THE POWER OF BUNDLE- & PER-NUMBERS UNLEASHED IN PRIMARY SCHOOL: CALCULUS IN GRADE ONE – WHAT ELSE?

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In middle school, fraction, percentage, ratio, rate, and proportion create problems to many students. So, why not teach it in primary school instead where they all may be examples of per-numbers coming from double-counting a total in two units. And bundle-numbers with units is what children develop when adapting to Many before school. Here children love counting, recounting, and double-counting before adding totals on-top or next-to as in calculus, also occurring when adding per-numbers. Why not accept, and learn from the mastery of Many that children possess until mathematics takes it away?

MATHEMATICS IS HARD, OR IS IT?

"Is mathematics hard by nature or by choice?" is a core sociological question inspired by the ancient Greek sophists warning against choice masked as nature. That mathematics seems to be hard is seen by the challenges left unsolved after 50 years of mathematics education research presented e.g. at the International Congress on Mathematics Education, ICME, taking place each 4 year since 1969. Likewise, increased funding used e.g. for a National Center for Mathematics Education in Sweden, seems to have little effect since this former model country saw its PISA result in mathematics decrease from 509 in 2003 to 478 in 2012, the lowest in the Nordic countries, and significantly below the OECD average at 494. This caused OECD (2015) to write the report 'Improving Schools in Sweden' describing the Swedish school system as being 'in need of urgent change'.

Also among the countries with poor PISA performance, Denmark has lowered the passing limit at the final exam to around 15% and 20 % in lower and upper secondary school. And, at conferences as e.g. The Third International Conference on Mathematics Textbook Research and Development, ICMT3 2019, high-ranking countries admit they have a high percentage of low scoring students. Likewise at conferences, discussing in the breaks what is the goal of mathematics education, the answer is almost always 'to learn mathematics'. When asked to define mathematics, some point to schoolbooks, others to universities; but all agree that learning it is important to master its outside applications.

So, we may ask, is the goal of mathematics education to master outside Many, or to first master inside mathematics as a means to later master outside Many. Here, institutionalizing mathematics as THE only inside means leading to the final outside goal may risk creating a goal displacement transforming the means to the goal instead (Bauman, 1990) leading on to the banality of evil (Arendt, 1963) by just following the orders of the tradition with little concern about its effect as to reaching the outside goal. To avoid this, this paper will answer the question about the hardness by working backwards, not from mathematics to Many, but from Many to mathematics. So here the focus is not to study why students have difficulties mastering inside mathematics, but to observe and investigate the mastery of outside Many that children bring to school before being forced to learn about inside mathematics instead.

RESEARCH METHOD

Difference research searching for differences has uncovered hidden differences (Tarp, 2018c). To see if the differences make a difference, phenomenology (Tarp, 2018a), experiential learning (Kolb, 1984), and design research (Bakker, 2018) may create cycles of observations, reflections, and designs of micro curricula to be tested in order to create a new cycle for testing the next generation of curricula.

OBSERVATIONS AND REFLECTIONS 01

Asked "How old next time?" a three-year-old will say four showing four fingers, but will react to seeing the fingers held together two by two: "That is not four. That is two twos!" The child thus describes what exists, bundles of 2s, and 2 of them. Likewise, counting a total of 8 sticks in bundles of 2s by pushing away 2s, a 5-year-old easily accepts iconizing this as 8 = (8/2)x2 using a stroke as an icon for a broom pushing away bundles, and a cross as an icon for a lift stacking the bundles. And laughs when seeing that a calculator confirms this independent of the total and the bundle thus giving a formula with unspecified numbers '$T = (T/B)\text{x}B$' saying "from T, T/B times, B may be pushed away and stacked". Consequently, search questions about 'bundle-numbers' and 'recounting' may be given to small groups of four preschool children to get ideas about how to design a generation-1 curriculum.

GUIDING QUESTIONS

The following guiding questions were used: "There seems to be five strokes in the symbol five. How about the other symbols?", "How many bundles of 2s are there in ten?", "How to count if including the bundle?", "How to count if using a cup for the bundles?", "Can bundles also be bundled, e.g. if counting ten in 3s?", "What happens if we bundle to little or too much?", "How to recount icon-numbers in tens?", "How to manually recount 8 in 2s, and recount 7 in 2s?", "What to do if a bundle is not full?", "How to bundle-count seconds, minutes, hours, and days?", "How to double-count lengths in centimeters and inches?", "A dice decided my share in a lottery ticket, how to share a gain?", "Which numbers can be folded in other numbers than 1s?", "Asking how many 2s in 8 may be written as u*2 = 8, how can this equation be solved?", "How to recount from tens to icons?", "How to add 2 3s and 4 5s next-to?", "How to add 2 3s and 4 5s on-top?", " 2 3s and some 5s gave 3 8s, how many?", "How to add totals bundle-counted in tens?", "How to subtract totals bundle-counted in tens?", "How to add per-numbers?", "How to enlarge or diminish bundle-bundle squares?", "What happens when recounted stacks are placed on a squared paper?", "What happens when turning or stacking stacks?"

OBSERVATIONS AND REFLECTIONS 02

Data and ideas allowed designing Micro Curricula (MC) with guiding questions and answers (Q, A).

MC 01: Digits as Icons

With strokes, sticks, dolls, and cars we observe that four 1s can be bundled into 1 fours that can be rearranged into a 4-icon if written less sloppy. So, for each 4 1s there is 1 4s, or there is 1 4s per 4 1s. In this way, all digits may be iconized, and used as units for bundle-counting (Tarp, 2018b).

MC 02: Bundle-counting Ten Fingers

A total of ten ones occurring as ten fingers, sticks or cubes may be counted in ones, in bundles, or with 'underloads' counting what must be borrowed to have a full bundle. Count ten in 5s, 4s, 3s, and 2s.

In 5s with bundles: 0*B*1, …, 0*B*4, 0*B*5 no 1*B*0, 1*B*1, …, 1*B*4, 1*B*5 no 2*B*0.

In 5s with bundles and underloads: 1*B*-4, 1*B*-3, …, 1*B*0, 2*B*-4, … , 2*B*0.

MC 03: Counting Sequences Using Tens and Hundreds

In oral counting-sequences the bundle is present as tens, hundreds, thousands, ten thousand (wan in Chinese) etc. By instead using bundles, bundles of bundles etc. it is possible to let power appear as the number of times, bundles have been bundled thus preparing the ground for later writing out a multi-digit number fully as a polynomial, T = 345 = 3*BB*4*B*5 = 3**B*^2 + 4**B* + 5*1.

Count 10, 20, 30, …, 90, 100 etc. Then 1*B*, 2*B*, …, 9*B*, ten*B* no 1*BB*.

Count 100, 200, 300, …, 900, ten-hundred no thousand. Then 1*BB*, 2*BB*, …, 9*BB*, ten*BB* no 1*BBB*.

Count 100, 110, 120, 130, …, 190, 200 etc. Then 1*BB*0*B*, 1*BB*1*B*, …, 1*BB*9*B*, 1*BB*ten*B* no 2*BB*0*B*.

A dice shows 3 then 4. Name it in five ways: thirty-four, three-ten-four, three-bundle-four, four-bundle-less6, and forty less 6. Travel on a chess board while saying 1*B*1, 2*B*1, 3*B*1, 3*B*2, …; 3*B*4.

MC04: Cup-counting and Bundle-bundles

When counting a total, a bundle may be changed to a single thing representing the bundle to go to a cup for bundles, later adding an extra cup for bundles of bundles. Writing down the result, bundles and unbundled may be separated by a bundle-letter, a bracket indicating the cups, or a decimal point.

Q. T = two hands, how many 3s?

A. With 1 3s per 3 1s we count 3 bundles and 1 unbundled, and write T = 3*B*1 3s = 3]1 3s = 3.1 3s showing 3 bundles inside the cup, and 1 unbundled outside. However, 3 bundles are 1 bundle-of-bundles, 1*BB*, so with bundle-bundles we write T = 1*BB*0*B*1 3s = 1]0]1 3s = 10.1 3s with an additional cup for the bundle-bundles.

Q. T = two hands, how many 2s?

A. With 1 2s per 2 1s we count 5 bundles, T = 5*B*0 2s = 5]0 2s = 5.0 2s. But, 2 bundles is 1 bundle-of-bundles, 1*BB*, so with bundle-bundles we write T = 2*BB*1*B*0 2s = 2]1]0 2s = 21.0 2s. However, 2 bundles-of-bundles is 1 bundle-of-bundles-of-bundles, 1*BBB*, so with bundle-bundle-bundles we write T = 1*BBB*0*BB*1*B*0 2s= 1]0]1]0 2s = 101.0 2s with an extra cup for the bundle-bundle-bundles.

MC05: Recounting in the Same Unit Creates Underloads and Overloads

Recounting 8 1s in 2s gives T = 4*B*0 2s. We may create an underload by borrowing 2 to get 5 2s. Then T = 5*B*-2 2s = 5]-2 2s = 5.-2 2s. Or, we may create an overload by leaving some bundles unbundled. Then T = 3*B*2 2s = 2*B*4 2s = 1*B*6 2s. Later, such ‘flexible bundle-numbers’ will ease calculations.

MC06: Recounting in Tens

With ten fingers, we typically use ten as the counting unit thus becoming 1*B*0 needing no icon.

Q. T = 3 4s, how many tens? Use sticks first, then cubes.

A. With 1 tens per ten 1s we count 1 bundle and 2, and write T = 3 4s = 1*B*2 tens = 1]2 tens = 1.2 tens, or T = 2*B*-8 tens = 2.-8 tens using flexible bundle-numbers. Using cubes or a pegboard we see that increasing the base from 4s to tens means decreasing the height of the stack. On a calculator we see that 3x4 = 12 = 1.2 tens, using a cross called multiplication as an icon for a lift stacking bundles. Only the calculator leaves out the unit and the decimal point. Often a star * replaces the cross x.

Q. *T* = 6 7s, how many tens?

A. With 1 tens per ten 1s we count 4 bundles and 2, and write *T* = 6 7s = 4*B*2 tens = 4]2 tens = 4.2 tens. Using flexible bundle-numbers we write *T* = 6 7s = 5*B*-8 tens = 5]-8 tens = 5.-8 tens = 3*B*12 tens. Using cubes or a pegboard we see that increasing the base from 7s to tens means decreasing the height of the stack. On a calculator we see that 6*7 = 42 = 4.2 tens.

Q. *T* = 6 7s, how many tens if using flexible bundle-numbers on a pegboard?

A. *T* = 6 7s = 6*7 = (*B*-4) * (*B*-3) = *BB*-3*B*-4*B*+4*3 = 10*B*-3*B*-4*B*+1*B*2 = 4*B*2 since the 4 3s must be added after being subtracted twice.

MC 07: Recounting Iconizes Operations and Creates a Recount-formula for Prediction

A cross called multiplication is an icon for a lift stacking bundles. Likewise, an uphill stroke called division is an icon for a broom pushing away bundles. Recounting 8 1s in 2s by pushing away 2-bundles may then be written as a 'recount-formula' 8 = (8/2)*2 = 8/2 2s, or *T* = (*T*/*B*)**B* = *T*/*B* *B*s, saying "From *T*, *T*/*B* times, we push away *B* to be stacked". Division followed by multiplication is called changing units or proportionality. Likewise, we may use a horizontal line called subtraction as an icon for a rope pulling away the stack to look for unbundled singles.

These operations allow a calculator predict recounting 7 1s in 2s. First entering '7/2' gives the answer '3.some' predicting that pushing away 2s from 7 can be done 3 times leaving some unbundled singles that are found by pulling away the stack of 3 2s from 7. Here, entering '7-3*2' gives the result '1', thus predicting that 7 recounts in 2s as 7 = 3*B*1 2s = 3]1 2s = 3.1 2s.

Recounting 8 1s in 3s gives a stack of 2 3s and 2 unbundled. The singles may be placed next-to the stack as a stack of unbundled 1s, written as *T* = 8 = 2.2 3s. Or they may be placed on-top of the stack counted in bundles as 2 = (2/3)*3, written as *T* = 8 = 2 2/3 3s thus introducing fractions. Or, as *T* = 8 = 3.-1 3s if counting what must be borrowed to have another bundle.

Q. *T* = 9, 8, 7; use the recount-formula to predict how many 2s, 3s, 4s, 5s before testing with cubes.

MC 08: Recounting in Time

Counting in time, a bundle of 7days is called a week, so 60days may be recounted as *T* = 60days = (60/7)*7days = 8*B*4 7days = 8weeks 4days. A bundle of 60 seconds is called a minute, and a bundle of 60 minutes is called an hour, so 1 hour is 1 bundle-of-bundles of seconds. A bundle of 12hours is called a half-day, and a bundle of 12months is called a year.

MC 09: Double-counting in Space Creates Per-Numbers or Rates

Counting in space has seen many units. Today centimeter and inches are common. 'Double-counting' a length in inches and centimeters approximately gives a 'per-number' or rate 2in/5cm shown with cubes forming an L. Out walking we may go 3 meters each 5 seconds, giving the per-number 3m/5sec. The two units may be bridged by recounting in the per-number, or by physically combining Ls.

Q. *T* = 12in = ?cm; and *T* = 20cm = ?in

A1. *T* = 12in = (12/2)*2in = (12/2)*5cm = 30cm; and A2. *T* = 20cm = (20/5)*5cm = (20/5)*2in = 8in

MC 10: Per-numbers Become Fractions

Double-counting in the same unit makes a per-number a fraction. Recounting 8 in 3s leaves 2 that on-top of the stack become part of a whole, and a fraction when counted in 3s: *T* = 2 = (2/3)*3 = 2/3 3s.

Q. Having 2 per 3 means having what per 12?

A. We recount 12 in 3s to find the number of 2s: *T* = 12 = (12/3)*3 giving (12/3) 2s = (12/3)*2 = 8. So, having 2/3 means having 8/12. Here we enlarge both numbers in the fraction by 12/3 = 4.

Q. Having 2 per 3 means having 12 per what?

A. We recount 12 in 2s to find the number of 3s: *T* = 12 = (12/2)*2 giving (12/2) 3s = (12/2)*3 = 18. So, having 2/3 means having 12/18. Here we enlarge both numbers in the fraction by 12/2 = 6.

MC 11: Per-numbers Become Ratios

Recounting a dozen in 5s gives 2 full bundles, and one bundle with 2 present, and 3 absent: *T* = 12 = 2*B*2 5s = 3*B*-3 5s. We say that the ratio between the present and the absent is 2:3 meaning that with 5 places there will be 2 present and 3 absent, so the present and the absent constitute 2/5 and 3/5 of a bundle. Likewise, if recounting 11 in 5s, the ratio between the present and the absent will be 1:4, since the present constitutes 1/5 of a bundle, and the absents constitute 4/5 of a bundle. So, splitting a total between two persons A and B in the ration 2:4 means that A gets 2, and B gets 4 per 6 parts, so that A gets the fraction 2/6, and B gets the fraction 4/6 of the total.

MC 12: Prime Units and Foldable Units

Bundle-counting in 2s has 4 as a bundle-bundle. 1s cannot be a unit since 1 bundle-bundle stays as 1. 2 and 3 are prime units that can be folded in 1s only. 4 is a foldable unit hiding a prime unit since 1 4s = 2 2s. Equal number can be folded in 2s, odd numbers cannot. Nine is an odd number that is foldable in 3s, 9 1s = 3 3s. Find prime units and foldable units up to two dozen.

MC 13: Recounting Changes Units and Solves Equations

Rephrasing the question “Recount 8 1s in 2s” to “How many 2s are there in 8?” creates the equation ‘*u**2 = 8’ that evidently is solved by recounting 8 in 2s since the job is the same:

If *u**2 = 8, then *u**2 = 8 = (8/2)*2, so *u* = 8/2 = 4.

The solution *u* = 8/2 to *u**2 = 8 thus comes from moving a number to the opposite side with the opposite calculation sign. The solution is verified by inserting it in the equation: *u**2 = 4*2 = 8, OK.

Recounting from tens to icons gives equations: “42 is how many 7s” becomes *u**7 = 42 = (42/7)*7.

MC 14: Next-to Addition of Bundle-Numbers Involves Integration

Once recounted into stacks, totals may be united next-to or on-top, iconized by a cross called addition.

To add bundle-numbers as 2 3s and 4 5s next-to means adding the areas 2*3 and 4*5, called integral calculus where multiplication is followed by addition.

Q. Next-to addition of 2 3s and 4 5s gives how many 8s?

A1. T = 2 3s + 4 5s = (2*3+4*5)/8 8s = 3.2 8s; or A2. T = 2 3s + 4 5s = 26 = (26/8) 8s = 3.2 8s

MC 15: On-top Addition of Bundle-Numbers Involves Proportionality

To add bundle-numbers as 2 3s and 4 5s on-top, the units must be made the same by recounting.

Q. On-top addition of 2 3s and 4 5s gives how many 3s and how many 5s?

A1. T = 2 3s = (2*3/5)*5 = 1.1 5s, so 2 3s and 4 5s gives 5.1 5s

A2. T = 2 3s + 4 5s = (2*3+4*5)/5 5s = 5.1 5s; or T = 2 3s + 4 5s = 26 = (26/5) 5s = 5.1 5s

MC 16: Reversed Addition of Bundle-Numbers Involves Differentiation

Reversed addition may be performed by a reverse operation, or by solving an equation.

Q. Next-to addition of 2 3s and how many 5s gives 3 8s?

A1: Removing the 2*3 stack from the 3*8 stack, and recounting the rest in 5s gives (3*8 – 2*3)/5 5s or 3.3 5s. Subtraction followed by division is called differentiation.

A2: The equation 2 3s + u*5 = 3 8s is solved by moving to opposite side with opposite calculation sign

u*5 = 3 8s – 2 3s = 3*8 – 2*3, so u = (3*8 – 2*3)/5 = 18/5 = 3 3/5, giving 3.3 5s.

MC 17: Adding and Subtracting Tens

Bundle-counting typically counts in tens, but leaves out the unit and the decimal point separating bundles and unbundled: T = 4*B*6 tens = 4.6 tens = 46. Except for e-notation with a decimal point after the first digit followed by an e with the number of times, bundles have been bundled: T = 468 = 4.68e2.

Calculations often leads to overloads or underloads that disappear when re-bundling:

Addition: 456 + 269 = 4*BB*5*B*6 + 2*BB*6*B*9 = 6*BB*11*B*15 = 7*BB*12*B*5 = 7*BB*2*B*5 = 725.

Subtraction: 456 – 269 = 4*BB*5*B*6 – 2*BB*6*B*9 = 2*BB*-1*B*-3 = 2*BB*-2*B*7 = 1*BB*8*B*7 = 187

Multiplication: 2 * 456 = 2 * 4*BB*5*B*6 = 8*BB*10*B*12 = 8*BB*11*B*2 = 9*B*1*B*2 = 912

Division: 154 /2 = 15*B*4 /2 = 14*B*12 /2 = 7*B*6 = 76

MC 18: Next-to Addition & Subtraction of Per-Numbers and Fractions is Calculus

Throwing a dice 8 times, the outcome 1 and 6 places 4 cubes on a chess board, and the rest 2 cubes. When ordered it may be 5 squares with 2 cubes per square, and 3 squares with 4 cubes per square. When adding, the square-numbers 5 and 3 add as single-numbers to 5+3 squares, but the per-numbers add as stack-numbers, i.e. as 2 5s + 4 3s = (2*5+4*3)/8*8 = 2.6 8s called the average: If alike, the per-numbers would be 2.6 cubes per square. Thus per-numbers add by areas, i.e. by integration. Reversing the question to 2 5s + ? 3s total 3 8s then leads to differentiation: 2 5s + ? 3s = 3 8s gives the equation

2*5 + u*3 = 3*8, so u*3 = 3*8 – 2*5, so u = (3*8 – 2*5)/3 = 4 2/3, or u = (T2-T1)/3 = ΔT/3

Likewise, with fractions. With 2 apples of which 1/2 is red, and 3 apples of which 2/3 are red, the total is 5 apples of which 3/5 are red. Again, the unit-numbers add as single numbers, and, as per-numbers, the fractions must be multiplied before adding thus creating areas added by integration.

MC 19: Having Fun with Bundle-Bundle Squares

On a pegboard we see that 5 5s + 2 5s + 1 = 6 6s, and 5 5s – 2 5s + 1 = 4 4s suggesting three formulas:

$n*n + 2*n + 1 = (n+1)*(n+1)$; and $n*n - 2*n + 1 = (n-1)*(n-1)$; and $(n-1)*(n+1) = n*n - 1$.

Two $s*s$ bundle-bundles form two squares that halved by their diagonal d gives four half-squares called right triangles. Rearranged, they form a diagonal-square $d*d$. Consequently, $d*d = 2*s*s$

Four $c*b$ playing cards with diagonal d are placed after each other to form a $(b+c)*(b+c)$ bundle-bundle square. Below to the left is a $c*c$ square, and to the right a $b*b$ square. On-top are 2 playing cards. Inside there is a $d*d$ square and 4 half-cards. Since 4 half-cards is the same as 2 cards, we have the formula $c*c + b*b = d*d$ making it easy the add squares, you just square the diagonal.

MC 20: Having Fun with Halving Stacks by its Diagonal to Create Trigonometry

Halving a stack by its diagonal creates two right triangles. Traveling around the triangle we turn three times before ending up in the same direction. Turning 360 degrees implies that the inside angles total 180 degree, and that a right angle is 90 degrees. Measuring a 5up_per_10out angle to 27 degrees we see that tan(27) is 0.5 approximately. So, the tan-number comes from recounting the height in the base.

MC 21: Having Fun with a Squared Paper

A dozen may be 12 1s, 6 2s, 4 3s, 3 4s, 2 6s, or 1 12s. Placed on a squared paper with the lower left corners coinciding, the upper right corners travel on a bending line called a hyperbola showing that a dozen may be transformed to a 3.5 3.5s bundle-bundle square approximately. Traveling by saying "3up_per_1out, 2up_per_1out, …, 3down_per_1out" allows the end points to follow a parabola. With a per-number 2G/3R, a dozen R can be changed to 2G+9R, 4G+6R, 6G+3R, and 8G. Plotted on a square paper with R horizontally and G vertically will give a line sloping down with the per-number.

MC 22: Having Fun with Turning and Combining Stacks

Turned over, a 3*5 stack becomes a 5*3 stack with the same total, so multiplication-numbers may commute (the commutative law). Adding 2 7s on-top of 4 7s totals (2+4) 7s, 2*7+4*7 = (2+4)*7 (the distributive law). Stacking stacks gives boxes. Thus 2 3s may be stacked 4 times to the box T = 4*(2*3) that turned over becomes a 3*(2*4) box. So, 2 may freely associate with 3 or 4 (the associative law).

DISCUSSION AND RECOMMENDATION

This paper asks: what mastery of Many does the child develop before school? The question comes from observing that mathematics education still seems to be hard after 50 years of research; and from wondering if it is hard by nature or by choice, and if it is needed to achieve its goal, mastery of Many.

To find an answer, phenomenology, experiential, and design research is used to create a cycle of observations, reflections, and testing of micro curricula designed from observing the reflections of preschoolers to guiding questions on mastering Many. The first observation is that children use two-dimensional bundle-numbers with units instead of the one-dimensional single numbers without units that is taught in school together with a place value system. Reflecting on this we see that units make counting, recounting, and double-counting core activities leading to proportionality by combining division and multiplication, thus reversing the order of operations: first division pulls away bundles to be lifted by multiplication into a stack that is pulled away by subtraction to identify unbundled singles that becomes decimal, fractional or negative numbers. And that recounting between icons and tens leads to equations when asking e.g. 'how many 5s are 3 tens?' And that units make addition ambiguous: shall totals add on-top after proportionality has made the units like, or shall they add next-to as an example of integral calculus adding areas, and leading to differentiation when reversed? Finally, we see that flexible bundle-numbers ease traditional calculations on ten-based numbers.

Testing the micro curricula will now show if mathematics is hard by nature or by choice. Of course, investments in traditional textbooks and teacher education, all teaching single numbers without units, will deport testing to the outskirts of education, to pre-school or post-school; or to special, adult, migrant, or refugee education; or to classes stuck in e.g. division, fractions, precalculus, etc. All that is needed is asking students to count fingers in bundles. Recounting 8 in 2s thus directly gives the proportionality recount-formula $8 = (8/2)*2$ or $T = (T/B)*B$ used in STEM, and to solve equations. Likewise, direct and reversed next-to addition leads directly to calculus. Furthermore, testing micro curricula will allow teachers to practice action learning and action research in their own classroom.

Phenomenologically, it is important to respect and develop the way Many presents itself to children thus providing them with the quantitative competence of a number-language. Teaching numbering instead of numbers thus creates a new and different Kuhnian paradigm (1962) that allows mathematics education to have its communicative turn as in foreign language education (Widdowson, 1978). The micro-curricula allow research to blossom in an educational setting where the goal of mathematics education is to master outside Many, and where inside schoolbook and university mathematics is treated as grammatical footnotes to bracket if blocking the way to the outside goal, mastery of Many.

To master mathematics may be hard, but to master Many is not. So, to reach this goal, why force upon students a detour over a mountain too difficult for them to climb? If the children already possess mastery of Many, why teach them otherwise? Why not lean from children instead?

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