COUNT&ADD IN TIME

T1

Question	Answer
How different are word- & number-language	They both use sentences with subjects, verbs and predicates.
How can counting & adding be reversed?	By calculating backward moving a number to the opposite side with its opposite calculation sign.
Some 3s with 2 added gave 14. How many?	x*3+2 = 14 is reversed to x = (14-2)/3
Can all calculations be reversed?	Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x =b/a, x^a =b is reversed to x =a√b, a^x =b is reversed to x =logb/loga

01 OUR TWO LANGUAGE HOUSES

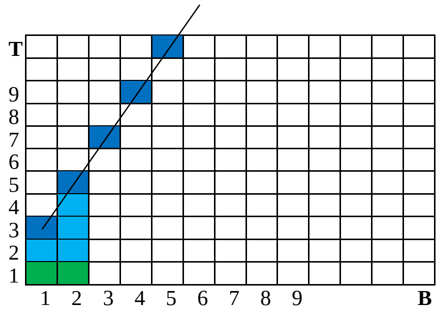
Question. How similar are the word-language and the number-language?
Answer. Both use sentences with a subject, a verb, and a predicate.
The WORD-language assigns words in sentences with a subject, a verb & a predicate. The NUMBER-language assigns numbers instead. Both languages have a META-language, a grammar, describing the language, that should be learned before the grammar. A number-language sentence is called a formula.

	WORD language	NUMBER language
META-language grammar	*'is' is a verb*	*'x' is an operation*
Language	This is a table	T = 3x4

02 FORWARD CALCULATION FORMULAS

Question. How can we visualize forward calculations?
Answer. By tables and curves.
Example.
Tables and curves can show how the size of a total counted as 2B1 change with the bundle-size B.

Formula: T = 2*B+1. Curve:

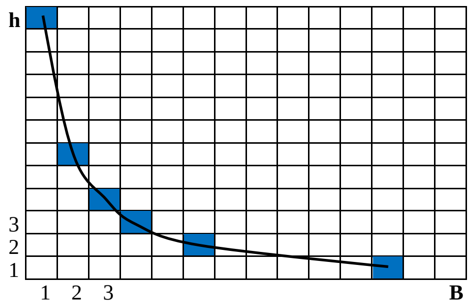


Table

B	1	2	3	4	5	6	…
T = 2*B+1	3	5	7	9	11	13	

03 REVERSED CALCULATION FORMULAS

Question. How can we predict reversed calculations?
Answer. By equations, tables and curves.
Example.
A total of 12 may be recounted in different bundles.
Equation: 12 = h*B, solved as a formula h = 12/B.
Curve:



Table

x	1	2	3	4	6	12	…
B	12	6	4	3	2	1	

04 THE 4 WAYS TO UNITE AND SPLIT TOTALS

The general number-formula T = a*B^2 + b*B + c*1 is called a polynomial. It shows the four different ways to unite, called algebra in Arabic: addition, multiplication, repeated multiplication or power, and block-addition or integration. Which is precisely the core of traditional mathematics education, teaching addition and multiplication together with their reverse operations subtraction and division in primary school; and power and integration together with their reverse operations factor-finding (root), factor-counting (logarithm) and per-number-finding (differentiation) in secondary school. Including the units, we see there can be only four ways to unite numbers: addition and multiplication unite changing and constant unit-numbers, and integration and power unite changing and constant per-numbers. We call this simplicity 'the algebra square'.

Operations **unite**/ *split* Totals in	Changing	Constant
Unit-numbers m, s, kg, $	***T = a + n*** *T – n = a*	***T = a*n*** *T/n = a*
Per-numbers m/s, $/kg, $/100$ = %	***T = ∫a*dn*** *dT/dn = a*	***T = a^n*** $\sqrt[n]{T} = a$ $\log_a T = n$

05 REVERSED VERTICAL CALCULATIONS

Question. How can a calculation be reversed vertically?
Answer. Use arrows to illustrate the forward and backward calculation steps (dance the equation).
Example. The equation 3*x + 2 = 14 is a story about two calculations that took place after each other. FIRST the number x was multiplied by 3, THEN 2 was added producing a total of 14.
This sequence can be reversed to produce x: FIRST 2 is subtracted from 14; THEN this is divided by 3. So 3*x + 2 = 14 makes x = (14-2)/3. Finally, to check, the forward calculation can be repeated.

x	*--(*3)-->*	3*x	*--(+ 2)-->*	3*x + 2		(forward)
4	*<--(/3)--*	12	*<--(- 2)--*	14		(backward)

Exercise1. Dance 2*x+4 = 10, 3*x+5 = 17, 4*x+1 = 9, 5*x+2 = 17. First on the floor, then write.
Exercise2. Dance 2*x-4 = 6, 3*x-5 = 7, 4*x-1 = 11, 5*x-2 = 18. First on the floor, then write.
Exercise3. Dance x*2 = 6, x*2 = 7, x*2+1 = 6, x*2-1 = 6, x*3+2 = 16, x*3-4 = 12. From floor, then write.

06 REVERSED HORIZONTAL CALCULATIONS

Question. How can a calculation be reversed horizontally?
Answer. Use arrows to illustrate upward and downward calculation steps (climb the equation)
Example. The equation 3*x + 2 = 14 is a story about two calculations that took place after each other. FIRST, on the forward side, the calculation is built up to give a total: x is multiplied by 3, and 2 is added giving 14. THEN, on the backward side, the result is broken down to produce the initial number: 2 is subtracted from 14 and the result is divided by 3. So 3*x + 2 = 14 makes x = (14-2)/3. Finally, to check, the upward calculation is repeated. If we leave out the arrows the opposite-side&sing method becomes visible.

Forward-side		*Backward-side*
3*x + 2	=	14
	+2 ↑↓ -2	
3*x	=	14-2 = 12
	**3 ↑↓ /3*	
x	=	12/3 = 4

Forward	*Backward*
3*x + 2	= 14
3*x	= 14-2 = 12
x	= 12/3 = 4

Forward	*Backward*
m*x + b	= c
m*x	= c-b
x	= (c-b)/m

Exercise1. Climb 2*x+4 = 10, 3*x+5 = 17, 4*x+1 = 9, 5*x+2 = 17. First from the floor, then write.
Exercise2. Climb 2*x-4 = 6, 3*x-5 = 7, 4*x-1 = 11, 5*x-2 = 18. First from the floor, then write.
Exercise3. Climb x*2 = 6, x*2 = 7, x*2+1 = 6, x*2-1 = 6, x*3+2 = 16, x*3-4 = 12. From floor, then write.

07 CALCULATION TABLES

Question. How can we report solving an equation?
Answer. Use a calculation-table showing both what we know and don't know and the equation to be solved.
Example. In the equation 3*x+2 = 14 the double-calculation 3*x+2 is split up into two calculations by the 'invisible' parenthesis: 3*x+2 = (3*x)+2.

Calculating numbers

x = ?	3*x + 2	= 14
	(3*x) + 2	= 14
	3*x	= 14-2
	x	= (14-2)/3
	x	= 12/3
	x	**= 4**
Check:	*3*4 + 2*	*= 14*
	14	*= 14* ☺

Calculating letters

x = ?	m*x + b	= c
m = 3	(m*x) + b	= c
b = 2	m*x	= c-b
c = 14	x	= (c-b)/m
	x	= (14-2)/3
	x	**= 4**
Check:	*3*4 + 2*	*= 14*
	14	*= 14* ☺

Exercise1. Solve in a calculation-table 2*x+4 = 10, 3*x+5 = 17, 4*x+1 = 9, 5*x+2 = 17.
Exercise2. Solve in a calculation-table 2*x-4 = 6, 3*x-5 = 7, 4*x-1 = 11, 5*x-2 = 18.
Exercise3. Solve in a calculation-table x*2 = 6, x*2 = 7, x*2+1 = 6, x*2-1 = 6, x*3+2 = 16, x*3-4 = 12.

08 APPLYING CALCULATION TABLES WITH FORMULAS

Question. Where can we use calculation-tables? **Answer**. Calculation-tables can be used with formulas.
Example1. Recounting units

3 $ for 4 pieces: 21$ for ? pieces

pieces = ?	**pieces = (pieces/$)*$**
pieces/$ = 4/3	pieces = 4/3*21
$ = 21	pieces = 28

3 $ for 4 pieces: ?$ for 24 pieces

$ = ?	**$ = ($/pieces)*pieces**
$/pieces = 3/4	$ = ¾*24
pieces = 24	$ = 18

Example2. Percentages I

30 $ = 40%, 21$ = ?%

% = ?	**% = (%/$)*$**
%/$ = 40/30	% = 40/30*21
$ = 21	% = 28

30 $ = 40%, ?$ = 24%

$ = ?	**$ = ($/%)*%**
$/% = 30/40	$ = 30/40*24
% = 24	$ = 18

Example2. Percentages II

25% af 200 $. = ? $.

A = ?	**p = a/T**
p = 25%	p*T = a
T = 200	25%*200 = a
	50 = a

25% af ? $. = 40 $

T = ?	**p = a/T**
p = 25%	p*T = a
a = 40	T = a/p
	T = 40/25%
	T = 160

?% af 200 $. = 40 $

p = ?	**p = a/T**
a = 40	p = 40/200
T = 200	p = 0.20
	p = 20/100
	p = 20%

Example3. Adding percentages

200 + 25% = ?

K = ?	**K = Ko*(1+r)**
Ko = 200	K = 200*(1+0.25)
r = 25%	**K = 250**
= 0.25	

? + 25% = 400

Ko = ?	**K = Ko*(1+r)**
K = 500	K/(1+r) = Ko
r = 25%	500/(1+0.25) = Ko
= 0.25	**400 = Ko**

200 + ?% = 280

r = ?	**K = Ko*(1+r)**
K = 280	K/Ko = 1+r
Ko = 200	(K/Ko)-1 = r
	(280/200)-1 = r
	0.40 = r
	40% = r

Example4. Per-numbers

25kg. à 4 $./kg. = ? $.

T = ?	**k*p = T**
k = 25	25*4 = T
p = 4	100 = T

25kg. à ? $./kg. = 200$.

p = ?	**k*p = T**
k = 25	p = T/k
T = 200	p = 200/25
	p = 8

?kg. à 4 $./kg. = 300$.

k = ?	**k*p = T**
p = 4	p = T/k
T = 300	p = 300/4
	p = 75

Exercise. Repeat the calculations with different numbers. Remember to check.

09 THE LEVER METHOD FOR NEUTRALISING

Question. Are there other ways to solve equations?
Answer. Modern mathematics introduced the lever-method to make solving equations understandable being under the false assumption that the opposite-side&sign method was only accessible through rote learning. The equation sign is considered an example of an equivalence relation. And an equation 3*x+2 = 14 is considered an example of an open statement expressing the equivalence between two numbers 3*x+2 and 14. The equation is solved by determining the statement's truth-set, i.e. the set of numbers that make the open statement a true statement. This is done by neutralising the numbers with their inverse numbers that has to be included on both sides of the equal sign to preserve the equivalence as indicated by the equivalent arrows.

L = {x| 3*x + 2 = 14}	L is the truth-set making the open statement 3*x + 2 = 14 true
3*x + 2 = 14	The open statement
↕ (3*x) + 2 = 14	A hidden parenthesis is added according to priority
↕ ((3*x) + 2)+(-2) = 14+(-2)	To neutralise +2 its inverse under +, –2, is added on both sides
↕ (3*x) + (2+(-2)) = 12	+ parentheses can be removed or added (the associative law)
↕ (3*x) + 0 = 12	+2 and –2 neutralise each other, and +'s neutral number is 0
↕ 3*x = 12	The definition of the neutral number says that a+0 = 0+a = a
↕ (3*x)*(1/3) = 12*(1/3)	To neutralise *3 its inverse under *, 1/3, is multiplied on both side
↕ (x*3)*(1/3) = 4	*numbers (and +numbers) may be commuted (the commutative law)
↕ x*(3*(1/3)) = 4	*parentheses can be removed or added (the associative law)
↕ x*1 = 4	*3 and 1/3 neutralise each other, and *'s neutral number is 1
↕ x = 4	The definition of the neutral number says that a*1 = 1*a = a
L = {x | 3*x + 2 = 14} = {4}	The truth-set L. Because of the arrows the check is not needed.

Exercise. Repeat the lever or neutralising method with a different equation as e.g. 3+2*x = 11.

10 BACKWARD CALCULATION WITH CODED STACKS

Question. What about coded stacks?
Answer. Backward calculation can also be used with coded stocks using the reversed FOIL-method.
Example1. T = (4*x+3*y)*? = 24*x^2 + 46*x*y + 21*y^2, using medieval multiplication #

<u>Dividing coded stacks</u>

(4*x + 3*y) * ? = 24*x^2 + 46*x*y + 21*y^2

	?	?	
+ 3*y			
4*x			
	24*x^2	**+46*x*y**	**+21*y^2**

4x * ? = 24*x^2 <u>*? = 6*x*</u>
3y * 6x = 18*x*y

	6*x	**?**	
+ 3*y	*18*x*y*		
4*x	*24*x^2*		
	24*x^2	**+46*x*y**	**+21*y^2**

18*x*y + ? = 46*x*y <u>*? = 28*x*y*</u>

	6*x	**?**	
+ 3*y	*18*x*y*		
4*x	*24*x^2*	*28*x*y*	
	24*x^2	**+46*x*y**	**+21*y^2**

4*x * ? = 28*x*y <u>*? = 7*y*</u>
3*y * 7*y = 21*y^2

	6*x	**+ 7*y**	
+ 3*y	*18*x*y*	*21*y^2*	
4*x	*24*x^2*	*28*x*y*	
	24*x^2	**+46*x*y**	**+21*y^2**

So 24*x^2 + 46*x*y + 21*y^2 = (4*x+3*y)*(6*x+7*y)

Example2. (x+a)(x-a) = x^2-a^2

Forward	*Backward*
(x + 6)(x - 6) = x^2 – 6^2 = x^2 – 36	x^2 – 36 = x^2 – 6^2 = (x + 6)(x - 6)
(x + √20)(x - √20) = x^2 – (√20)^2 = x^2 – 20	x^2 – 20 = x^2 – (√20)^2 = (x + √20)(x - √20)

Example3. (x±a)^2 = x^2 + a^2±2*a*x

Forward	*Backward*
(x + 6)^2 = x^2 + 6^2 + 2*6*x = x^2 + 36 + 12x	x^2 + 36 + 12x = x^2 + 6^2 + 2*6*x = (x + 6)^2
(x - 6)^2 = x^2 + 6^2 - 2*6*x = x^2 + 36 - 12x	x^2 + 36 - 12x = x^2 + 6^2 - 2*6*x = (x - 6)^2

Example4. A second degree equation: If a*x^2 + b*x + c = 0 then x = ?

Forward	*Backward*	*Summary*
T = (x + k)^2 T = x^2 + k^2 + 2*k*x 0 = x^2 + (2*k)*x + (k^2 – T) 0 = x^2 + p*x + q p = 2*k og q = k^2 – T p/2 = k og T = k^2 – q = (p/2)^2 – q	(x + k)^2 = T x + k = ± √T x = -k ± √T ------- If x^2 + p*x + q = 0 then **x = - p/2 ± √((p/2)^2 – q)**	a*x^2 + b*x + c = 0 x^2 + (b/a)*x + (c/a) = 0 x^2 + p*x + q = 0 p = b/a og q = c/a x = - b/(2a) ± √((b/(2a))^2 – c/a) **x = (-b ± √D)/(2*a) , D = b^2 – 4*a*c**

Exercise1. Do (8x^2 + 22 x*y + 15y^2)/(2x+3y). Do (6x^2 + 26 x*y + 24y^2)/(3x+4y)
Exercise2. Factorise x^2-25, 3x^2-48, x^2+6x+8, 2x^2-4x-30. Solve x^2-8x+12 = 0, 4x^2-24x+20 = 0

11 A SUMMARY

Question. Is there a common principle when solving equations?
Answer. Move a number to the other side of the equation sign and change its calculation sign.

	5+3 = 5+1+1+1	5*3 = 5+5+5	5^3 = 5*5*5	
Forward	5 + 3 = ?	5 * 3 = ?	5^3 = ?	5^3 = ?
Backward	? + 3 = 8 *? = 8-3*	? * 3 = 15 *? = 15/3*	?^3 = 64 *? = 3√64*	5^? = 64 *? = log64/log5*
Definitions	8-3 is the +number that together with 3 gives 8	15/3 is the *number that together with 3 gives 15	3√64 factor-finder: Find the factor, so 3 give the total factor 64	log64/log5 factor-counter: How many 5-factors give the total factor 64
opposite side&sign	x + 3 = 8 x = 8 – 3	x * 3 = 15 x = 15/3	x^3 = 64 x = 3√64	x^2 = 25 x = √25
Move-rules	+numbers <--> -numbers	*numbers <--> /numbers	^3 <--> 3√	^2 <--> √

Exercise1. Solve the following equations using both the opposite-side&sign and the neutralising-method.

1	T = a+(1+b)*c	T = a+(1-b)*c	T = a+b*(1+c)	T = a+b*(1-c)
2	T = a-(1+b)*c	T = a-(1-b)*c	T = a-b*(1+c)	T = a-b*(1-c)
3	T = a+(1+b)/c	T = a+(1-b)/c	T = a+b/(1+c)	T = a+b/(1-c)
4	T = a-(1+b)/c	T = a-(1-b)/c	T = a-b/(1+c)	T = a-b/(1-c)

Exercise2. Solve the following equations using both the opposite-side&sign and the neutralising-method.

1	T = a+b*c^d	T = a+(1+b)*c^d	T = a+(1-b)*c^d	T = a+b*(1-c)^d
2	T = a-b*c^d	T = a-(1+b)*c^d	T = a-(1-b)*c^d	T = a-b*(1-c)^d
3	T = a+b/c^d	T = a+(1+b)/c^d	T = a+(1-b)/c^d	T = a+b/(1-c)^d
4	T = a-b/c^d	T = a-(1+b)/c^d	T = a-(1-b)/c^d	T = a-b/(1-c)^d

Exercise3. Solve the following equations.

	*Find **a, b** and **c*** *Answer:*	a	b	c
1	$T = a + b\cdot c$	$T - b\cdot c$	$\frac{T-a}{c}$	$\frac{T-a}{b}$
2	$T = a - b\cdot c$	$T + b\cdot c$	$\frac{a-T}{c}$	$\frac{a-T}{b}$
3	$T = a + \frac{b}{c}$	$T - \frac{b}{c}$	$(T-a)\cdot c$	$\frac{b}{T-a}$
4	$T = a - \frac{b}{c}$	$T + \frac{b}{c}$	$(a-T)\cdot c$	$\frac{b}{a-T}$
5	$T = (a + b)\cdot c$	$\frac{T}{c} - b$	$\frac{T}{c} - a$	$\frac{T}{a+b}$
6	$T = (a - b)\cdot c$	$\frac{T}{c} + b$	$a - \frac{T}{c}$	$\frac{T}{a-b}$
7	$T = \frac{a+b}{c}$	$T\cdot c - b$	$T\cdot c - a$	$\frac{a+b}{T}$
8	$T = \frac{a-b}{c}$	$T\cdot c + b$	$a - T\cdot c$	$\frac{a-b}{T}$
9	$T = \frac{a}{b+c}$	$T\cdot (b+c)$	$\frac{a}{T} - c$	$\frac{a}{T} - b$
10	$T = \frac{a}{b-c}$	$T\cdot (b-c)$	$\frac{a}{T} + c$	$b - \frac{a}{T}$
11	$T = \frac{a}{b} + c$	$(T-c)\cdot b$	$\frac{a}{T-c}$	$T - \frac{a}{b}$
12	$T = \frac{a}{b} - c$	$(T+c)\cdot b$	$\frac{a}{T+c}$	$\frac{a}{b} - T$
13	$T = a\cdot b^{c}$	$\frac{T}{b^{c}}$	$\sqrt[c]{\frac{T}{a}}$	$\frac{\log(\frac{T}{a})}{\log b}$
14	$T = \frac{a}{b^{c}}$	$T\cdot b^{c}$	$\sqrt[c]{\frac{a}{T}}$	$\frac{\log(\frac{a}{T})}{\log b}$
15	$T = (a\cdot b)^{c}$	$\frac{\sqrt[c]{T}}{b}$	$\frac{\sqrt[c]{T}}{a}$	$\frac{\log T}{\log(a\cdot b)}$
16	$T = (\frac{a}{b})^{c}$	$\sqrt[c]{T}\cdot b$	$\frac{a}{\sqrt[c]{T}}$	$\frac{\log T}{\log(\frac{a}{b})}$
17	$T = (a + b)^{c}$	$\sqrt[c]{T} - b$	$\sqrt[c]{T} - a$	$\frac{\log T}{\log(a+b)}$
18	$T = (a - b)^{c}$	$\sqrt[c]{T} + b$	$a - \sqrt[c]{T}$	$\frac{\log T}{\log(a-b)}$
19	$T = a + b^{c}$	$T - b^{c}$	$\sqrt[c]{T-a}$	$\frac{\log(T-a)}{\log b}$
20	$T = a - b^{c}$	$T + b^{c}$	$\sqrt[c]{a-T}$	$\frac{\log(a-T)}{\log b}$
21	$T = a^{(b+c)}$	$\sqrt[(b+c)]{T}$	$\frac{\log T}{\log a} - c$	$\frac{\log T}{\log a} - b$
22	$T = a^{(b-c)}$	$\sqrt[(b-c)]{T}$	$\frac{\log T}{\log a} + c$	$b - \frac{\log T}{\log a}$