Self-explanatory Learning Material to Improve your Mastery of Many

Allan Tarp

MATHeCADEMY.net
Allan.Tarp@gmail.com

TOPIC 02, Research-based poster

1. Abstract

This poster illustrates a hands-on experience with educating math educators from the child's perspective, relating to the topic 2 question 4 about designing innovative self-explanatory material that has large potentials for scaling-up.

It is based on the observation that when asked 'How old next time?', a 3year old will say 4 showing 4 fingers; but will protest when held together two by two by saying 'That is not 4. That is 2 2s', thus rejecting the predication 'four' by insisting on describing what exists, bundles of 2s and 2 of them. Meeting Many, children develop a number-language with full sentences including a subject and a verb and a predicate as in the word-language, as well as 2-dimensional block-numbers with units, neglected by the school's 1-dimensional line-names, making some children count-over by saying 'twenty-ten'. So, the goal of the workshop is to inquire into the mastery of Many children bring to school to see what kind of mathematics occur if allowing the children to develop their already existing quantitative competence under proper guidance.

2. Digits and operations as icons bridging inside signs and outside existence

Matches and a folding ruler show that digits are, not symbols as the alphabet, but sloppy writings of icons having in them as many sticks as they represent: five sticks in the 5-icon, etc.

Operations also are inside icons reflecting outside actions: a division broom pushes away bundles, to be stacked by a multiplication lift, to be pulled by a subtraction rope to identify unbundles ones, to be placed next-to the stack as decimals, or on-top as fractions or negatives, predicted by a 'recount-formula', $T = (T/B)*B$, saying 'from T, T/B times, B is taken away'.

3. Bundle-counting fingers roots negative numbers and polynomials

To emphasize bundles, the fingers may be bundle-counted as: 0Bundle1, 0*B*2, 0*B*3, 0*B*4, ½*B*, 0*B*6, 0*B*7, and then 0*B*8, 0*B*9, 1*B*0; or 1*B* less2, 1*B*-1, 1*B*0, continuing with 'Viking-counting' one-left (eleven), two left (twelve), and finally BundleBundle as 100. Two-digit numbers are named by their two neighbours: *T* = 68 = 6*B*8 tens = 7*B*-2 tens = 6ten8 = 7ten-2.

Counting ten fingers in 3s introduces bundles of bundles: *T* = ten = 3*B*1 3s = 1*BB*1 3s, leading on to the general number-formula or polynomial *T* = ten = 1**B*^2 + 0**B* + 1*1 3s. Likewise counting in tens, *T* = 345 = 3**BB* + 4**B* + 5*1 = 3**B*^2 + 4**B* + 5*1, showing the four ways to unite numbers (the Arabic meaning of Algebra): on-top addition, multiplication, power and next-to block-addition called integration, all with reverse splitting operations: subtraction, division, factor-finding (root), factor-counting (logarithm), and differentiation.

4. Block-counting cubes roots decimals, fractions and negative numbers

Block-counting 8 cubes in 5s gives 1 5s and 3 unbundled 1s as predicted: *T* = 8 = (8/5)*5 = 1*5 & 3. Placing the 3 1s after the 1 5s roots decimal-writing, *T* = 1.3 5s = 2.-2 5s. Placing the unbundled instead on-top of the block of bundles roots fractions and decimal numbers, *T* = 8 = 1 3/5 5s = 2 -2/5 5s = 2 5s less 2. Counting in tens, *T* = 68 = 6 8/10 tens = 6.8 tens = 7.-2 tens.

5. Recounting roots flexible numbers and proportionality and per-numbers

Recounting in the same unit creates flexible numbers: T = 68 = 6.8 tens = 7.-2 tens

Recounting in another unit by asking e.g. 'T = 3 4s = ? 5s', the recount-formula allows a calculator to predict the answer. Entering 3*4/5, the answer '2.some' shows that a stack of 2 5s can be taken away. Entering 3*4 – 2*5, the answer '2' shows that 3 4s recounts in 5s as 2*B*2 5s or 2.2 5s. Counting 3 in 5s gives a fraction: T = 3 = (3/5)*5. Recounting in physical units creates 'per-numbers' as e.g. 2$ per 3kg, or 2$/3kg, bridging the units by recounting in the per-number: Asking '6$ = ?kg', we recount 6 in 2s: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg; and vice versa.

6. Recounting from tens and to tens roots equations and multiplication tables

Recounting from tens to icons by asking e.g. 'T = 2.4 tens = 24 = ? 8s' becomes an equation, u*8 = 24, that is easily solved by recounting 24 in 8s as 24 = (24/8)*8 so that the unknown number is u = 24/8, attained by moving 8 to the opposite side with the opposite calculation sign.

Recounting from icons to tens by asking e.g. 'T = 3 7s = ? tens' we notice that with no ten-button on a calculator, the recount-formula cannot predict the answer. But, it is programmed to give the answer directly by using multiplication alone: T = 3 7s = 3*7 = 21 = 2.1 tens, only it leaves out the unit and misplaces the decimal point. The multiplication tables may use 'less-numbers', geometrically on an abacus, or algebraically with brackets: T = 3*7 = 3 * (ten, less 3) = 3 * ten, less 3*3 = 3ten, less 9 = 3ten, less (ten less1) = 2ten, less less 1 = 2ten & 1 = 21. And, 7*9 = (ten, less 3)*(ten, less 1) = ten ten, less 3ten, less 1ten, lessless3 = 6ten & 3 = 63.

7. Recounting is exemplified in STEM-formulas

STEM contains multiplication formulas with per-numbers: meter = (meter/sec)*sec = velocity*sec, kg = (kg/cubic-meter)*cubic-meter = density*cubic-meter; force = (force/square-meter)*square-meter = pressure*square-meter; energy = (energy/sec)*sec = Watt*sec; energy = (energy/kg)*kg = heat * kg. Lego-bricks: number = (number/meter)*meter = density*meter.

8. Recounting sides in a block halved by its diagonal roots angles, trigonometry and pi

Recounting a block with base b and height a, halved by its diagonal c, creates per-numbers: $a = (a/c)*c = \sin A*c$; $b = (b/c)*c = \cos A*c$; $a = (a/b)*b = \tan A*b$; and $\pi \approx n*\sin(180/n)$.

9. Adding totals on-top and its reverse roots proportionality and differential calculus

Adding 2 3s and 4 5s on-top, the units must be harmonized by recounting. Adding next-to means adding areas, called integral calculus; as when adding per-numbers and fraction that must change to unit-numbers by multiplication, thus creating areas to be added.

Reversing addition by asking e.g. '2 3s and ? 5s total 4 5s or 2 8s' will become equations, 2*3 + u*5 = 4*5 or 2*3 + u*5 = 2*8, solved by moving to opposite side with opposite sign.

10. Grand theory holds conflicting conceptions on concepts

Within philosophy, Platonism and Existentialism argue if concepts are examples of abstractions or abstractions from examples. Within psychology, Vygotsky and Piaget argue if concepts are constructions mediated socially or experienced individually. Within sociology, the agent-structure debate is about establishing inclusion by accepting the agent's own concepts or establishing exclusion by insisting on teaching and learning institutionalized concepts.

11. References

Tarp, A, 2018, "Mastering Many", Journal of Mathematics Education, vol 11(1), pp. 103-117.