COUNTING MANY

C1

Questions	Answers
How to count Many?	By Bundling and stacking the total T, predicted by T = (T/B)*B
How to recount 7 in 3s, T = 7 = ? 3s	T = 7 = ?3s, T = 7 = (7/3)*3 = 2B1 3s = 2.1 3s = 2 1/3 3s = 3.-2 3s
How to recount 6 kg in $: T= 6 kg=?$	With the per-number 4kg per 2$, 6kg = (6/4)*4kg = (6/4)*2$ = 3$
How to count in standard bundles?	Bundling bundles gives a multiple stack, a polynomial: T= 423 = 4BundleBundle2Bundle3 = 4tenten2ten3 = 4*B^2+2*B+3

01 MANY IN TIME AND SPACE

Question. How can repetition in time become Many in space?
Answer. By putting a finger to the throat and adding a stick or a stroke for each beat of the heart.

Example . . . . -> I I I I I I I I

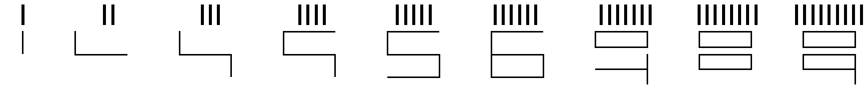
Exercise. Find other examples of spatial representation of temporal repetition.

02 MANY IS COUNTED IN BUNDLES

Question. How can we name Many?
Answer. By creating counting sequences naming the different degrees until the bundle allows starting over.
Examples. Counting a dozen in 5s: 0B1, 0B2, 0B3, 0B4, 0B5 no 1B0; 1B1, …, 1B5 no 2B0; 2B1, 2B2.
Again, counting a dozen in 5s: 0B1, 0B2, 1Bless2, 1B-1, 1B0; 1B1, 1B2, 2B-2, 2B-1, 2B0; 2B1, 2B2.
And in 3s: 0B1, 0B2, 0B3 no 1B0, 1B1, …, 2B2, 2B3 no 3B0 no 1BB, 1BB1, 1BB2, 1BB3 no 1BB1B (110)
Exercise. Count a dozen in 4s, 6s, 7s, and 2s. Eleven and twelve are Viking-numbers: 1left, 2left, 3ten, 4ten.

03 BUNDLES BECOME ICONS

Question. How can we show the first degrees of Many?
Answer. By iconising them to digits: change five ones to one fives to be rearranged as an icon, thus seeing that there are five strokes in the digit-icon 5 if written in less sloppy. Likewise, with the other digits. The Romans used the symbol X for the number ten. Instead, we say ten = 1 bundle and no unbundled = 1B0 = 10.
Example: I II III IIII IIIII IIIIII IIIIIII IIIIIIII IIIIIIIII



1 2 3 4 5 6 7 8 9

Exercise. Find other ways to build icons as above. What could be another name for 100, and for 1000?

04 MANY IS COUNTED AS A STACK PREDICTED BY A RECOUNT-FORMULA

Question. How can we count Many?
Answer. By bundling and stacking: line up the total, push away bundles, stack bundles, pull away the stack.

Example. I I I I I I I I -> II II II II or I I I I I I I I -> III III II or I I I I I I I I -> IIII IIII or …

We count 8 in 2s by pushing away 2s, iconized by a division-broom /: 8 = (8/2)*2 = 4*2. Here 4 is the counter and 2 is the unit. With unspecified numbers we get a 'RECOUNT-FORMULA' **T = (T/B)*B** saying 'From the total T, T/B times, B taken away, and stacked', predicting recounting T in B-bundles. Division thus means recounting, not sharing, which is an application. Pushed away, bundles are stacked, iconized by a multiplication-lift x, often written as *. Pulling the stack away to find unbundled is iconized by a subtraction-rope –. Thus, the process 'from T pull away 4' may be iconized as 'T-4' and worded as 'T minus 4'.
The 4 pulled away do not disappear, they are just put next-to so the original total T is split into two totals, one containing T-4 and the other containing 4 as predicted by the 'RESTACK-FORMULA' **T = (T–B) + B.**

$\boxed{IIIIIIIII}$ -> $\boxed{IIIII}$ $\boxed{IIII}$ -> $\boxed{IIIII}$ $\boxed{IIII}$
9 = (9-4) + 4 = 5 + 4

Placed next-to the stack, the unbundled are reported as a decimal number, 8 = 2B2 3s = 2.2 3s.
Placed on-top the stack it is counted in bundles, 2 = (2/3)*3 = 2/3 3s, reported as a fraction, T = 2 2/3 B 3s.
Or, if counting what is needed for an extra bundle, reported by a negative number, T = 3B-1 3s = 3.-1 3s.

Examples IIIIIIIIIIII -> IIIIIIII $\boxed{IIII}$ -> IIII $\boxed{IIII}$ $\boxed{IIII}$ -> $\boxed{IIII}$ $\boxed{IIII}$ $\boxed{IIII}$ ->	$\boxed{IIII}$ $\boxed{IIII}$ $\boxed{IIII}$	T = 3 4s = 3*4 (a stack)
Leftovers are placed next-to the stack as a stack of its own IIIIIIIIIIIII -> IIIIIIIII $\boxed{IIII}$ -> IIIIIII $\boxed{IIII}$ $\boxed{IIII}$ -> III $\boxed{IIII}$ $\boxed{IIII}$ $\boxed{IIII}$ ->	$\boxed{IIII}$ I $\boxed{IIII}$ I $\boxed{IIII}$ I	T = 3B3 4s = 3.3 4s decimals separate the unbundled

Or the leftovers are placed on-top and counted in 4s, 3 = (3/4)*4 Or we count what is needed to have an extra bundle IIIIIIIIIIII -> IIIIIIIII IIII -> IIIIII IIII IIII -> III IIII IIII IIII -> IIII	IIII IIII IIII IIII	T = 3B3 4s = 3 ¾ 4s T = 4B-1 4s = 4.-1 4s

Exercise. Bundle ten sticks in 3s, then in 4s, then in 2s, etc. Predict the result. The same with a dozen sticks.

05 STACKS ARE RECOUNTED IN A NEW UNIT

Question. How can we change the unit of a stack, e.g. T = 3 4s = ? 5s.
Answer. By de-stacking, de-bundling, re-bundling and re-stacking: First the stack is de-stacked into separate bundles, then the bundles are de-bundled into a total, then the total is bundled, then the bundles are stacked, and finally the heights are counted. Recounted in 2s, **even** numbers give a stack, and **odd** numbers leaves 1.
Foldable-units may fold: 8 = 4fold2. **Prime**-units may not: 5 = 5fold1. 1 is not a prime unit since B = BB = 1.

Example

IIII IIII IIII -> IIII IIII IIII -> IIIIIIIIIIII -> IIIIIII IIIII -> II IIIII IIIII -> IIIII IIIII I I

3*4 = 4 4 4 = T = (T-5)+5 = (T/5)*5 = 2*5 + 2*1 = 2.2 5s

Again the recounting result can be predicted by the recount-formula: T = (T/5)*5 = (3*4/5)*5 = 2*5 + 2*1.
A calculator shows the result: Entering '3*4/5' gives '2.some'. Entering '3*4 – 2*5' gives '2'. So T = 2.2 5s.
Exercise1. Recount a 2-stack in 3s, in 4s, in 5s, etc. Recount a 3-stack in 2s, in 4s, in 5s, etc. Always predict.
Exercise2. Recount a 2-stack in ½ s, in 1/3 s, in ¼ s, etc. Show that T = (T*n)*1/n.

06 USING CUPS FOR BUNDLE-COUNTING

Question. How can we describe using physical cups for counting bundles?
Answer. By using a cup-symbol ']' instead of the bundle symbol: T = 2B3 = 2]3 Bs.
In this way, an empty cup means 0: T = ten = 3B1 3s = 1BB1 3s, or with cups: T = ten = 3]1 = 1]]1 = 1]0]1.
With tens, we leave out the cup in multi-digit numbers: T = 2]3 tens = 23, and T = 2] = 2]0 tens = 20.
Example

If B = 3, then T = eleven = 1B8 = 2]5 = 3]2 = 4]-1 3s
B IIIIIIII -> B III IIIII -> BB IIIII -> BB III II -> BBB II -> BBBB I
T = 1 8 = 2 5 = 3 2 = 4 -1
overload, *overload,* *normal,* *underload,*

If B = 4, twelve = 3B0 = 2]4 = 1]8
BBB -> BB IIII -> B IIIIIIII
T = 30 = 2 4 = 1 8
overload, *overload*

Or with cups: T = 18 = 1]8
T = 1]8 = 1+1]-3+8 = 2]5 = 25 3s
T = 2]5 = 2+1]-3+5 = 3]2 = 32 3s

I	III IIIII
II	III II
III	II

T = 30 = 3] = 3]0
T = 3]0 = 3-1]+4+0 = 2]4 = 24 4s
T = 2]4 = 2-1]+4+4 = 1]8 = 18 4s

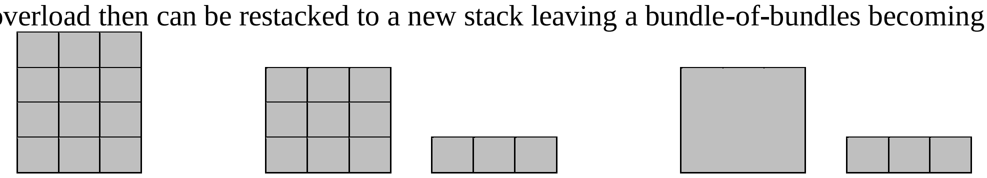
III	
II	IIII
I	IIII IIII

Exercise1. Recount ten, eleven and twelve in 5s, 4s, 3s, and 2s. First use cups, then use fingers + arms.
Exercise2. With a cup for 5s, remove overloads and create an underload, e.g. 3]6]8 = 3]7]3 = 4]2]3 = 4]3]-2.
Exercise3. What is the effect of adding or removing a cup? T = 2]3 -> 2]3], and T = 4]5]6] -> 4]5]6

07 SQUARES AS BUNDLE-BUNDLES

Question. What do we do with an overload where a stack is higher than its unit?
Answer. The overload then can be restacked to a new stack leaving a bundle-of-bundles becoming a square.

Example



T = 4*3 (overload) = (4*3 – 1*3) + 1*3 = 1 BB + 1B

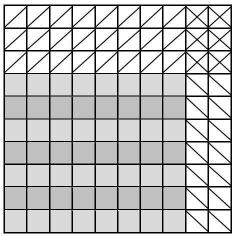
T = 234 = 2 bundles-of-bundles + 3 bundles + 4 unbundled (T = 2tenten 3ten 4)
In short, a given degree of Many may be rearranged as a multiple stack, a polynomial:
T = 2345 = 2]3]4]5 = 2]]]+3]]+4]+5 = 2*BBB + 3*BB + 4*B + 5*1 = 2*B^3 + 3*B^2 + 4*B + 5*1
Exercise1. Recount overloads: T= 7 5s, T= 18 8s, T= 34 tens, T = 562 hundreds, T = 562 tens, T = 562 1s.

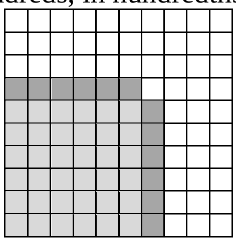
08 TOTALS ARE RECOUNTED FROM TENS TO ICONS

Question. How can we change the unit from tens to icons?
Answer. We formulate an equation that is solved by recounting.
Example. 'How many 2s in 34?' gives the equation u*2 = 34 with the solution u = 34/2 since 34 = (34/2)*2.
Thus, a multiplication equation is solved by moving a number to opposite side with opposite sign, as in an addition equation u+2 = 34 having the solution u = 34 – 2 since 34 = (34 – 2) + 2.
Exercise. Recount a ten-stack in 2s, in 3s, in 4s, etc.

09 TOTALS ARE RECOUNTED FROM ICONS TO TENS

Question. How can we change the unit from icons to tens?
Answer. Instead of recounting in tens, we simply multiply to get the answer without unit and decimal point.
Example. T = 3 6s = 3*6 = 18 = 1B8 tens = 1]8 tens = 1.8 tens.
Recounting from icons to tens, as e.g. 4 7s = ? tens, leads to multiplication tables eased by showing numbers with underloads on a pegboard: 7 8s = 7*8 = (B-3)*(B-2) = 10B-3B-2B+6 (subtracted twice) = 5B6 = 56.
Recounting in tens leads to decimals and percentages:
T = 3 6s = 3*6 = 18 = 1B8 tens = 1 8/10 tens = 1.8 tens = 1.8*10 = (18/100)*100 = 18%*100 = 0.18 tentens
T = 23.6 = (23.6/10)*10 = 2.36*10 = 2.36 tens, and T = 23.6 = (23.6/100)*100 = 0.236*100 = 0.236 tentens
T = 3.6 = (3.6/(1/10))*1/10 = 36*1/10 = 36 tenths = (3.6*10)*1/10, so T/(1/B) = T*B
Remark. T = 3*6 = 3 6s and T = 3*6 = 18 only if recounted in tens. Multiplication thus is a special division.
Exercise1. Use a pegboard with rubber bands to work through the whole table.
Exercise2. Use a pegboard, focus on squares. What is needed to go from on square to the next?
Exercise3. Recount a 3-stack in tens, in tenths, in hundreds, in hundredths, in thousands, in thousandths.


7 8s = (B-3)*(B-2) = 10B-3B-2B+6


7 7s = 6 6s + 2 6s + 1, or (n+1)^2 = n^2 + 2*n + 1

10 TOTALS ARE DOUBLE-COUNTED IN TWO UNITS

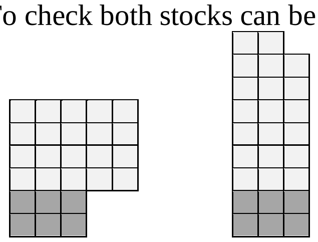
Question. How can we recount a total in different units?
Answer. Recount a number in the per-number; or recount the unit in the per-number.
Example1. Sugar counted in kilos, litres, dollars, % give the per-numbers 2 kg per 5 $ per 6 litres per 100 %

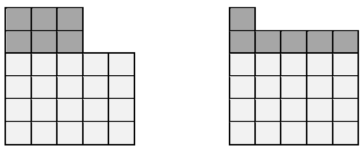
T = 7 kg = ? Recount the number	Recount the unit
T = 7 kg = (7/2)*2 kg = (7/2)*5 $ = 17.50 $ T = 7 kg = (7/2)*2 kg = (7/2)*6 litres = 21 litres T = 7 kg = (7/2)*2 kg = (7/2)*100 % = 350 % P = 5% = (5/100)*100% = (5/100)*2 kg = 0.1 kg	$ = ($/kg)*kg = (5/2)*7 = 17.5 litres = (litres/kg)*kg = (6/2)*7 = 21 % = (%/kg)*kg = (100/2)*7 = 350 kg = (kg/%)*% = (2/100)*5 = 0.1

Example2. Double-counting with per-numbers occurs all over math and science: $\Delta y = (\Delta y/\Delta x)*\Delta x = m*\Delta x$, (meter = meter/sec)*sec = speed*sec, kg = (kg/cubic-meter) * cubic-meter = density * cubic-meter.
Double-counted in the same unit, per-numbers are fractions, both operators needing numbers to be numbers.
Exercise. 4 kg = 6 $ = 7 litres = 100 %, T = 10 kg = ?, T = 8 $ = ?; T = 30 litres = ?; T = 40 % = ?,

11 STACKS ADDED ON-TOP GIVES PROPORTIONALITY

Question. How can we add stacks on-top?
Answer. Stacks add on-top after recounting creates like units. Changing unit is called proportionality.
Examples. T = 2 3s + 4 5s = ? 3s, T = 2 3s + 4 5s = ? 5s. The recount-formula predicts the result.
The 4 5s are recounted in 3s: T = 2 3s + 4 5s = 2 3s + (4*5/3)*3 = 2 3s + 6*3 +2 = 2 3s + 6 3s + 2 = 8 3s + 2
The 2 3s are recounted in 5s: T = 2 3s + 4 5s = (2*3)/5*5 + 4 5s = 1*5 + 1 + 4 5s = 1 5s + 1 + 4 5s = 5 5s + 1
To check both stacks can be recounted in tens: T = 8 3s + 2 = 8*3+2 = 26 and T = 5 5s + 1 = 5*5+1 = 26


T = 2 3s + 4 5s = 8.2 3s

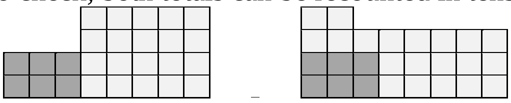

T = 4 5s + 2 3s = 5.1 5s

Exercise. Add 1 2s + 3 4s on-top and predict the result. Add 2 3s + 2 4s on-top and predict the result, etc.

12 STACKS ADDED NEXT-TO GIVES INTEGRATION

Question. How can we add stacks next-to?
Answer. Adding stacks next-to is called integration of areas. It is done by recounting and restacking.
Example. T = 2 3s + 4 5s = ? 8s. The result can be predicted by the recount-formula.

The 3s and 5s are added as 3+5 = 8s: T = 2 3s + 4 5s = (2*3 + 4*5)/8*8 = 3*8 +2 = 3 8s + 2
To check, both totals can be recounted in tens: T = 2 3s + 4 5s = 2*3 + 4*5 = 26 and T = 3*8 +2 = 26

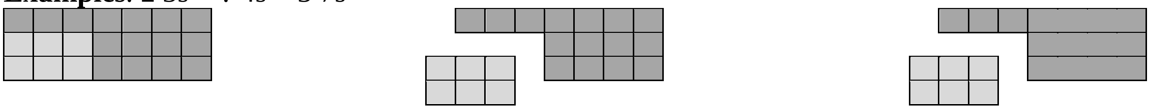


T = 2 3s + 4 5s = 3.2 8s

Thus, 2 + 5 gives different results when added on-top and next-to.
T = 2*3 + 5*3 = 7*3 if added on-top; and T = 2*3 + 5*3 = (2*3 + 5*3)/6*6 = 3 3/6 *6 if added next-to.
Saying '(3s of-which 2) + (5s of-which 4) = (8s of-which 3 2/8)' show that of-which-numbers add by areas.
Remark. Reverse integration leads to differentiation: T = 2 3s + u 5s = 3 8s, u = (3*8-2*3)/5 = ΔT/5.
Exercise. Add 1 2s + 3 4s on-top and next-to, predict the result. Use dices to create other exercises.

13 REVERSED NEXT-TO ADDITION GIVES DIFFERENTIATION

Question. How can we reverse next-to addition?
Answer. First push the initial stack away, then recount the rest.
Examples. 2 3s + ? 4s = 3 7s



T = 2 3s + u 4s = 3 7s   T1   T – T1 = ΔT   u = ΔT/4 = 3B3 4s

14 REVERSED ADDITION GIVES EQUATIONS

Question. How can we reverse addition?
Answer. Restack or recount to solve equations. Rule: Move numbers to opposite side with opposite sign.
Examples. If u+3 = 8 then u = 8-3; if 2*u = 6 then u = 6/2; if 2*u+1 = 9 then u = (9-1)/2

● u III = IIIIIIII = IIIII III
u + 3 = 8 = (8-3) + 3   So u = 8-3 = 5 as predicted by restacking u+3 = 8 = (8-3)+3 = 5+3

II
II
uu = IIIIII -> II II II -> II
● 2*u = 6 = (6/2)*2 = 3*2   So u = 6/2 = 3 as predicted by recounting 2*u = 6 = (6/2)*2 = 3*2
uu I = IIIIIIIII = IIIIIIII I = II II II II I
● 2*u + 1 = 9 = (9-1)+1 = 8+1 = (8/2)*2 +1 = 4*2 + 1
So u = 4 as predicted by 2*u + 1 = 9 = (9-1)+1 = 8+1, so 2*u = 8 = (8/2)*2 = 4*2 .
Exercise1. Solve the equation u + 1 = 3, u + 2 = 5, etc. Construct more equations by using dices.
Exercise2. Solve the equation 2*u = 4, 3*u = 12, etc. Construct more equations by using dices.
Exercise3. Solve the equation 2*u + 1 = 7, 3*u + 2 = 14, etc. Construct more equations by using dices.

15 MEETING ALGEBRA IN A TILE-SYSTEM

Question. How can we add and subtract playing cards?
Answer. Moving h*b playing cards around makes visible several block formulas called algebra.
Examples. (h – b)^2 = h^2– 2*a*b + b^2, and (h+b)^2 = h*h + b*b + 2*h*b, and (h+b)*(h-b) = h^2 – b^2
Half of a b*b tile extends a tile to a h*b playing card. Removing from a h*h square two playing cards, and adding the bottom tile that has been removed twice will leave the square (h – b)^2 = h^2– 2*h*b + b^2. And, removing from a h*b playing card the bottom b*b tile will leave the top (h – b)*b = h*b – b*b. Four playing cards are arranged to form a (h+b)*(h+b) square. Inside we find a h*h square, a b*b square and two playing cards, so, (h+b)*(h+b) = (h+b)^2 = h*h + b*b + 2*h*b. Pulling away a b*b tile from the h*h square leaves a (h–b)*h and a (h–b)*b rectangles that add up to a (h–b)*(h+b) rectangle. So, (h+b)*(h–b) = h^2 – b^2.
To solve the quadratic equation x^2+6*x+8 = 0, two cards form a square with the side x+6/2, containing two 6/2*x rectangles and two squares, x^2, and (6/2)^2 split in two parts, 8 below and (6/2)^2-8 above if possible, all disappearing except for upper part. So (x+6/2)^2 = (6/2)^2– 8 = 1, which gives x = -6/2 ± 1 = -2 and -4.
Looking instead at x^2+b*x+c = 0, gives the solution x = -b/2 ± √((b/2)^2 –c).