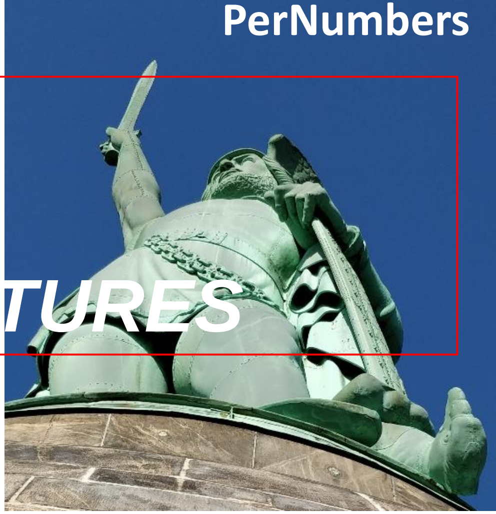
The best way to predict **THE FUTURE** is to invent it (Alan Kay) …. *Not me, I am Allan Tarp, I invent* ***FUTURES***



The Roman Fraction eagle

Welcome to a FractionFreeFuture - **Welcome** to 2040

Where mathe-matics is MANY-math, a natural science about MANY

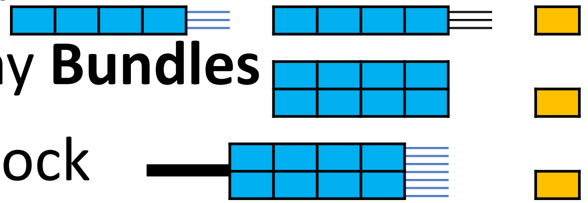
- that respects and develops childrens' own number-languge with

- Childrens' own double-numbers, T = 2 **3s** & childrens' own operations:

  Division: 9/4 means (~~9 split by 4~~) 9 recounted in **4s** by pushing away **Bundles**

  Multiplication: 2x4 means (~~8~~) 2 times lifting **Bundles** of **4s** into a block

  Subtraction: 9 – 2x4 is a rope pulling away the block to find the unbundled ones



- Flexible **Bundle**-numbers: T = 9 = 1**B**5 **4s** = 2**B**1 **4s** = 3**B**-3 **4s** = 2.1 **4s** = 2 1/4 **4s**
- Double-numbers in secondary school: **Per-numbers** & fractions

  (2$/3kg, 2$/3$ = 2/3) both adding by their areas, i.e. as integral calculus

The Child's Own Mastery of Many

Count & ReCount & DoubleCount

before Adding NextTo & OnTop

master many with manymath

Allan.Tarp@MATHeCADEMY.net, September 2019

MATHeCADEMY

Education is a cure: Does it work

In Sweden, OECD says that it excludes 1 of 4 socially

"PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (page 3)

http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm

Why teach children if they already know?

With education curing un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

Core Questions:

- What Mastery does children develop when adapting to Many?
- What could be a Question-guided ChildCenteredCurriculum in Quantitative Competence?

Adaption creates two language houses

The **WORD language** assigns words in sentences with The **NUMBER language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. But does mathematics respect teaching language before grammar?

	WORD language	NUMBER language
META-language, grammar	*'is' is a verb*	*'x' is an operation* Mathematics
Language	This is a chair Mother tongue	T = 3x4

WORLD

The **Communicative Turn** in language ed.

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: “A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us.”
Likewise, Widdowson adopts a “communicative approach to the teaching of language” allowing more students to learn a language through its use for communication about outside things and actions.

Could mathematics also have its Communicative turn?

	WORD language	NUMBER language
(META-language, grammar)		
Inside Language	This is a chair Mother tongue	T = 3 4s Mathematics
Outside world		

Children see Many as bundles with units

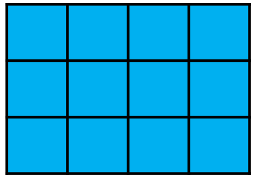
“How old next time?” A 3year old says “Four” showing 4 fingers: I I I I

But, the child reacts strongly to 4 fingers held together 2 by 2: II II

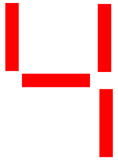
“That is not four, that is two twos” (T ≠ 4, T = 2 **2s**)

The child sees what exists, and with units: bundles of **2s**, and 2 of them.

- The block 3 **4s** has two numbers: 3 (the counting-number) and **4** (the unit-number)



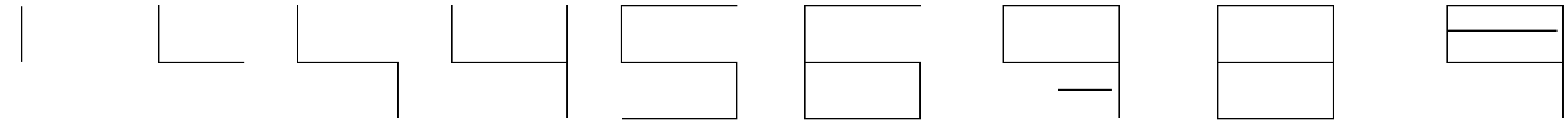
Let us invent a **Q**uestion **D**riven **C**urriculum to minimize the ‘mediated correctness’ effect of textbooks and teachers

Q01. Create icons: I I I I → IIII →  →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.

An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become units to use when counting in bundles.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII



1	2	3	4	5	6	7	8	9

Q02, counting sequences

"How to count fingers?"

Using **5s** as the bundle-size, fingers can be counted as

"0**B**1, 0**B**2, 0**B**3, 0**B**4, 0**B**5 – sorry, **Bundle**"

and the rest can be counted in as

"**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B**, **1left, 2left** (a-leven, twe- leven)".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1**B**3 **7s** = 2**B**2 **4s** = 3**B**1 **3s** = 1**BB**1 **3s**.

Q03, bundle-counting in icon-units

"How to count by bundling?"

Five fingers can be bundle-counted in pairs or triplets, allowing both an OVERLOAD and an UNDERLOAD; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

I I I I I ● ǂ I I I ● ǂ ǂ I ● ǂ ǂ ǂ ● ǂ ǂ I

T = 5 = 1**Bundle**3 **2s** = 2**B**1 **2s** = 3**B**-1 **2s** = 1**BB**1 **2s**

T = 5 = 1.3 **2s** = 2.1 **2s** = 3.-1 **2s** = 10.1 **2s**

Likewise, if counting in **ten**-bundles:
T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

Q04, unbundled as decimals or negatives or fractions

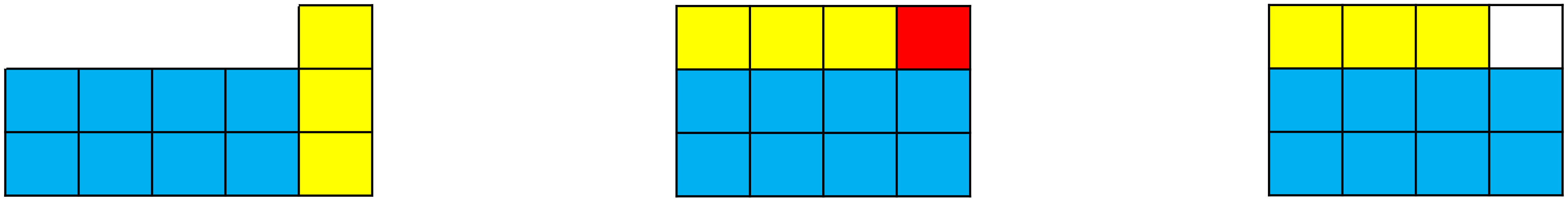
0.3 **4s** or 1.-1 **4s** or 3/4 **4s**

"Where to put the unbundled singles?"

When counting by bundling, the unbundled singles can be placed

NextTo the block | **OnTop** of the block

counted as a block of **1s** | counted as a bundle | counted in bundles



T = 2B3 **4s** = 2.3 **4s**
A decimal number

T = 3B-1 **4s** = 3.-1 **4s**
A negative number

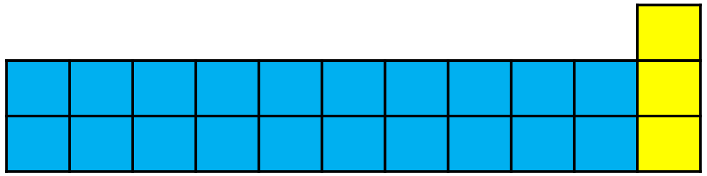
T = 2 3/4 **4s**
A fraction

Q04, counting in tens

“Where to put the unbundled singles with **tens**?”

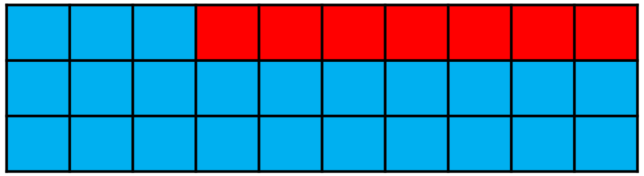
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit and the decimal point,

- or as:



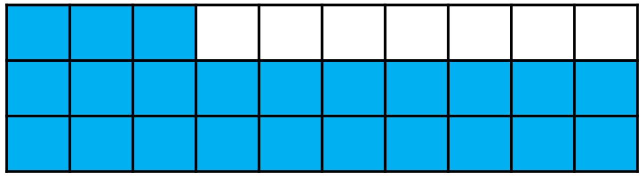
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

T = 3B-7 **tens**



T = 2 3/10 **tens**

T = 2 3/10 B **tens**

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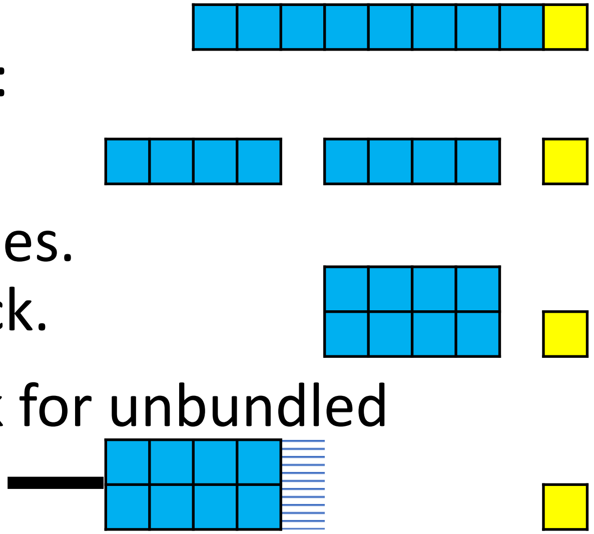
Q05, calculators predict

“Can a calculator predict a counting result?”

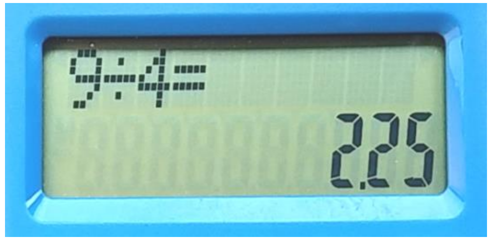


We may see <u>division</u> as an icon for a broom pushing away bundles:
9/4 means ‘from 9, push away bundles of 4s’.

- The calculator says ‘2.some’, thus predicting it can be done 2 times.
  Now <u>multiplication</u> iconizes a lift stacking the bundles into a block.

- Finally, <u>subtraction</u> iconizes a rope pulling away the block to look for unbundled singles.
- With ‘9-2x4 = 1’ the calculator predicts that 9 can be recounted as 2**B**1 **4s**.

9/4	**2.some**
9 – 2x4	**1**



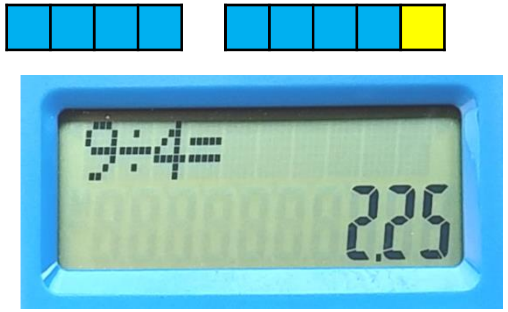
Q05, counting creates a ReCounting formula

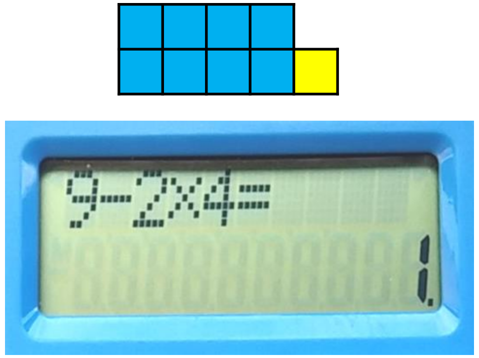
ReCount **T = (T/B) x B**	from a total **T**, **T/B** times, **Bs** is taken away and stacked

As sentences of the number language, **Formulas Predict:**

Predicting that **T** = 9 = 2.1 **4s:**

9/4	**2.some**
9 – 2x4	**1**

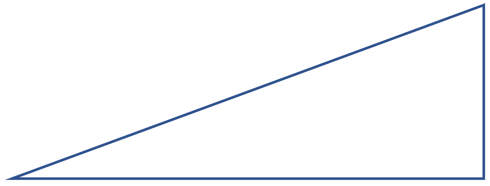


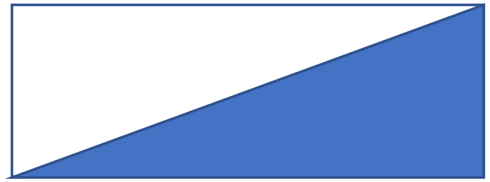


Q05, the recounting formula is a core formula

T = (T/B)*B is all over STEM (Science, Technology, Engineering, Mathematics):

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y’** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second





Q06, recounting in a different unit

“How to change a unit?”

The recount-formula allows changing the unit.

Asking T = 3 **4s** = ? **5s**, the recount-formula gives T = 3 **4s** = (3x4/5) **5s**.

Entering 3x4/5, the answer ‘2.some’ shows that a block of 2 **5s** can be taken away.

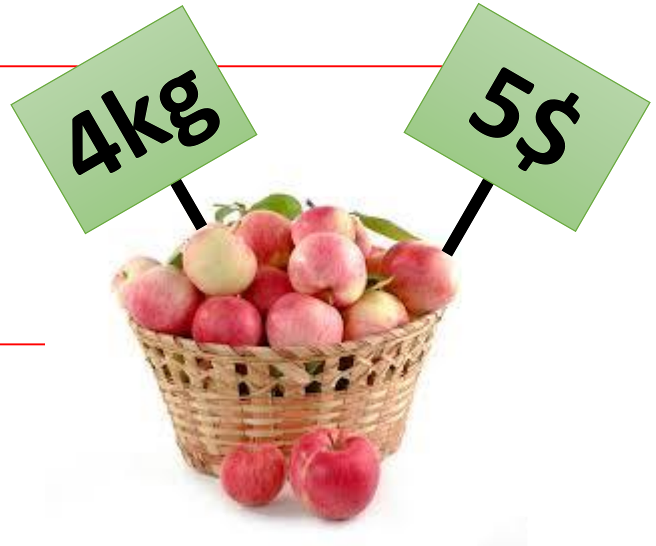
With 3x4–2x5, the answer ‘2’ shows that 3 **4s** can be recounted as 2B2 **5s** or 2.2 **5s**.

3 **4**s = IIII IIII IIII = IIII I III II II = IIIII IIIII II = 2B2 **5**s = 2.2 **5s**

3x4/5	**2.**some
3x4 – 2x5	**2**

Change Unit = Proportionality

Q06, double-counting in two units creates DoubleNumbers or PerNumbers



"How to double-count in two units?"

DoubleCounting in kg & $, we get **4kg = 5$** or

4kg **per** 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber.

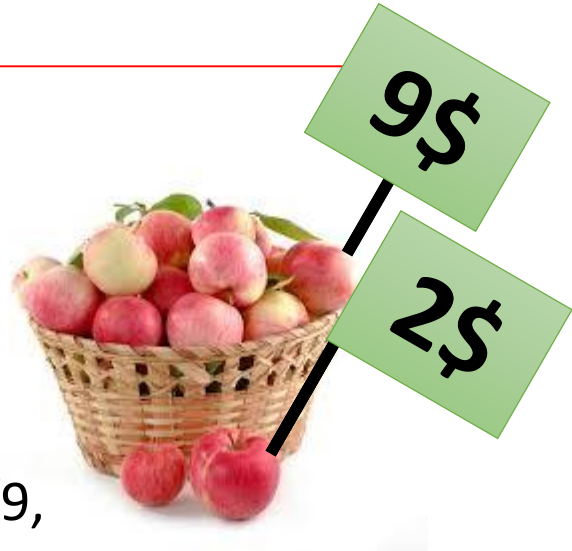
With 4kg bridged to 5$ we answer questions by recounting in the per-number.

Questions:

7kg = ?$	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

Answer: *Recount in the PerNumber (Proportionality)*

Q06, double-counting in the same unit creates fractions



"How to double-count in the same unit?"

Double-counted in the same unit, per-numbers are fractions, 2$ per 9$ = 2/9, or percentages, 2 per 100 = 2/100 = 2%.

To find a fraction or a percentage of a total, again we just recount in the per-number.

- **Taking 3 per 4 = taking ? per 100.** With 3 bridged to 4, we recount 100 in 4s:

100 = (100/4)*4 giving (100/4)*3 = 75, and 75 per 100 = 75%.

- **Taking 3 per 4 of 60 gives ?.** With 3 bridged to 4, we recount 60 in 4s:

60 = (60/4)*4 giving (60/4)*3 = 45.

- **Taking 20 per 100 of 60 gives ?.** With 20 bridged to 100, we recount 60 in 100s:

60 = (60/100)*100 giving (60/100)*20 = 12 .

We observe that per-numbers and fractions are not numbers but OPERATORS needing a number to become a number.

Proportionality shows the instability of ‘School Math’ I

Proportionality, **Q1**: “2kg costs 5$, what does 7kg cost”; **Q2**: “What does 12$ buy?”

➔1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: ‘2kg cost 5$, 7kg cost ?$’. Multiply-then-divide gives the $-number 7x5/2 = 17.5.

Q2: ‘5$ buys 2kg, 12$ buys ?kg’. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

➔2) Find the unit

Q1: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

➔3) Cross multiplication

Q1: 2/5 = 7/*u*, so 2**u* = 7*5, *u* = (7*5)/2 = 17.5. **Q2**: 2/5 = *u*/12, so 5**u* = 12*2, *u* = (12*2)/5 = 4.8

➔4) ‘Re-counting’ in the ‘per-number’ 2kg/5$ coming from ‘double-counting’ the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

Proportionality shows the instability of 'School Math' II

➔ 5) <u>Modeling</u> with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive $-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ ● $(c*2)*½ = 5*½$ ● $c*(2*½) = 5/2$ ● $c*1 = 5/2$ ● $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$$.
- With 12$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

Double-counting gives per-numbers in STEM multiplication formulas I

STEM typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

Double-counting gives per-numbers in STEM multiplication formulas II

Extra STEM examples:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec;
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = **(energy/charge)** x **(charge/sec)** or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.