = ? = 65 – 27 = ? = 7* 48 = ? = 336 /7 = ? =	6**B**5 + 2**B**7 = 8**B**12 = 9**B**2 = 92 6**B**5 – 2**B**7 = 4**B**-2 = 3**B**8 = 38 7* 4**B**8 = 28**B**56 = 33**B**6 = 336 33**B**6 /7 = 28**B**56 /7 = 4**B**8 = 48

E07. With cubes, transform the three OUTSIDE parts of a counting process, PUSH & LIFT & PULL, into three INSIDE operation-icons: division & multiplication & subtraction.

Five counted in **2s**: I I I I I (push away **2s**) II II I (lift to stack) II II I (pull to find unbundles ones) II II I.

E08. OUTSIDE BundleCounting with icons as units may be predicted INSIDE by a **recount-formula**
T = (T/B)*B, (from T, T/B times, take Bs away) using a full number-language sentence with a subject, a verb and a predicate.

OUTSIDE: T = I I I I I; T counted in **2s**: II II I; T – 2x2: = II II I ; INSIDE: 5/2 2. some; 5 – 2x2 1

E09. Recount in a new unit to change units, predicted by the recount-formula

OUTSIDE, use sticks or cubes to answer 3 **4s** = ? **5s**. INSIDE, use the recount-formula to predict: 3x4/5

E10. Recount from tens to icons

OUTSIDE, to answer the question '40 = ? **5s**', on squared paper transform the block 4.0 **tens** to **5s**.

INSIDE, formulate an equation to be solved by recounting 40 in **5s**: *u**5 = 40 = (40/5)*5, so *u* = 40/5.

Notice that recounting gives the solution rule 'move to opposite side with opposite calculation sign'.

E11. Recount from icons to tens

OUTSIDE, to answer '3 **7s** = ? **tens**' on squared paper transform the block 3 **7s** to **tens**.

INSIDE: oops, with no ten-button on a calculator we can't use the recount-formula? Oh, we just multiply!

E12. ReCounting in two physical units

Recounting in two physical units gives a 'per-number' as e.g. 2m per 3sec, or 2m/3sec.
To answer the question 'T = 6m = ?sec', we just recount 6 in the per-number **2s**: T = 6m = (6/2)*2m = (6/2)*3sec = 9sec.

E13. Solving STEM proportionality heating problems with recounting
With a heater giving 20 J in 30 sec, what does 40 seconds give, and how many seconds is needed for 50J?
With 40 Joules melting 5kg, what will 60 Joules melt and what will 7 kg need?
With 3 degrees needs 50 Joules, what does 7 degrees need; and what does 70 Joules give?
With 4 deg. in 20kg needing 50 Joules, what does 9 deg. in 30 kg need? What does 70 Joules give in 40 kg?

E14. Mutual ReCounting the sides in a block halved by its diagonal creates trigonometry: a = (a/b)*b = tanA*b
Draw a vertical tangent to a circle with radius r. With a protractor, mark the intersection points on the tangent for angles from 10 to 80. Compare the per-number intersection/radius with tangent of the angle on a calculator.

E15. Engineering
A 12 x 12 square ABCD has AB on the ground and is inclined 20 degrees. From B, a straight road inclined 5 degrees is to be constructed intersecting the borderline AD in the point E. Find the length DE.
Hint: Show that if DE = 2, then the incline of the road is 3.2 degrees.

E16. Traveling
With 4 meters taking 5 seconds, what does 6 meters take; and what does 7 seconds give?
With distance d and speed v and time t related as d = v*t, what time is needed to go 20m with velocity 4m/s?
With distance d and time related as d = 5*t^2, what time is needed to go 30m?

Hint: Use that if p^2 < N < $(p+1)^2$, then √N ≈ $\frac{N+p^2}{2p}$

10**B**0	10**B**1	10**B**2	10**B**3	10**B**4	10**B**5	10**B**6	10**B**7	10**B**8	10**B**10	~~10**B**10~~
9**B**0	9**B**1	9**B**2	9**B**3	9**B**4	9**B**5	9**B**6	9**B**7	9**B**8	9**B**9	~~9**B**10~~
8**B**0	8**B**1	8**B**2	8**B**3	8**B**4	8**B**5	8**B**6	8**B**7	8**B**8	8**B**9	~~8**B**10~~
7**B**0	7**B**1	7**B**2	7**B**3	7**B**4	7**B**5	7**B**6	7**B**7	7**B**8	7**B**9	~~7**B**10~~
6**B**0	6**B**1	6**B**2	6**B**3	6**B**4	6**B**5	6**B**6	6**B**7	6**B**8	6**B**9	~~6**B**10~~
5**B**0	5**B**1	5**B**2	5**B**3	5**B**4	5**B**5	5**B**6	5**B**7	5**B**8	5**B**9	~~5**B**10~~
4**B**0	4**B**1	4**B**2	4**B**3	4**B**4	4**B**5	4**B**6	4**B**7	4**B**8	4**B**9	~~4**B**10~~
3**B**0	3**B**1	3**B**2	3**B**3	3**B**4	3**B**5	3**B**6	3**B**7	3**B**8	3**B**9	~~3**B**10~~
2**B**0	2**B**1	2**B**2	2**B**3	2**B**4	2**B**5	2**B**6	2**B**7	2**B**8	2**B**9	~~2**B**10~~
1**B**0	1**B**1	1**B**2	1**B**3	1**B**4	1**B**5	1**B**6	1**B**7	1**B**8	1**B**9	~~1**B**10~~
0**B**0	0**B**1	0**B**2	0**B**3	0**B**4	0**B**5	0**B**6	0**B**7	0**B**8	0**B**9	~~0**B**10~~

Addition-free STEM-based Math for Migrants

using the Child's own Mastery of Many, saying:

Count & *ReCount* & *DoubleCount*

before you Add NextTo & OnTop

- that's how you

master many

with manymath

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teaching teachers to teach Math as ManyMath

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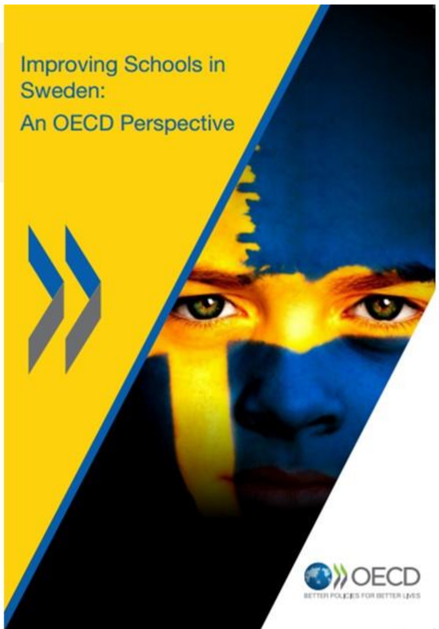
In Sweden, OECD says that 'Math' excludes 1 of 4 socially

"PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (page 3)



http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm

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How to **P**ower **P**oor **P**ISA **P**erformance

To find an unorthodox way, a university in Sweden, challenged by many young male migrants, may arrange a **C**urriculum **A**rchitect **C**ontest:

- Philosophize the low success of 50 years of math education research
- Derive from this a STEM-based curriculum so young migrants soon become STEM pre-teachers and pre-engineers to help develop or rebuild their home country

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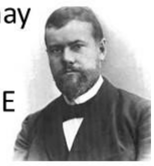
Philosophy: INSIDE representations of the OUTSIDE

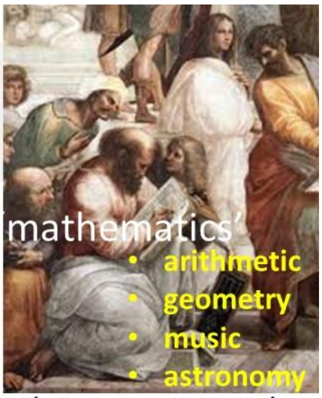
Seeing OUTSIDE **existence** represented by INSIDE **essence**, or *ontology* represented by *epistemology*.

- Sartre on existentialism: OUTSIDE **existence** should precede INSIDE **essence**.
- Heidegger: The sentence „**subject IS predicate"** bridges the OUT- and INSIDE. Trust the OUTSIDE subject, it exists; but question the INSIDE predicate: it may be an institutionalized verdict - that should be doubted and appealed.
  - Weber: If carried too far, rational institutions may become an INSIDE **iron cage** that **disenchants** its OUTSIDE subject.
  - Arendt: Blindly Institutionalizing may lead to '**the banality of evil**'.
  - Sociology: Beware of institutions, they monopolize!



MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY

What IS mathematics & education? In ancient Greece: a natural science



In ancient Greece, Pythagoreans used the word 'mathematics' meaning 'knowledge', as a common label for four descriptions of Many by itself and in space & time

Together they formed the '**quadrivium**' recommended by Plato as a general curriculum after the '**trivium**' consisting of grammar & logic & rhetoric.
Geometry & algebra are both grounded in Many as shown by names:

- In Greek, **Geometry** means to measure earth
- In Arabic, **Algebra** means to reunite numbers

Later, math chose SET self-reference

But Russell said : "INSIDE self-reference leads to the classical liar paradox '**this sentence is false**', being true if false & opposite.
Let M be the set of sets not belonging to itself, $M = \{A \mid A \notin A\}$.
Then $M \in M \Leftrightarrow M \notin M$. Forget about sets. Use type theory instead.
So, by self-reference & without units, fractions are not numbers."

But mathematics insisted on being a self-refering **MetaMatics** by saying: "Forget about Russell, he is not a mathematician. We just institutionalize fractions as so-called rational numbers."

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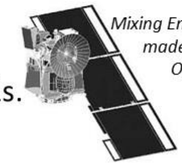
Adding without units creates MatheMatism *true INSIDE but seldom OUTSIDE*

The Teacher	**The Students** (the fraction paradox)
What is 1/2 + 2/3?	Well, 1 of 2 + 2 of 3 gives (1+2) of (2+3), or 3 of 5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But if the browns are 1/2 of 2 cakes, and 2/3 of 3 cakes, then they are 1+2 of 2+3 cakes, i.e. 3/5 of 5 cakes! How can the browns be 7 cakes out of 6 cakes?
INSIDE this classroom 1/2 + 2/3 **IS** 7/6 !	

Without units, fractions & digits are operators, needing numbers to become numbers.

2+3 IS 5? No, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.

2x3 IS 6? Yes, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.



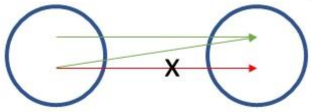
*Mixing English and metric units made NASA's Mars Climate Orbiter **CRASH** in 1999.*

MATHeCADEMY

3 kinds of math, **pre-**, **present** & **post**-setcentric mathematics, defining a 'function' differently

Pre-setcentric math: a function is a CALCULATION with both specified and unspecified numbers, e.g. **2+*u***.

Present setcentric math: a function is a SUB**SET** OF **SET**-PRODUCT where first-component identity implies second-component identity.



Post-setcentric math: a function is a NUMBER-LANGUAGE SENTENCE, e.g. **T = 2+*u***, bridging an OUTSIDE existence to an INSIDE chosen essence.

MATHeCADEMY

Post-setcentric math: through its use, as with the other language in our two language houses

The **WORD-language** assigns words in sentences with The **NUMBER-language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

	WORD-language	NUMBER-language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
INSIDE Language	This is a chair	T = 3x4
OUTSIDE World		

Mathematics

Mother tongue

MATHeCADEMY

Two different kinds of education

The Enlightenment Century rooted education, but in different forms in its two republics, the North American in 1776 and the French in 1789.

- In North America, education enlightens children about their OUTSIDE world, and enlightens teenagers about their INSIDE individual talent, to be uncovered and developed through self-chosen ½year **BLOCKS** with teachers teaching only one subject in the teacher's own classroom.
- To protect its republic from its German speaking neighbors, France was forced to create institutions controlled by a strong central administration with public servants trained at elite schools with multi-year **LINES**, later copied by European Bildung-education.

MATHeCADEMY

3x2 different kinds of math education

Mathematics in	**self-chosen ½year BLOCKS**	**forced multi-year LINES**
Pre-SETcentric	North America	UK Commonwealth
Present SETcentric	-	Continental Europe
Post-SETcentric	MATH**e**CADEMY.net	

MATHeCADEMY

Philosophizing the low success of 50 years of mathematics education research

Mathematics also needs a <u>COMMUNICATIVE TURN</u> where

- instead of learned **INSIDE-INSIDE** through its grammar, it is learned **OUTSIDE-INSIDE** as a INSIDE number-language communicating about OUTSIDE things and actions, thus learned through its use, and not before its use
- instead of learning about numbers, students learn how to number, and how to communicate about Many in full sentences containing:

  1) an **OUTSIDE** <u>subject</u>, 2) a linking <u>verb</u>, and 3) an **INSIDE** <u>predicate</u>: **T = 2x3**

So, now we look for an **OUTSIDE-INSIDE post-SETcentric mathematics** to replace the present **INSIDE-INSIDE meta-matism** by asking:

What kind of mathematics grows from the Mastery of Many that children develop through use, and before school?

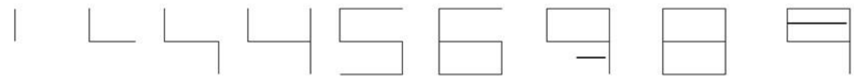
Pablo Picasso: It took me four years to paint like Raphael, but a lifetime to paint like a child

Creating icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

This is not 4
*it is 2 **2s***

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9



Division & multiplication & subtraction are icons also

- From 9 **PUSH** away **4s** we write 9/4 iconized by a broom, called *division*.

- 2 times **LIFTING** the **4s** to a stack we write 2x4 iconized by a lift called *multiplication*.

- From 9 **PULL** away 2 **4s**' to find un-bundled we write 9 – 2x4 iconized by a rope, called *subtraction*.

MATHeCADEMY

Counting is predicted by a ReCount formula

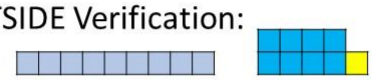
T = (T/B) x B	From a total **T**, **T/B** times, **B** is pushed away

A formula is an INSIDE **prediction**, making the number-language a language for prediction.

INSIDE Prediction: ReCounting 9 in **4s** gives 2**B**1 **4s**:

9/4	**2.some**
9 – 2x4	**1**

OUTSIDE Verification:

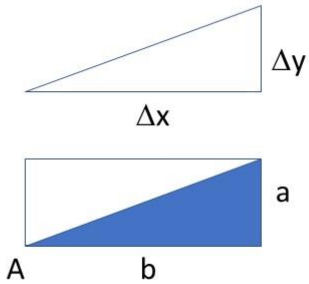


MATHeCADEMY

The **ReCount formula** is the core of math & science

T = (T/B)*B expresses proportionality when changing unit, and is all over:

Proportionality	y = **c** * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y’** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **speed** * second



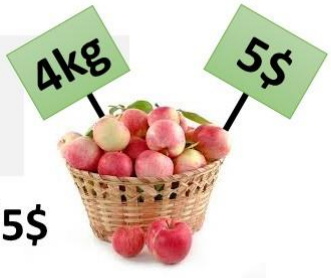
MATHeCADEMY

ReCounting from tens to icons

T = ? **8s** = 24 = (24/8)x8 ? = 24/8 = 3	*Asking how many **8s** in 24,* *we just recount 24 in **8s***
u x 8 = 24 = (24/8) x 8 **u** = 24/8 **u** = 3	Formulated as an **equation**, we find the unknown **u** by moving 8 **to opposite side - with opposite sign**
As geometry: A block decreasing its unit must increase its height to keep its size	

MATHeCADEMY

DoubleCounting gives PerNumbers



DoubleCounting in kg & $, we may get **4kg/5$**

With **4kg** bridged to **5$** we answer questions by recounting in the per-number (per-numbers = proportionality = change units)

Questions: **7kg = ?$**	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

With like units, PerNumbers are Fractions: 4$/5$ = 4/5 & 4$/100$ = 4/100 = 4%

MATHeCADEMY

STEM multiplication formulas come from DoubleCounting and PerNumbers I

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x sq.-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole

MATHeCADEMY

STEM multiplication formulas come from DoubleCounting and PerNumbers II

More STEM examples:

- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec;
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = (**energy/charge**) x (**charge/sec**) or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

MATHeCADEMY

5 ways to master proportionality I

Proportionality, **Q1**: “2kg costs 5$, what does 7kg cost”; **Q2**: “What does 12$ buy?”

1) **Regula de Tri** (Rule of Three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

- **Q1**: ‘2kg cost 5$, 7kg cost ?$’. Multiply-then-divide gives the $-number 7x5/2 = 17.5.
- **Q2**: ‘5$ buys 2kg, 12$ buys ?kg’. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

2) **Find the unit by dividing before multiplying**

- **Q1**: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

3) **Cross multiplication when equating the per-numbers**

- **Q1**: $2/5 = 7/u$, so $2*u = 7*5$, $u = (7*5)/2 = 17.5$. **Q2**: $2/5 = u/12$, so $5*u = 12*2$, $u = (12*2)/5 = 4.8$

4) **ReCount in the per-number** 2kg/5$ coming from ‘double-counting’ the total T.

- **Q1**: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

MATHeCADEMY

5 ways to master proportionality II

5) **Modern Math Modeling** with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive $-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$$.
- With 12$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

MATHeCADEMY

ReCounting sides in a block: Trigonometry

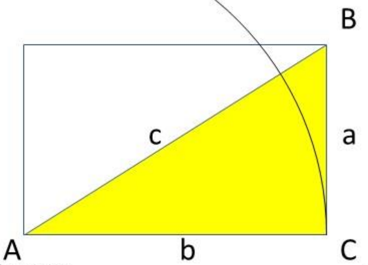
A right triangle is <u>a block halved by its diagonal</u> giving 3 sides: <u>base b</u>, <u>height a</u> and <u>diagonal c</u> connected with the angles when recounting one side in the other side or in the diagonal

a = (a/c)*c = sinA * c

b = (b/c)*c = cosA * c

tanA = a/b = Δy/Δx = gradient

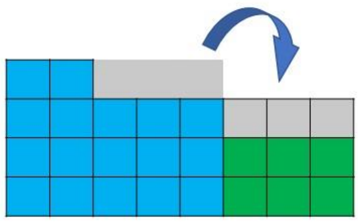
Circle: circum./diam. = π ≈ **n*tan(180/n)** for n large

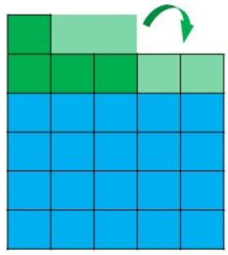


MATHeCADEMY

Counted & recounted, Totals can be added

BUT: NextTo	**or OnTop**
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 4 **5s** + 1**B**1 **5s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





PerNumbers add by areas (integration)

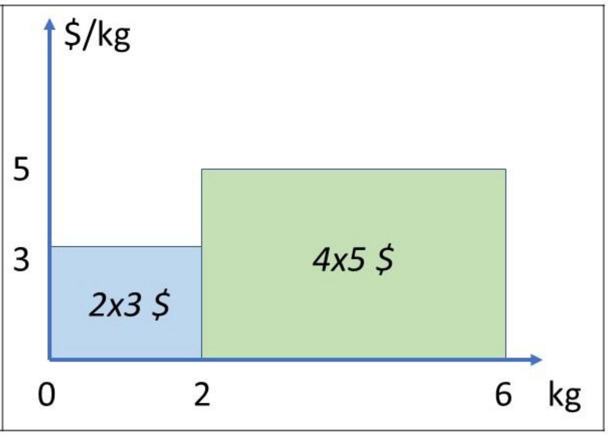
"2kg at 3$/kg + 4kg at 5$/kg = 6kg at **?** $/kg?"

2 kg at 3 $/kg

+ 4 kg at 5 $/kg

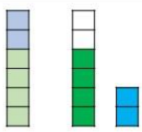
(2+4) kg at **?** $/kg

- Unit-numbers add on-top.
- Per-numbers must be multiplied so they add by **areas** under the per-number graph (integration).

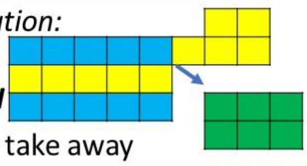


Reversed addition = solving equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 6 = (6/2) x 2	**2 + ? = 6** = (6-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 6/2 *Solved by ReCounting*	? = 6-2 *Solved by ReStacking*	? = (3.2 **8s** – 2 **3s**)/5 = *(T-T1)/5 = ΔT/5*



Differentiation:
first ***PULL***
then ***PUSH***



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

MATHeCADEMY

Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: **O**pposite **S**ide with **O**ppoSite **S**ign

MetaMath (Don't test, but DO remember the bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
↕	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
↕	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
↕	*u* x (2 x (½)) = 4	Applying the **associative** law
↕	*u* x 1 = 4	Applying the definition of an inverse element
↕	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

MATHeCADEMY

The simplicity of mathematics I: There are just four ways to unite into a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration).
Addition and multiplication add changing and constant unit-numbers.
Integration and power add changing and constant per-numbers.
We might call this beautiful simplicity the '*Algebra Square*'.

Operations unite	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n**	**T = a * n**
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn**	**T = a^n**

The simplicity of mathematics II: There are just five ways to split a Total

The 4 uniting operations (+, *, ^, ∫) each has a reverse splitting operation:
Addition has subtraction (–), and multiplication has division (/).
Power has factor-finding (root, √) and factor-counting (logarithm, log).
Integration has per-number finding (differentiation dT/dn = T‘).

*Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.*

Operations unite / *split into*	**changing**	**constant**
unit-numbers *m, s, \$, kg*	**$T = a + n$** $T – a = n$	**$T = a * n$** $T/n = a$
per-numbers *m/s, \$/kg, m/(100m) = %*	**$T = \int a\, dn$** $dT/dn = a$	**$T = a$^n** $\log_a T = n$, $\sqrt[n]{T} = a$

Conclusion and recommendations

● Problems come, when presenting math through **INSIDE** self-reference instead of as a natural science communicating about the **OUTSIDE** fact Many.

● As a foreign language, the number-language may be learned through its use where core mathematics as proportionality, equations, per-numbers and calculus grow directly from the Mastery of Many that children develop in **OUTSIDE** use and before school. *Digits & fractions are operators, not numbers.*

● Math is not a self-referring goal in itself, but an **INSIDE** means to an **OUTSIDE** goal that is reached better and by more through communicating about Many.

● Recounting is rooting **INSIDE** math in **OUTSIDE** STEM-examples. This allows young migrants learn core STEM subjects also, to become STEM pre-teachers or pre-engineers that can help develop or rebuild their own country.

References

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- **Tarp**, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.
- **Tarp**, A. (2019). Addition-free stem-based math for migrants. Retrieved from: http://mathecademy.net/ctras-2019-contributions/
- **Widdowson**, H. G. (1978). *Teaching language as communication.* Oxford, UK: Oxford University Press.

Teach language AFTER grammar! No, BEFORE *the Communicative Turn* *around 1970*

Halliday (1973, p. 7) defines **a functional approach** to language:

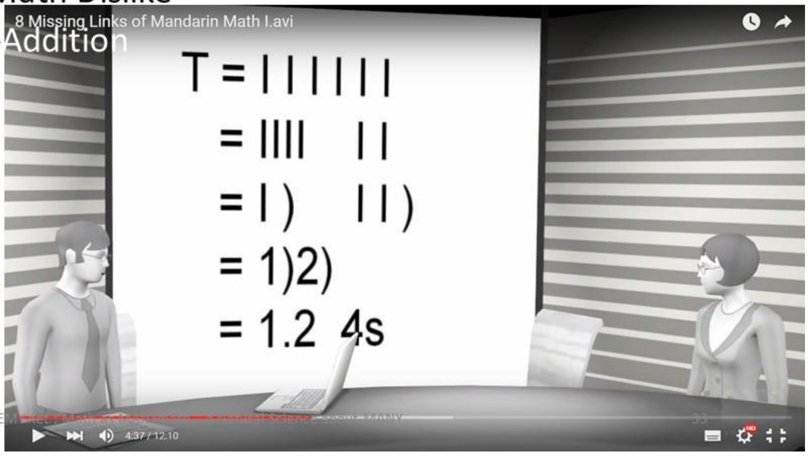
"A functional approach to language means, first of all, investigating how language is used: trying to find out what are the purposes that language serves for us, and how we are able to achieve these purposes through speaking and listening, reading and writing. But it also means more than this. It means seeking to explain the nature of language in functional terms: seeing whether language itself has been shaped by use, and if so, in what ways - how the form of language has been determined by the functions it has evolved to serve."

Likewise, Widdowson (1978) adopts a **communicative approach** to the teaching of language' allowing more students to learn INSIDE language through its OUTSIDE use for communication about things and actions.

Some MrAlTarp YouTube Videos

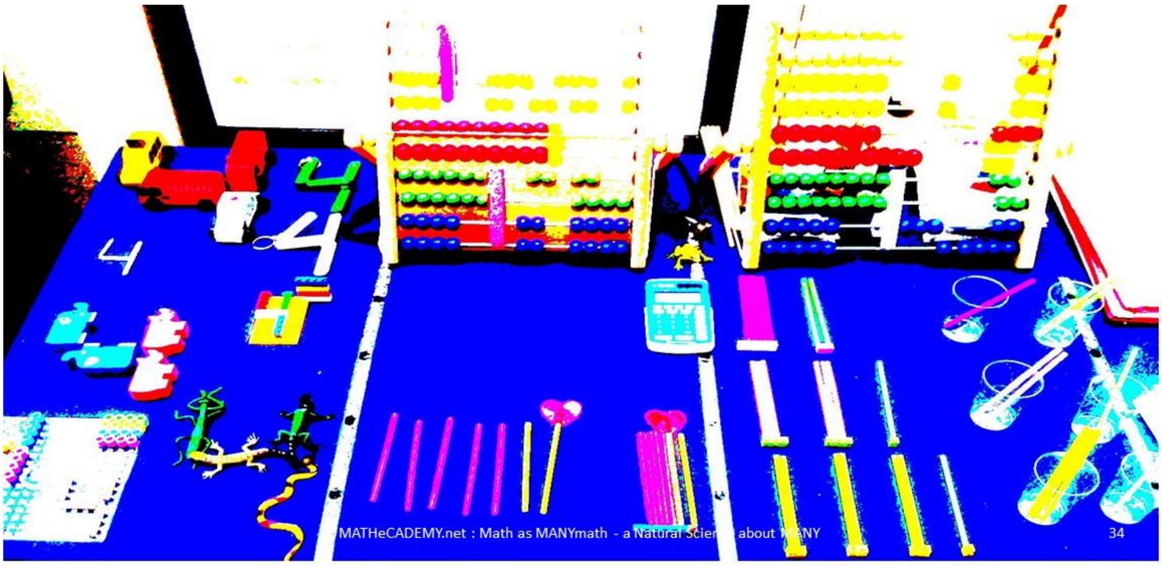
DrAlTarp YoKu

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



Number icons

ReCounting **7** in **5s** & **3s** & **2s**

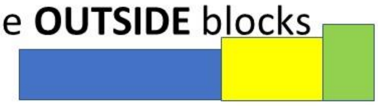


What Mastery of Many do children get through use before school?

Children typically see Many as **OUTSIDE** blocks described by **INSIDE** numbres, counting bundles, bundles of bundles etc.

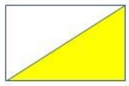
So, to children, BLOCKS are fundamental:

- in **INSIDE** numbers: 456 = 4***BB***5***B***6 = three **OUTSIDE** blocks

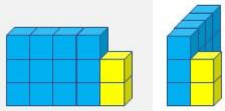


- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks



The child's own twin math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving **OUTSIDE** geometrical multi-blocks, & (when turned to hide the units behind) **INSIDE** algebraic bundle-numbers.

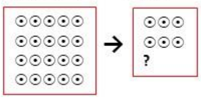
3) Operations are **INSIDE** icons, showing the 3 **OUTSIDE** counting steps: PUSHING bundles & LIFTING bundles & PULLING stacks to find the unbundled ones.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

Question: T = 4 ***5s*** *= ?* ***3s*** ● *Answer: T = 4* ***5s*** *= 6****B****2* ***3s*** ● *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



MATHeCADEMY

Main parts of a ManyMath curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage

- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- BundleCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: divide, multiply, subtract, add on-top & next-to

Middle school – integrating algebra and geometry, the content of the label 'math'

- DoubleCounting produces PerNumbers and fractions as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always, so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE

- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

MATHeCADEMY

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MATHeCADEMY

Question guided teacher education

MATHeCADEMY.net

Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

MATHeCADEMY

PYRAMIDeDUCATION

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting their Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams