Math Ed & Research 2019

Power Point Presentations

Addition-free STEM-based Math for Migrants

The Child's Own Mastery of Many

Making Grounded Math, Education & Research Relevant to Learners

De-Model Numbers, Operations and Equations

Allan.Tarp@MATHeCADEMY.net

Contents

Preface

● The texts 01 and 02 concern the ICMI Study 24, School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities, held in Tsukuba, Japan, 26-30 November 2018. My paper, ‘A Twin Curriculum Since Contemporary Mathematics May Block the Road to its Educational Goal, Mastery of Many’, is included in the 2018 Articles. The discussion document theme B, “Analysing school mathematics curriculum reforms for coherence and relevance” had five sub-questions, and I was asked to contribute to writing a chapter addressing key-question B2, “How are mathematics content and pedagogical approaches in reforms determined for different groups of students (for e.g. in different curriculum levels or tracks) and by whom? How do curriculum reforms establish new structures in content, stakeholders (e.g. students and teachers), and school organisations; and what are their effects?”, in short called mathematics for all.

The first outline was scheduled to February 15, 2019. Having almost finished the outline, on February 12 we received a mail saying “The deadline for the chapter outlines has been extended until at least the end of February (from 15 February). I would like to open discussion between us on several matters.” On February 20 I sent in my response (see 02).

I never got any reaction so on March 30 I sent a mail to the organizing committee saying “To stimulate our work, would it be an idea to send out a monthly or quarterly newsletter reporting on the progress and challenges being made and met?” No response came so I began writing a proposal for a contribution (se 01). Then on May 10 I got a mail saying “There will not be a separate chapter on key question 2”, to which I responded “I think that the question ‘math for all’ as focused on in the question B2 is so important that it deserves an answer. When mentioned at the conference that no paper addressed this I objected since my paper is addressing the question if it with a different way of organizing math education will be able to include all. Furthermore, I have written a first draft that I send on February 12. Moreover, I have collected a substantial amount of material to include, just waiting for an answer to my mail on February 20 and March 28. So I am going to write a chapter with the focus originally decided upon since I think the research question deserves an answer as mentioned above. I will send it to you as planned before at the end of June. You might then decide not to include it, it will be your choice, then I will publish it elsewhere since the question is very important and since the material, I have collected shows that it is indeed possible to have mathematics for all in different ways.”

I then sent in my proposal before July 1 but heard nothing then or after the time limit for a reaction on August 15. So apparently my contribution will be the chapter that was commissioned and rejected without even being read.

01. The same Mathematics curriculum for different students.

The paper has the following chapters: 01. A need for curricula for all students 02. Addressing the need 03. Coherence and relevance 04. Parallel tracks to the main curriculum, examples 05. Pre-calculus, typically the last mandatory curriculum 06. Precalculus in the Danish parallel high school, a case study 07. A refugee camp curriculum 08. Do we really need parallel curricula 09. Conclusion.

02. Comments to a discussion paper

● At the Genoa University April 8-11, Paolo Boero held an international workshop called ‘Habermas’ elaboration on rationality and mathematics education’ over the Habermas text ‘Some further clarifications of the concept of communicative rationality’. I was allowed to give a short presentation.

03. A Mathematics Teacher Using Communicative Rationality Towards Children

● At Beijing Normal University June 28-30 2019, the 2019 Classroom Teaching Research for All Students Conference (CTRAS) took place with the conference theme ‘innovative practices and research initiatives in STEM integration that supports all students’ mathematics learning (..) The conference will provide participants from around the world with the opportunity to share: 1) best practices of STEM integration; 2) the most contemporary STEM research initiatives; 3) innovative curriculum of STEM integration; and 4) professional development approaches for STEM educators.' I contributed with a paper with a power point presentation, a proposal for a short presentation, a poster and a workshop, 04-08

● At Paderborn University September 16-19 the third international conference on mathematics textbooks research and development, icmt3, will take place. Invitations were sent out to contribute with oral communications, workshops, posters, papers and symposia. I sent in one oral presentation (09), one paper (14), one workshop (11) and three posters (10, 12, 13). All were rejected except for the paper that was accepted for a ten minutes oral presentation.

09. Developing the Child's Own Mastery of Many, oral presentation

10. Math Dislike Cured with Inside-Outside Deconstruction, poster

11. Learning from The Child's Own Mathematics, workshop

12. Five Alternative Ways to Teach Proportionality, poster

13. New Textbooks, but for Which of the 3x2 Kinds of Mathematics Education, poster

14. Developing the Child's Own Mastery of Many, paper. The abstract says: Sociological imagination sees continuing educational problems as possibly caused by a goal displacement making mathematics see itself as the goal instead of its outside root, mastery of Many. Typically, the number-language is taught inside-inside as examples of its meta-language. However, as the word-language, it can also be taught inside-outside, thus bridging it to the outside world it describes. So, textbooks should not reject, but further guide the mastery of Many that children bring to school.

The chapters are called 'is one curriculum and textbook for all students possible, meeting many, children bundle to count and share, textbooks for a question guided counting curriculum, textbook for a question guided adding curriculum, discussion and future research.'

15. The PowerPointPresentation is called 'The Child's Own Mastery of Many, Count & ReCount & DoubleCount, before Adding NextTo & OnTop' and contains 43 slides.

● At Freiburg Pädagogische Hochschule October 7-8 the third Educating the Educators International Conference on approaches to scaling-up professional development in maths and science education will take place. Invitations were sent out to contribute with oral presentation sessions in the three dimensions (personal, material and structural) to report on projects, approaches and research, workshop sessions actively involving all participants, discussion group sessions also actively involving all participants, poster sessions and materials market, allowing participants to exhibit interesting professional development materials (including classroom materials) and learn about other materials.

The conference focused on three topics wanting to 'serve as a lever and platform for international exchange about concepts and experiences. The aim is to present and discuss different approaches which ensure a high quality of the education of educators:

* Personal dimension: Which roles, contents and activities have to be considered in the professional development courses for PD course leaders and facilitators in professional learning?

* Material dimension: Which role can materials play in professional development for maths and science teachers (classroom materials, face-to-face PD materials and e-learning PD materials)?

* Structural dimension: How can projects or initiatives for scaling up professional development look like and how can they be evaluated?

I sent in four proposals. One was rejected (16, sent as a poster for topic 3), two were accepted as posters (17 sent as a presentation for topic 1, 18 sent as a workshop for topic 2), one was accepted for presentation (19 sent as a discussion group 3). The proposal for a material market (21) was accepted.

16. Addition-Free Math Make Migrants and Refugees Stem Educators

17. Recounting Before Adding Makes Teachers Course Leaders and Facilitators

18. Self-explanatory Learning Material to Improve your Mastery of Many

19. Can Grounded Math and Education and Research Become Relevant to Learners

20. The PowerPointPresentation is called ‘Can Grounded Mathematics & Education & Research become Relevant to Learners?’ and contains 54 slides.

21. Recounting in Icon-Units and in Tens Before Adding Totals Next-To and On-Top, together with the posters presented at the stand.

● The following note is handed out to students and to teacher to have a basic discussion of the need and form of mathematics education.

22. What is Math - and Why Learn it?

● This material is meant for high school to illustrate how algebra and geometry should be always together and never apart.

23. Mathematics with Playing Cards

● This math compendium is meant for a high school pre-calculus course to illustrate the point made in (01) that it is possible to start all over from the bottom in a pre-calculus course, and also to give an introduction to calculus presenting integral calculus before differential calculus. The compendium also includes several projects modeling real world problems.

24. Mathematics Predicts, PreCalculus

● In Växjö January 14-15, 2020, the Swedish Society for Research in Mathematics Education welcome to Madif 12, its twelfth research seminar in connection with the Matematikbiennalen 2020. The theme of the seminar is ‘Sustainable mathematics education in a digitalized world’. I sent in three papers inspired by (01), one on early childhood education (25), and one on middle school (26), and one on precalculus (27) as well as two proposals for a workshop (28, 29). All were rejected.

25. Sustainable Adaption to Quantity: From Number Sense to Many Sense

The abstract says: Their biological capacity to adapt to their environment make children develop a number-language based upon two-dimensional block- and bundle-numbers, later to be colonized by one-dimensional place-value numbers with operations derived from a self-referring setcentric grammar, forced upon them by institutional education. The result is widespread innumeracy making OECD write the report ‘Improving Schools in Sweden’. To create a sustainable quantitative competence, the setcentric one-dimensional number-language must be replaced by allowing children develop their own native two-dimensional language. And math education must accept that its goal is not to mediate the truth regime of setcentric university math, but to develop the child’s already existing adaption to Many.

The chapters are called: Decreased PISA Performance Despite Increased Research Mathematics and its Education, Biology Looks at Education, Philosophy Looks at Education, Psychology Looks at Education, Sociology Looks at Education, Meeting Many, Children Bundle to Count and Share, A Contemporary Mathematics Curriculum, The Difference to a Typical Contemporary Mathematics Curriculum, Mathematics as a Number-Language, Discussing Number Sense and Number Nonsense, Conclusion and Recommendation.

26. Per-numbers connect Fractions and Proportionality and Calculus and Equations Sustainable Adaption to Quantity: From Number Sense to Many Sense

The abstract says: In middle school, fractions and proportionality are core subjects creating troubles to many students, thus raising the question: can fractions and proportionality be seen and taught differently? Searching for differences making a difference, difference-research suggests widening the word ‘percent’ to also talk about other ‘per-numbers’ as e.g. ‘per-five’ thus using the bundle-size five as a unit. Combined with a formula for recounting units, per-numbers will connect fractions, quotients, ratios, rates and proportionality as well as and calculus when adding per-numbers by their areas, and equations when recounting in e.g. fives.

The chapters are called: Mathematics is Hard, or is it, The ICMT3 Conference, Different Ways of Seeing Fractions, Ratios and Rates, Per-numbers Occur when Double-counting a Total in two Units, Fractions as Per-numbers, Expanding and Shortening Fractions, Taking Fractions of Fractions, the Per-number Way, Direct and Inverse Proportionality, Adding Fractions, the Per-number Way, Solving Proportionality Equations by Recounting , Seven Ways to Solve the two Proportionality Questions, A Case: Peter, about to Peter Out of Teaching, Discussion and Recommendation

27. Sustainable Adaption to Double-Quantity: From Pre-Calculus to Per-Number Calculations

The abstract says: Their biological capacity to adapt make children develop a number-language based upon two-dimensional block-numbers. Education could profit from this to teach primary school calculus that adds blocks. Instead it teaches one-dimensional line-numbers, claiming that numbers must be learned before they can be applied. Likewise, calculus must wait until precalculus has introduced the functions to operate on. This inside-perspective makes both hard to learn. In contrast to an outside-perspective presenting both as means to unite and split into per-numbers that are globally or piecewise or locally constant, by utilizing that after being multiplied to unit-numbers, per-numbers add by their area blocks.

The chapters are called: A need for curricula for all students, A Traditional Precalculus Curriculum, A Different Precalculus Curriculum, Precalculus, building on or rebuilding?, Using Sociological Imagination to Create a Paradigm Shift, A Grounded Outside-Inside Fresh-start Precalculus from Scratch, Solving Equations by Moving to Opposite Side with Opposite Sign, Recounting Grounds Proportionality, Double-counting Grounds Per-numbers and Fractions, The Change Formulas, Precalculus Deals with Uniting Constant Per-Numbers as Factors, Calculus Deals with Uniting Changing Per-Numbers as Areas, Statistics Deals with Unpredictable Change, Modeling in Precalculus Exemplifies Quantitative Literature, A Literature Based Compendium, An Example of a Fresh/start Precalculus Curriculum, An Example of an Exam Question, Discussion and Conclusion.

28. A Lyotardian dissension to the early childhood consensus on numbers and operations. The chapter are called: Can sociological imagination improve mathematics education? Consensus and Dissension on Early Childhood Numbers & Operations. Time Table for the Workshop.

29. Salon des Refusés, a Way to Assure Quality in the Peer Review Caused Replication Crisis? The chapter are called: Does Mathematics Education Research have an Irrelevance Paradox? The Replication Crisis in Science. Time Table for the Workshop.

30. Bundle Counting Table. A guide to bundle-counting in pre-school. Written for the stand at the Matematikbiennale.

31. Proposals for the 2020 Swedish Math Biennale. All were rejected.

● At the Ho Chi Minh City University of Education December 7 a conference was held called ‘Psychology and Mathematics education’. I was invited to give the plenary talk Saturday, which I named after the paper I send in (32), together with a Power Point Presentation (33). Sunday, I gave a talk on modeling to a group of master students. Monday, I gave a talk to a class of senior students on a poster presentation from the ‘Educating the Educators’ conference in Freiburg, Germany, in October, and handed out the notes ‘What is Math - and Why Learn it?’ and ‘Bundle Counting Table’. Tuesday, I gave a talk to the staff on research in mathematics education and networks to join and design research as a methodology to use when researching the implementation of the new activity-based curriculum inspired by Kolb’s experimental learning theory.

32. De-Modeling Numbers, Operations and Equations: From Inside-Inside to Outside-Inside Understanding

The abstract says: Adapting to the outside fact Many, children internalize social number-names, but how do they externalize them when communicating about outside numerosity? Mastering Many, children use bundle-numbers with units; and flexibly use fractions and decimals and negative numbers to account for the unbundled singles. This suggests designing a curriculum that by replacing abstract-based with concrete-based psychology mediates understanding through de-modeling core mathematics, thus allowing children to expand the number-language they bring to school.

The chapters are: 1. Introduction, 2. Materials/ Subjects and Methods, 2.1. Reflections on Different forms of Mathematics, 2.2. Reflections on Different forms of Psychology, 2.3. Merging Mathematics and Psychology, 2.4. De-modelling Digits, 2.4.1. Designing and Implementing a micro-curriculum, 2.5. Reflections on how to De-model Bundle-counting Sequences, 2.5.1. Designing and Implementing a micro-curriculum, 2.6. Reflections on how to De-model Operations, 2.7. Reflections on how to Recount into Tens, 2.7.1. Designing and Implementing a micro-curriculum, 2.8. Reflections on how to Model Double-counting with Per-numbers and Fractions, 2.9. Reflections on how to De-model Trigonometry, 3. Results and Discussion, 4. Conclusion.

33. De-Model Numbers, Operations and Equations, PPP.

34. Visit to Ho Chi Minh City University of Education December 7-13 2019.

● The ICMT3 and Educating Educators conferences used peer-reviews, and in the first you were allowed to comment on the reviews

35. Review 01 ICMT3

36. Review 02 ICMT3

37. Comments to ICMT3 Reviewers

38. Educating Educators Reviews

Aarhus, December 2019, Allan Tarp

08. ADDITION-FREE STEM-BASED MATH FOR MIGRANTS, PPP

Addition-free STEM-based Math for Migrants

using the Child's own Mastery of Many, saying:

Count & *ReCount* & *DoubleCount*

before you Add NextTo & OnTop

- that's how you

master many
with ManyMath

Allan.Tarp@MATHeCADEMY.net
teaching teachers to teach Math as ManyMath

MATHeCADEMY

In Sweden, OECD says that 'Math' excludes 1 of 4 socially

"PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (page 3)

http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY 2

MATHeCADEMY

How to **P**ower **P**oor **P**ISA **P**erformance

To find an unorthodox way, a university in Sweden, challenged by many young male migrants, may arrange a **C**urriculum **A**rchitect **C**ontest:

- Philosophize the low success of 50 years of math education research
- Derive from this a STEM-based curriculum so young migrants soon become STEM pre-teachers and pre-engineers to help develop or rebuild their home country

MATHeCADEMY

Philosophy: INSIDE representations of the OUTSIDE

Seeing OUTSIDE **existence** represented by INSIDE **essence**, or *ontology* represented by *epistemology*.

- Sartre on existentialism: OUTSIDE **existence** should precede INSIDE **essence**.
- Heidegger: The sentence „**subject IS predicate**" bridges the OUT- and INSIDE. Trust the OUTSIDE subject, it exists; but question the INSIDE predicate: it may be an institutionalized verdict - that should be doubted and appealed.
    - Weber: If carried too far, rational institutions may become an INSIDE **iron cage** that **disenchants** its OUTSIDE subject.
    - Arendt: Blindly Institutionalizing may lead to '**the banality of evil**'.
    - Sociology: Beware of institutions, they monopolize!

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY

MATHeCADEMY

What IS mathematics & education? In ancient Greece: a natural science

In ancient Greece, Pythagoreans used the word meaning 'knowledge', as a common label for four descriptions of Many by itself and in space & time

Together they formed the '**quadrivium**' recommended by Plato as a general curriculum after the '**trivium**' consisting of grammar & logic & rhetoric.
Geometry & algebra are both grounded in Many as shown by names:

- In Greek, **Geometry** means to measure earth
- In Arabic, **Algebra** means to reunite numbers

MATHeCADEMY

Later, math chose SET self-reference

But Russell said : "INSIDE self-reference leads to the classical liar paradox '**this sentence is false**', being true if false & opposite.
Let M be the set of sets not belonging to itself, $M = \{A \mid A \notin A\}$.
Then $M \in M \Leftrightarrow M \notin M$. Forget about sets. Use type theory instead.
So, by self-reference & without units, fractions are not numbers."
But mathematics insisted on being a self-refering **MetaMatics** by saying: "Forget about Russell, he is not a mathematician. We just institutionalize fractions as so-called rational numbers."

MATHeCADEMY

Adding without units creates MatheMatism *true INSIDE but seldom OUTSIDE*

The Teacher	**The Students** (the fraction paradox)
What is 1/2 + 2/3?	Well, 1 of 2 + 2 of 3 gives (1+2) of (2+3), or 3 of 5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But if the browns are 1/2 of 2 cakes, and 2/3 of 3 cakes, then they are 1+2 of 2+3 cakes, i.e. 3/5 of 5 cakes! How can the browns be 7 cakes out of 6 cakes?
INSIDE this classroom 1/2 + 2/3 **IS** 7/6 !	

Without units, fractions & digits are operators, needing numbers to become numbers.

2+3 IS 5? No, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.

2x3 IS 6? Yes, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.

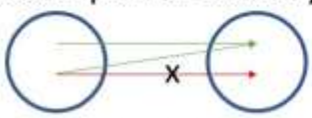
*Mixing English and metric units made NASA's Mars Climate Orbiter **CRASH** in 1999.*

MATHeCADEMY

3 kinds of math, pre-, present & post-setcentric mathematics, defining a 'function' differently

Pre-setcentric math: a function is a CALCULATION with both specified and unspecified numbers, e.g. **2+*u***.

Present setcentric math: a function is a SUB**SET** OF **SET**-PRODUCT where first-component identity implies second-component identity.



Post-setcentric math: a function is a NUMBER-LANGUAGE SENTENCE, e.g. **T = 2+*u***, bridging an OUTSIDE existence to an INSIDE chosen essence.

MATHeCADEMY

Post-setcentric math: through its use, as with the other language in our two language houses

The **WORD-language** assigns words in sentences with The **NUMBER-language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

	WORD-language	NUMBER-language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
INSIDE Language	This is a chair	T = 3x4
OUTSIDE World		

Mathematics

Mother tongue

MATHeCADEMY

Two different kinds of education

The Enlightenment Century rooted education, but in different forms in its two republics, the North American in 1776 and the French in 1789.

- In North America, education enlightens children about their OUTSIDE world, and enlightens teenagers about their INSIDE individual talent, to be uncovered and developed through self-chosen ½year **BLOCKS** with teachers teaching only one subject in the teacher's own classroom.
- To protect its republic from its German speaking neighbors, France was forced to create institutions controlled by a strong central administration with public servants trained at elite schools with multi-year **LINES**, later copied by European Bildung-education.

MATHeCADEMY

3x2 different kinds of math education

Mathematics in	self-chosen ½year BLOCKS	forced multi-year LINES
Pre-SETcentric	North America	UK Commonwealth
Present SETcentric	-	Continental Europe
Post-SETcentric	MATHeCADEMY.net	

MATHeCADEMY

Philosophizing the low success of 50 years of mathematics education research

Mathematics also needs a COMMUNICATIVE TURN where

- instead of learned **INSIDE-INSIDE** through its grammar, it is learned **OUTSIDE-INSIDE** as a INSIDE number-language communicating about OUTSIDE things and actions, thus learned through its use, and not before its use
- instead of learning about numbers, students learn how to number, and how to communicate about Many in full sentences containing:

  1) an **OUTSIDE** subject, 2) a linking verb, and 3) an **INSIDE** predicate: **T = 2x3**

So, now we look for an **OUTSIDE-INSIDE post-SETcentric mathematics** to replace the present **INSIDE-INSIDE meta-matism** by asking:

What kind of mathematics grows from the Mastery of Many that children develop through use, and before school?

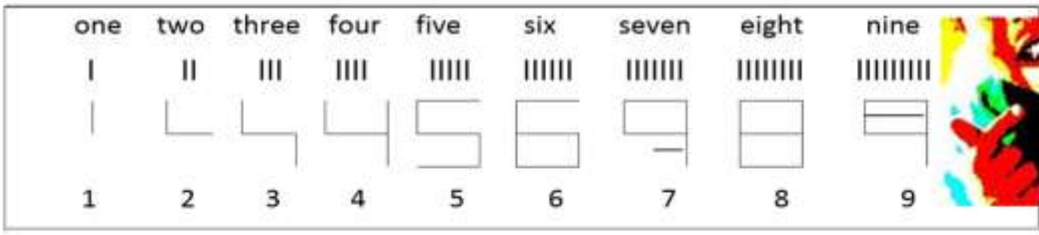
Pablo Picasso: It took me four years to paint like Raphael, but a lifetime to paint like a child

MATHeCADEMY

Creating icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

This is not 4
it is 2 ***2s***



MATHeCADEMY

Division & multiplication & subtraction are icons also

- From 9 **PUSH** away **4s** we write 9/4 iconized by a broom, called *division*.

- 2 times **LIFTING** the **4s** to a stack we write 2x4 iconized by a lift called *multiplication*.

- From 9 **PULL** away 2 **4s'** to find un-bundled we write 9 – 2x4 iconized by a rope, called *subtraction*.

MATHeCADEMY

Counting is predicted by a ReCount formula

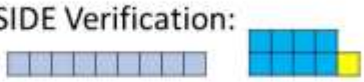
T = (T/B) x B	From a total **T**, **T/B** times, **B** is pushed away

A formula is an INSIDE **prediction**, making the number-language a language for prediction.

INSIDE Prediction: ReCounting 9 in **4s** gives 2**B**1 **4s**:

9/4	**2.some**
9 – 2x4	**1**

OUTSIDE Verification:

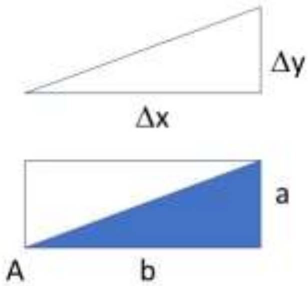


MATHeCADEMY

The **ReCount formula** is the core of math & science

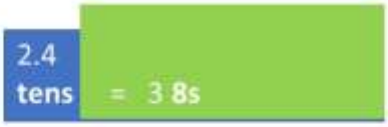
T = (T/B)*B expresses proportionality when changing unit, and is all over:

Proportionality	y = **c** * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **speed** * second



MATHeCADEMY

ReCounting from tens to icons

T = ? **8s** = 24 = (24/8)x8 ? = 24/8 = 3	*Asking how many **8s** in 24,* *we just recount 24 in **8s***
u x 8 = 24 = (24/8) x 8 **u** = 24/8 **u** = 3	Formulated as an **equation**, we find the unknown **u** by moving 8 **to opposite side - with opposite sign**
As geometry: A block decreasing its unit must increase its height to keep its size	

MATHeCADEMY

DoubleCounting gives PerNumbers

DoubleCounting in kg & $, we may get **4kg/5$**

With **4kg** bridged to **5$** we answer questions by recounting in the per-number (per-numbers = proportionality = change units)

Questions: **7kg = ?$**	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

With like units, PerNumbers are Fractions: 4$/5$ = 4/5 & 4$/100$ = 4/100 = 4%

MATHeCADEMY

STEM multiplication formulas come from DoubleCounting and PerNumbers I

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x sq.-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole

MATHeCADEMY

STEM multiplication formulas come from DoubleCounting and PerNumbers II

More STEM examples:

- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec;
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = (energy/charge) x (charge/sec) or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

MATHeCADEMY

5 ways to master proportionality I

Proportionality, **Q1**: "2kg costs 5$, what does 7kg cost"; **Q2**: "What does 12$ buy?"

1) **Regula de Tri** (Rule of Three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

- **Q1**: '2kg cost 5$, 7kg cost ?$'. Multiply-then-divide gives the $-number 7x5/2 = 17.5.
- **Q2**: '5$ buys 2kg, 12$ buys ?kg'. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

2) **Find the unit by dividing before multiplying**

- **Q1**: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

3) **Cross multiplication when equating the per-numbers**

- **Q1**: $2/5 = 7/u$, so $2*u = 7*5$, $u = (7*5)/2 = 17.5$. **Q2**: $2/5 = u/12$, so $5*u = 12*2$, $u = (12*2)/5 = 4.8$

4) **ReCount in the per-number** 2kg/5$ coming from 'double-counting' the total T.

- **Q1**: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

MATHeCADEMY

5 ways to master proportionality II

5) **Modern Math Modeling** with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive $-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$$.
- With 12$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

MATHeCADEMY

ReCounting sides in a block: Trigonometry

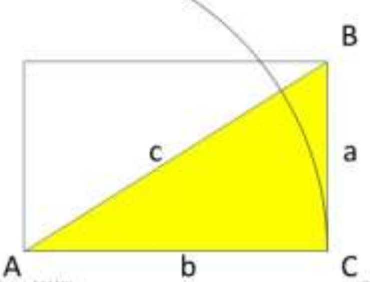
A right triangle is <u>a block halved by its diagonal</u> giving 3 sides: <u>base b</u>, <u>height a</u> and <u>diagonal c</u> connected with the angles when recounting one side in the other side or in the diagonal

a = (a/c)*c = sinA * c

b = (b/c)*c = cosA * c

tanA = a/b = Δy/Δx = gradient

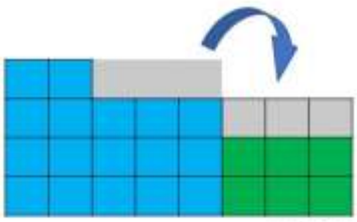
Circle: circum./diam. = π ≈ **n*tan(180/n)** for n large

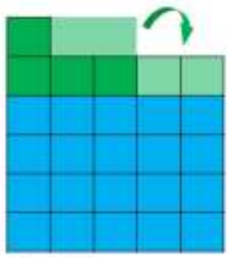


MATHeCADEMY

Counted & recounted, Totals can be added

BUT: NextTo	**or OnTop**
4 5s + 2 3s = 3B2 8s	4 5s + 2 3s = 4 5s + 1B1 5s = 5B1 5s
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





MATHeCADEMY

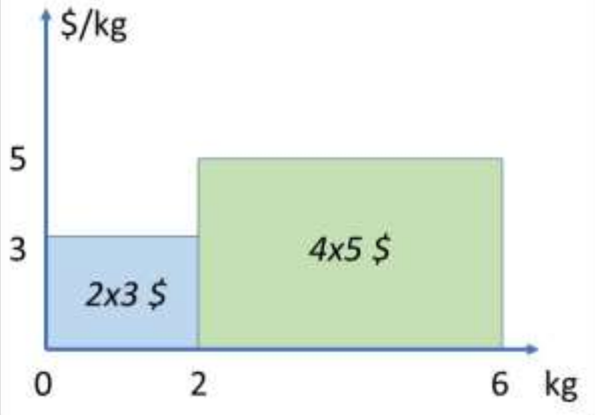
PerNumbers add by areas (integration)

"2kg at 3$/kg + 4kg at 5$/kg = 6kg at ? $/kg?"

2 kg at 3 $/kg
\+ 4 kg at 5 $/kg

(2+4) kg at ? $/kg

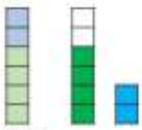
- Unit-numbers add on-top.
- Per-numbers must be multiplied so they add by **areas** under the per-number graph (integration).



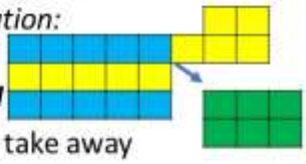
MATHeCADEMY

Reversed addition = solving equations

OppoSite Side with OppoSite Sign		NextTo
2 x ? = 6 = (6/2) x 2	**2 + ? = 6** = (6-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 6/2 *Solved by ReCounting*	? = 6-2 *Solved by ReStacking*	? = (3.2 **8s** – 2 **3s**)/5 = *(T-T1)/5 = ΔT/5*



Differentiation:
first ***PULL***
then ***PUSH***



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

$2 \times u = 8 = (8/2) \times 2$	Solved by re-counting 8 in 2s
$u = 8/2 = 4$	Move: **O**pposite **S**ide with **O**ppo**S**ite **S**ign

MetaMath (Don't test, but DO remember the bi-implication arrows)

	$2 \times u = 8$	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
⇕	$(2 \times u) \times (½) = 8 \times (½)$	Multiplying 2's inverse element ½ to both number-names
⇕	$(u \times 2) \times (½) = 4$	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕	$u \times (2 \times (½)) = 4$	Applying the **associative** law
⇕	$u \times 1 = 4$	Applying the definition of an inverse element
⇕	$u = 4$	Applying the definition of a neutral element. *With arrows a test is not needed.*

The simplicity of mathematics I: There are just four ways to unite into a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration).
Addition and multiplication add changing and constant unit-numbers.
Integration and power add changing and constant per-numbers.
We might call this beautiful simplicity the '*Algebra Square*'.

Operations unite	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n**	**T = a * n**
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn**	**T = a^n**

MATHeCADEMY

The simplicity of mathematics II: There are just five ways to split a Total

The 4 uniting operations (+, *, ^, ∫) each has a reverse splitting operation:
Addition has <u>subtraction</u> (–), and multiplication has <u>division</u> (/).
Power has factor-finding (<u>root</u>, √) and factor-counting (<u>logarithm</u>, log).
Integration has per-number finding (<u>differentiation</u> dT/dn = T').

Reversing operations is solving equations, done by moving to ***opposite side*** *with* ***opposite sign****.*

Operations unite / *split into*	**changing**	**constant**
unit-numbers *m, s, $, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $log_a T = n$, $\sqrt[n]{T} = a$

MATHeCADEMY

Conclusion and recommendations

- Problems come, when presenting math through **INSIDE** self-reference instead of as a natural science communicating about the **OUTSIDE** fact Many.
- As a foreign language, the number-language may be learned through its use where core mathematics as proportionality, equations, per-numbers and calculus grow directly from the Mastery of Many that children develop in **OUTSIDE** use and before school. *Digits & fractions are operators, not numbers.*
- Math is not a self-referring goal in itself, but an **INSIDE** means to an **OUTSIDE** goal that is reached better and by more through communicating about Many.
- Recounting is rooting **INSIDE** math in **OUTSIDE** STEM-examples. This allows young migrants learn core STEM subjects also, to become STEM pre-teachers or pre-engineers that can help develop or rebuild their own country.

MATHeCADEMY

References

- **Halliday**, M. A. K. (1973). *Explorations in the function of language*. London, UK: Edward Arnold.
- **OECD**. (2015). *Improving schools in Sweden: An OECD Perspective*. Retrieved from: www.oecd.org/edu/school/improving-schools-in-sweden-an-oecd-perspective.htm.
- **Tarp**, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.
- **Tarp**, A. (2019). Addition-free stem-based math for migrants. Retrieved from: http://mathecademy.net/ctras-2019-contributions/
- **Widdowson**, H. G. (1978). *Teaching language as communication*. Oxford, UK: Oxford University Press.

MATHeCADEMY

Teach language AFTER grammar! No, BEFORE *the Communicative Turn around 1970*

Halliday (1973, p. 7) defines **a functional approach** to language:

"A functional approach to language means, first of all, investigating how language is used: trying to find out what are the purposes that language serves for us, and how we are able to achieve these purposes through speaking and listening, reading and writing. But it also means more than this. It means seeking to explain the nature of language in functional terms: seeing whether language itself has been shaped by use, and if so, in what ways - how the form of language has been determined by the functions it has evolved to serve."

Likewise, Widdowson (1978) adopts a **communicative approach** to the teaching of language' allowing more students to learn INSIDE language through its OUTSIDE use for communication about things and actions.

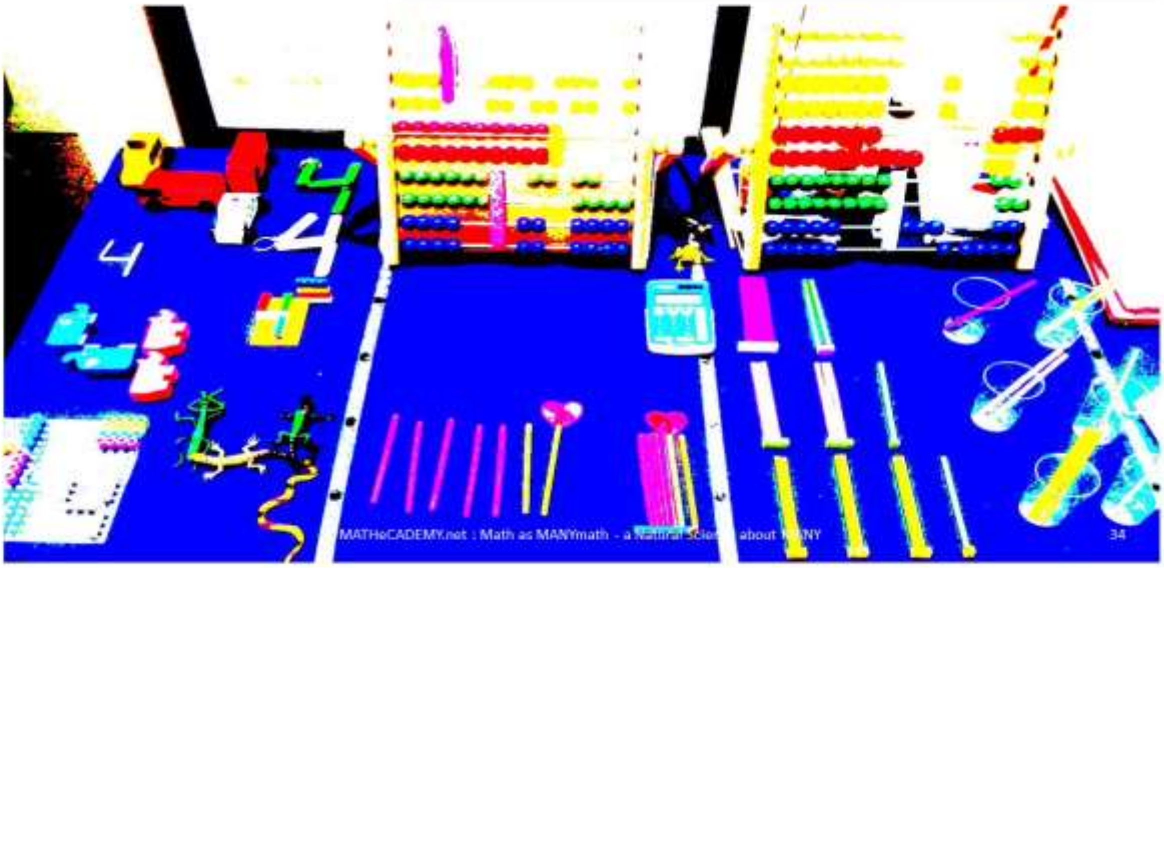
Some MrAlTarp YouTube Videos

DrAlTarp YoKu

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History

Number icons

ReCounting **7** in **5s** & **3s** & **2s**



MATHeCADEMY

What Mastery of Many do children get through use before school?

Children typically see Many as **OUTSIDE** blocks described by **INSIDE** numbres, counting bundles, bundles of bundles etc.

So, to children, BLOCKS are fundamental:

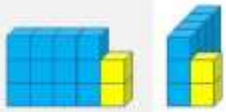
- in **INSIDE** numbers: 456 = 4*BB*5*B*6 = three **OUTSIDE** blocks

- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks

MATHeCADEMY

The child's own twin math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving **OUTSIDE** geometrical multi-blocks, & (when turned to hide the units behind) **INSIDE** algebraic bundle-numbers.

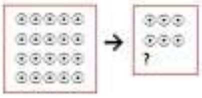
3) Operations are **INSIDE** icons, showing the 3 **OUTSIDE** counting steps: PUSHING bundles & LIFTING bundles & PULLING stacks to find the unbundled ones.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

Question: T = 4 ***5s*** *= ?* ***3s*** • *Answer: T = 4* ***5s*** *= 6B2* ***3s*** • *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



Main parts of a ManyMath curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage
- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- BundleCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: divide, multiply, subtract, add on-top & next-to

Middle school – integrating algebra and geometry, the content of the label 'math'
- DoubleCounting produces PerNumbers and fractions as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always, so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE
- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY 35

Main parts of a ManyMath curriculum

Primary School – respecting and developing the Child's own 2D NumberLanguage
- Digits are Icons and Natural numbers are 2dimensional block-numbers with units
- BundleCounting & ReCounting before Adding
- NextTo Addition (PreSchool Calculus) before OnTop Addition
- Natural order of operations: divide, multiply, subtract, add on-top & next-to

Middle school – integrating algebra and geometry, the content of the label 'math'
- DoubleCounting produces PerNumbers and fractions as operators needing numbers to become numbers, thus being added as areas (MiddleSchool Calculus)
- Geometry and Algebra go hand in hand always, so length becomes change and vv.

High School – integrating algebra and geometry to master CHANGE
- Change as the core concept: constant, predictable and unpredictable change
- Integral Calculus before Differential Calculus

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY 37

MATHeCADEMY

Question guided teacher education

MATHeCADEMY.net

Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= 7*3= 7 3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system. If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

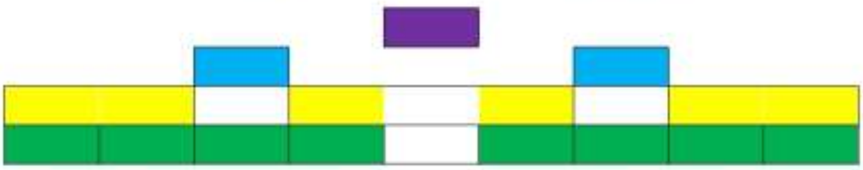
MATHeCADEMY

PYRAMIDeDUCATION

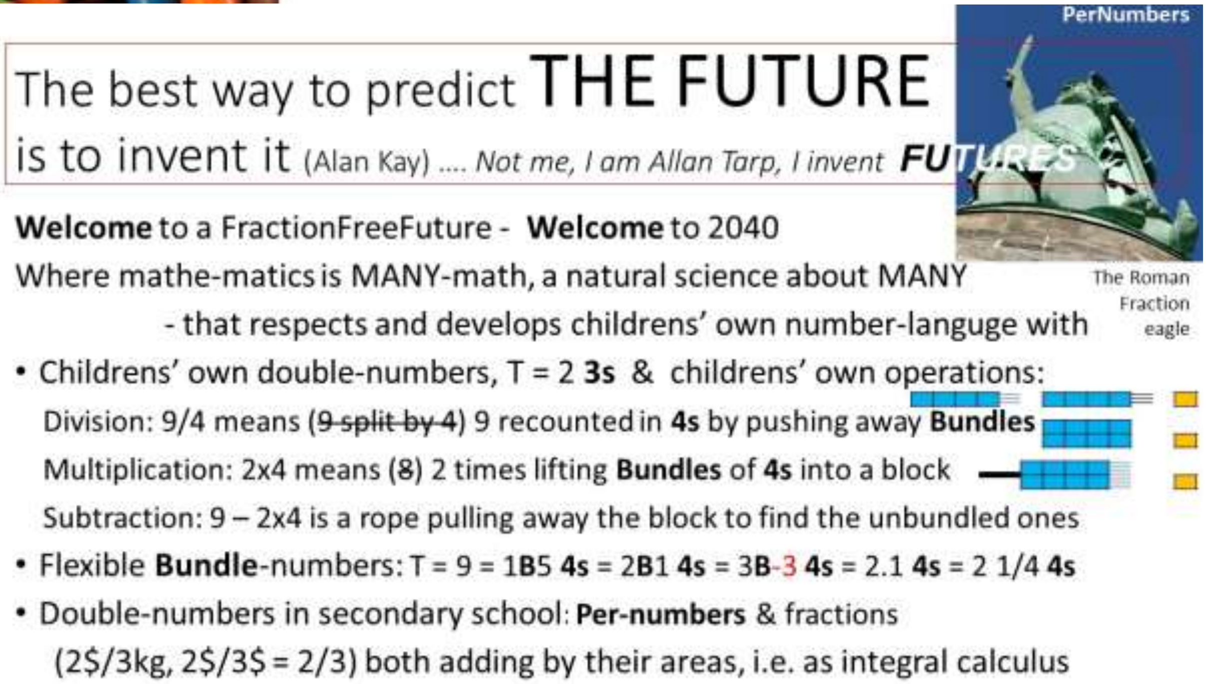
In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting their Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams

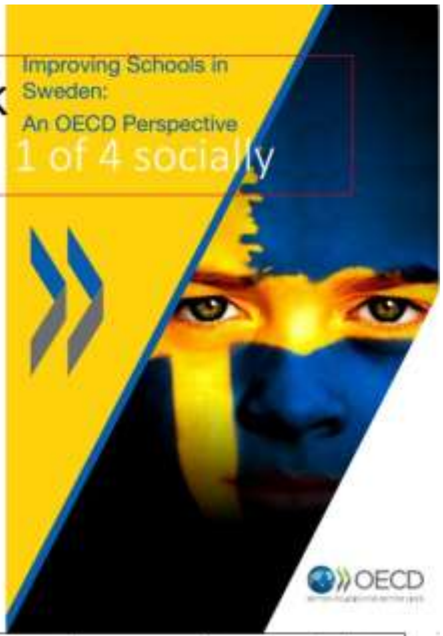


15. THE CHILD'S OWN MASTERY OF MANY, PPP



MATHeCADEMY

Education is a cure: Does it work
In Sweden, OECD says that it excludes 1 of 4 socially



"PISA 2012, however, showed a stark decline in the performance of 15-year-old students in all three core subjects (reading, mathematics and science) during the last decade, with more than one out of four students not even achieving the baseline Level 2 in mathematics at which students begin to demonstrate competencies to actively participate in life." (page 3)

http://www.oecd.org/sweden/sweden-should-urgently-reform-its-school-system-to-improve-quality-and-equity.htm

MATHeCADEMY

Why teach children if they already know?

With education curing un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

Core Questions:

- What Mastery does children develop when adapting to Many?
- What could be a Question-guided ChildCenteredCurriculum in Quantitative Competence?

MATHeCADEMY

Adaption creates two language houses

The **WORD language** assigns words in sentences with The **NUMBER language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. But does mathematics respect teaching language before grammar?

	WORD language	NUMBER language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
		Mathematics
Language	This is a chair	T = 3x4
	Mother tongue	

WORLD

MATHeCADEMY

The **Communicative Turn** in language ed.

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: "A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us."
Likewise, Widdowson adopts a "communicative approach to the teaching of language" allowing more students to learn a language through its use for communication about outside things and actions.

Could mathematics also have its Communicative turn?

	WORD language	NUMBER language
(META-language, grammar)		
Inside Language	This is a chair	T = 3 4s
	Mother tongue	Mathematics
Outside world		

MATHeCADEMY

Children see Many as bundles with units

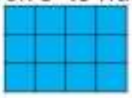
"How old next time?" A 3year old says "Four" showing 4 fingers: I I I I

But, the child reacts strongly to 4 fingers held together 2 by 2: II II

"That is not four, that is two twos" (T ≠ 4, T = 2 **2s**)

The child sees what exists, and with units: bundles of **2s**, and 2 of them.

- The block 3 **4s** has two numbers: 3 (the counting-number) and **4** (the unit-number)



Let us invent a **Q**uestion **D**riven **C**urriculum to minimize the 'mediated correctness' effect of textbooks and teachers?

MATHeCADEMY

Q01. Create icons: I I I I → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.

An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become units to use when counting in bundles.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9

MATHeCADEMY

Q02, counting sequences

"How to count fingers?"

Using **5s** as the bundle-size, fingers can be counted as

"0**B**1, 0**B**2, 0**B**3, 0**B**4, 0**B**5 – sorry, **Bundle**"

and the rest can be counted in as

"**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B**, **1left, 2left** (a-leven, twe- leven)".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1**B**3 **7s** = 2**B**2 **4s** = 3**B**1 **3s** = 1**BB**1 **3s**.

MATHeCADEMY

Q03, bundle-counting in icon-units

"How to count by bundling?"

Five fingers can be bundle-counted in pairs or triplets, allowing both an OVERLOAD and an UNDERLOAD; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

IIIII	•	ǂ III	•	ǂ ǂ I	•	ǂ ǂ ǂ	•	ǂ ǂ I
T = 5	=	1Bundle3 **2s**	=	2B1 **2s**	=	3B-1 **2s**	=	1BB1 **2s**
T = 5	=	1.3 **2s**	=	2.1 **2s**	=	3.-1 **2s**	=	10.1 **2s**

Likewise, if counting in **ten**-bundles:
T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

MATHeCADEMY

Q04, unbundled as decimals or negatives or fractions

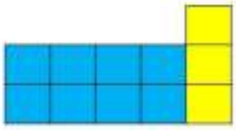
0.3 **4s** or 1.-1 **4s** or 3/4 **4s**

“Where to put the unbundled singles?”

When counting by bundling, the unbundled singles can be placed

NextTo the block

counted as a block of **1s**

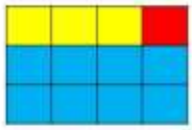


T = 2B3 **4s** = 2.3 **4s**

A decimal number

OnTop of the block

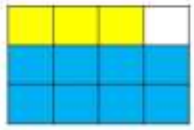
counted as a bundle



T = 3B-1 **4s** = 3.-1 **4s**

A negative number

counted in bundles



T = 2 3/4 **4s**

A fraction

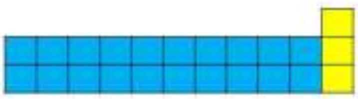
MATHeCADEMY

Q04, counting in tens

“Where to put the unbundled singles with **tens**?”

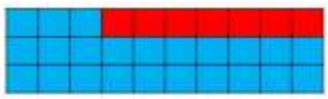
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit and the decimal point,

- or as:



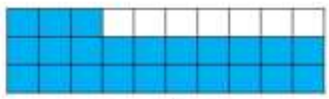
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

T = 3B-7 **tens**



T = 2 3/10 **tens**

T = 2 3/10 B **tens**

MATHeCADEMY

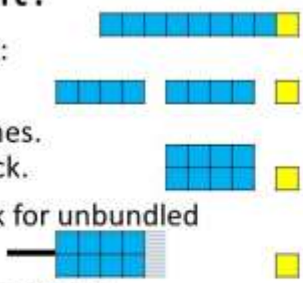
Q05, calculators predict

"Can a calculator predict a counting result?"

We may see division as an icon for a broom pushing away bundles: 9/4 means 'from 9, push away bundles of 4s'.

- The calculator says '2.some', thus predicting it can be done 2 times. Now multiplication iconizes a lift stacking the bundles into a block.

- Finally, subtraction iconizes a rope pulling away the block to look for unbundled singles.
- With '9-2x4 = 1' the calculator predicts that 9 can be recounted as 2B1 **4s**.



9/4	**2.some**
9 – 2x4	**1**

MATHeCADEMY

Q05, counting creates a ReCounting formula

ReCount **T = (T/B) x B**	from a total **T**, **T/B** times, **Bs** is taken away and stacked

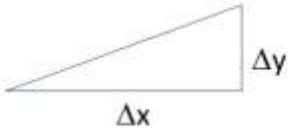
As sentences of the number language, **Formulas Predict:**
Predicting that **T** = 9 = 2.1 **4s:**

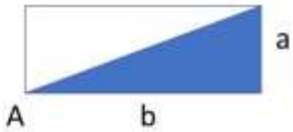
9/4	**2.some**
9 – 2x4	**1**

Q05, the recounting formula is a core formula

T = (T/B)*B is all over STEM (Science, Technology, Engineering, Mathematics):

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second





Q06, recounting in a different unit

"How to change a unit?"

The recount-formula allows changing the unit.

Asking T = 3 **4s** = ? **5s**, the recount-formula gives T = 3 **4s** = (3x4/5) **5s**.

Entering 3x4/5, the answer '2.some' shows that a block of 2 **5s** can be taken away.

With 3x4–2x5, the answer '2' shows that 3 **4s** can be recounted as 2B2 **5s** or 2.2 **5s**.

3 **4**s = IIII IIII IIII = IIII I III II II = IIIII IIIII II = 2B2 **5**s = 2.2 **5s**

3x4/5	**2.**some
3x4 – 2x5	**2**

Change Unit = Proportionality

MATHeCADEMY

Q06, double-counting in two units creates DoubleNumbers or PerNumbers

"How to double-count in two units?"

DoubleCounting in kg & $, we get **4kg = 5$** or
4kg **per** 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber.

With 4kg bridged to 5$ we answer questions by recounting in the per-number.
Questions:

7kg = ?$	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

Answer: *Recount in the PerNumber (Proportionality)*

MATHeCADEMY

Q06, double-counting in the same unit creates fractions

"How to double-count in the same unit?"

Double-counted in the same unit, per-numbers are fractions, 2$ per 9$ = 2/9, or percentages, 2 per 100 = 2/100 = 2%.

To find a fraction or a percentage of a total, again we just recount in the per-number.

- **Taking 3 per 4 = taking ? per 100.** With 3 bridged to 4, we recount 100 in 4s:
100 = (100/4)*4 giving (100/4)*3 = 75, and 75 per 100 = 75%.
- **Taking 3 per 4 of 60 gives ?.** With 3 bridged to 4, we recount 60 in 4s:
60 = (60/4)*4 giving (60/4)*3 = 45.
- **Taking 20 per 100 of 60 gives ?.** With 20 bridged to 100, we recount 60 in 100s:
60 = (60/100)*100 giving (60/100)*20 = 12 .

> We observe that per-numbers and fractions are not numbers but OPERATORS needing a number to become a number.

MATHeCADEMY

Proportionality shows the instability of 'School Math' I

Proportionality, **Q1**: "2kg costs 5\$, what does 7kg cost"; **Q2**: "What does 12\$ buy?"

→1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: '2kg cost 5\$, 7kg cost ?\$'. Multiply-then-divide gives the \$-number 7x5/2 = 17.5.

Q2: '5\$ buys 2kg, 12\$ buys ?kg'. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

→2) Find the unit

Q1: 1kg costs 5/2\$, so 7kg cost 7x(5/2) = 17.5\$. **Q2**: 1\$ buys 2/5kg, so 12\$ buys 12x(2/5) = 4.8kg

→3) Cross multiplication

Q1: $2/5 = 7/u$, so $2*u = 7*5$, $u = (7*5)/2 = 17.5$. **Q2**: $2/5 = u/12$, so $5*u = 12*2$, $u = (12*2)/5 = 4.8$

→4) 'Re-counting' in the 'per-number' 2kg/5\$ coming from 'double-counting' the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5\$ = 17.5\$; **Q2**: T = 12\$ = (12/5)x5\$ = (12/5)x2kg = 4.8kg.

MATHeCADEMY

Proportionality shows the instability of 'School Math' II

→ 5) Modeling with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive \$-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$\$.
- With 12\$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

MATHeCADEMY

Double-counting gives per-numbers in STEM multiplication formulas I

STEM typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

MATHeCADEMY

Double-counting gives per-numbers in STEM multiplication formulas II

Extra STEM examples:

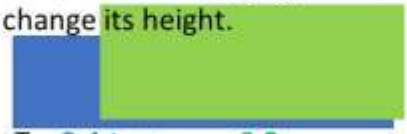
- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec;
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = (energy/charge) x (charge/sec) or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

MATHeCADEMY

Q07, recounting from tens to icons

"How to change unit from tens to icons?"

Asking 'T = 2.4 **tens** = 24 = ? **8s**', we just recount 24 in **8s**:
T = 24 = (24/8)x8 = 3x8 = 3 **8s.**

Formulated as an **equation** we use ***u*** for the unknown number, ***u***x8 = 24. Recounting 24 in 8s shows that ***u*** is 24/8. So, equations are solved by moving **to opposite side - with opposite sign**	To keep its size, a block changing its unit must also change its height.  T = 2.4 **tens** = 3 **8s**

u x 8 = 24 = (24/8)x8
u = 24/8 = 3

MATHeCADEMY

Q8, recounting from icons to tens (multiplication) 3 7s = ? tens

"How to change unit from icons to tens?"

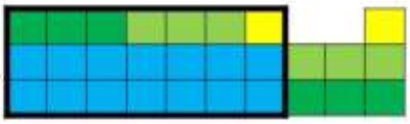
Asking 'T = 3 **7s** = ? **tens**', the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone: T = 3 **7s** = 3x7 = 21 = 2.1 **tens**, only it leaves out the unit and the decimal point.

Alternatively, we may use 'less-numbers', so 7 = **ten** less 3

T = 3x7 = 3 x (**ten** less 3) = 3 x **ten** less 3x3 = 3**ten** less 9 = 2**ten**1 = 21,

or with 9 = **ten** less 1:

T = 3**ten** less (**ten** less1) = 2**ten** lessless 1 = 2**ten** & 1 = 21.



showing that 'lessless' cancel out

MATHeCADEMY

Recounting large numbers in or from tens: *same size, but new form*

Recounting 6 47s in **tens** Recounting 476 in **7s**

*BundleWriting seprates **INSIDE** bundles from **OUTSIDE** singles*

T = 6 x 47 = 6 x 4**B**7	T = 476 = 47.6 **tens**
= 24**B**42	= 47**B**6
= 28**B**2	= 42**B**56
= 28.2	= 6x7**B**8x7
tens	= 68 x 7

MATHeCADEMY

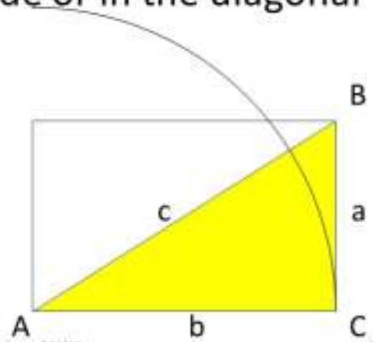
Q09, ReCounting sides in a block: Trigonometry

A right triangle is <u>a block halved by its diagonal</u> giving 3 sides: <u>base b</u>, <u>height a</u> and <u>diagonal c</u> connected with the angles when recounting one side in the other side or in the diagonal

a = (a/c)*c = sinA * c

b = (b/c)*c = cosA * c

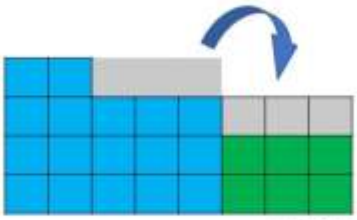
tanA = a/b = $\Delta y/\Delta x$ = gradient

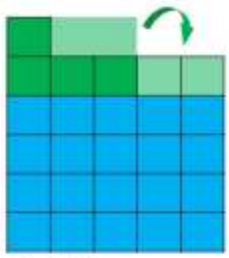


Circle: circum./diam. = $\pi \approx$ **n*tan(180/n)** for n large

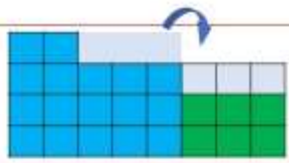
Once counted & recounted, Totals can be added

BUT: **NextTo** ➡	**or** **OnTop** ⬆
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 4 **5s** + 1**B**1 **5s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





Q11, next-to addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added next-to as **8s**?"

Outside, next-to addition geometrically means adding areas.
Next-to addition is also called integral calculus.

Inside, the recount formula algebraically predicts the result.
Here multiplication precedes addition.

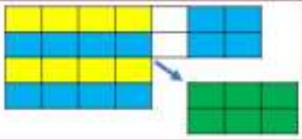
T = (T/**B**) x **B**

= ((4x5 + 2x3)/8) x 8 = 3.2 **8s**

(4x5 + 2x3)/8	**3.some**
(4x5 + 2x3) – 3x8	**2**

MATHeCADEMY

Q12, reversed next-to addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

Outside, we remove the initial block T1 and recount the rest in **4s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here subtraction precedes division; which is natural as reversed integration.

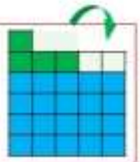
T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

MATHeCADEMY

Q13, on-top addition

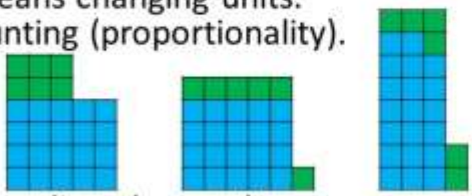


"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added on-top?"

Outside, on-top addition geometrically means changing units.
On-top addition thus often involves recounting (proportionality).

T = 4 **5s** + 2 **3s** = 4 **5s** + 1.1 **5s** = 5.1 **5s**

T = 4 **5s** + 2 **3s** = 6.2 **3s** + 2 **3s** = 8.2 **3s**



Inside, the recount formula algebraically predicts the result.
Here again, multiplication precedes addition.

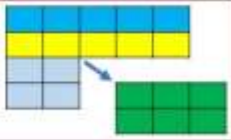
T = (T/**B**) x **B**

= ((4x5 + 2x3)/5) x 5 = 5.1 **5s**

(4x5 + 2x3)/5	**5.some**
(4x5 + 2x3) – 5x5	**1**

MATHeCADEMY

Q14, reversed on-top addition



"T1 = 2 **3s** and how many **5s** (T2) add on-top as 4 **5s**?"

Outside, we remove the initial block T1 and recount the rest in **5s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed on-top addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

T2 = (T2/**B**) x **B**

= ((4x5 – 2x3)/5) x 5 = 2.4 **5s**

(4x5 – 2x3)/5	**2.some**
(4x5 – 2x3) – 2x5	**4**

MATHeCADEMY

Q29, adding PerNumbers as areas (integration)

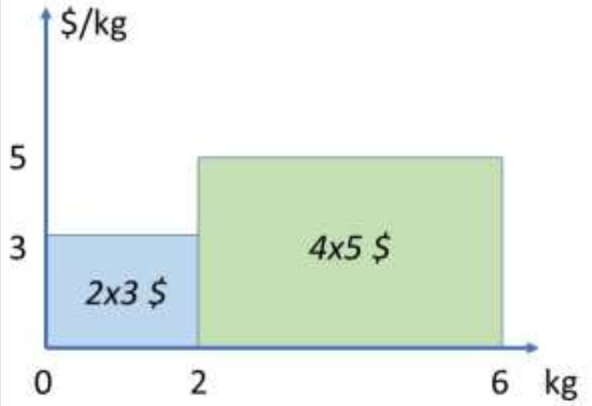
"2kg at 3$/kg + 4kg at 5$/kg = 6kg at ? $/kg?"

2 kg at 3 $/kg

+ 4 kg at 5 $/kg

(2+4) kg at ? $/kg

- Unit-numbers add on-top.
- Per-numbers add next-to as **areas** under the per-number graph. Here multiplication precedes addition.



MATHeCADEMY

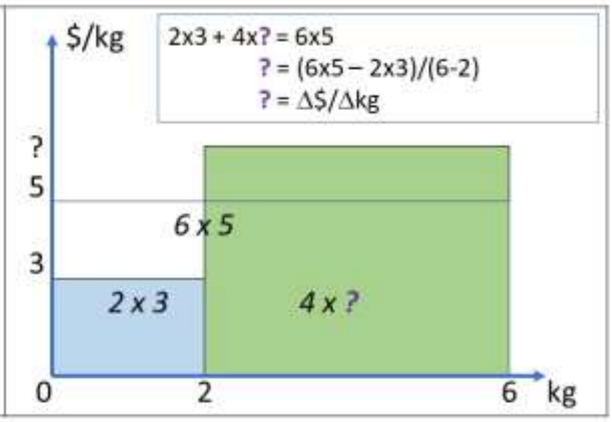
Q30, subtracting PerNumbers (differentiation)

"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

2 kg	at	3 $/kg
+ 4 kg	at	? $/kg
6 kg	at	5 $/kg

Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) precedes division.



MATHeCADEMY

Never add without units, the fraction paradox

The Teacher	The Students
What is 1/2 + 2/3?	Well, 1/2 + 2/3 = (1+2)/(2+3) = 3/5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But 1/2 of 2 cakes + 2/3 of 3 cakes is 1+2 of 2+3 cakes, i.e. 3/5 of 5 cakes! How can it be 7 cakes out of 6 cakes?
Inside this classroom 1/2 + 2/3 **IS** 7/6 !	

Fractions are not numbers, but operators, needing numbers to become numbers.

2+3 IS 5! <u>No</u>, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.

2*3 IS 6! <u>Yes</u>, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.

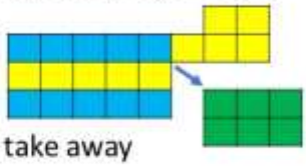
Adding without units: MatheMatism.

Mixing English and metric units made NASA's Mars Climate Orbiter fail in 1999.

MATHeCADEMY

Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		NextTo
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 8/2	? = 8-2	? = (3.2 **8s** – 2 **3s**)/5
Solved by ReCounting	*Solved by ReStacking*	*Solved by differentiation: (T-T1)/5 = ΔT/5*



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

MATHeCADEMY

Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: Opposite Side with OppoSite Sign

MetaMath (Don't test, but DO remember the bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
$\Updownarrow$	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
$\Updownarrow$	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
$\Updownarrow$	*u* x (2 x (½)) = 4	Applying the **associative** law
$\Updownarrow$	*u* x 1 = 4	Applying the definition of an inverse element
$\Updownarrow$	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

MATHeCADEMY

Conclusions

What Mastery of Many does the child have already?

- Children typically see Many as blocks with a number af bundles, and use flexible numbers with units and with over- or underloads

In ManyMath, BLOCKS are fundamental:

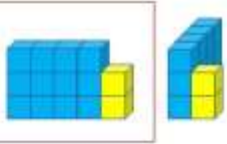
- in numbers: 456 = three blocks

- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks

MATHeCADEMY

The child's own math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving <u>outside</u> geometrical multi-blocks, & (when turned to hide the units behind) <u>inside</u> algebraic bundle-numbers.

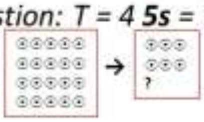
3) Operations are icons, showing the 3 counting steps: Removing bundles & stacking bundles & removing stacks to find the unbundled.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

*Question: T = 4 **5s** = ? **3s*** • *Answer: T = 4 **5s** = 6**B**2 **3s*** • *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



MATHeCADEMY

Four ways to unite into a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration). Addition and multiplication add changing and constant unit-numbers. Integration and power add changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square'.

Operations unite	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n**	**T = a * n**
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn**	**T = a^n**

MATHeCADEMY

Five ways to split a Total

The 4 uniting operations (+, *, ^, ∫) each has a reverse splitting operation: Addition has subtraction (–), and multiplication has division (/). Power has factor-finding (root, √) and factor-counting (logarithm, log). Integration has per-number finding (differentiation dT/dn = T').

Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / *split into*	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $\log_a T = n$, $\sqrt[n]{T} = a$

MATHeCADEMY

What is 3/5 of 2/4

What is 3 per 5 of 2 per 4?

Since we are going to recount in both 5s and 4s, we might take 20 as the total.

2per4 of 20 is what?

Well, recounting 20 in 4s we get 20 = (20/4)*4 giving (20/4)*2 = 10

3per 5 of ten is what?

Well, recounting 10 in 5s we get 10 = (10/5)*5 giving (10/5)*3 = 6

So 3 per 5 of 2 per 4 of 20 gives 6 per 20.

*Test: 6 = (6/20)*20 = 6 per 20 of 20*

So 3 per 5 of 2 per 4 is 6 per 20

MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY

MATHeCADEMY

Theoretical background

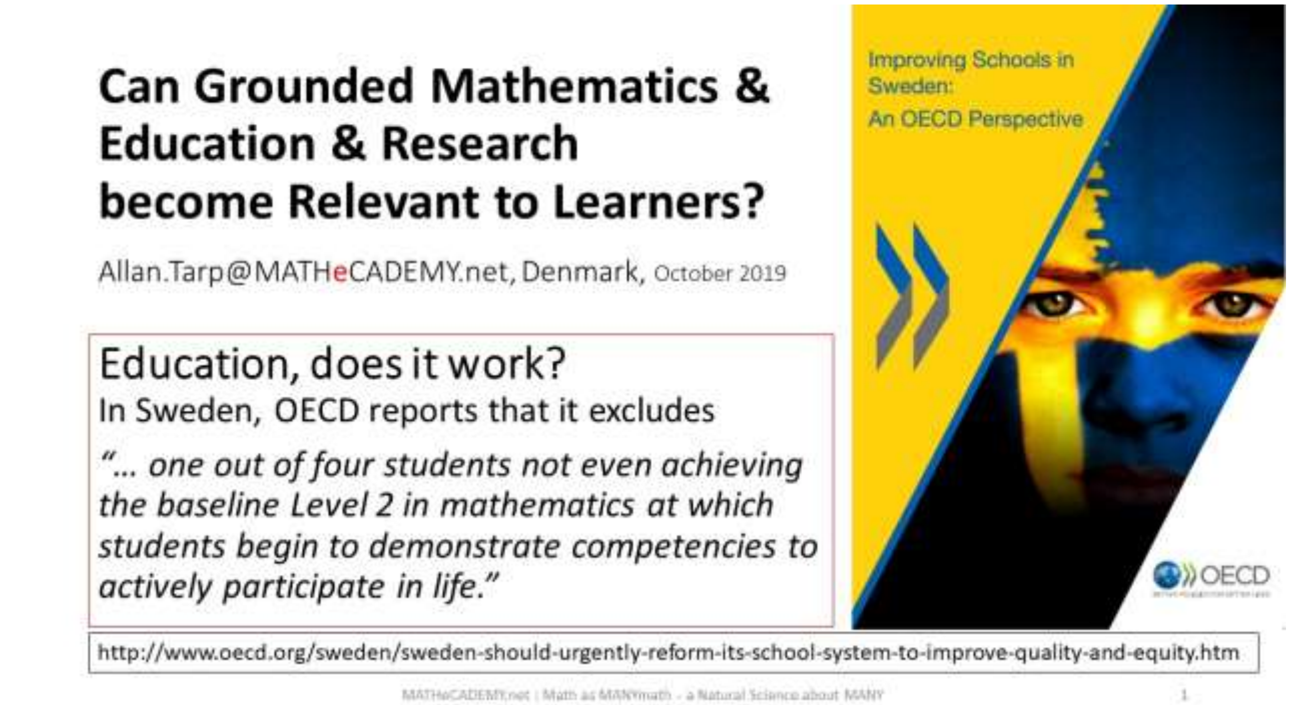
Tarp, A. (2018). Mastering Many by counting, recounting and double-counting before adding on-top and next-to.
Journal of Mathematics Education, March 2018, 11(1), 103-117.

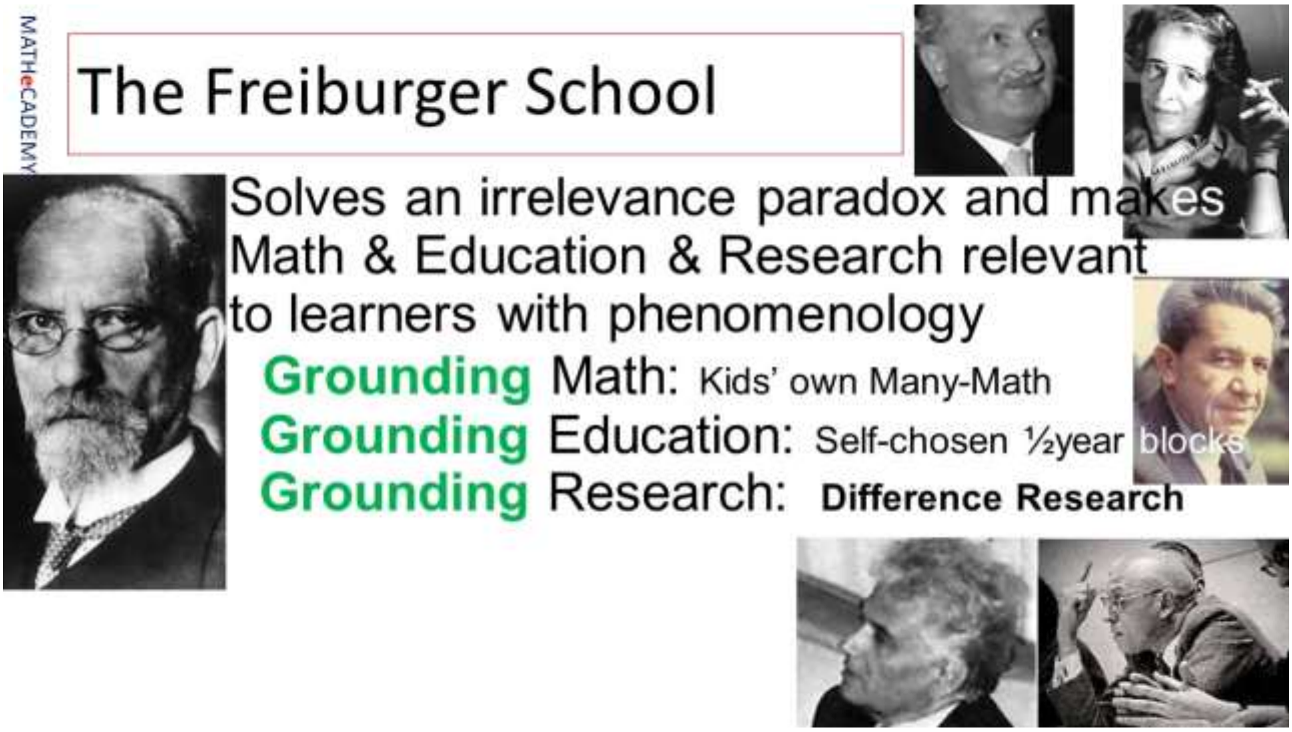
The Child's Own Mastery of Many
Count & ReCount & DoubleCount
before Adding NextTo & OnTop

master many
manymath

MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY

20. CAN GROUNDED MATHEMATICS & EDUCATION & RESEARCH BECOME RELEVANT TO LEARNERS?, PPP





Creating phenomenology, The Freiburg school looks at INSIDE appearance of OUTSIDE phenomena

Husserl: Not things in themselves, but how they appear matters.

Heidegger: The sentence „**subject IS predicate"** bridges an OUTSIDE subject with an INSIDE predicate. Trust the OUTSIDE subject, it exists; but question the INSIDE predicate: it may be an institutionalized verdict - that should be doubted and appealed.

- Arendt, looking at the job-situation in a KZ-camp: By its monoploly, an institution forces you to follow orders, which may lead to '**the banality of evil**'.
- Schütz, influencing US Pragmatism & Grounded Theory: "Does man's social being determine his consciousness, or does his consciousness determine his social being?"

Foucault, inspired by Heidegger: Education is a Pris(on)-(hos)pital

„It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them." (Chomsky & Foucault debate).

Sociologically, education is a pris-pital mixing power techniques of a prison and a hospital: The 'pati-mates' are forced to return to the same class day after day where they are treated for a self-referring diagnose (teach MATH to learn MATH) making them accept a Kafkadian verdict: "I am no good."

Derrida: **DECONSTRUCT** predicates that, by installing what they mention, create **CENTRISM**

MATHeCDMY

Freiburger School questions to math

It is not how universities define it, that matters.
It is how its root, Many, presents itself to humans working to adpat to the outside world.
”Existence precedes essence” (Existentialism)

So:

- How do children experience Many?
- How well-defined is math education after all?

MATHeCDMY

How well-defined is mathematics?

This statement is true	Always	Never	Sometimes
2 + 3 = 5			
2 x 3 = 6			
1/2 + 2/3 = 7/6			
1/2 + 2/3 = 3/5			

A function is …

MATHeCDMY

Five Questions Answered

This is true	Always	Never	Sometimes
2 + 3 = 5	Only with the same unit; 2weeks + 3days = 17days		x
2 x 3 = 6	x	2x3 is 2 **3s** that can always be recounted as 6 **1s**	
$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$	1 of 2 apples + 2 of 3 apples gives 3 of 5 apples, and not 7 of 6		x
$\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$			x Only if taken of the same total

a **FUNCTION** is	for example 2+x, but not 2+3 *(pre-setcentric)*
	an example of a many-1 set relation *(setcentric)*
	a number-language sentence *(post-setcentric)*

MATHeCADEMY

Post-setcentric math: math through its use, as with the other language in our 2 language houses

The **WORD-language** assigns words in sentences with The **NUMBER-language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

	WORD-language	NUMBER-language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
INSIDE Language	This is a chair	T = 3x4
OUTSIDE World		

Mathematics

Mother tongue

MATHeCADEMY

The **Communicative Turn** in language ed.

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: "A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us."
Likewise, Widdowson adopts a "communicative approach to the teaching of language" allowing more students to learn a language through its use for communication about outside things and actions.

Could mathematics also have its Communicative turn?

	WORD language	NUMBER language
(META-language, grammar)		
Inside Language	This is a chair Mother tongue	T = 3 4s Mathematics
Outside world		

MATHeCADEMY

Defining <u>MetaMatism</u> = MetaMatics+MatheMatism

MetaMatics is defining a concept, not as a ~~BottomUp abstraction from many examples~~ but as a <u>TopDown example of one abstraction</u>, derived from the meta-physical abstraction **SET**, made meaningless by self-reference as shown by Russell's version of the liar paradox: M belongs, only if it does not, to the set of sets not belonging to itself:

$$\boxed{\text{With } \mathbf{M} = \{\mathbf{A} \mid \mathbf{A} \notin \mathbf{A}\}: \quad \mathbf{M} \in \mathbf{M} \Leftrightarrow \mathbf{M} \notin \mathbf{M}}$$

MatheMatism is a statement that is correct inside, but seldom outside a classroom , as e.g. adding numbers without units as <u>2+3 = 5</u>, where e.g. 2**w**+3**d**=17**d**. In contrast to <u>2x3 = 6</u> saying that 2 **3s** can be recounted as 6 **1s**.

Neglecting English and metric units made NASA's Mars Climate Orbiter CRASH in 1999.

Education? Two different kinds

The 1700 Enlightenment Century rooted education, but in different forms in its two republics, in North America in 1776 and in France in 1789.

- In North America, education enlightens children about their OUTSIDE world, and enlightens teenagers about their INSIDE individual talent, to be uncovered and developed through self-chosen ½year **BLOCKS** with teachers teaching only one subject in the teacher's own classroom.
- To protect its republic from its German speaking neighbors, France was forced to create institutions controlled by a strong central administration with public servants trained at elite schools with forced multi-year **LINES**, later copied by the German Bildung-education (and by the rest of Europe).

3x2 different kinds of math education

Mathematics in	self-chosen ½year BLOCKS	forced multi-year LINES
Pre-SETcentric	North America	UK Commonwealth
Present SETcentric	-	Continental Europe
Post-SETcentric	MATHeCADEMY.net	

MATHeCADEMY

Why teach children if they already know?

With education curing the diagnose un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

Core Questions:

- What Mastery does children develop when adapting to Many?
- What could be a Question-guided <u>Child-Grounded-Curriculum</u> in Quantitative Competence?

MATHeCADEMY

Children see Many as double-numbers, as bundles with units

"How old next time?" A 3year old says "Four" showing 4 fingers: I I I I

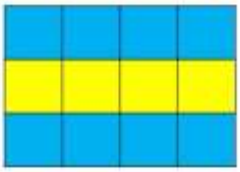
But, the child reacts strongly to 4 fingers held together 2 by 2: II II

"That is not four, that is two twos" (T ≠ 4, T = 2 **2s**)

The child sees what exists, and with units: bundles of **2s**, and 2 of them.

The block 3 **4s** has two numbers:

3 (the counting-number) and **4** (the unit-number)



MATHeCADEMY

Exploring children's double-numbers

Tarp, A. (2018). Mastering Many by counting, recounting and double-counting before adding on-top and next-to.
Journal of Mathematics Education, March 2018, 11(1), 103-117.

The Child's Own Mastery of Many
BundleCount & ReCount & DoubleCount
before Adding NextTo & OnTop

master many
manymath

MATHeCADEMY

Q01. Create icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.

An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become units to use when counting in bundles.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9

MATHeCADEMY

Q02. Bundle-counting sequences

"How to bundle-count fingers?"

Using **5s** as the bundle-size, fingers can be counted as

"0**B**1, 0**B**2, 0**B**3, 0**B**4, 0**B**5 – sorry, **Bundle**"

and the rest can be counted in as

"**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B, 1left, 2left** (a-leven, twe- leven)".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1B3 **7s** = 2B2 **4s** = 3B1 **3s** = 1BB1 **3s**.

MATHeCADEMY

Q03. Flexible bundle-numbers

"How to count by bundling?"

Five fingers can be bundle-counted in pairs or triplets, allowing both an OVERLOAD and an UNDERLOAD; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

IIIII	•	II III	•	II II I	•	II II II	•	II II I
T = 5	=	1Bundle3 **2s**	=	2B1 **2s**	=	3B-1 **2s**	=	1BB0B1 **2s**
T = 5	=	1.3 **2s**	=	2.1 **2s**	=	3.-1 **2s**	=	10.1 **2s**

Likewise, if counting in **ten**-bundles:
T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

MATHeCADEMY

Math Dislike CURED with flexible bundle-numbers

When counting in tens, before calculating, we bundle-write the number to separate the **INSIDE** bundles from the **OUTSIDE** singles. Later we recount.

- **65 + 27** = 6)5 + 2)7 = 8)12 = 9)2 = **92**
- **65 – 27** = 6)5 – 2)7 = 4)-2 = 3)8 = **38**
- **7x 48** = 7x 4)8 = 28)56 = 33)6 = **336**
- **336 /7** = 33)6 /7 = 28)56 /7 = 4)8 = **48**

*With 336 we have 33 **INSIDE**, so to get 28, so we move 5 **OUTSIDE** as 50.*
Now try 456 / 7.

- **456 /7** = 45)6 /7 = 42)36 /7 = 6)5 + 1 = **65 1/7**

19

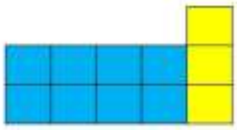
MATHeCADEMY

Q04. Unbundled as decimals or negatives or fractions
0.3 **4s** or 1.-1 **4s** or 3/4 **4s**

"Where to put the unbundled singles?"

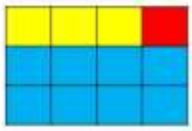
When counting by bundling, the unbundled singles can be placed

NextTo the block — counted as a block of **1s**



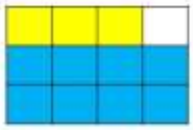
T = 2B3 **4s** = 2.3 **4s**
A decimal number

OnTop of the block — counted as a bundle



T = 3B-1 **4s** = 3.-1 **4s**
A negative number

OnTop of the block — counted in bundles



T = 2 3/4 **4s**
A fraction

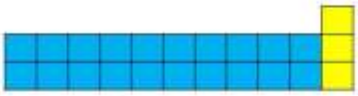
MATHeCADEMY

Q04. Counting in tens

"Where to put the unbundled singles with **tens**?"

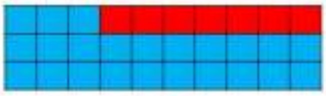
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit and the decimal point,

- or as:



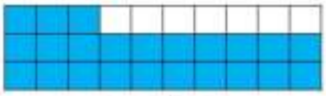
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

T = 3B-7 **tens**



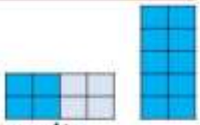
T = 2 3/10 **tens**

T = 2 3/10 **B** **tens**

MATHeCADEMY

Footnote I: Prime & foldable bundle-units

"When can blocks be folded in like bundles?"



The block T = 2 **4s** = 2 x 4 has 4 as the bundle-unit.

Turning over gives T = 4 **2s** = 4 x 2, now with 2 as the bundle-unit.

4s can be folded in another bundle as 2 **2s**, whereas 2s cannot.

(1 is not a bundle, nor a unit since a bundle-of-bundles stays as 1).

We call 2 a **prime bundle-unit** and 4 a **foldable bundle-unit**, 4 = 2 **2s**.

A block of 3 **2s** cannot be folded.

A block of 3 **4s** can be folded: T = 3 **4s** = 3 x (2 x 2) = (3 x 2) x 2 = 2 **3x2s**.

A number is called **even** if it can be written with 2 as the unit, else **odd**.

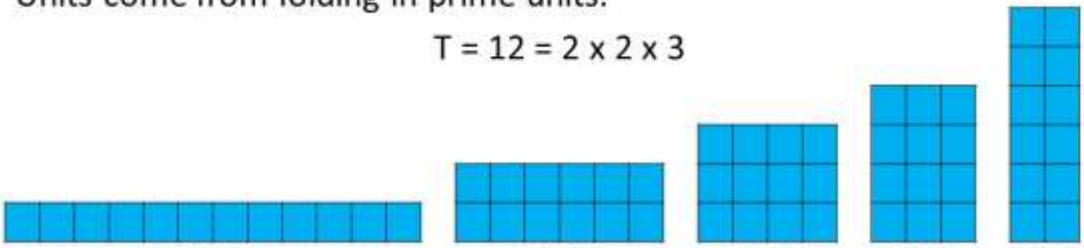
MATHeCADEMY

Footnote II: Finding possible units

"What are possible units in T = 12?"

Units come from folding in prime units:

T = 12 = 2 x 2 x 3



T = 12 **1s** = 2 **2x3s** = 3 **2x2s** = 2x2 **3s** = 2x3 **2s**

MATHeCADEMY

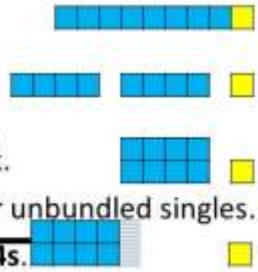
Q05. Calculators predict

"Can a calculator predict a counting result?"

We may see <u>division</u> as an icon for a broom **PUSHING** away bundles: 9/4 means 'from 9, push away bundles of 4s'.

- The calculator says '2.some', thus predicting it can be done 2 times. Now <u>multiplication</u> iconizes a lift **STACKING** the bundles into a block.

- And, <u>subtraction</u> iconizes a rope **PULLING** away the block to look for unbundled singles.
- With '9-2x4 = 1' the calculator predicts: 9 can be recounted as 2B1 **4s**.



9/4	**2.some**
9 – 2x4	**1**

MATHeCADEMY

Q05. Counting creates a ReCount-formula

Bundle-counting T pushes away B-bundles to stack T/B times:

T = (T/B) x B	from a total **T**, **T/B** times, we push **B** away

As sentences of the number language, **FORMULAS PREDICT** that **T** = 9 = 2.1 **4s:**

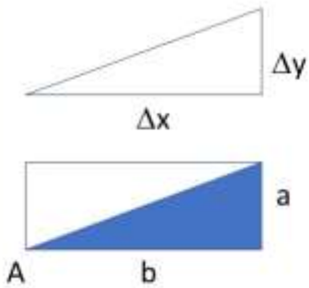
9/4	**2.some**
9 – 2x4	**1**

MATHeCADEMY

Q05. The recount-formula is a core formula

T = (T/B)*B is all over STEM (Science, Technology, Engineering, Mathematics):

Proportionality	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **velocity** * second



MATHeCADEMY

Footnote I. Recounting in STEM formulas

STEM typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

MATHeCADEMY

Footnote II. Recounting in STEM formulas

Extra STEM examples:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec;
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = (energy/charge) x (charge/sec) or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

MATHeCADEMY

Footnote III: Proportionality shows the diversity of 'School Math'

Proportionality, **Q1**: "2kg costs 5$, what does 7kg cost"; **Q2**: "What does 12$ buy?"

→1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: '2kg cost 5$, 7kg cost ?$'. Multiply-then-divide gives the $-number 7x5/2 = 17.5.

Q2: '5$ buys 2kg, 12$ buys ?kg'. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

→2) Find the unit

Q1: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

→3) Cross multiplication

Q1: $2/5 = 7/u$, so $2*u = 7*5$, $u = (7*5)/2 = 17.5$. **Q2**: $2/5 = u/12$, so $5*u = 12*2$, $u = (12*2)/5 = 4.8$

→4) 'Re-counting' in the 'per-number' 2kg/5$ coming from 'double-counting' the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

MATHeCADEMY

Footnote IV: Proportionality shows the diversity of 'School Math'

→ 5) Modeling with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive $-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$$.
- With 12$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

MATHeCADEMY

Q06. Recounting in a different unit

"How to change a unit?"

The recount-formula allows changing the unit.

Asking T = 3 **4s** = ? **5s**, the recount-formula gives T = 3 **4s** = (3x4/5) **5s**.

Entering 3x4/5, the answer '2.some' shows that a block of 2 **5s** can be pushed away.

With 3x4–2x5, the answer '2' shows that 3 **4s** can be recounted as 2B2 **5s** or 2.2 **5s**.

3 **4**s = IIII IIII IIII = IIII I III II II = IIIII IIIII II = 2B2 **5**s = 2.2 **5s**

3x4/5	**2**.some
3x4 – 2x5	**2**

Change Unit = Proportionality

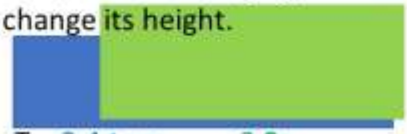
MATHeCADEMY

Q07. ReCounting from tens to icons

"How to change unit from tens to icons?"

Asking 'T = 2.4 **tens** = 24 = ? **8s**', we just recount 24 in **8s**:

T = 24 = (24/8)x8 = 3x8 = 3 **8s.**

Formulated as an **equation** we use ***u*** for the unknown number, ***u***x8 = 24. Recounting 24 in 8s shows that ***u*** is 24/8. So, equations are solved by moving **to opposite side - with opposite sign**	To keep its size, a block changing its unit must also change its height.  T = 2.4 tens = 3 8s

u x 8 = 24 = (24/8)x8

u = 24/8 = 3

MATHeCADEMY

Q08. ReCounting from icons to tens (multiplication) 3 7s = ? tens

"How to change unit from icons to tens?"

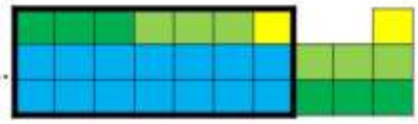
Asking 'T = 3 **7s** = ? **tens**', the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone: T = 3 **7s** = 3x7 = 21 = 2.1 **tens**, only it leaves out the unit and the decimal point.

Alternatively, we may use 'less-numbers', so 7 = **ten** less 3

T = 3x7 = 3 x (**ten** less 3) = 3 x **ten** less 3x3 = 3**ten** less 9 = 2**ten**1 = 21,

or with 9 = **ten** less 1:

T = 3**ten** less (**ten** less1) = 2**ten** lessless 1 = 2**ten** & 1 = 21.



showing that 'lessless' cancel out

MATHeCADEMY

Recounting large numbers in or from tens: *same size, but new form*

Recounting 6 47s in tens Recounting 476 in 7s

*BundleWriting seprates **INSIDE** bundles from **OUTSIDE** singles*

T = 6 x 47 = 6 x 4B7	T = 476 = 47.6 tens
= 24B42	= 47B6
= 28B2	= 42B56
= 28.2	= 6x7B8x7
tens	= 68 x 7

MATHeCADEMY

Q09. Double-counting in two units creates DoubleNumbers or PerNumbers

"How to double-count in two units?"

DoubleCounting in kg & $, we get **4kg = 5$** or
4kg **per** 5$ = 4kg/5$ = 4/5 kg/$ = a PerNumber.

With 4kg bridged to 5$ we answer questions by recounting in the per-number.

Questions:

7kg = ?$	**8$ = ?kg**
7kg = (7/4) x 4kg = (7/4) x 5$ = 8.75$	8$ = (8/5) x 5$ = (8/5) x 4kg = 6.4kg

Answer: *Recount in the PerNumber (Proportionality)*

MATHeCADEMY

Q09. Double-counting in the same unit creates fractions

"How to double-count in the same unit?"

Double-counted in the same unit, per-numbers are fractions, 2$ per 9$ = 2/9, or percentages, 2 per 100 = 2/100 = 2%.

To find a fraction or a percentage of a total, again we just recount in the per-number.

- **Taking 3 per 4 = taking ? per 100.** With 3 bridged to 4, we recount 100 in 4s:
100 = (100/4)*4 giving (100/4)*3 = 75, and 75 per 100 = 75%.
- **Taking 3 per 4 of 60 gives ?** With 3 bridged to 4, we recount 60 in 4s:
60 = (60/4)*4 giving (60/4)*3 = 45.
- **Taking 20 per 100 of 60 gives ?** With 20 bridged to 100, we recount 60 in 100s:
60 = (60/100)*100 giving (60/100)*20 = 12 .

We observe that per-numbers and fractions are not numbers but OPERATORS needing a number to become a number.

MATHeCADEMY

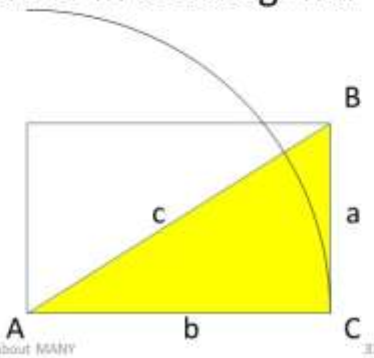
Q10. ReCounting sides in a block: Trigonometry

A right triangle is <u>a block halved by its diagonal</u> giving 3 sides: <u>base b</u>, <u>height a</u> and <u>diagonal c</u> connected with the angles when recounting one side in the other side or in the diagonal

T = (T/B) * B

a = (a/c)*c = sinA * c
b = (b/c)*c = cosA * c
tanA = a/b = Δy/Δx = gradient

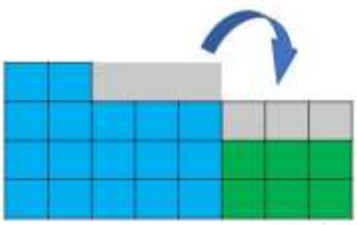
Circle: circum./diam. = $\pi \approx$ **n*tan(180/n)** for n large

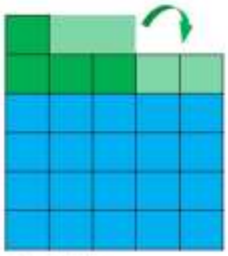


MATHeCADEMY

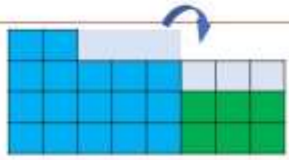
Once bundle-counted & recounted, Totals can add

BUT: NextTo	or OnTop
4 5s + 2 3s = 3B2 8s	4 5s + 2 3s = 4 5s + 1B1 5s = 5B1 5s
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





Q11. NextTo addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added next-to as **8s**?"

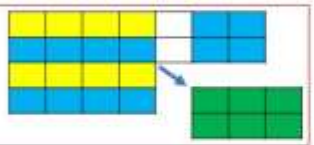
Outside, next-to addition geometrically means adding areas.
Next-to addition is also called integral calculus.

Inside, the recount formula algebraically predicts the result.
Here multiplication precedes addition.

T = (T/**B**) x **B**
= ((4x5 + 2x3)/8) x 8 = 3.2 **8s**

(4x5 + 2x3)/8	**3.some**
(4x5 + 2x3) – 3x8	**2**

Q12. Reversed NextTo addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

Outside, we remove the initial block T1 and recount the rest in **4s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

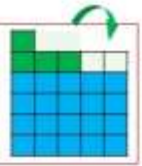
Here subtraction precedes division; which is natural as reversed integration.

T2 = (T2/**B**) x **B**
= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

MATHeCADEMY

Q13. OnTop addition

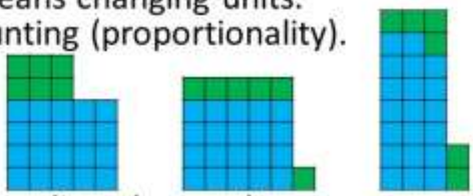


"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added on-top?"

Outside, on-top addition geometrically means changing units.
On-top addition thus often involves recounting (proportionality).

T = 4 **5s** + 2 **3s** = 4 **5s** + 1.1 **5s** = 5.1 **5s**

T = 4 **5s** + 2 **3s** = 6.2 **3s** + 2 **3s** = 8.2 **3s**



Inside, the recount formula algebraically predicts the result.
Here again, multiplication precedes addition.

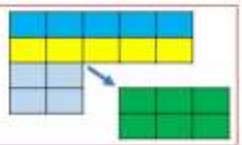
T = (T/**B**) x **B**

= ((4x5 + 2x3)/5) x 5 = 5.1 **5s**

(4x5 + 2x3)/5	**5.some**
(4x5 + 2x3) – 5x5	**1**

MATHeCADEMY

Q14. Reversed OnTop addition



"T1 = 2 **3s** and how many **5s** (T2) add on-top as 4 **5s**?"

Outside, we remove the initial block T1 and recount the rest in **5s**.

Thus reversed next-to addition geometrically means subtracting areas.

Reversed on-top addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

T2 = (T2/**B**) x **B**

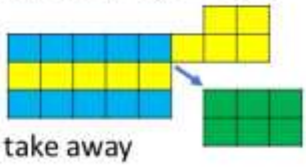
= ((4x5 – 2x3)/5) x 5 = 2.4 **5s**

(4x5 – 2x3)/5	**2.some**
(4x5 – 2x3) – 2x5	**4**

MATHeCADEMY

Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 8/2	? = 8-2	? = (3.2 **8s** – 2 **3s**)/5
Solved by ReCounting	*Solved by ReStacking*	*Solved by differentiation: (T-T1)/5 = ΔT/5*



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

MATHeCADEMY

Solving equations by recounting, we may bracket Group Theory from Abstract Algebra

ManyMath

2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: Opposite Side with OppoSite Sign

MetaMath (Don't test, but DO remember the bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
⇕	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
⇕	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕	*u* x (2 x (½)) = 4	Applying the **associative** law
⇕	*u* x 1 = 4	Applying the definition of an inverse element
⇕	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

MATHeCADEMY

Four ways to unite and split Totals

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the four ways to add: +, *, ^, next-to block-addition (integration).
Addition and multiplication add changing and constant unit-numbers.
Integration and power add changing and constant per-numbers.
We might call this beautiful simplicity the 'Algebra Square'.

Operations unite/ *split into*	**Changing**	**Constant**
Unit-numbers *m, s, \$, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, \$/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $log_aT = n$, $\sqrt[n]{T} = a$

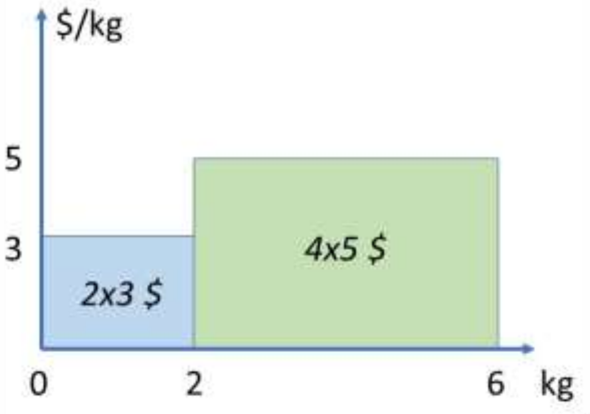
MATHeCADEMY

Q15. Adding PerNumbers as areas (integration)

"2kg at 3\$/kg + 4kg at 5\$/kg = 6kg at ? \$/kg?"

2 kg at 3 \$/kg
+ 4 kg at 5 \$/kg
(2+4) kg at ? \$/kg

- Unit-numbers add on-top.
- Per-numbers add next-to as **areas** under the per-number graph. Here multiplication precedes addition.



MATHeCADEMY

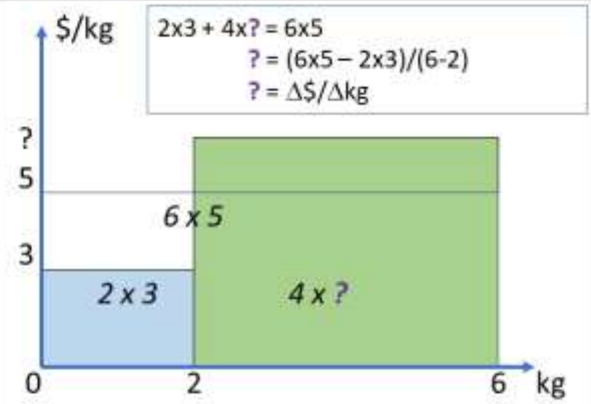
Q16. Subtracting PerNumbers (differentiation)

"2kg at 3\$/kg + 4kg at **what** = 6kg at 5\$/kg?"

2 kg	at	3 \$/kg
+ 4 kg	at	? \$/kg
6 kg	at	5 \$/kg

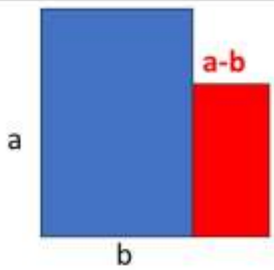
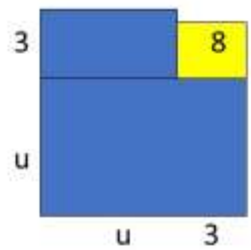
Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) precedes division.



MATHeCADEMY

Q17. Geometry & Algebra, hand in hand

Quadratic Rule with 2 Cards	**Quadratic Equations with 3 Cards**
	u^2 + 6u + 8 = 0 
Corner = (a-b)^2 = a^2 – 2 cards + b^2 So (a-b)^2 = a^2 – 2 x a x b + b^2	(u+3)^2 = u^2 + 6u + 8 + 1 (u+3)^2 = 0 + 1 u = -3 ± 1 <u>u = -4 & u = -2</u>

MATHeCADEMY

Conclusion

What Mastery of Many does the child have already?

- Children typically see Many as blocks with a number af bundles, and use flexible numbers with units and with over- or underloads

In ManyMath, BLOCKS are fundamental:

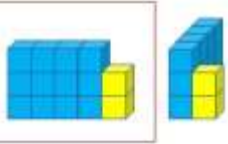
- in numbers: 456 = three blocks

- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks

MATHeCADEMY

The child's own Many-math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving <u>outside</u> geometrical multi-blocks, & (when turned to hide the units behind) <u>inside</u> algebraic bundle-numbers.

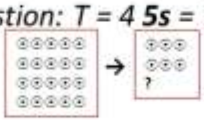
3) Operations are icons, showing the 3 counting steps: Removing bundles & stacking bundles & removing stacks to find the unbundled.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting & double-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** (from T, T/B times, Bs can be taken away)

Question: T = 4 ***5s*** *= ?* ***3s*** • *Answer: T = 4* ***5s*** *= 6B2* ***3s*** • *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



A final footnote from the Frankfurter School

- With Habermas we may ask: How can a math teacher use communicative rationality to establish a non-patronizing power-free rational dialogue with grade one children about the objective fact Many, present in both the children and the teacher's life-world; thus accepting four fingers held together two by two being rationalized as (as do the children) 'the total I two twos' and not just as 'four'?
- With Marx quoted in the Berlin Humboldt University we may agree:

Recommendation from the Freiburger School

STOP teaching and researching wrong numbers and wrong operations

- **START** to accept and develop the child's own flexible bundle-numbers and (re)counting operations

STOP using education as pris-pitals, fixing and diagnosing humans

- **START** guiding the child when exploring its outside world, and the teenager when exploring the inside self via self-chosen ½year blocks

STOP forcing the outside world to adapt to inside rigid theory

- **START** forcing inside theory to adapt flexibly to outside existence

MATHeCADEMY

Philosophizing the low success of 50 years of mathematics education research

Mathematics also needs a COMMUNICATIVE TURN where

- instead of learned **INSIDE-INSIDE** through its grammar, it is learned **OUTSIDE-INSIDE** as a INSIDE number-language communicating about OUTSIDE things and actions, thus learned through its use, and not before its use
- instead of learning about numbers, students learn how to number, and how to communicate about Many in full sentences containing:

  1) an **OUTSIDE** subject, 2) a linking verb, and 3) an **INSIDE** predicate: **T = 2x3**

So, maybe we need an **OUTSIDE-INSIDE post-SETcentric mathematics** to replace the present **INSIDE-INSIDE meta-matism** by asking:

What kind of mathematics grows from the Mastery of Many that children develop through use, and before school?

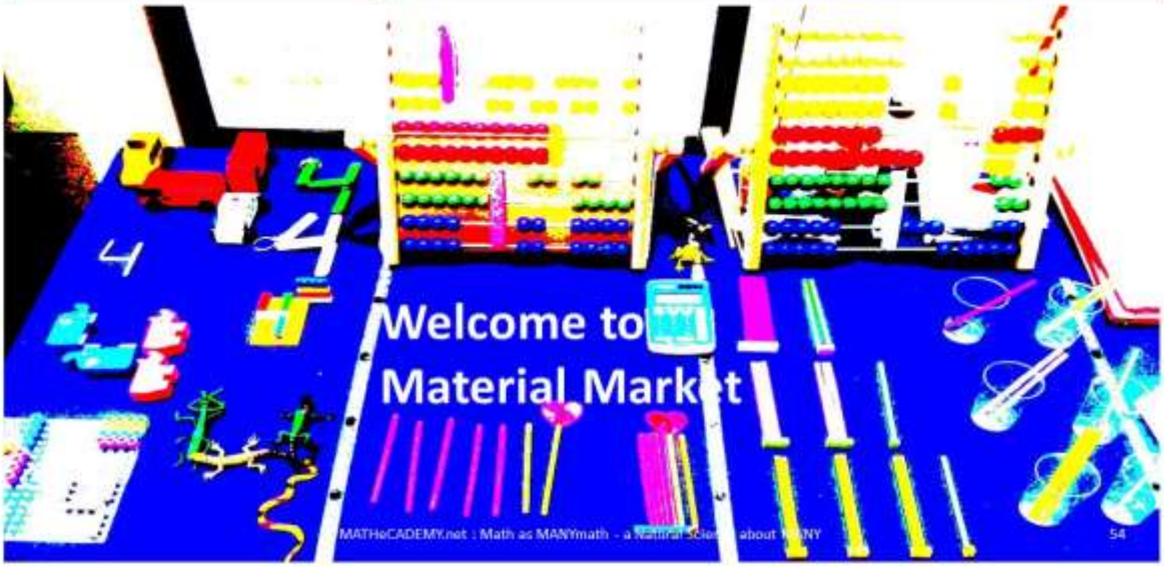
Pablo Picasso: It took me four years to paint like Raphael, but a lifetime to paint like a child

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY

MATHeCADEMY

Number Icons

ReCounting 7 in **5s** & **3s** & **2s**



33. DE-MODEL NUMBERS, OPERATIONS AND EQUATIONS, PPP

DE-MODEL
Numbers & Operations & Equations

Understanding? INSIDE-INSIDE or OUTSIDE-INSIDE

Adaptive Mathematics: Kids' own DoubleNumbers

Bundle- & Per-Numbers in Primary & Secondary School

The Goal of Math Education is to:

master outside many, or

master inside math (to later master **outside** many)

Allan.Tarp@MATHeCADEMY.net, Denmark, December 2019

MATHeCADEMY.net

Xin Chào, Welcome, Velkommen

I am glad to be in Vietnam with its rich ancient culture.
I am sorry, I do not speak Vietnamese.
In Denmark/Greenland, we speak ancient English, or 'Anglish', a western Danish dialect from the Viking times.
As coast-people, the Danes sailed all over, and brought their language when they settled in the nearby lands:
England, Scotland, Ireland, Iceland, Greenland.

(An extended PPP is on the MATHeCADEMY.net site)

MATHeCADEMY.net

Observation 01: Is Mathematics WellDefined?

No, three Versions: MetaMatics, MatheMatism, ManyMath

This is true	Always Never Sometimes
2 + 3 = 5	Only with the same unit; 2weeks + 3days = 17days **x** (MatheMatsim)
2 x 3 = 6	**x** 2x3 is 2 **3s** that can always be recounted as 6 **1s** (ManyMath)
$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$	 **x** (ManyMath) 1 of 2 apples + 2 of 3 apples gives 3 of 5 apples, and not 7 of 6
$\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$	Only if taken of the same total **x** (MatheMatsim) *Fractions are not numbers, but operators, needing numbers to become numbers*
C1: a **FUNCTION** is	**For example** 2+*x*, but not 2+3 i.e. a name for a calculation with an unspecified number **(1750-1900)** (ManyMath)
C2:	**An example of** a SET-relation where first component identity implies second component identity **(after 1900)** (MetaMatics)

MATHeCADEMY.net

Observation 02: Adapting to Many, Children create **Flexible BundleNumbers** with Units

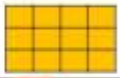
"How old next time?" A 3year old says "Four" showing 4 fingers: **I I I I**

But, reacts strongly to 4 fingers held together 2 by 2: **II II**

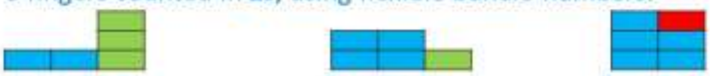
"That is not four, that is two **twos**"

Adapting to Many, a child uses BUNDLE-NUMBERS to describe what exists, and with units: bundles of **2s**, and 2 of them.

The block 3 **4s** has two numbers: 3 (the counting-number) & 4 (the unit-number)



5 fingers counted in **2s**, using flexible bundle-numbers:



5 = 1B3 **2s** | 5 = 2B1 **2s** | 5 = 3B-1 **2s**

Over-load | Normal | Under-load

MATHeCADEMY.net

Bundle-Numbers can Shift Unit and create a RecountFormula

8 = (8/2)x2
T = (T/B)xB

Bundle-counting 8 in 2s: 8 = ? **2s**

Fingers: 8 = 4 **2s**

Bricks bundle-counted: 8 = 8/2 **2s**

Bundles stacked: 4 **2s** = 4x2

Bundle-counting: 8 = (8/2)x2

Recount-Formula: **T = (T/B)xB**

8/2: From 8 **PUSH** away 2

4 times **LIFT** 2

*u*x2 = 8 = (8/2)x2

***u* = 8/2**

OPPOSITE side & sign

Equations Proportionality

Shifting unit	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
STEM	meter = (**meter/sec**) * sec = **speed** * sec

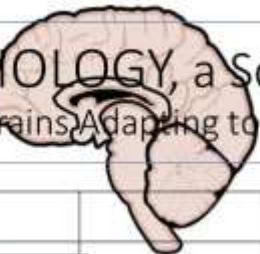
Finding: ReCounting in BundleNumbers contains Core Mathematics & STEM

MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY 5

MATHeCADEMY.net

PSYCHOLOGY, a Science about Behavior & Brain

0,1,2,3 Brains Adapting to Life on the 3rd Rock from the Sun

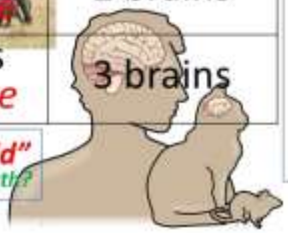


Sun		Earth	Life	Brains
		Green Cells	Plants *Quantity*	No brain
Matter + Antimatter	Energy Radiation		Reptiles *Mutation*	1 brain
		Grey Cells	Mammals *ChildCare*	2 brains
			Humans *Language*	3 brains

Holes in the head for food and information

Darwin: "To *SURVIVE*, you must ***ADAPT to the outside world***"

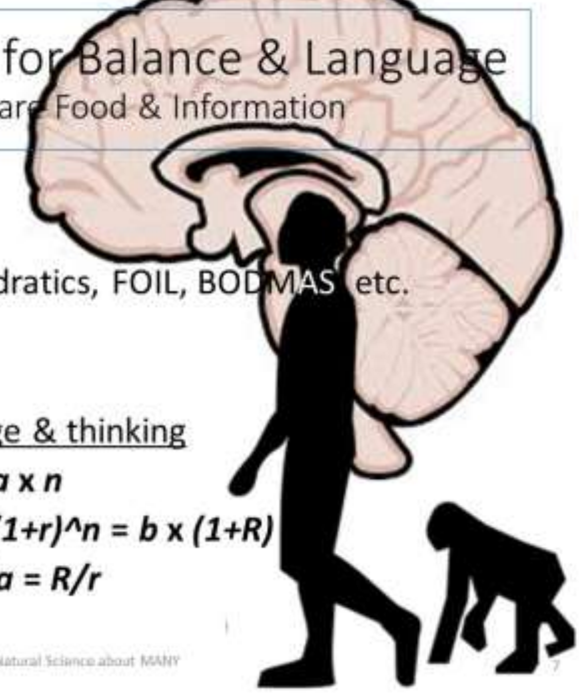
And so must math?



MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY 6

Standing up Created a Brain for Balance & Language

Forelegs became Hands to Grasp and Share Food & Information



Humans have 3 brains:

<u>**A Reptile Brain** for routines</u>

- By heart, I know tables, formulas, quadratics, FOIL, BODMAS etc.

<u>**A Mammal Brain** for feelings</u>

- I **LIKE** math; I **DISLIKE** math

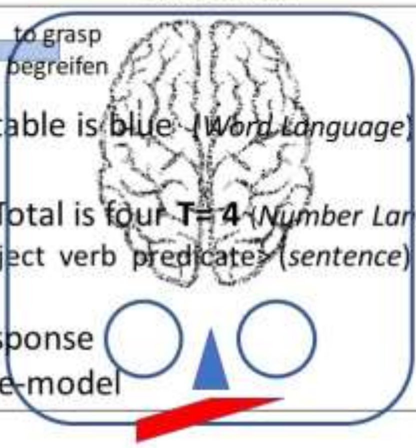
<u>**A Human Brain** for information, language & thinking</u>

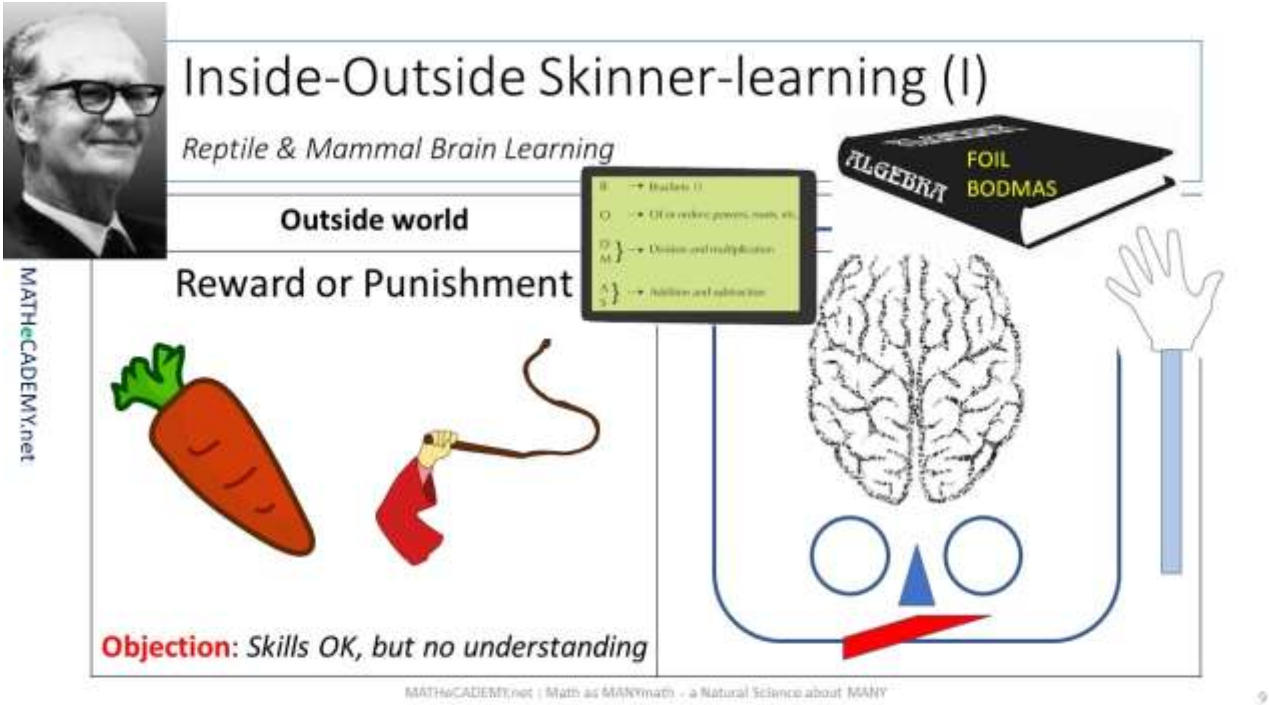
- Total after *n* times adding *a*\$: $T = b + a \times n$
- Total after *n* times adding *r*% : $T = b \times (1+r)^n = b \times (1+R)$
- Total after *n* times adding *a*\$ & *r*% : $T/a = R/r$

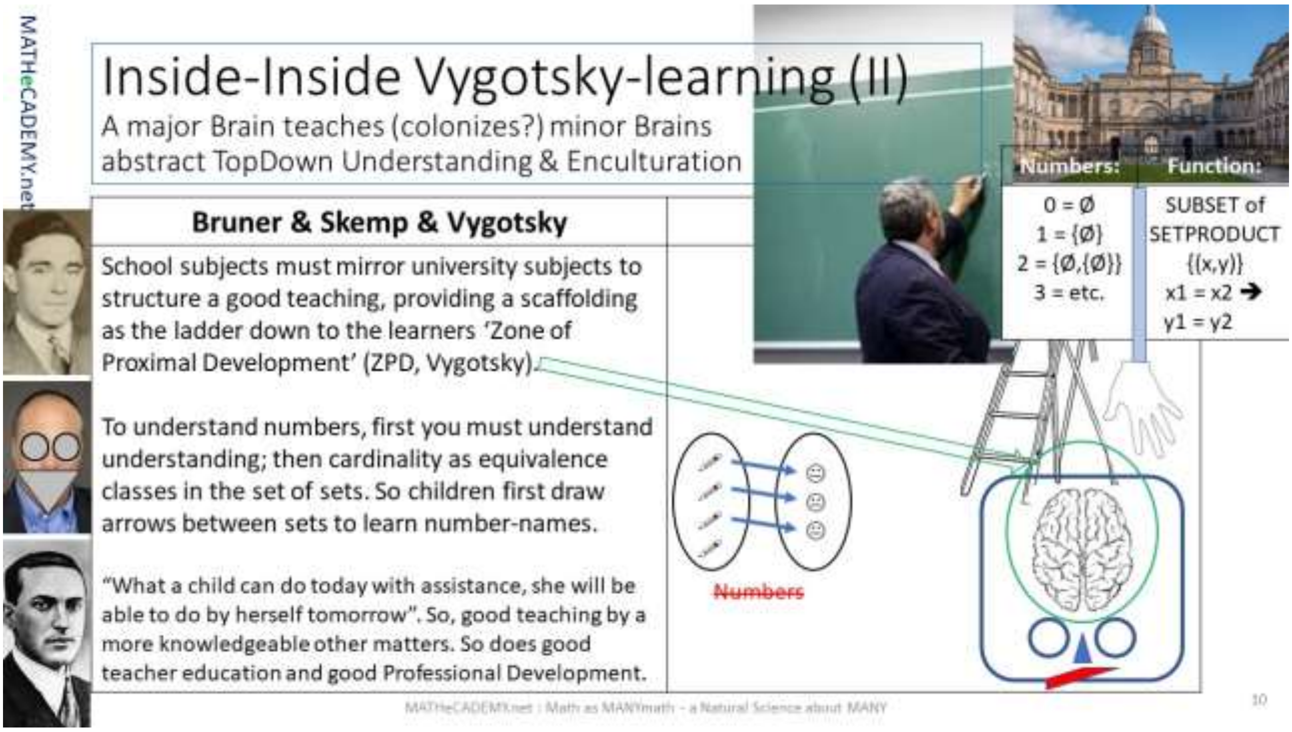
A Brain Adapts through Stimuli & Response

OUTSIDE Stimulus ⏩ ⏪ INSIDE Response

Outside world	Inside brain
 Grasp Greifen	to grasp begreifen
	The table is blue (*Word Language*)
IIII	The Total is four **T = 4** (*Number Language*) <Subject verb predicate> (*sentence*)
Stimulus ⏩	⏪ Response
Model ⏩	⏪ De-model




Inside-Outside Skinner-learning (I)
Reptile & Mammal Brain Learning
Outside world
Reward or Punishment
ALGEBRA
FOIL
BODMAS
Objection: Skills OK, but no understanding
MATHeCADEMY.net
MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY
9


Inside-Inside Vygotsky-learning (II)
A major Brain teaches (colonizes?) minor Brains
abstract TopDown Understanding & Enculturation
Bruner & Skemp & Vygotsky
School subjects must mirror university subjects to structure a good teaching, providing a scaffolding as the ladder down to the learners 'Zone of Proximal Development' (ZPD, Vygotsky).
To understand numbers, first you must understand understanding; then cardinality as equivalence classes in the set of sets. So children first draw arrows between sets to learn number-names.
"What a child can do today with assistance, she will be able to do by herself tomorrow". So, good teaching by a more knowledgeable other matters. So does good teacher education and good Professional Development.
Numbers:
0 = Ø
1 = {Ø}
2 = {Ø,{Ø}}
3 = etc.
Function:
SUBSET of SETPRODUCT
{(x,y)}
x1 = x2 ➔
y1 = y2
~~Numbers~~
MATHeCADEMY.net
MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY
10

Outside-Inside Piaget-learning (III)

Bottom-up Understanding through Concrete Examples
Human Brain Learning

MATHeCADEMY.net

Outside world	Inside brain

Numbers, Operations *manual*	Functions are Sentences *verbal*
	T = 2+3 No T = 2+? Yes

Adaption ⏩ Schemata
Validate ⏪ Assimilate
Resistance ⏩ Accomodate

"Every time we teach a child something, we keep him from inventing it himself. On the other hand, that which we allow him to discover for himself will remain with him visible for the rest of his life"

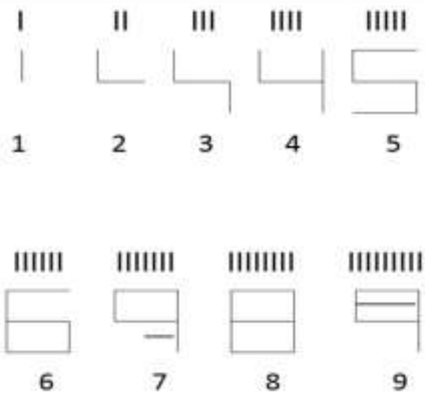
Research:
Grounded Theory =
Collective Schemata

MATHeCADEMY.net

DeModel Digits as Icons

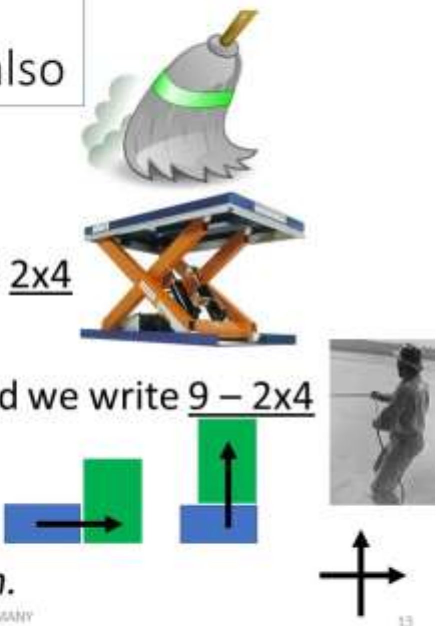
with as Many Sticks as they Represent: 4 sticks in the 4-icon etc.

Outside world	Inside brain



What? Digits are icons with as many sticks as they represent!
But, why does ten not have an icon?
Oh, it is a **Bundle**, so ten is 1**Bundle**0.
So eleven is 1**B**1, and twelve is 1**B**2.
Ah, the Vikings liked to shorten:
1**B**1 became 1-left (one-leven),
1**B**2 became 2-left (twe-leven).

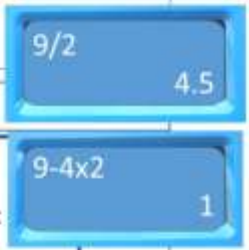
DeModel Division & Multiplication & Subtraction & Addition as Icons also

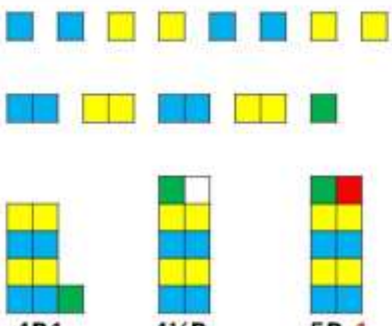


- From 9 **PUSH** away **4s** we write 9/4 iconized by a broom, called *division*.
- 2 times **LIFTING** the **4s** to a stack we write 2x4 iconized by a lift called *multiplication*.
- From 9 **PULL** away 2 **4s**' to find un-bundled we write 9 – 2x4 iconized by a rope, called *subtraction*.
- **UNITING** next-to or on-top we write A+C iconized by two directions, called *addition*.

BundleCounting a Total of 9 in 2s

9/2	**4.some**
9 – 4x2	**1**

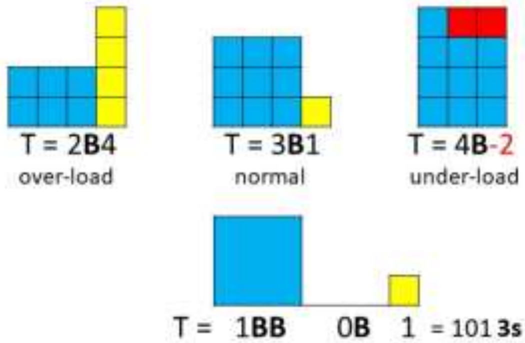


Outside World		Inside Brain
 **9**		From 9, 9/2 times push away 2. From 9, pull away 4 **2s**, leaving 1. Prediction by the recount-formula: T = 9 = 4**B**1 **2s**
	bundled in ***2s*** *with 1 unbundled*	The unbundled can be placed • next-to the stack iconized by a dot named a decimal point; 4.1 **2s**; or on-top of the stack • counted in bundles as 1 = (1/2) x 2 giving 4½**B 2s**, • counting what is missing in a full bundle, 5**B**-1 **2s**.
4**B**1 4½**B** **2s** 5**B**-1	*stacked as 4x2 with 1 unbundled* *placed next-to or on-top*	This de-models decimals, fractions & negatives.

MATHeCADEMY.net

BundleCounting Fingers in **3s**

Over-load, Normal, Under-load	Singles, Bundles, Bundle-Bundles
Two hands bundle-counted in **3s**:	Counting-sequence bundle-counting in **3s**:



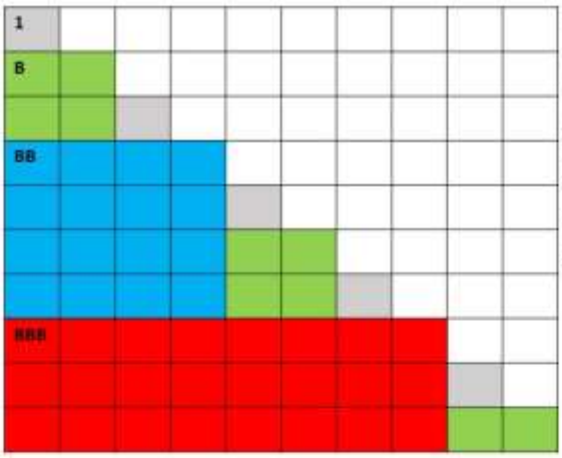
0**B**1, 0**B**2, ~~0**B**3~~ no 1**B**0,
1**B**1, 1**B**2, ~~1**B**3~~ no 2**B**0,
2**B**1, 2**B**2, ~~2**B**3~~ no 3**B**0,
3**B**1 or ten

But 3 **Bundles**, is 1 **Bundle-of-Bundles.**
So T = 9 = 1**BB** **3s** or
T = ten = 3**B**1 **3s** = 1**BB**1 **3s** or
T = ten = 1**BB**0**B**1 **3s** or 1**BB**1**B**-2 **3s** or
T = ten = 101 **3s**

MATHeCADEMY.net

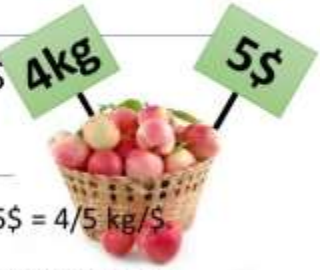
BundleCounting Fingers in 2s

1	1	1
2	1**B**0	10
3	1**B**1	11
4	1**BB**00	100
5	1**BB**01	101
6	1**BB**1**B**0	110
7	1**BB**1**B**1	111
8	1**BBB**000	1000
9	1**BBB**001	1001
Ten	1**BBB**01**B**0	1010



This can be shown with Lego bricks having different colors:
a green 2-brick is **B**
a blue 4-brick is **BB**
a red 8-brick is **BBB**

DoubleCounting in two Units creates PerNumbers & Proportionality



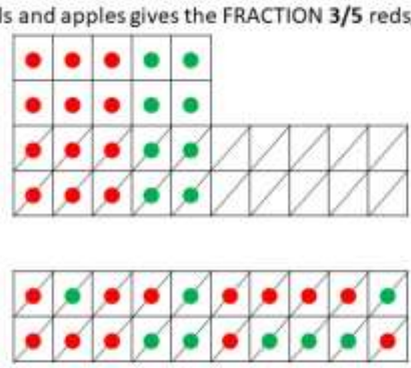
DoubleCounting in kg & \$, we get a PerNumber 4kg **per** 5\$ = 4kg/5\$ = 4/5 kg/\$.

With 4kg bridged to 5\$, we recount in the per-number. Or we recount the units directly. Or we equate the per-numbers. Or we use the before 1900 'Rule of 3' (regula de tri) alternating the units, and, from behind, first multiply, then divide.

Questions:

12kg = ?\$	**20\$ = ?kg**
12kg = (12/4) x 4kg = (12/4) x 5\$ = 15\$	20\$ = (20/5) x 5\$ = (20/5) x 4kg = 16kg
\$ = (\$/kg) x kg = 5/4 x 12 = 15	kg = (kg/\$) x \$ = 4/5 x 20 = 16
u/12 = 5/4, so *u* = 5/4 x 12 = 15	*u*/20 = 4/5, so *u* = 4/5 x 20 = 16
If 4kg is 5\$, then 12kg is ?\$; answer: 12x5/4 = 15	If 5\$ is 4kg, then 20\$ is ?kg; answer: 20x4/5 = 16

With like Units, PerNumbers become Fractions, both Operators Needing Numbers to Become Numbers

Outside World	Inside Brain
In a box filled with 3 red per 5 apples, double-counting reds and apples gives the FRACTION **3/5** reds/apples. 	Q: ? red in 20 apples. A: Recount 20 in **5s** (the per-number) T = 20 a = (20/5) x 5a gives (20/5) x 3r = 12 red apples Or, we equal the per-numbers: *u*/20 = 3/5; so *u* = 3/5 x 20 = 12. *Moving 20 to opposite side with opposite sign*

DoubleCounting the Sides in a Block

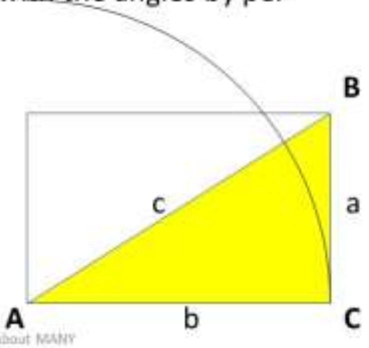
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras formula. And connected with the angles by per-number formulas double-counting the sides pairwise.

A + **B** + **C** = 180

a*a + b*b = c*c (the Pythagoras formula)

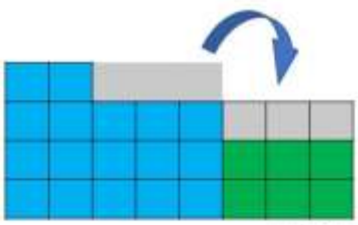
a = (a/c) x c = sin**A** x c; tan**A** = a/b = Δy/Δx = gradient

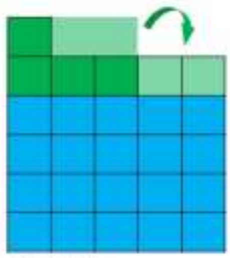
Circle: circum./diam. = π = **n*tan(180/n)** for n large



Counted & Recounted, Totals may be Added

BUT: **NextTo**	**or** **OnTop**
4 5s + 2 3s = 3B2 8s	4 5s + 2 3s = 4 5s + 1B1 5s = 5B1 5s
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = Proportionality*





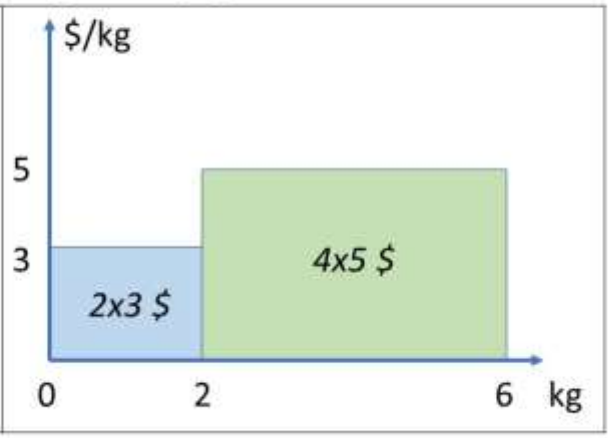
MATHeCADEMY.net

Adding PerNumbers as Areas (Integration)

"2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at ? **$/kg**?"

2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	**? $/kg**

- Unit-numbers add on-top.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number graph.
- Here, multiplication before addition



MATHeCADEMY.net

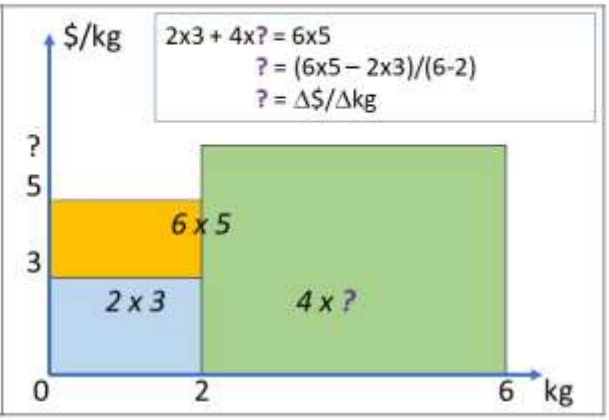
Subtracting PerNumbers (Differentiation)

"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

2 kg	at	3 $/kg
+ 4 kg	at	? $/kg
6 kg	at	5 $/kg

Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) before division.



MATHeCADEMY.net

Adding or Subtracting Unspecified Numbers

"Only add like units, so how to add $T = 4ab^2 + 6abc$?"

Here units come from folding (factoring):

$T = 4ab^2 + 6abc = T1 + T2$

$= 2 \times 2 \times a \times b \times b \quad + \quad 2 \times 3 \times a \times b \times c$

$= 2 \times b \times (2 \times a \times b) \quad + \quad 3 \times c \times (2 \times a \times b)$

$= (2b+3c) \times \mathbf{2ab}$

$= 2b+3c$ **2ab**s

a factor-filter

T1	2	2	*a*	*b*	*b*
T2	2	3	*a*	*b*	*c*
unit	**2**		***a***	***b***	
T1 left		2			*b*
T2 left		3			*c*

MATHeCADEMY.net

Conclusion

We ask: Can children discover/invent mathematics themselves to obtain a concrete exemplified understanding?

The answer is YES, if we

- de-model digits as icons with as many sticks as they represent
- use the flexible bundle-numbers children develop when adapting to Many
- de-model operations as means for bundle-counting 8 as 8/2 **2s**, leading directly to the recount-formula $T = (T/B) \times B$, used to change units, and to
- solve equations as 'How many 2s in 8?' by recounting 8 in 2s
- use double-counting to construct per-numbers, fractions and trigonometry
- add both next-to and on-top, making calculus be addition of per-numbers

$u \times 2 = 8 = 8/2 \times 2$
so $u = 8/2$

Discussion: What is the Difference?

		Traditional math	**Adaptive math**
Digits	4	Symbol	Icon with four strokes
Numbers	456	One number	Three numberings, 4**BB**5**B**6
Division	8/2	8 split in 2	8 counted in **2s**
Multiplication	6 x 7	42	6 **7s** or 4**B**2 **tens**
Addition	2+3	2+3 = 5	2 **4s** + 3 **5s** = 2**B**3 **9s** 2 **4s** + 3 **5s** = 4**B**1 **5s**
Equations	3 x *u* = 12	Neutralize (3 x *u*) x 1/3 = 12 x 1/3 (*u* x 3) x 1/3 = 4 *u* x (3 x 1/3) = 4 *u* x 1 = 4 *u* = 4	Opposite side & sign *u* x 3 = 12 = (12/3) x 3 *u* = 12/3 = 4
Fractions	2/3	Numbers 1/2 + 2/3 **IS** 7/6	Per-numbers, i.e. operators, needing numbers to become numbers: 1/2 of 2 + 2/3 of 3 **IS** 3/5 of 5

Flexible BundleNumbers Ease Operations

Over-load and under-load come in handy:

T = 65 + 27 = 6**B**5 + 2**B**7 = 8**B**12 = 9**B**2 = 92

T = 65 - 27 = 6**B**5 - 2**B**7 = 4**B**-2 = 3**B**8 = 38

T = 7 x 48 = 7 x 4**B**8 = 28**B** 56 = 33**B**6 = 336

T = 336 /7 = 33**B**6 /7 = 28**B**56 /7 = 4**B**8 = 48

T = 336 /7 = 33**B**6 /7 = 35**B**-14 /7 = 5**B**-2 = 48

Four Ways to Unite and Split a Total

A number-formula T = 345 = 3BB4B5 = 3*B^2 + 4*B + 5 (a polynomial) shows the 4 ways to add: +, *, ^, next-to block-addition (integration). Addition and multiplication add changing and constant unit-numbers. Integration and power add changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite. • The 4 uniting operations each has a reverse splitting operation: Addition has subtraction (–), and multiplication has division (/). Power has factor-finding (root, √) and factor-counting (logarithm, log). Integration has per-number finding (differentiation dT/dn = T').
Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / *split into*	**changing**	**constant**
Unit-numbers *m, s, \$, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, \$/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $log_a T = n$, $\sqrt[n]{T} = a$

Recommendation: Learners should be Researchers, Extending their Already Existing Adaption to Many

- To survive, also math must adapt to the outside world . So it should adopt the double-numbers children develop before school; and accept fractions as per-numbers, both operators needing numbers to become numbers.
- Hence to survive math must learn from children, not the other way around.
- Designing a micro- or macro-curriculum we should always ask: What is it out there that the learners need to adapt to?
- When adapting, learners should use grounded theory to answer the guiding learning questions listed in the curriculum.
- Teaching should be minimized to supplying concrete material and extra guiding questions, and to be opponents on the learners' findings.

MATHeCADEMY.net

Question Guided Teacher Education

MATHeCADEMY.net

Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

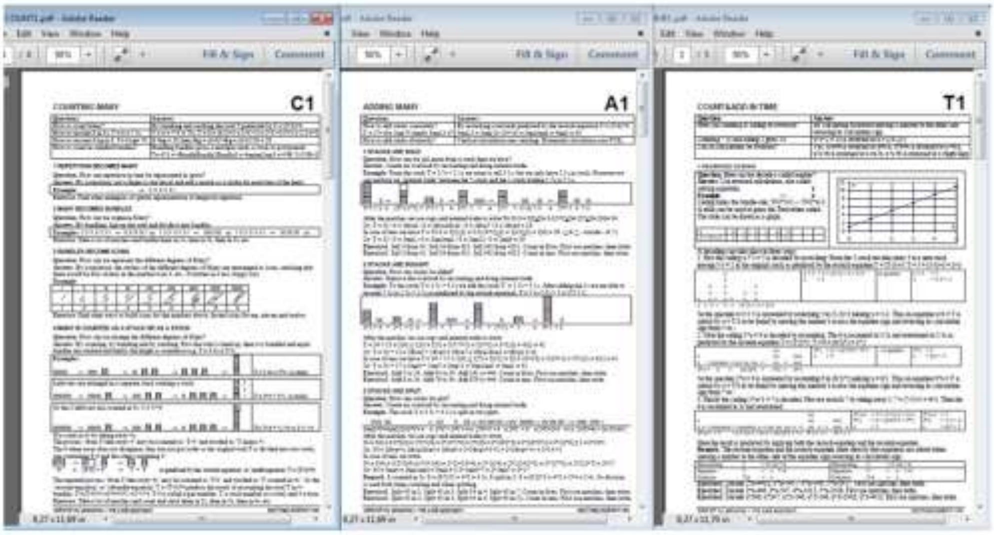
	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system. If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

MATHeCADEMY.net | Math as MANYmath - a Natural Science about MANY

29

MATHeCADEMY.net

Teacher Training in CATS ManyMath
Count & Add in Time & Space



30

MATHeCADEMY.net

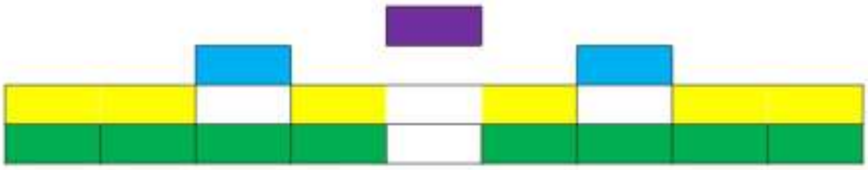
PYRAMIDeDUCATION

To learn MATH: Count&Add MANY
Always ask Many, not the Instructor
MATHeCADEMY.net - a VIRUSeCADEMY

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams



MATHeCADEMY.net

Theoretical Background

Tarp, A. (2018). Mastering Many by counting, recounting and double-counting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.