Can Grounded Mathematics & Education & Research become Relevant to Learners?

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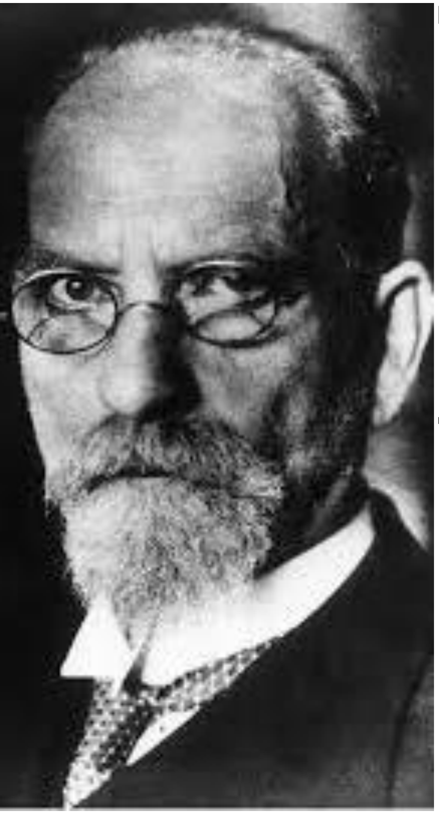
Education, does it work?

In Sweden, OECD reports that it excludes

"… one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life."

http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm



The Freiburger School

Solves an irrelevance paradox and makes Math & Education & Research relevant to learners with phenomenology

Grounding Math: Kids' own Many-Math

Grounding Education: Self-chosen ½year blocks

Grounding Research: **Difference Research**

Creating phenomenology, The Freiburg school looks at INSIDE appearance of OUTSIDE phenomena

Husserl: Not things in themselves, but how they appear matters.

Heidegger: The sentence „**subject IS predicate**“ bridges an OUTSIDE subject with an INSIDE predicate. Trust the OUTSIDE subject, it exists; but question the INSIDE predicate: it may be an institutionalized verdict - that should be doubted and appealed.

- Arendt, looking at the job-situation in a KZ-camp: By its monoploly, an institution forces you to follow orders, which may lead to ‘**the banality of evil**’.
- Schütz, influencing US Pragmatism & Grounded Theory: “Does man’s social being determine his consciousness, or does his consciousness determine his social being?”

Foucault, inspired by Heidegger: Education is a Pris(on)-(hos)pital

„It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them." (Chomsky & Foucault debate).

Sociologically, education is a pris-pital mixing power techniques of a prison and a hospital: The 'pati-mates' are forced to return to the same class day after day where they are treated for a self-referring diagnose (teach MATH to learn MATH) making them accept a Kafkadian verdict: "I am no good."

Derrida: DECONSTRUCT predicates that, by installing what they mention, create **CENTRISM**

Freiburger School questions to math

It is not how universities define it, that matters.
It is how its root, Many, presents itself to humans working to adpat to the outside world.
”Existence precedes essence” (Existentialism)

So:

- How do children experience Many?
- How well-defined is math education after all?

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How well-defined is mathematics?

This statement is true	Always	Never	Sometimes
2 + 3 = 5			
2 x 3 = 6			
1/2 + 2/3 = 7/6			
1/2 + 2/3 = 3/5			

A function is …

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Five Questions Answered

This is true	Always	Never	Sometimes
2 + 3 = 5	Only with the same unit; 2weeks + 3days = 17days		x
2 x 3 = 6	x	2x3 is 2 **3s** that can always be recounted as 6 **1s**	
$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$		1 of 2 apples + 2 of 3 apples gives 3 of 5 apples, and not 7 of 6	x
$\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$			x Only if taken of the same total
a **FUNCTION** is	for example 2+x, but not 2+3		*(pre-setcentric)*
	an example of a many-1 set relation		*(setcentric)*
	a number-language sentence		*(post-setcentric)*

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Post-setcentric math: math through its use, as with the other language in our 2 language houses

The **WORD-language** assigns words in sentences with The **NUMBER-language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

	WORD-language	NUMBER-language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
INSIDE Language	This is a chair	T = 3x4
OUTSIDE World		

Mathematics

Mother tongue

The **Communicative Turn** in language ed.

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: “A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us.”
Likewise, Widdowson adopts a “communicative approach to the teaching of language” allowing more students to learn a language through its use for communication about outside things and actions.

Could mathematics also have its Communicative turn?

	WORD language	NUMBER language
(META-language, grammar)		
Inside Language	This is a chair	T = 3 4s
Outside world		

Mother tongue

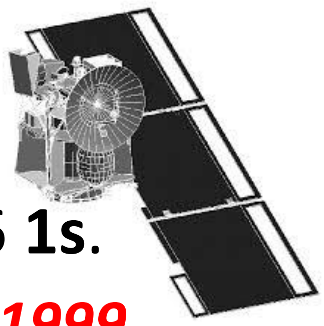
Mathematics

Defining <u>MetaMatism</u> = MetaMatics+MatheMatism

MetaMatics is defining a concept, not as a ~~BottomUp abstraction from many examples~~ but as a <u>TopDown example of one abstraction</u>, derived from the meta-physical abstraction **SET**, made meaningless by self-reference as shown by Russell's version of the liar paradox: M belongs, only if it does not, to the set of sets not belonging to itself:

$$\text{With } \mathbf{M} = \{\mathbf{A} \mid \mathbf{A} \notin \mathbf{A}\}: \quad \mathbf{M} \in \mathbf{M} \Leftrightarrow \mathbf{M} \notin \mathbf{M}$$

MatheMatism is a statement that is correct inside, but seldom outside a classroom , as e.g. adding numbers without units as <u>2+3 = 5</u>, where e.g. 2**w**+3**d**=17**d**. In contrast to <u>2x3 = 6</u> saying that 2 **3s** can be recounted as 6 **1s**.



Neglecting English and metric units made NASA's Mars Climate Orbiter CRASH in 1999.

Education? Two different kinds

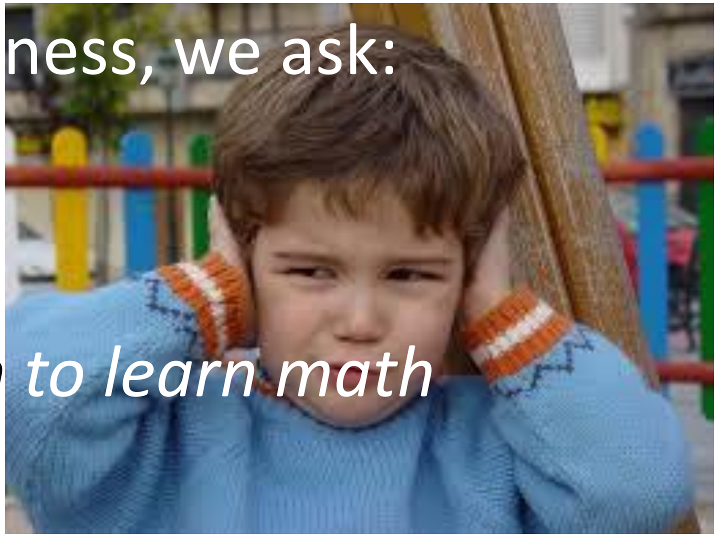
The 1700 Enlightenment Century rooted education, but in different forms in its two republics, in North America in 1776 and in France in 1789.

- In North America, education enlightens children about their OUTSIDE world, and enlightens teenagers about their INSIDE individual talent, to be uncovered and developed through self-chosen ½year **BLOCKS** with teachers teaching only one subject in the teacher's own classroom.
- To protect its republic from its German speaking neighbors, France was forced to create institutions controlled by a strong central administration with public servants trained at elite schools with forced multi-year **LINES**, later copied by the German Bildung-education (and by the rest of Europe).

3x2 different kinds of math education

Mathematics in	**self-chosen ½year BLOCKS**	**forced multi-year LINES**
Pre-SETcentric	North America	UK Commonwealth
Present SETcentric	-	Continental Europe
Post-SETcentric	MATHeCADEMY.net	

Why teach children if they already know?



With education curing the diagnose un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

Core Questions:

- What Mastery does children develop when adapting to Many?
- What could be a Question-guided <u>Child-Grounded-Curriculum</u> in Quantitative Competence?

Children see Many as double-numbers, as bundles with units

“How old next time?” A 3year old says “Four” showing 4 fingers: I I I I

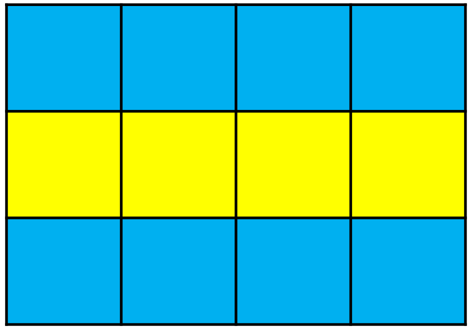
But, the child reacts strongly to 4 fingers held together 2 by 2: II II

“That is not four, that is two twos” (T ≠ 4, T = 2 **2s**)

The child sees what exists, and with units: bundles of **2s**, and 2 of them.

The block 3 **4s** has two numbers:

3 (the counting-number) and **4** (the unit-number)



Exploring children's double-numbers

Tarp, A. (2018). Mastering Many by counting, recounting and double-counting before adding on-top and next-to.
Journal of Mathematics Education, March 2018, 11(1), 103-117.

The Child's Own Mastery of Many
BundleCount & ReCount & DoubleCount
before Adding NextTo & OnTop

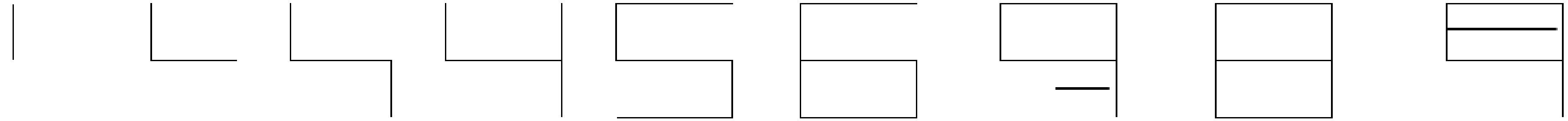
master many
manymath

Q01. Create icons: I I I I → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.

An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become units to use when counting in bundles.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9



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Q02. Bundle-counting sequences

"How to bundle-count fingers?"

Using **5s** as the bundle-size, fingers can be counted as

"0**B**1, 0**B**2, 0**B**3, 0**B**4, 0**B**5 – sorry, **Bundle**"

and the rest can be counted in as

"**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B, 1left, 2left** (a-leven, twe- leven)".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1**B**3 **7s** = 2**B**2 **4s** = 3**B**1 **3s** = 1**BB**1 **3s**.

Q03. Flexible bundle-numbers

"How to count by bundling?"

Five fingers can be bundle-counted in pairs or triplets, allowing both an OVERLOAD and an UNDERLOAD; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

IIIII • ~~II~~ III • ~~II~~ ~~II~~ I • ~~II~~ ~~II~~ ~~II~~ • ~~II~~ ~~II~~ I

T = 5 = 1**Bundle**3 **2s** = 2**B**1 **2s** = 3**B**-1 **2s** = 1**BB**0**B**1 **2s**

T = 5 = 1.3 **2s** = 2.1 **2s** = 3.-1 **2s** = 10.1 **2s**

Likewise, if counting in **ten**-bundles:
T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

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Math Dislike CURED with flexible bundle-numbers

When counting in tens, before calculating, we bundle-write the number to separate the **INSIDE** bundles from the **OUTSIDE** singles. Later we recount.

- **65 + 27** = 6)5 + 2)7 = 8)12 = 9)2 = **92**
- **65 – 27** = 6)5 – 2)7 = 4)-2 = 3)8 = **38**
- **7x 48** = 7x 4)8 = 28)56 = 33)6 = **336**
- **336 /7** = 33)6 /7 = 28)56 /7 = 4)8 = **48**

*With 336 we have 33 **INSIDE**, so to get 28, so we move 5 **OUTSIDE** as 50.*
Now try 456 / 7.

- **456 /7** = 45)6 /7 = 42)36 /7 = 6)5 + 1 = **65 1/7**

Q04. Unbundled as decimals or negatives or fractions

0.3 **4s** or 1.-1 **4s** or 3/4 **4s**

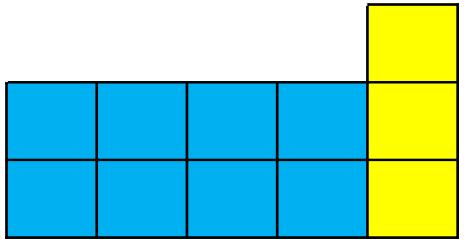
"Where to put the unbundled singles?"

When counting by bundling, the unbundled singles can be placed

NextTo the block

OnTop of the block

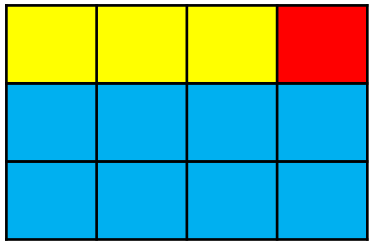
counted as a block of **1s**



T = 2B3 **4s** = 2.3 **4s**

A decimal number

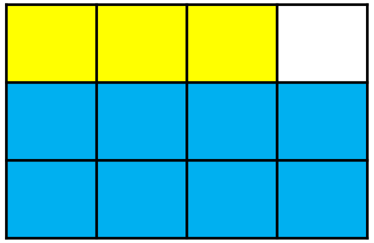
counted as a bundle



T = 3B-1 **4s** = 3.-1 **4s**

A negative number

counted in bundles



T = 2 3/4 **4s**

A fraction

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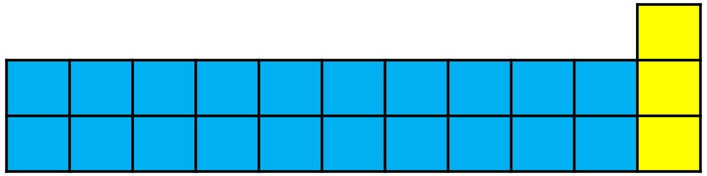
Q04. Counting in tens

"Where to put the unbundled singles with **tens**?"

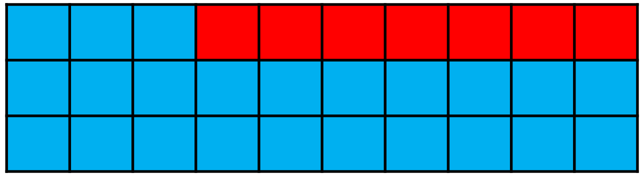
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit and the decimal point,

- or as:



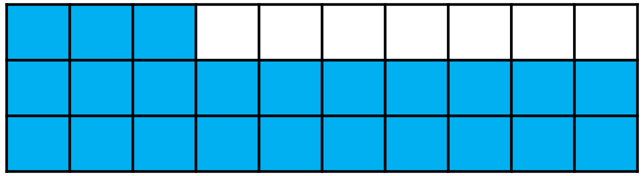
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

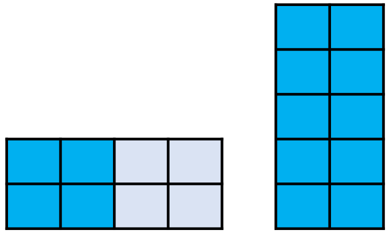
T = 3B-7 **tens**



T = 2 3/10 **tens**

T = 2 3/10 B **tens**

Footnote I: Prime & foldable bundle-units



"When can blocks be folded in like bundles?"

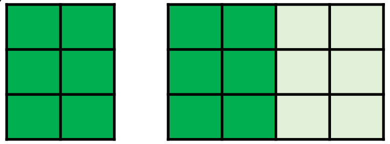
The block T = 2 **4s** = 2 x 4 has 4 as the bundle-unit.

Turning over gives T = 4 **2s** = 4 x 2, now with 2 as the bundle-unit.

4s can be folded in another bundle as 2 **2s**, whereas 2s cannot.

(1 is not a bundle, nor a unit since a bundle-of-bundles stays as 1).

We call 2 a **prime bundle-unit** and 4 a **foldable bundle-unit**, 4 = 2 **2s**.



A block of 3 **2s** cannot be folded.

A block of 3 **4s** can be folded: T = 3 **4s** = 3 x (2 x 2) = (3 x 2) x 2 = 2 **3x2s**.

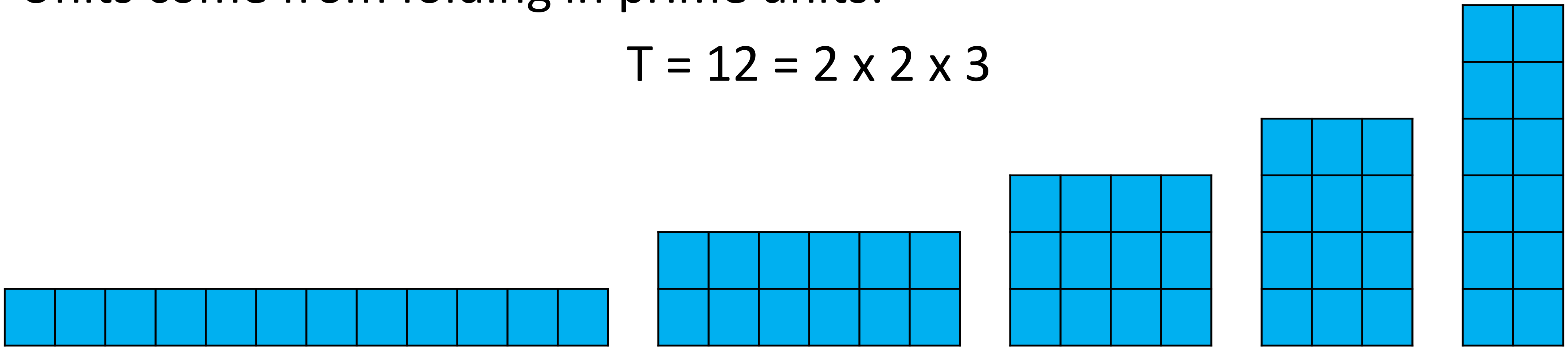
A number is called **even** if it can be written with 2 as the unit, else **odd**.

Footnote II: Finding possible units

“What are possible units in T = 12?”

Units come from folding in prime units:

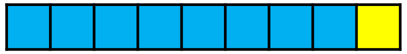
T = 12 = 2 x 2 x 3



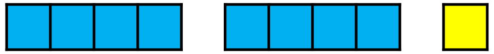
T = 12 **1s** = 2 **2x3s** = 3 **2x2s** = 2x2 **3s** = 2x3 **2s**

Q05. Calculators predict

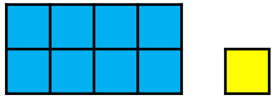
"Can a calculator predict a counting result?"



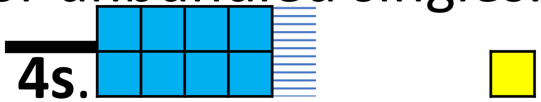
We may see division as an icon for a broom **PUSHING** away bundles:
9/4 means 'from 9, push away bundles of 4s'.



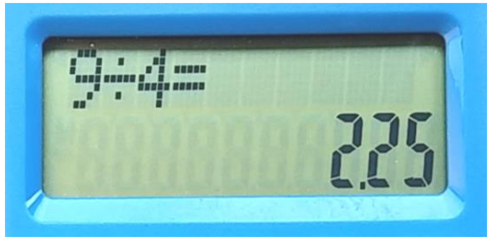
- The calculator says '2.some', thus predicting it can be done 2 times.
  Now multiplication iconizes a lift **STACKING** the bundles into a block.



- And, subtraction iconizes a rope **PULLING** away the block to look for unbundled singles.
- With '9-2x4 = 1' the calculator predicts: 9 can be recounted as 2B1 **4s**.



9/4	**2.some**
9 – 2x4	**1**



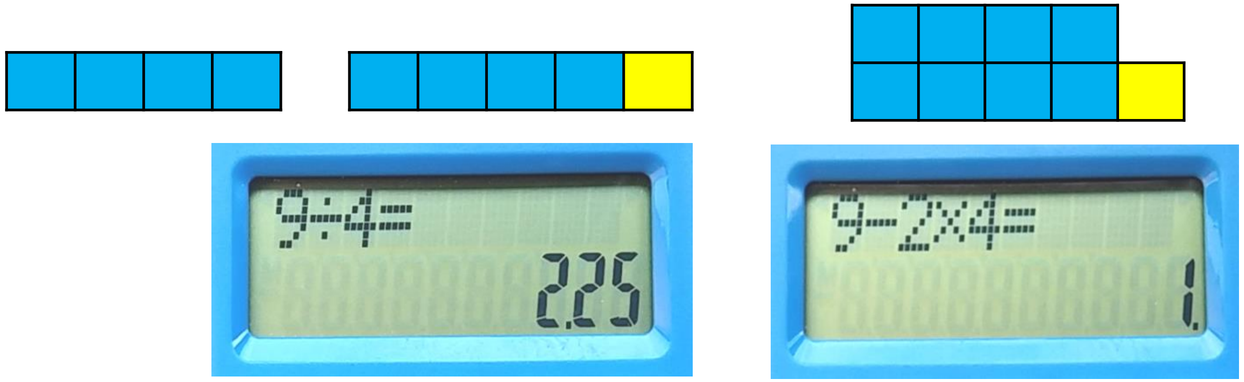
Q05. Counting creates a ReCount-formula

Bundle-counting T pushes away B-bundles to stack T/B times:

T = (T/B) x B	from a total **T**, **T/B** times, we push **B** away

As sentences of the number language, **FORMULAS PREDICT** that **T** = 9 = 2.1 **4s:**

9/4	**2.some**
9 – 2x4	**1**



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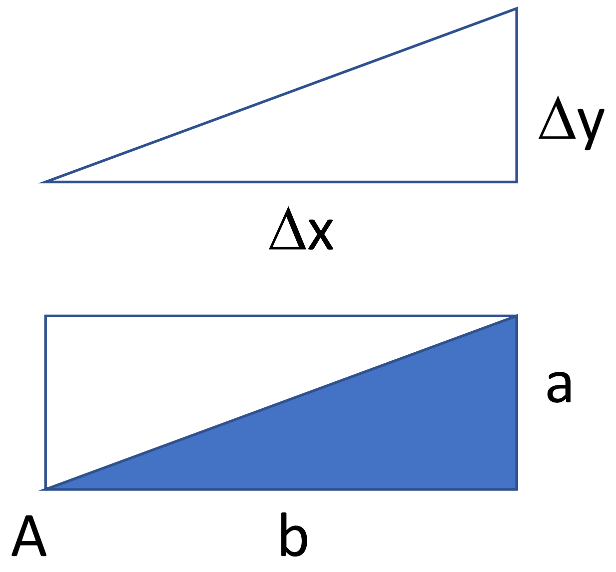
Q05. The recount-formula is a core formula

T = (T/B)*B is all over STEM (Science, Technology, Engineering, Mathematics):

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y’** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second



Footnote I. Recounting in STEM formulas

STEM typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

Footnote II. Recounting in STEM formulas

Extra STEM examples:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec;
- **Δ** energy = (**Δ energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = **(energy/charge)** x **(charge/sec)** or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

Footnote III:
Proportionality shows the diversity of 'School Math'

Proportionality, **Q1**: "2kg costs 5$, what does 7kg cost"; **Q2**: "What does 12$ buy?"

➔1) <u>Regula de Tri</u> (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: '<u>2kg cost 5$, 7kg cost ?$</u>'. Multiply-then-divide gives the $-number 7x5/2 = 17.5.

Q2: '<u>5$ buys 2kg, 12$ buys ?kg</u>'. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

➔2) <u>Find the unit</u>

Q1: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

➔3) <u>Cross multiplication</u>

Q1: 2/5 = 7/*u*, so 2**u* = 7*5, *u* = (7*5)/2 = 17.5. **Q2**: 2/5 = *u*/12, so 5**u* = 12*2, *u* = (12*2)/5 = 4.8

➔4) '<u>Re-counting</u>' in the 'per-number' 2kg/5$ coming from 'double-counting' the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

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Footnote IV: Proportionality shows the diversity of 'School Math'

➔ 5) <u>Modeling</u> with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive \$-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$\$.
- With 12\$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

Q06. Recounting in a different unit

"How to change a unit?"

The recount-formula allows changing the unit.

Asking T = 3 **4s** = ? **5s**, the recount-formula gives T = 3 **4s** = (3x4/5) **5s**.

Entering 3x4/5, the answer '2.some' shows that a block of 2 **5s** can be pushed away.

With 3x4–2x5, the answer '2' shows that 3 **4s** can be recounted as 2B2 **5s** or 2.2 **5s**.

3 **4**s = IIII IIII IIII = IIII I III II II = IIIII IIIII II = 2B2 **5**s = 2.2 **5s**

3x4/5	**2.**some
3x4 – 2x5	**2**

Change Unit = Proportionality

Q07. ReCounting from tens to icons

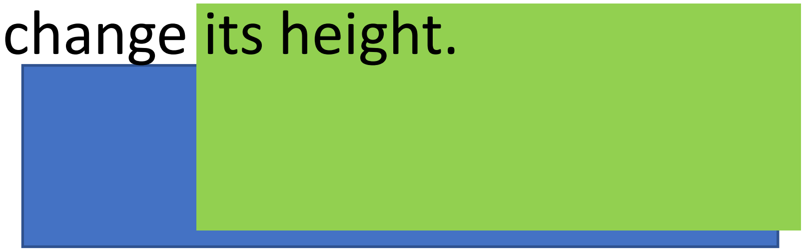
"How to change unit from tens to icons?"

Asking 'T = 2.4 **tens** = 24 = ? **8s**', we just recount 24 in **8s**:

T = 24 = (24/8)x8 = 3x8 = 3 **8s.**

Formulated as an **equation** we use ***u*** for the unknown number, ***u***x8 = 24.
Recounting 24 in 8s shows that ***u*** is 24/8.
So, equations are solved by moving
to opposite side - with opposite sign

To keep its size, a block changing its unit must also change its height.



T = 2.4 **tens** = 3 **8s**

$$u \times 8 = 24 = (24/8)\times 8$$
$$u = 24/8 = 3$$

Q08. ReCounting from icons to tens (multiplication) 3 **7s** = ? **tens**

"How to change unit from icons to tens?"

Asking 'T = 3 **7s** = ? **tens**', the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone: T = 3 **7s** = 3x7 = 21 = 2.1 **tens**, only it leaves out the unit and the decimal point.

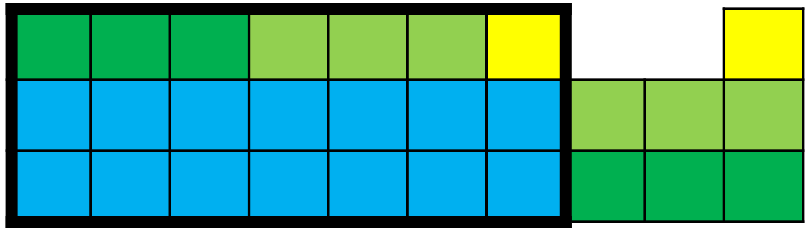
Alternatively, we may use 'less-numbers', so 7 = **ten** less 3

T = 3x7 = 3 x (**ten** less 3) = 3 x **ten** less 3x3 = 3**ten** less 9 = 2**ten**1 = 21,

or with 9 = **ten** less 1:

T = 3**ten** less (**ten** less1) = 2**ten** lessless 1 = 2**ten** & 1 = 21.

showing that 'lessless' cancel out



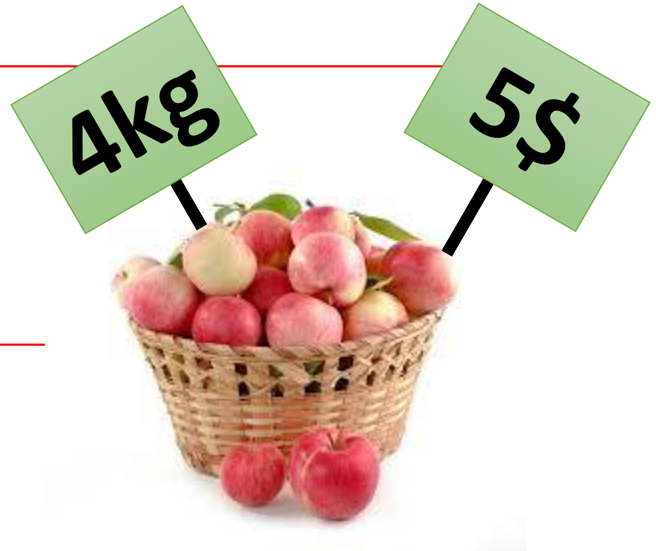
Recounting large numbers in or from tens: *same size, but new form*

Recounting 6 **47s** in **tens** — Recounting 476 in **7s**

*BundleWriting seprates **INSIDE** bundles from **OUTSIDE** singles*

T = 6 x 47 = 6 x 4**B**7	T = 476 = 47.6 **tens**
= 24**B**42	= 47**B**6
= 28**B**2	= 42**B**56
= 28.2	= 6x7**B**8x7
tens	= 68 x 7

Q09. Double-counting in two units creates DoubleNumbers or PerNumbers



"How to double-count in two units?"

DoubleCounting in kg & $, we get **4kg = 5$** or

4kg **per** 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber.

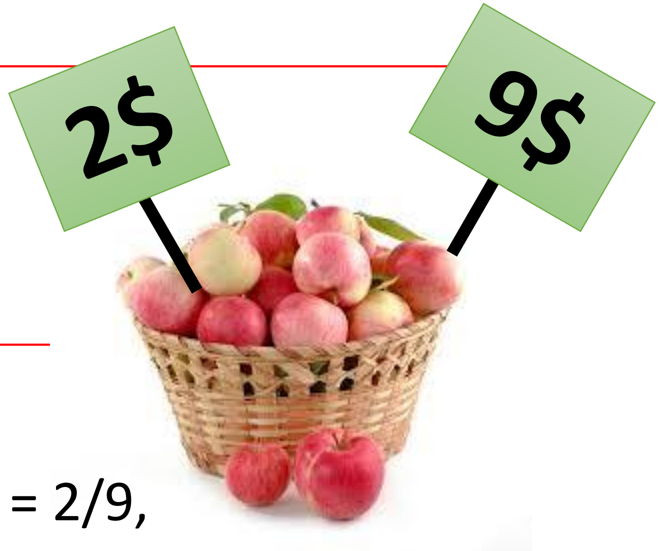
With 4kg bridged to 5$ we answer questions by recounting in the per-number.

Questions:

7kg = ?$	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

Answer: *Recount in the PerNumber (Proportionality)*

Q09. Double-counting in the same unit creates fractions



“How to double-count in the same unit?”

Double-counted in the same unit, per-numbers are fractions, 2$ per 9$ = 2/9,
or percentages, 2 per 100 = 2/100 = 2%.

To find a fraction or a percentage of a total, again we just recount in the per-number.

- **Taking 3 per 4 = taking ? per 100.** With 3 bridged to 4, we recount 100 in 4s:

100 = (100/4)*4 giving (100/4)*3 = 75, and 75 per 100 = 75%.

- **Taking 3 per 4 of 60 gives ?** With 3 bridged to 4, we recount 60 in 4s:

60 = (60/4)*4 giving (60/4)*3 = 45.

- **Taking 20 per 100 of 60 gives ?** With 20 bridged to 100, we recount 60 in 100s:

60 = (60/100)*100 giving (60/100)*20 = 12 .

We observe that per-numbers and fractions are not numbers but OPERATORS needing a number to become a number.

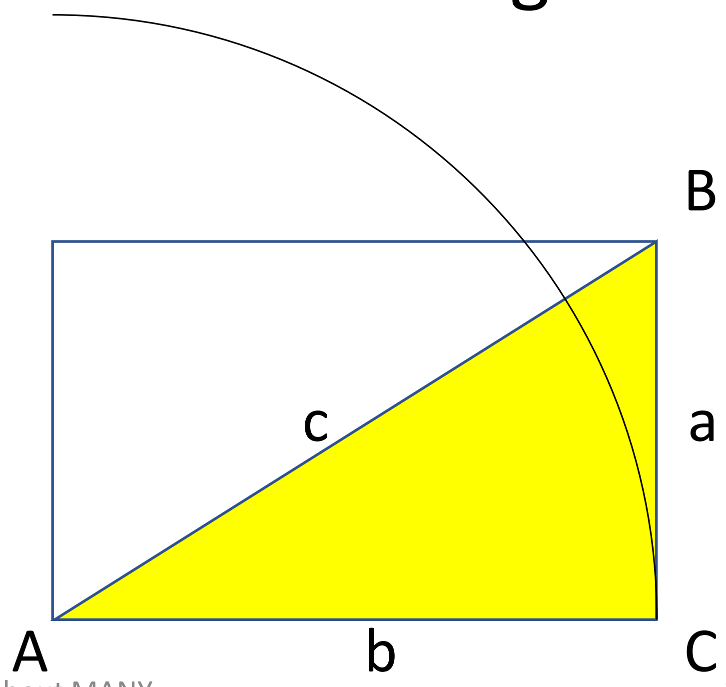
Q10. ReCounting sides in a block: Trigonometry

A right triangle is a block halved by its diagonal giving 3 sides: base b, height a and diagonal c connected with the angles when recounting one side in the other side or in the diagonal

T = (T/B) * B

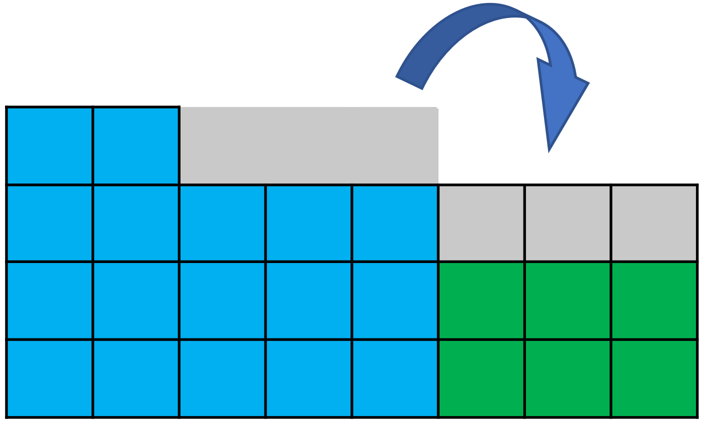
$a = (a/c)*c = \sin A * c$
$b = (b/c)*c = \cos A * c$
$\tan A = a/b = \Delta y/\Delta x = \text{gradient}$

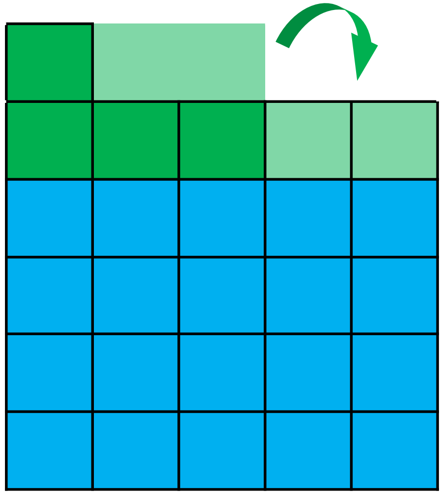
Circle: circum./diam. = $\pi \approx$ **n*tan(180/n)** for n large



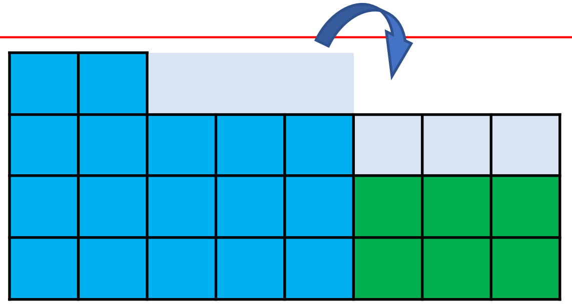
Once bundle-counted & recounted, Totals can add

BUT: NextTo	**or OnTop**
4 5s + 2 3s = 3B2 8s	4 5s + 2 3s = 4 5s + 1B1 5s = 5B1 5s
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





Q11. NextTo addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added next-to as **8s**?"

<u>Outside</u>, next-to addition geometrically means adding areas.
Next-to addition is also called integral calculus.

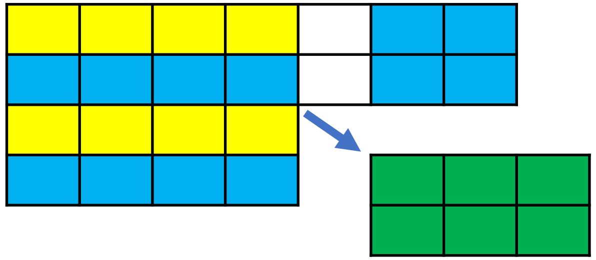
<u>Inside</u>, the recount formula algebraically predicts the result.
Here multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/8) x 8 = 3.2 **8s**

(4x5 + 2x3)/8	**3.some**
(4x5 + 2x3) – 3x8	**2**

Q12. Reversed NextTo addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

Outside, we remove the initial block T1 and recount the rest in **4s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here subtraction precedes division; which is natural as reversed integration.

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

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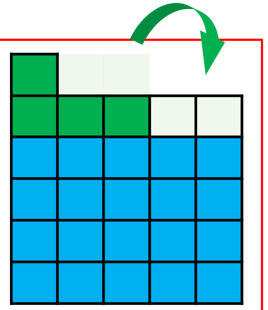
Q13. OnTop addition

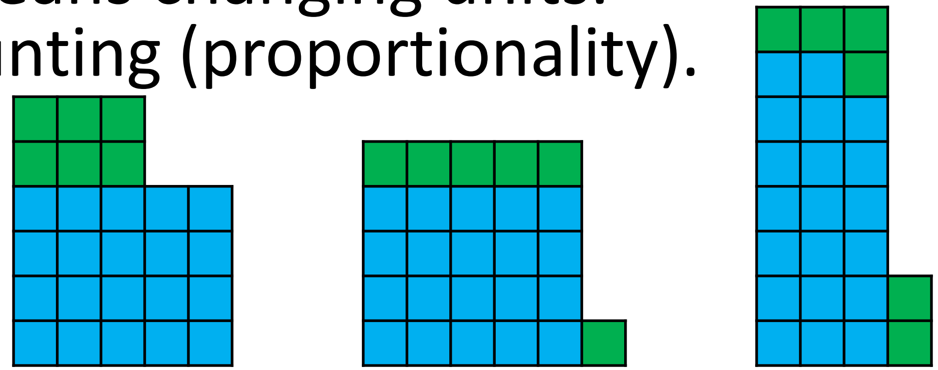


"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added on-top?"

<u>Outside</u>, on-top addition geometrically means changing units.
On-top addition thus often involves recounting (proportionality).

T = 4 **5s** + 2 **3s** = 4 **5s** + 1.1 **5s** = 5.1 **5s**

T = 4 **5s** + 2 **3s** = 6.2 **3s** + 2 **3s** = 8.2 **3s**



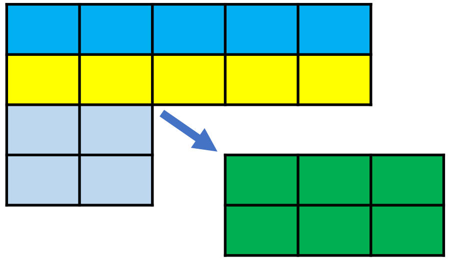
<u>Inside</u>, the recount formula algebraically predicts the result.
Here again, multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/5) x 5 = 5.1 **5s**

(4x5 + 2x3)/5	**5.some**
(4x5 + 2x3) – 5x5	**1**

Q14. Reversed OnTop addition



"T1 = 2 **3s** and how many **5s** (T2) add on-top as 4 **5s**?"

Outside, we remove the initial block T1 and recount the rest in **5s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed on-top addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

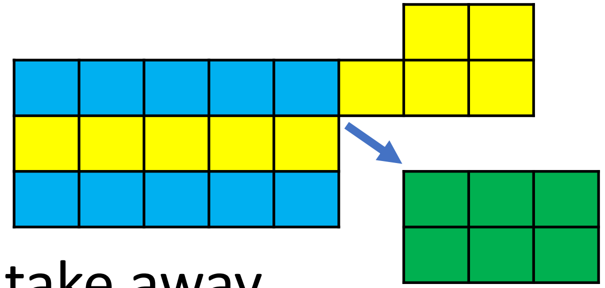
T2 = (T2/**B**) x **B**

= ((4x5 – 2x3)/5) x 5 = 2.4 **5s**

(4x5 – 2x3)/5	**2.some**
(4x5 – 2x3) – 2x5	**4**

Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 8/2 *Solved by ReCounting*	? = 8-2 *Solved by ReStacking*	? = (3.2 **8s** – 2 **3s**)/5 *Solved by differentiation: (T-T1)/5 = $\Delta T/5$*



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

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Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: **O**pposite **S**ide with **O**ppo**S**ite **S**ign

MetaMath (Don't test, but DO remember the bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
⇕	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
⇕	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕	*u* x (2 x (½)) = 4	Applying the **associative** law
⇕	*u* x 1 = 4	Applying the definition of an inverse element
⇕	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

Four ways to unite and split Totals

A number-formula T = 345 = 3**BB**4**B**5 = 3*$\mathbf{B}$^2 + 4*$\mathbf{B}$ + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration).

Addition and multiplication add changing and constant unit-numbers.

Integration and power add changing and constant per-numbers.

We might call this beautiful simplicity the 'Algebra Square'.

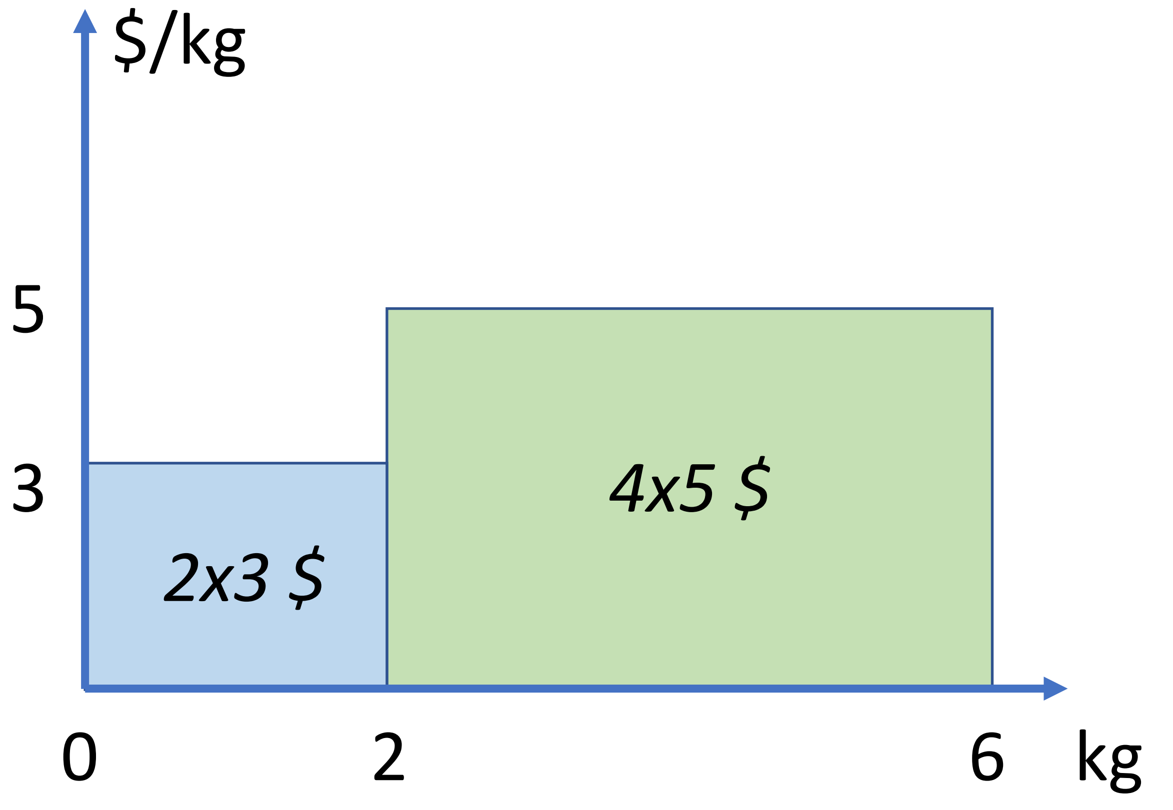
Operations unite/ *split into*	**Changing**	**Constant**
Unit-numbers *m, s, \$, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, \$/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $log_a T = n$, $\sqrt[n]{T} = a$

Q15. Adding PerNumbers as areas (integration)

"2kg at 3$/kg + 4kg at 5$/kg = 6kg at ? $/kg?"

	2 kg	at	3 $/kg
+	4 kg	at	5 $/kg
	(2+4) kg	at	? $/kg

- Unit-numbers add on-top.
- Per-numbers add next-to as **areas** under the per-number graph. Here multiplication precedes addition.



Q16. Subtracting PerNumbers (differentiation)

"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

2 kg	at	3 $/kg	
+ 4 kg	at	? $/kg	
6 kg	at	5 $/kg	

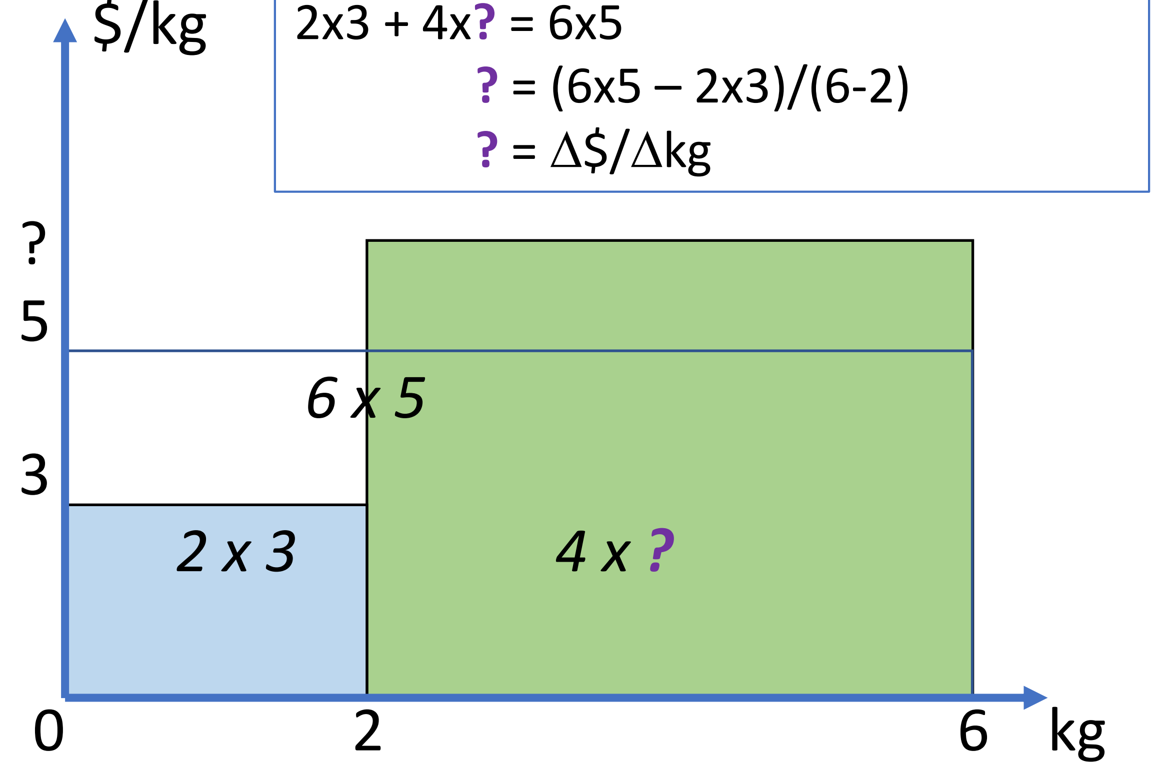
Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) precedes division.

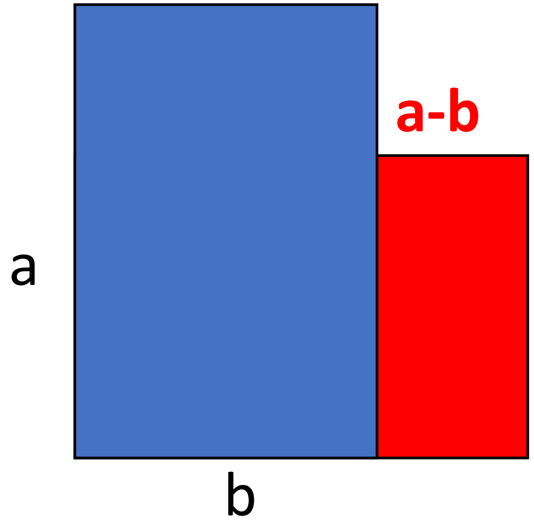
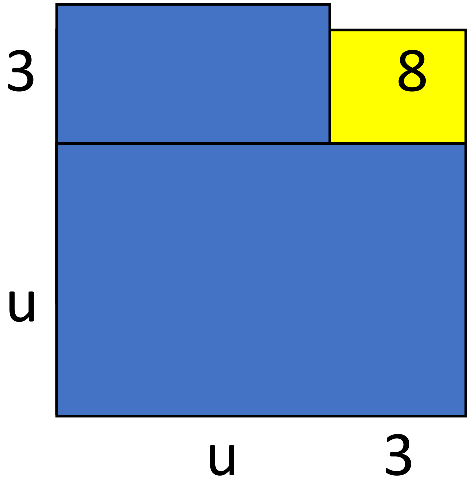
2x3 + 4x? = 6x5

? = (6x5 – 2x3)/(6-2)

? = Δ\$/Δkg



Q17. Geometry & Algebra, hand in hand

Quadratic Rule with 2 Cards	**Quadratic Equations with 3 Cards**
	**u^2 + 6u + 8 = 0** 
Corner = (a-b)^2 = a^2 – 2 cards + b^2 So (a-b)^2 = a^2 – 2 x a x b + b^2	(u+3)^2 = u^2 + 6u + 8 + 1 (u+3)^2 = 0 + 1 u = -3 ± 1 **u = -4 & u = -2**

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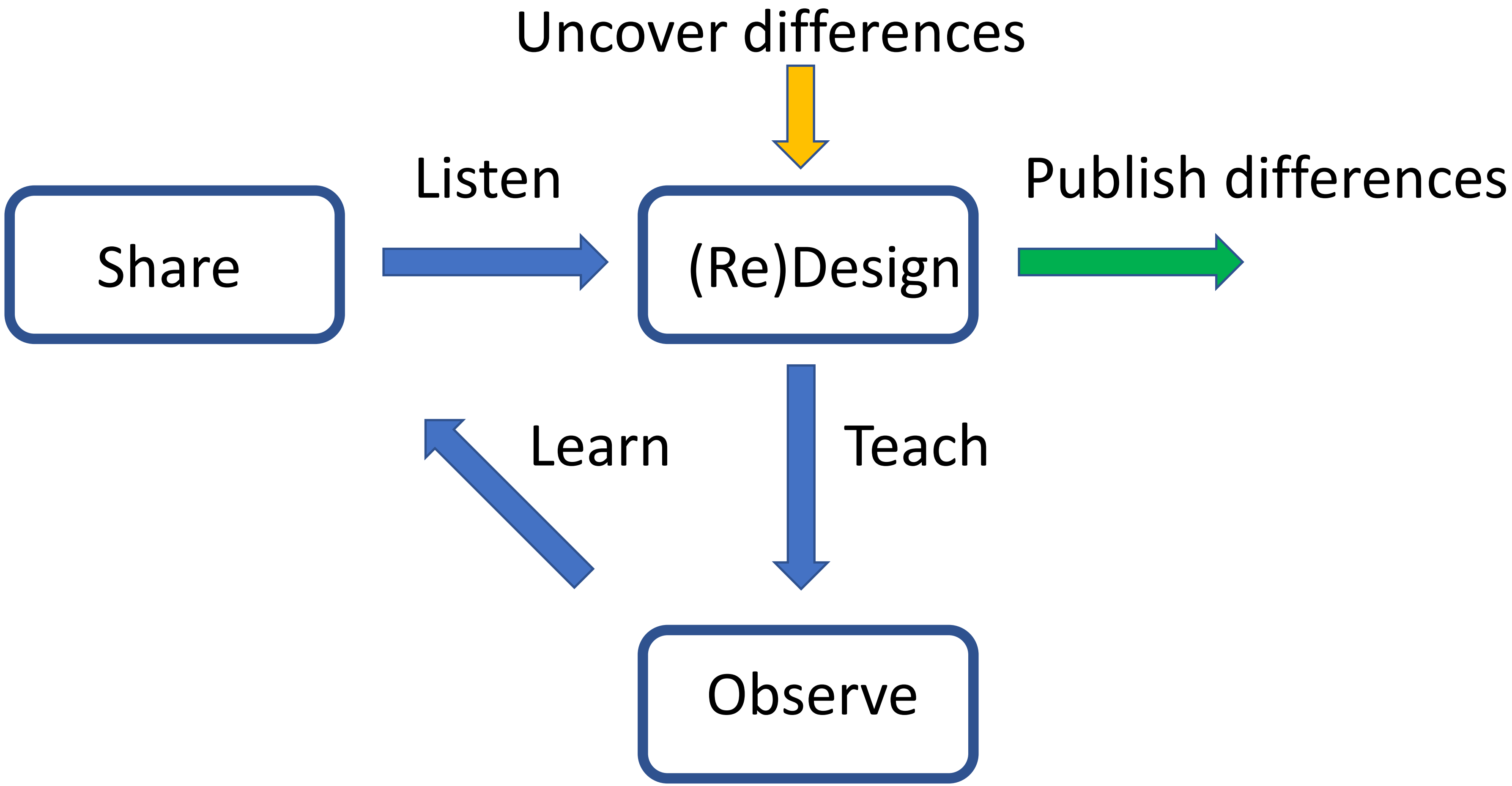
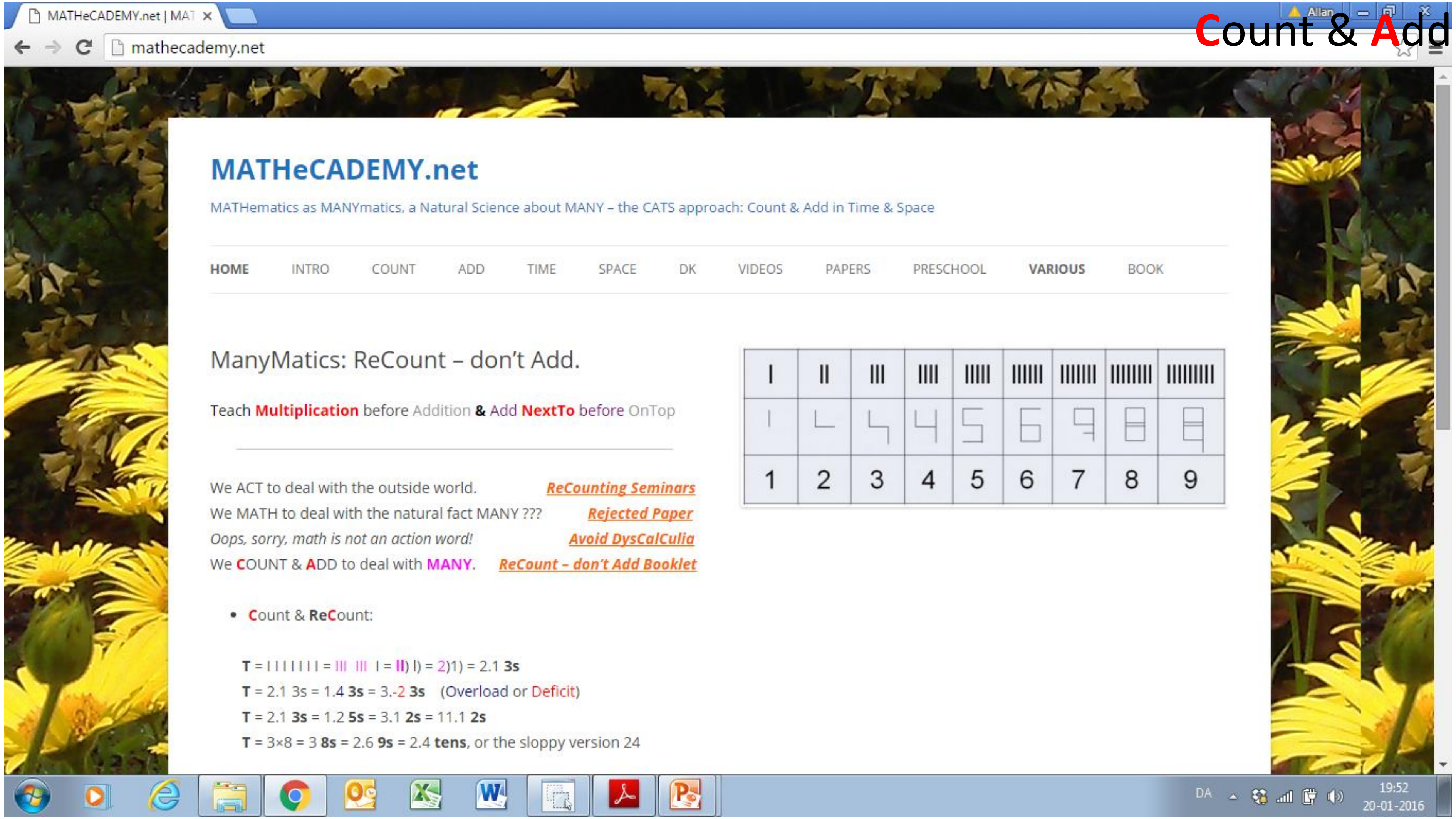
Difference-research finds differences making a difference
Action Learning & Action Research



MATHeCDMY

Inspiration from MATHeCADEMY.net

Teaches Teachers to Teach
MATHEmatics as MANYmatics,
a Natural Science about MANY.
The CATS method: To learn Math
Count & Add in Time & Space



MATHeCDMY

8 MicroCurricula for Action Learning & Research

C1. Create Icons

C2. Count in Icons (Rational Numbers)

C3. ReCount in the Same Icon (Negative Numbers)

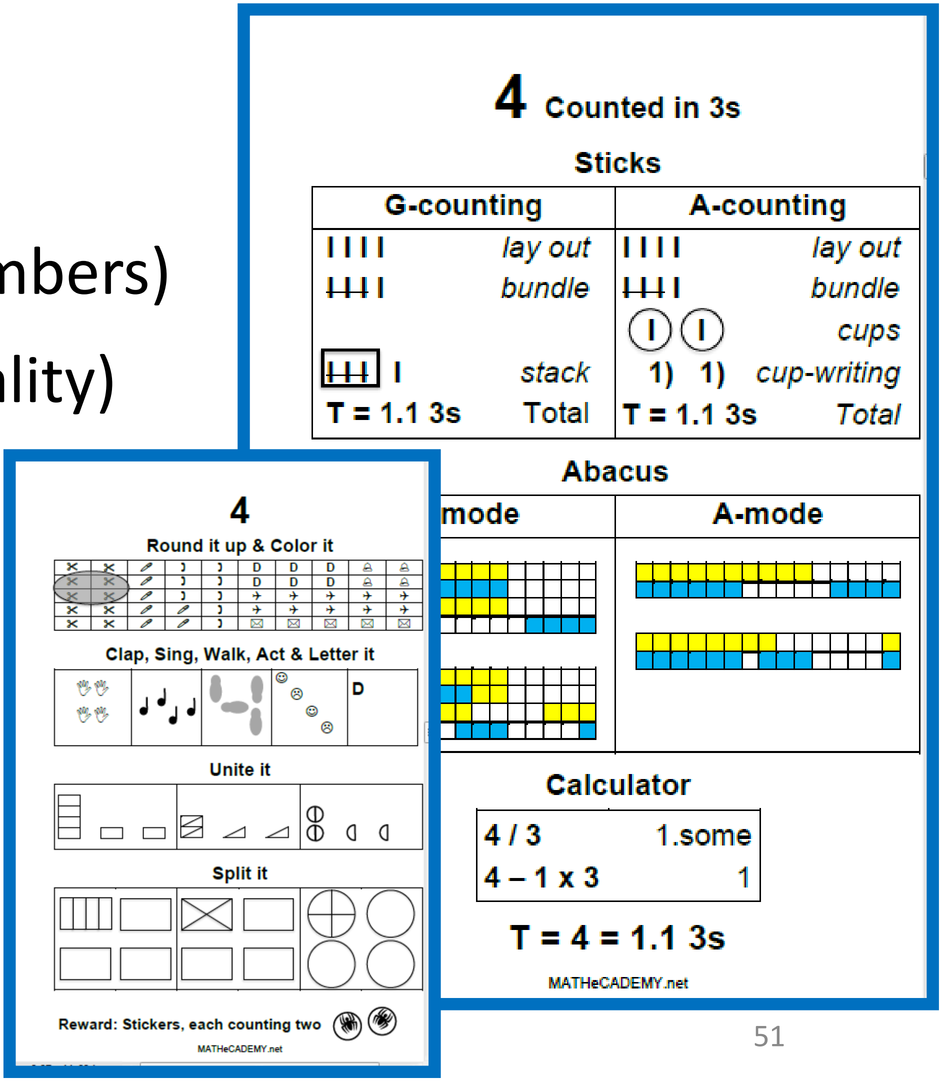
C4. ReCount in a Different Icon (Proportionality)

A1. Add OnTop (Proportionality)

A2. Add NextTo (Integrate)

A3. Reverse Adding OnTop (Solve Equations)

A4. Reverse Adding NextTo (Differentiate)



ReCount – don't Add Booklet, free to Download

ReCount

don't Add

MatheMatics as ManyMatics
for NewComers & LateComers & Migrants
to Avoid DysCalCulia

The Direct Way to Core Mathematics:
Proportionality & Fractions & Calculus & Solving Equations

Allan.Tarp
MATHeCADEMY.net

Contents

03. ReCounting in Icons

Q?		Do	Calculator	
9 in 5s	Line	T = I I I I I I I I I		
	Count	1, 2, 3, 4, B, 1B1, 1B2, 1B3, **1B4**		
	Bundle	T = ~~IIIII~~ I I I I	9/5	1.some
	Stack		9 – 1*5	4
	Cup	T = 1)4		
	Answer	T = 9 = 1.4 5s		
9 in 4s	Line	T = I I I I I I I I I		
	Count	1, 2, 3, B, 1B1, 1B2, 1B3, 2B, **2B1**		
	Bundle	T = ~~IIII~~ ~~IIII~~ I	9/4	2.some
	Cup	T = 2)1	9 – 2*4	1
	Stack			
	Answer	T = 9 = 2.1 4s		
9 in 3s	Line			
	Count			
	Bundle		9/	
	Cup		9 –	
	Stack			
	Answer			
8 in 4s	Line			
	Count			
	Bundle		8	
	Cup		8	
	Stack			
	Answer			
8 in 3s	Line			
	Count			
	Bundle		8	
	Cup		8	
	Stack			
	Answer			

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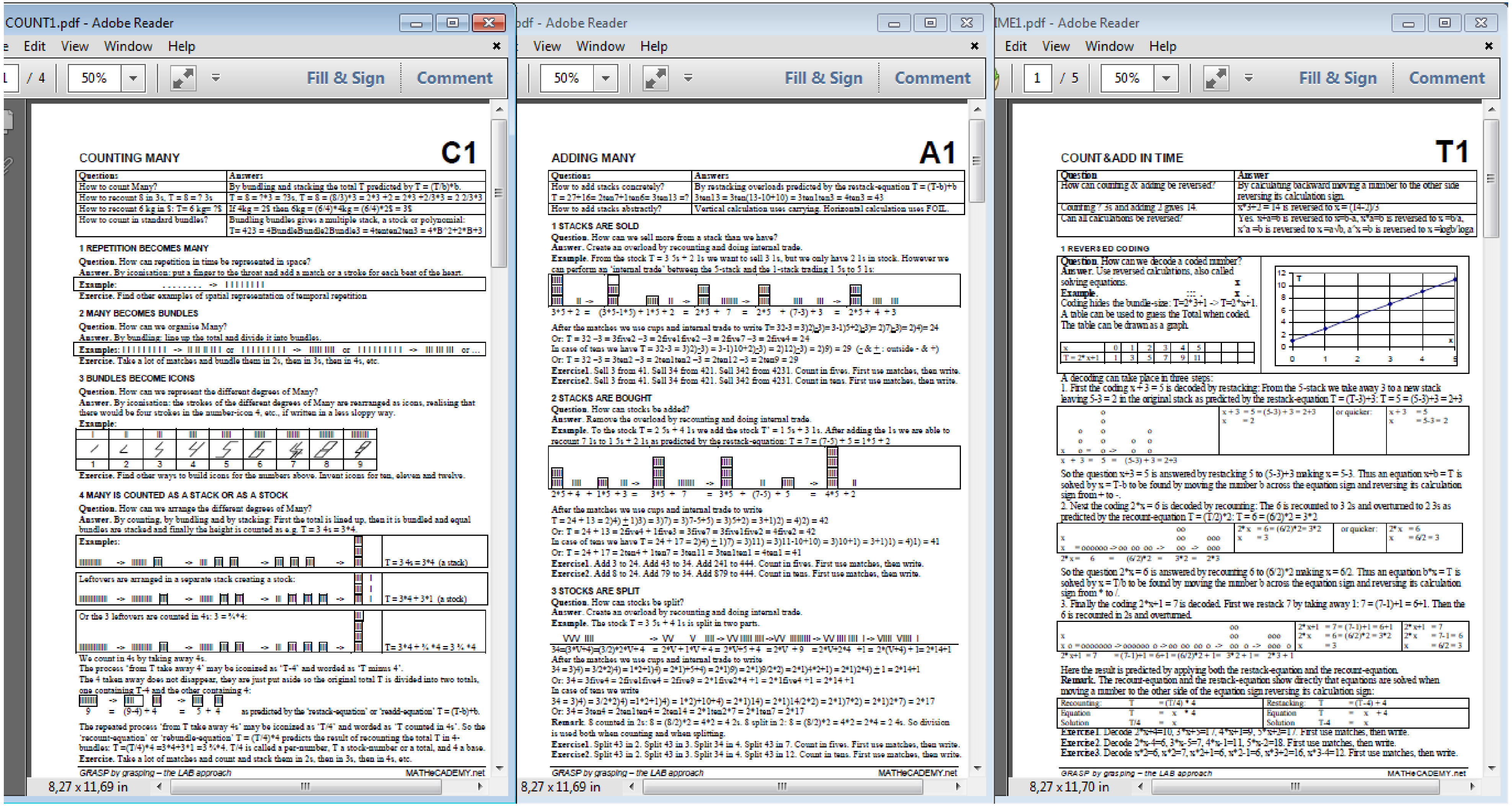
Teacher Training in CATS ManyMatics



1day Skype Seminar: To avoid Math Dislike,
ReCount in flexible BundleNumbers

Action Learning on the child's own 2D NumberLanguage as observed when showing 4 fingers together 2 by 2 makes a 3-year-old child say 'No, that is not 4, that is 2 2s.'

09-11. Listening and Discussing: Good & Bad & Evil MatheMatics
Bad MatheMatism is true inside but rarely outside classrooms.
Evil MetaMatics presents a concept TopDown as an example instead of BottomUp as an abstraction.
Good ManyMatics, a natural science Many mastering Many by ReCounting & adding OnTop/NextTo.
2D Bundle-Numbers with units as a hidden alternative to the traditional 1D Line Numbers without
Adding 1D Line Numbers without units may create Math Dislike.

11-13. Skype Conference. Lunch.

13-15. Doing: Trying out the 'ReCount – don't Add' booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in ReCounting and NextTo Addition.

15-16. Coffee. Skype Conference.

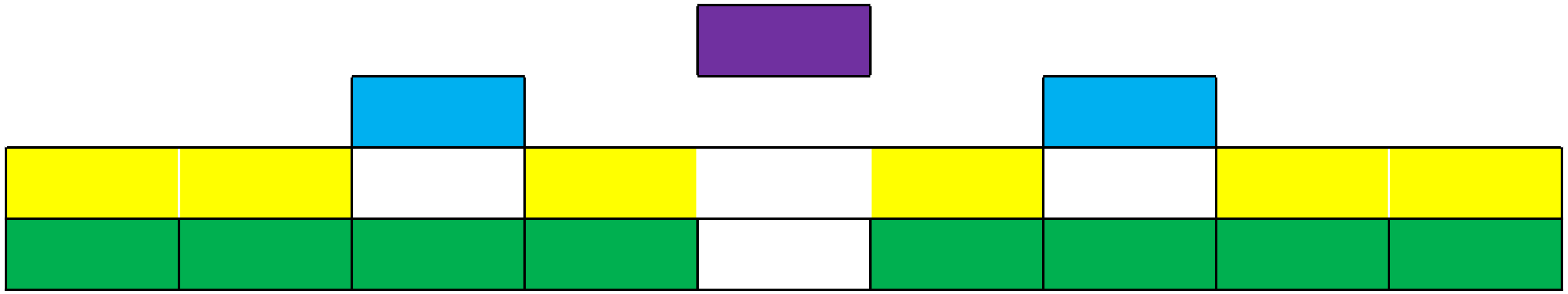
PYRAMIDeDUCATION

To learn MATH:
Count&Add MANY
But ReCount before you Add

In PYRAMIDeDUCATION a group of 8 learners are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

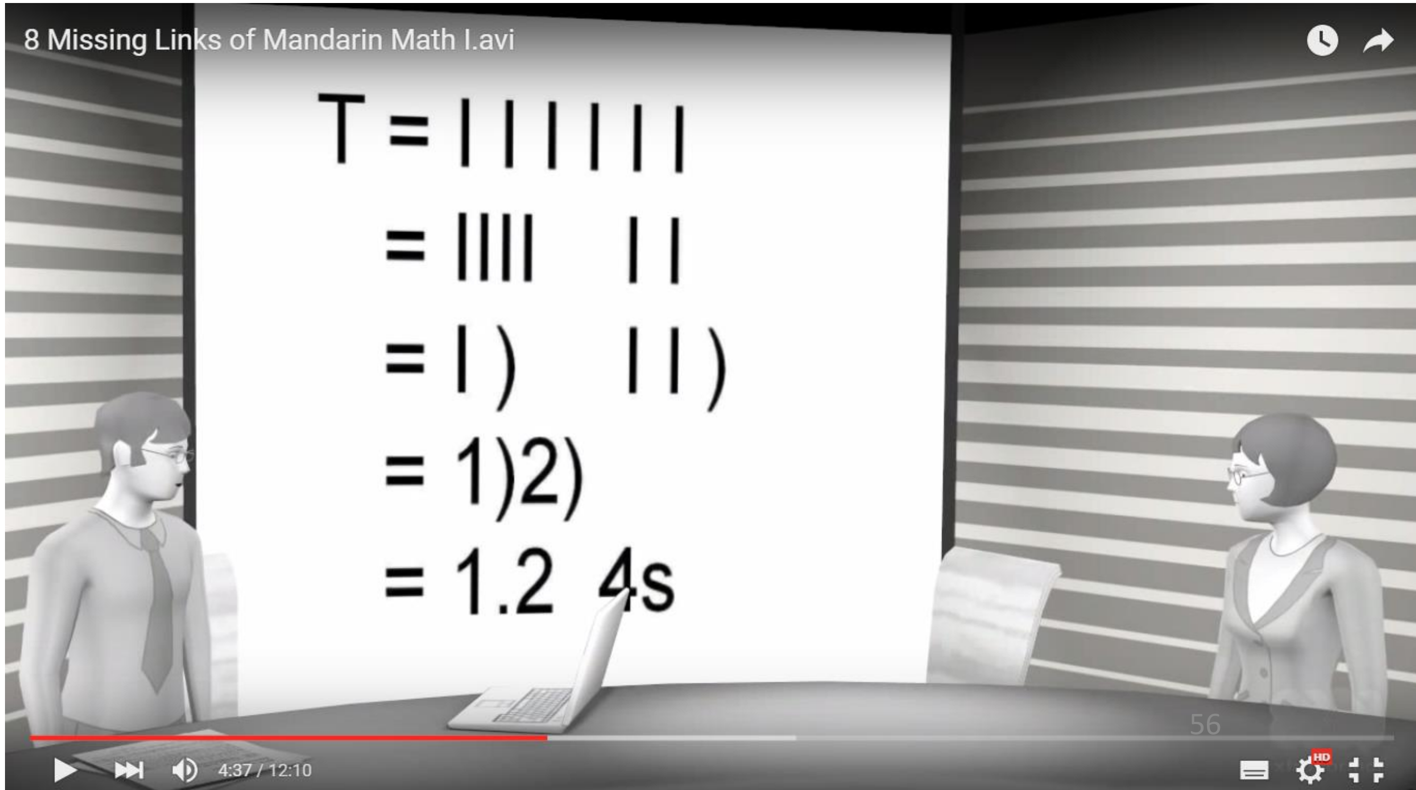
- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each learner pays by coaching a new group of 8 learners.

1 Coach
2 Instructors
3 Pairs
2 Teams



Watch MrAlTarp YouTube Videos

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



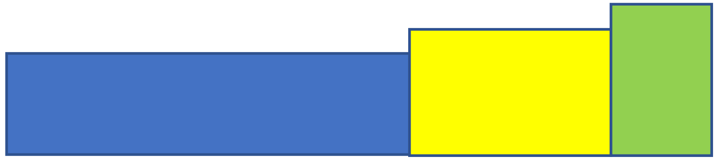
Conclusion

What Mastery of Many does the child have already?

- Children typically see Many as blocks with a number af bundles, and use flexible numbers with units and with over- or underloads

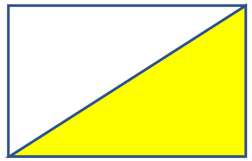
In ManyMath, BLOCKS are fundamental:

- in numbers: 456 = three blocks

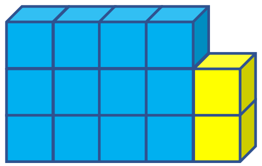
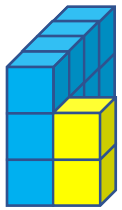


- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks



The child's own Many-math curriculum

1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving <u>outside</u> geometrical multi-blocks, & (when turned to hide the units behind) <u>inside</u> algebraic bundle-numbers.

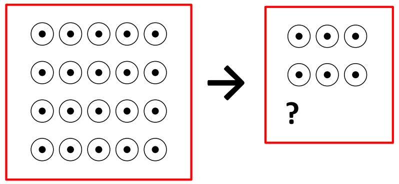
3) Operations are icons, showing the 3 counting steps: Removing bundles & stacking bundles & removing stacks to find the unbundled.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

Question: T = 4 ***5s*** *= ?* ***3s*** ● *Answer: T = 4* ***5s*** *= 6B2* ***3s*** ● *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



A final footnote from the Frankfurter School

- With Habermas we may ask: How can a math teacher use communicative rationality to establish a non-patronizing power-free rational dialogue with grade one children about the objective fact Many, present in both the children and the teacher’s life-world; thus accepting four fingers held together two by two being rationalized as (as do the children) ‘the total I two twos’ and not just as ‘four’?
- With Marx quoted in the Berlin Humboldt University we may agree:

Recommendation from the Freiburger School

STOP teaching and researching wrong numbers and wrong operations

- **START** to accept and develop the child's own flexible bundle-numbers and (re)counting operations

STOP using education as pris-pitals, fixing and diagnosing humans

- **START** guiding the child when exploring its outside world, and the teenager when exploring the inside self via self-chosen ½year blocks

STOP forcing the outside world to adapt to inside rigid theory

- **START** forcing inside theory to adapt flexibly to outside existence