What is Math - and Why Learn it?

Allan.Tarp@MATHeCADEMY.net, summer 2019

"What is math - and why learn it?" Two questions you want me to answer, my dear nephew.

0. What does the word mathematics mean?
In Greek, 'mathematics' means 'knowledge'. The Pythagoreans used it as a common label for their four knowledge areas: Stars, music, forms and numbers. Later stars and music left, so today it only includes the study of forms, in Greek called geometry meaning earth-measuring; and the study of numbers, in Arabic called algebra, meaning to reunite. With a coordinate-system coordinating the two, algebra is now the important part giving us a number-language, which together with our word-language allows us to assign numbers and words to things and actions by using sentences with a subject, a verb and a predicate or object:

"The table is green" and "The total is 3 4s" or "T = 3*4". Our number-language thus describes Many by numbers and operations.

1. Numbers and operations are icons picturing how we transform Many into symbols
The first ten degrees of Many we unite five sticks into one five icon, etc. The icons then become units when counting Many by uniting unbundles singles, bundles, bundles of bundles. Operations are icons also:

- Counting 8 in **2s** can be predicted by <u>division</u>, iconized by a broom pushing away **2s**: 8/2 = 4, so 8 = 4 **2s**.
- Stacking the **2s** into a block can be predicted by <u>multiplication</u>, iconized by a lift pushing up the **2s**: 8 = 4x2.
- Looking for unbundled can be predicted by <u>subtraction</u>, iconized by a rope pulling away the 4 **2s**: 8 – 4x2.
- Uniting bundles and singles is predicted by <u>addition</u>, iconized by a cross, +, placing blocks next-to or on-top.

Recounting a total T in B-bundles is predicted by a 'recount-formula':	**T = (T/B)*B**	
saying 'From T, T/B times, B can be pushed away'.	9/2	4.some
Recounting 9 in **2s**, the calculator predicts the result 9 = 4B1= 4.1 **2s** = 4 **2s** + 1	9 – 4x2	1

Now, let us write out the total 345 as we say it when bundling in ones, tens, and ten-tens, or hundreds, we get T = 3*B^2 + 4*B + 5*1. This shows that uniting takes place with four operations: number-addition unite unlike numbers, multiplication unite like numbers, power unite like factors, and block-addition (integration) unite unlike areas. So, one number is really many numberings united by calculations.

Thus, mathematics may also be called calculation on specified and unspecified numbers and formulas.

2. Placeholders
A letter like x is a placeholder for an unspecified number. A letter like f is a placeholder for an unspecified calculation formula. Writing 'y = f(x)' means that the y-number can be found by specifying the x-number in the f-formula. Thus, specifying f(x) = 2 + x will give y = 2+3 = 5 if x = 3, and y = 2+4 = 6 if x = 4. Writing y = f(2) is meaningless, since 2 is not an unspecified number. The first letters of the alphabet are used for unspecified numbers that do not vary.

3. Calculation formula predict
The addition calculation T = 5+3 predicts the result without having to count on. So, instead of adding 5 and 3 by 3 times counting on from 5, we can predict the result by the calculation 5+3 = 8.

Likewise, with the other calculations:

- The multiplication calculation 5*3 predicts the result of 3 times adding 5 to itself.
- The power calculation 5^3 predicts the result of 3 times multiplying 5 with itself.

4. Reverse calculations may also be predicted
'5 + 3 = ?' is an example of a forward calculation. '5 + ? = 8' is an example of a reversed calculation, often written as 5 + x = 8, called an equation that asks: which is the number that added to 5 gives 8?

An equation may be solved by guessing, or by inventing a reverse operation called subtraction, x = 8 – 5; so, by definition, 8-5 is the number x that added to 5 gives 8. The calculator says that 8-5 is 3.

We now test to see if this is the solution by calculating separately the left and right side of the equation. The left side gives 5 + x = 5 + 3 = 8. The right side is already calculated as 8.

When the left side is equal to the right side, the test shows that x =3 is indeed a solution to the equation.

Likewise, with the other examples of reverse calculations:

- $\frac{8}{5}$ is the number x, that multiplied with 5 gives 8. So, it solves the equation 5*x = 8.
- $\sqrt[5]{8}$ is the number x, that multiplied with itself 5 times gives 8. So, it solves the equation x^5 = 8.
- $\log_5(8)$ is the number x of times to multiply 5 with itself to give 8. So, it solves the equation 5^x = 8.

Thus, where the root is a factor-finder, the logarithm is a factor-counter.

Together we see that an equation is solved by 'moving to <u>opposite side</u> with <u>opposite sign</u>'

5 + x = 8	5*x = 8	x^5 = 8	5^x = 8
x = 8 – 5	x = $\frac{8}{5}$	x = $\sqrt[5]{8}$	x = $\log_5(8)$

5. Double-counting creates per-numbers and fractions
Double-counting in two units creates per-numbers as e.g. 3$ per 4kg or 3$/4kg or 3/4 $/kg.

To bridge the units, we just recount the per-number: 15$ = (15/3)*3$ = (15/3)*4kg = 20kg.

With the same unit, a per-number becomes a fractions or percent: 3$/4$ = 3/4, 3$/100$ = 3%.

Again, the per-number is a bridge: To find 3/4 of 20, we recount 20 in 4s. 20 = (20/4)*4 gives (20/4)*3 = 15.

6. Change formulas
The unspecified number-formula T = a*x^2 + c*x + d contains basic change-formulas:

- T = c*x; proportionality, linearity
- T = c*x+d; linear formula, change by adding, constant change-number, degree1 polynomial
- T = a*x^2 + c*x + d; parabola-formula, change by acceleration, constant changing change-number, degree2 polynomial
- T = a*b^x; exponential formula, change by multiplying, constant change-percent
- T = a*x^b; power formula, percent-percent change, constant elasticity

7. Use
- Asking '3kg at 5$ per kg gives what?', the answer can be predicted by the formula T = 3 * 5 = 15$.
- Asking '10 years at 5% per year gives what?', the answer can be predicted by the formula T = 105%^10 – 100% = 62.9% = 50% in plain interest plus 12.9% in compound interest.
- Asking 'If an x-change of 1% gives a y-change of 3%, what will an x-change of 7% give?', the answer can be predicted by the approximate formula T = 1.07^3 – 100% = 22.5% = 21% plus 1.5% extra elasticity.
- Asking 'Will 2kg at 3$/kg plus 4kg at 5$/kg total (2+4)kg at (3+5)$/kg?', the answer is 'yes and no'.

The unit-numbers 2 and 4 can be added directly, whereas the per-numbers 3 and 5 must first be multiplied to unit-numbers 2*3 and 4*5 before they can be added as areas. Thus, geometrically per-numbers add by the area below the per-number curve, also called by integral calculus. A piecewise (or local) constant p-curve means adding many area strips, each seen as the change of the area, p*Δx = ΔA, which allows the area to be found from the equation A = Δp/Δx, or A = dp/dx in case of local constancy, called a differential equation since changes are found as differences. We therefore invent d/dx-calculation also called differential calculus.

Geometrically, dy/dx is the local slope of a locally linear y-curve. It can be used to calculate a curve's geometric top or bottom points where the curve and its tangent are horizontal with a zero slope.

8. Conclusion. So, my dear Nephew, Mathematics is a foreign word for calculation, called algebra in Arabic. It allows us to unite and split totals into constant and changing unit- and per-numbers. *Love, your uncle Allan.*

Algebra **unites**/*splits into*	**Changing**	**Constant**
Unit-numbers (meter, second, dollar)	**T = a + b** $T - b = a$	**T = a*b** $\frac{T}{b} = a$
Per-numbers (m/sec, m/100m = %)	**T = ∫ f dx** $\frac{dT}{dx} = f$	**T =** a^b $\sqrt[b]{T} = a$ $\quad$ $\log_a(T) = b$