

# Q07, recounting from tens to icons

“How to change unit from tens to icons?”

Asking ‘ $T = 2.4 \text{ tens} = 24 = ? \text{ 8s}$ ’, we just recount 24 in 8s:

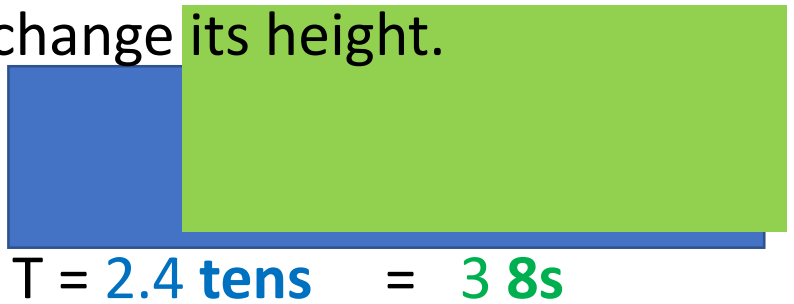
$$T = 24 = (24/8) \times 8 = 3 \times 8 = 3 \text{ 8s.}$$

Formulated as an **equation** we use *u* for the unknown number,  $u \times 8 = 24$ .

Recounting 24 in 8s shows that *u* is 24/8.

So, equations are solved by moving **to opposite side - with opposite sign**

To keep its size, a block changing its unit must also change its height.



$$u \times 8 = 24 = (24/8) \times 8$$

$$u = 24/8 = 3$$

# Q8, recounting from icons to tens (multiplication) $3 \text{ 7s} = ? \text{ tens}$



“How to change unit from icons to tens?”

Asking ‘ $T = 3 \text{ 7s} = ? \text{ tens}$ ’, the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone:  $T = 3 \text{ 7s} = 3 \times 7 = 21 = 2.1 \text{ tens}$ , only it leaves out the unit and the decimal point.

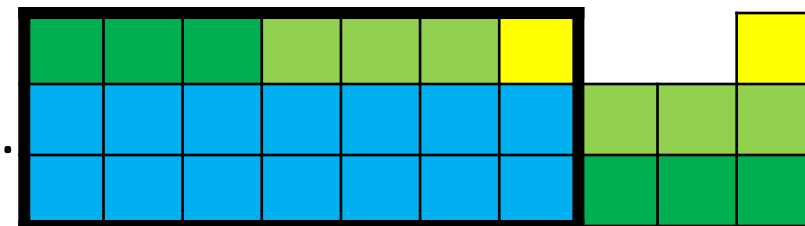
Alternatively, we may use ‘less-numbers’, so  $7 = \text{ten less } 3$

$$T = 3 \times 7 = 3 \times (\text{ten less } 3) = 3 \times \text{ten less } 3 \times 3 = 3 \text{ten less } 9 = 2 \text{ten } 1 = 21,$$

or with  $9 = \text{ten less } 1$ :

$$T = 3 \text{ten less } (\text{ten less } 1) = 2 \text{ten less } 1 = 2 \text{ten } \& 1 = 21.$$

*showing that ‘lessless’ cancel out*





Recounting large numbers in or from tens:  
*same size, but new form*

Recounting 6 47s in tens

Recounting 476 in 7s

*Bundle Writing separates INSIDE bundles from OUTSIDE singles*

|  |  |
|--|--|
| <p><math>T = 6 \times 47 = 6 \times 47</math></p>  <p><math>= 247</math></p> <p><math>= 282</math></p> <p><math>= 28.2</math></p> <p><b>tens</b></p> | <p><math>T = 476 = 47.6 \text{ tens}</math></p> <p><math>= 476</math></p> <p><math>= 4256</math></p> <p><math>= 6 \times 78 \times 7</math></p> <p><math>= 68 \times 7</math></p>  |
|--|--|

# Q09, ReCounting sides in a block: Trigonometry

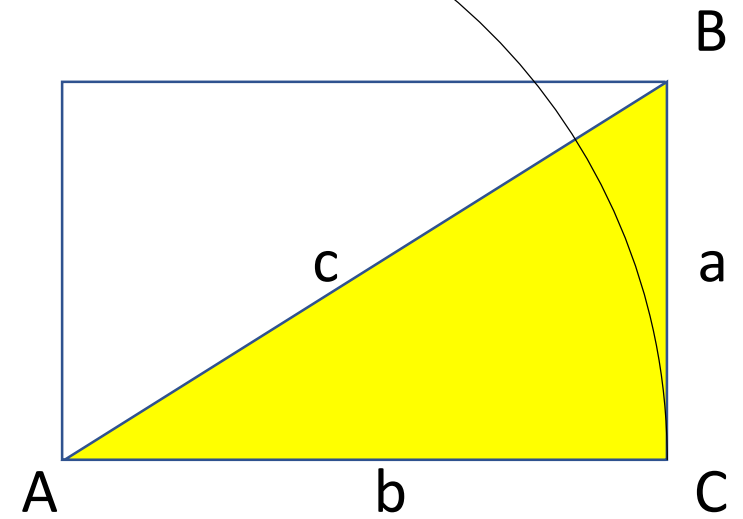
A right triangle is a block halved by its diagonal giving 3 sides: base b, height a and diagonal c connected with the angles when recounting one side in the other side or in the diagonal

$$a = (a/c) * c = \sin A * c$$



$$b = (b/c) * c = \cos A * c$$

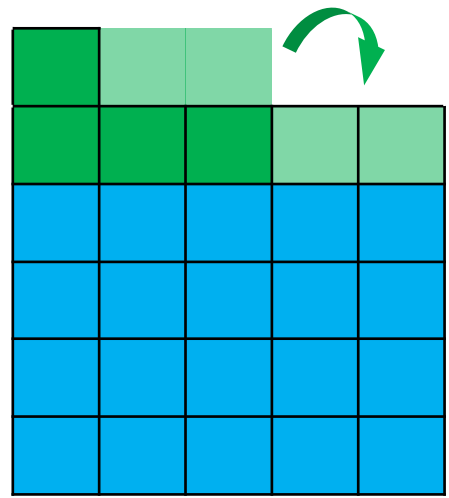
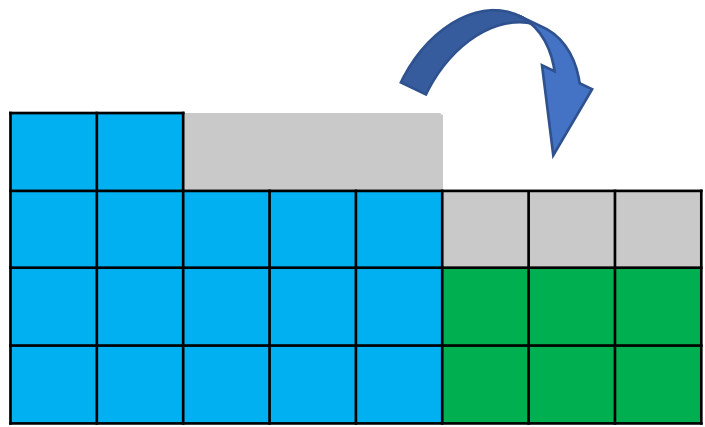
$$\tan A = a/b = \Delta y / \Delta x = \text{gradient}$$

Circle: circum./diam. =  $\pi \approx n * \tan(180/n)$  for n large

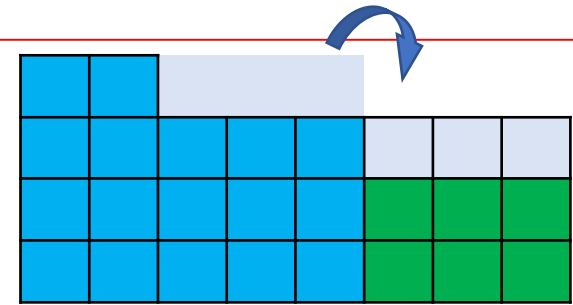


Once counted & recounted, Totals can be added

| <b>BUT:</b> <b>NextTo</b>  | <b>or</b> <b>OnTop</b>  |
|---|--|
| $4 \text{ } 5s + 2 \text{ } 3s = 3B2 \text{ } 8s$   | $4 \text{ } 5s + 2 \text{ } 3s = 4 \text{ } 5s + 1B1 \text{ } 5s = 5B1 \text{ } 5s$                        |
| The areas are integrated<br><i>Adding areas = Integration</i>   | The units are changed to be the same<br><i>Change unit = Proportionality</i>                               |



# Q11, next-to addition



“With  $T1 = 4 \text{ 5s}$  and  $T2 = 2 \text{ 3s}$ , what is  $T1+T2$  when added next-to as  $8\text{s}$ ?”

Outside, next-to addition geometrically means adding areas. Next-to addition is also called integral calculus.

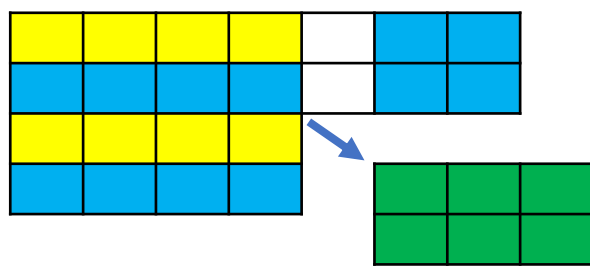
Inside, the recount formula algebraically predicts the result. Here multiplication precedes addition.

$$T = (T/B) \times B$$

$$= ( (4 \times 5 + 2 \times 3) / 8 ) \times 8 = 3.2 \text{ 8s}$$

|  |        |
|--|--------|
| $(4 \times 5 + 2 \times 3) / 8$          | 3.some |
| $(4 \times 5 + 2 \times 3) - 3 \times 8$ | 2      |

# Q12, reversed next-to addition



“If  $T1 = 2\ 3s$  and  $T2$  add next-to as  $4\ 7s$ , what is  $T2$ ?”

Outside, we remove the initial block  $T1$  and recount the rest in  $4s$ .

Thus reversed next-to addition geometrically means subtracting areas.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

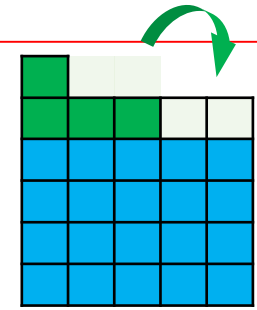
Here subtraction precedes division; which is natural as reversed integration.

$$T2 = (T2/B) \times B$$

$$= ( (4 \times 7 - 2 \times 3) / 4 ) \times 4 = 5.2\ 4s$$

|  |               |
|--|---------------|
| $(4 \times 7 - 2 \times 3) / 4$          | <b>5.some</b> |
| $(4 \times 7 - 2 \times 3) - 5 \times 4$ | <b>2</b>      |

# Q13, on-top addition

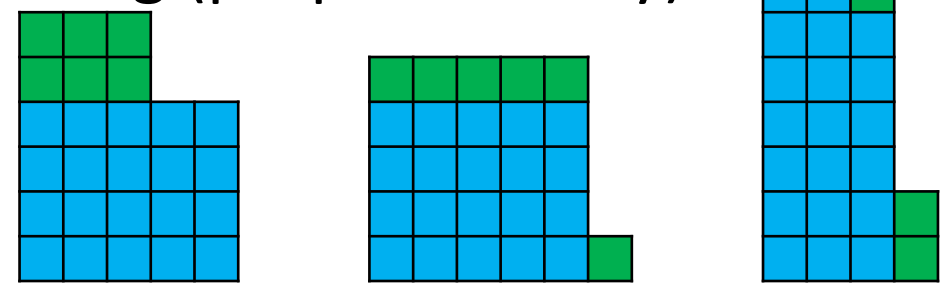


“With  $T1 = 4 \text{ 5s}$  and  $T2 = 2 \text{ 3s}$ , what is  $T1+T2$  when added on-top?”

Outside, on-top addition geometrically means changing units. On-top addition thus often involves recounting (proportionality).

$$T = 4 \text{ 5s} + 2 \text{ 3s} = 4 \text{ 5s} + 1.1 \text{ 5s} = 5.1 \text{ 5s}$$

$$T = 4 \text{ 5s} + 2 \text{ 3s} = 6.2 \text{ 3s} + 2 \text{ 3s} = 8.2 \text{ 3s}$$



Inside, the recount formula algebraically predicts the result. Here again, multiplication precedes addition.

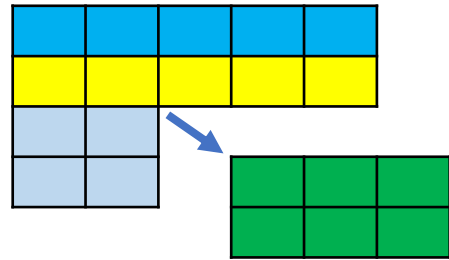
$$T = (T/B) \times B$$

$$= ( (4 \times 5 + 2 \times 3) / 5 ) \times 5 = 5.1 \text{ 5s}$$

|  |        |
|--|--------|
| $(4 \times 5 + 2 \times 3) / 5$          | 5.some |
| $(4 \times 5 + 2 \times 3) - 5 \times 5$ | 1      |



# Q14, reversed on-top addition



“T1 = 2 3s and how many 5s (T2) add on-top as 4 5s?”

Outside, we remove the initial block T1 and recount the rest in 5s.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed on-top addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

$$T2 = (T2/B) \times B$$

$$= ( (4 \times 5 - 2 \times 3) / 5 ) \times 5 = 2.4 \text{ 5s}$$

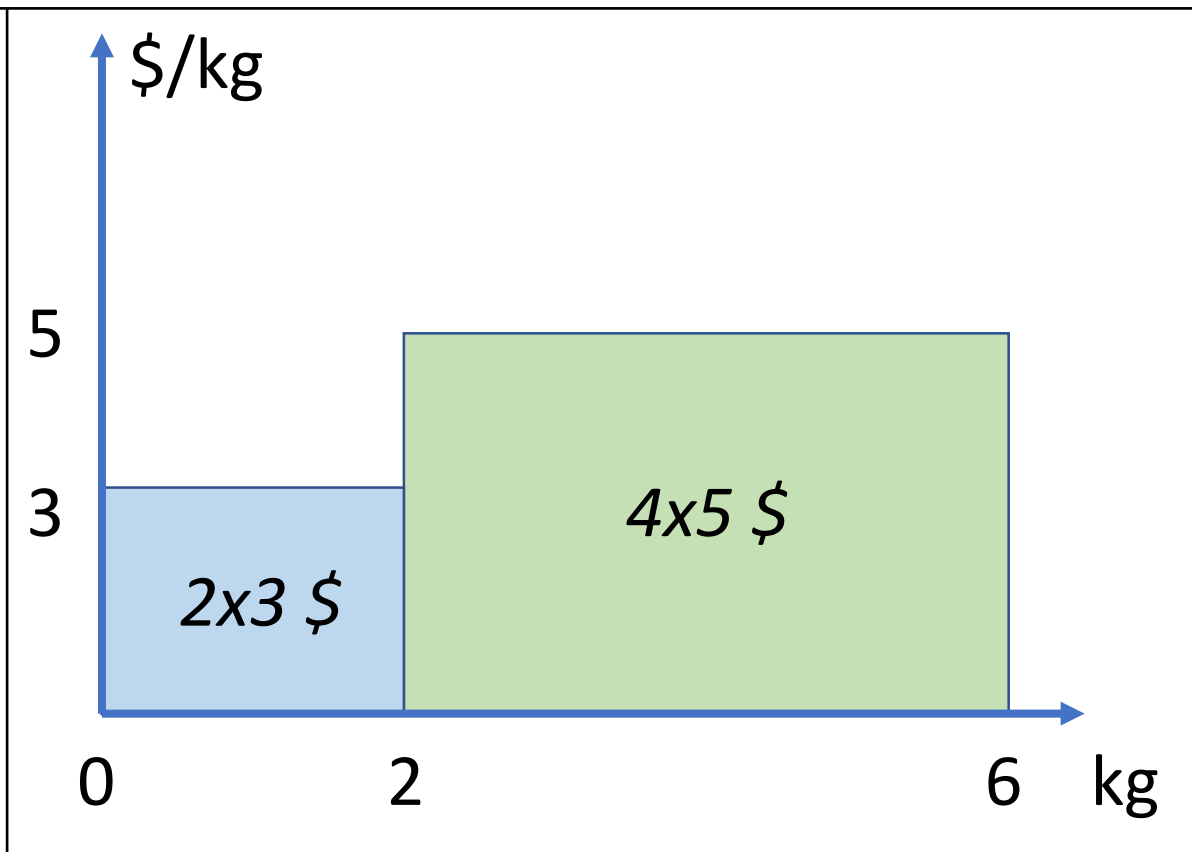
|  |        |
|--|--------|
| $(4 \times 5 - 2 \times 3) / 5$          | 2.some |
| $(4 \times 5 - 2 \times 3) - 2 \times 5$ | 4      |

# Q29, adding PerNumbers as areas (integration)

“2kg at 3\$/kg + 4kg at 5\$/kg = 6kg at ? \$/kg?”

$$\begin{array}{r}
 2 \text{ kg at } 3 \text{ \$/kg} \\
 + 4 \text{ kg at } 5 \text{ \$/kg} \\
 \hline
 (2+4) \text{ kg at } ? \text{ \$/kg}
 \end{array}$$

- Unit-numbers add on-top.
- Per-numbers add next-to as **areas** under the per-number graph. Here multiplication precedes addition.



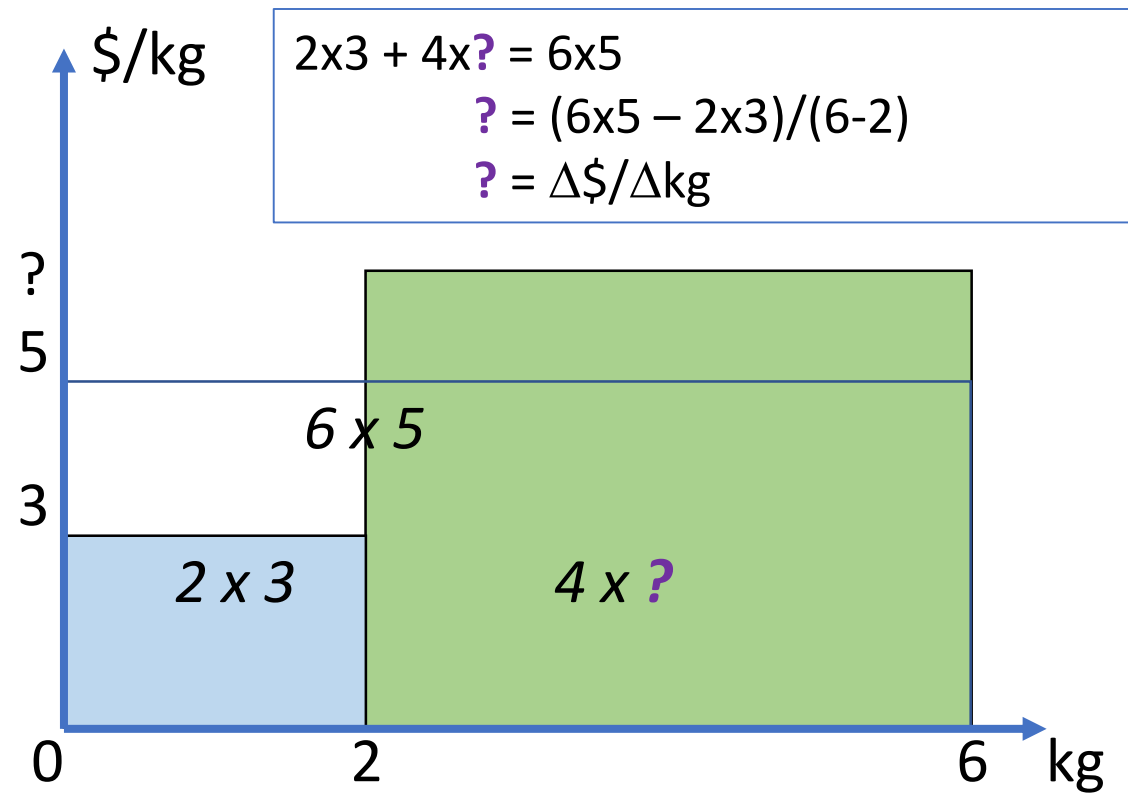
# Q30, subtracting PerNumbers (differentiation)

“2kg at 3\$/kg + 4kg at **what** = 6kg at 5\$/kg?”

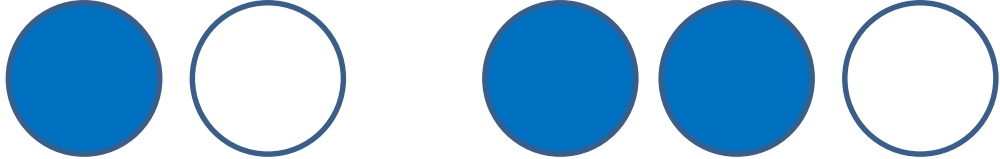
$$\begin{array}{r}
 2 \text{ kg at } 3 \text{ \$/kg} \\
 + 4 \text{ kg at } ? \text{ \$/kg} \\
 \hline
 6 \text{ kg at } 5 \text{ \$/kg}
 \end{array}$$

Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change,  $\Delta$ ) precedes division.



# Never add without units, the fraction paradox

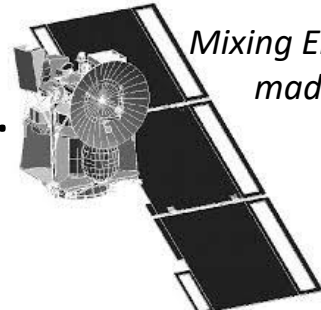
| The Teacher  | The Students  |
|--|---|
| What is $1/2 + 2/3$ ?                                  | Well, $1/2 + 2/3 = (1+2)/(2+3) = 3/5$   |
| No! $1/2 + 2/3$<br>$= 3/6 + 4/6$<br>$= 7/6$            | But $1/2$ of 2 cakes + $2/3$ of 3 cakes<br>is $1+2$ of $2+3$ cakes, i.e. $3/5$ of 5 cakes!<br>How can it be 7 cakes out of 6 cakes? |
| Inside this classroom<br>$1/2 + 2/3$ <b>IS</b> $7/6$ ! |    |

Fractions are not numbers, but operators, needing numbers to become numbers.

**2+3 IS 5!** No, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.

**2\*3 IS 6!** Yes, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.

*Adding without units: MatheMatism.*



*Mixing English and metric units made NASA's Mars Climate Orbiter fail in 1999.*