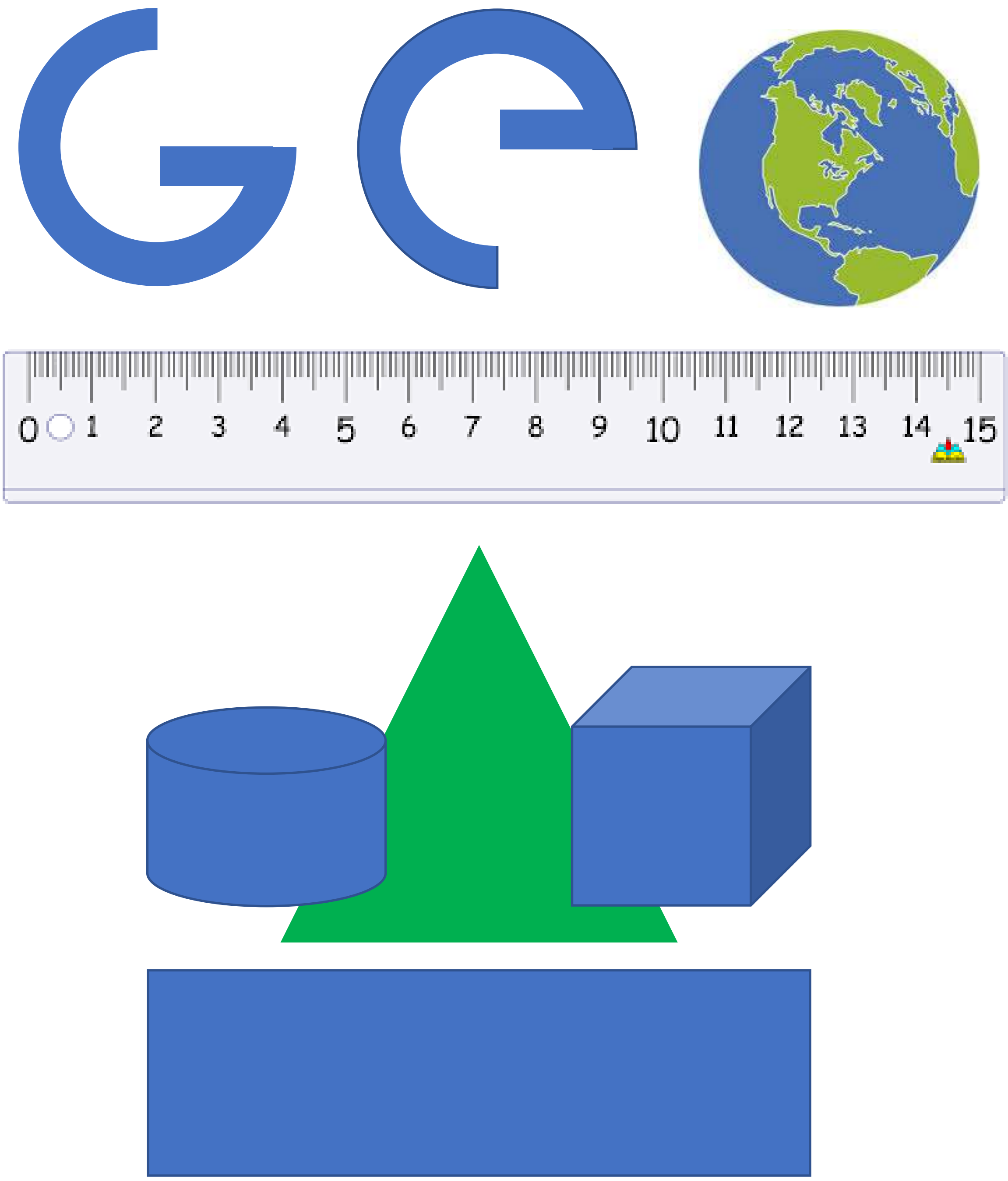

GeoMetry

Earth Measurement

from below

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GEOMETRY - FROM BELOW

‘Geometry from below’ means geometry as tales about a social practice, in this case about ‘earth-measurement’, to which the Greek word ’geo-metry’ can be directly translated.

The earth is what we live on and what we live on. We divide the earth between us by drawing dividing lines. This divides the soil into areas limited by lines, multi-edges, polygons.

If these boundaries disappear, it is important to be able to re-establish them, and this restoration of lines and corners requires that these can be measured.

In ancient Egypt, the Nile thus crossed its banks once a year and brought manure to the fields. After retiring, the divisions had to be re-established.

Geometry from below can be understood as the opposite of geometry from above, deducing geometry from metaphysical truths, axioms.

The following material is not a traditional textbook, but rather an activity guide with suggestions for a range of activities that the reader can perform and report.

So, the idea is that the reader builds his own textbook.

However, some proposals for definitions and rules are included.

Rules can be proved either through evidence or conviction. It is recommended to work with the last based on the task: “Try to convince someone else of the validity of the rule.” Likewise, the idea is that the reader performs his own illustrations, which is why very few illustrations are included in the material.

The following exercises should be performed both on paper, on the floor and on earth, as well as, if possible, on computer programs.

We work out a report of questions, techniques, inventions, and discoveries we meet along the way.

Jutland, Denmark, Allan Tarp

DK version January 2000, US version May 2020

http://mathecademy.net/geometry-from-below/

Contents

GE00 LAND SHARING

In this chapter we will look at a number of traditional land-sharing problems. No solutions are provided, but the problems can hopefully serve as motivation for the following chapters.

Basic problem: How to share an area?

Exercise 1. Two people stand in random places on a restricted floor surface and are tasked with dividing it between them. represent groups of different sizes. What are the different sharing principles? Note: Random location in a room may be possible. achieved with a cube: I stand with my back and right shoulder against the wall in a corner. Then I roll a dice 4-6 times. The cube tells me how many steps I need to walk alternately forward and to the left. Same with the other participants.

Exercise 2. Perform a sharing based on the sharing principle: No matter to everyone.

Exercise 3. Perform a sharing based on the sharing principle: Just far to the limit.

Exercise 4. Perform a division based on the sharing principle: The distances to the border must be 1:2, as there are twice as many people living in the other area.

Exercise 5. Repeat exercise 1-4, but now with three people.

Exercise 6. Repeat exercise 1-4, but now with more than three people.

GE01 POLYGONS

Basic problem: How to talk about and divide areas.

Exercise 1. Set up a triangle. Measure the triangle in order to re-establish it. Delete the triangle. Re-establish the triangle on the basis of the objectives, on the one hand at any point and partly in the same place. Insert names (definitions) based on this exercise.

Exercise 2. Construct a square with skewed angles. Measure the square in order to re-establish it. Delete the square. Re-establish the square on the basis of the dimensions, on the one hand, in any place and partly in the same place.

Exercise 3. Set up a triangle. Divide it into two right triangles. Measure the right triangles in order to restore them. Re-establish the triangles on the basis of the objectives, on the one hand, at any point and partly in the same place.

Exercise 4. Set two parallel lines. Measure the distance of the lines.

Exercise 5. What is A4 paper? And what are A1, A2, A3, A5 etc?

Exercise 6. Length can be measured in many ways. Today, meters are usually used. Other targets were used in the past. Which? Is the meter system used all over the world? What is the link between meters and other targets?

Exercise 7. Can angles be measured in ways other than degrees? Why is a right angle 90 degrees and not 100 degrees?

Exercise 8. Find rules for the defined concepts and convince someone else that these rules are true.

DEFINITION 1. One point is

DEFINITION 2. A straight line is

DEFINITION 3. A polygon is

DEFINITION 4. The circumference of a polygon is

DEFINITION 5. An angle is

DEFINITION 6. A polygon's parts are

DEFINITION 7. A diagonal is

DEFINITION 8. A rectangle is

DEFINITION 9. A right triangle is

DEFINITION 10. The distance between a point and a line is

DEFINITION 11. Parallel lines are

DEFINITION 12. A normal is

DEFINITION 13. A convex polygon is

Rule 1. A polygon can be divided into

Rule 2. A triangle can be divided into

GE02 TRIANGLES

Basic problem: How are triangles designated and calculated?

Exercise 1. Construct a triangle and enter names for the different components of a triangle. How many targets can we only achieve in order to uniquely construct a triangle? How many different triangle types are there?

Exercise 2. A triangle can be divided by different lines. Name some of these and discuss what sharing principles the different lines might embodies.

Exercise 3. Find rules for the named lines in practice 2 and convince someone else that these rules are true.

Exercise 4. Construct an equilateral triangle. Do special rules apply to straight-legged triangles?

Exercise 5. Construct an equilateral triangle. Do special rules apply to equilateral triangles?

Exercise 6. Cut out a triangle and hang it in a corner. Draw the load line, how is it? Switch to the other corners. Where is the center of gravity of the triangle?

Exercise 7. The foot point of a dividing line in a triangle is the intersection of the line with a side. Do special rules apply to foot points for heights, medians, angular halves and bisectors?

Exercise 8. A triangle can be wrapped in a rectangle in different ways. Which rectangle has the smallest circumference?

Exercise 9. Construct a triangle ABC. Move A to A* without changing the circumference. What will be the new points?

DEFINITION 1. A triangle or a three-corner is

DEFINITION 2. The three angles of a triangle are

DEFINITION 3. The three sides of a triangle are

DEFINITION 4. An acute triangle is

DEFINITION 5. An obtuse triangle is

DEFINITION 6. In triangle ABC the height ha is

DEFINITION 7. In triangle ABC the median ma is

DEFINITION 8. In triangle ABC, the angular bisector bA is

DEFINITION 9. In triangle ABC the bisector na is

DEFINITION 10. An SSA triangle is a triangle in which two sides and one angle are known. Similarly, an SAA and an SSS triangle are defined.

Rule 1. In a triangle is the angle sum.

Rule 2. In a triangle, the heights intersect

Rule 3. In a triangle, the angular half-lines intersect

Rule 4. In a triangle, the medians intersect

Rule 5. In a triangle, the bisectors intersect

Rule 6. In a triangle, the intersection of the heights has the following property:

Rule 7. In a triangle, the intersection of the angular halves has the following property:

Rule 8. In a triangle, the intersection of the medians has the following property:

Rule 9. In a triangle, the intersection of the bisectors has the following property:

GE03 RIGHT TRIANGLES

Basic problem: How is a right triangle designated and calculated?

Exercise 1. Construct a rectangle and divide it into two right triangles using a diagonal. Introduce names for the different components of the right triangle.

Exercise 2. Construct a rectangle and divide it into two right triangles using a diagonal. Measure the angles. Measure the diagonal and side lengths, partly in cm, and partly by diagonal lengths (as a percentage of the diagonal). Compare to the calculator's sin and cos button.

Exercise 3. Construct a rectangle and divide it into two right triangles using a diagonal. Measure the angles. Measure the diagonal and side lengths, partly in cm, partly in floor lengths (as a percentage of the floor, i.e. the horizontal side). Compare to the calculator's tan button.

Exercise 4. Set on millimeter paper a quartz circle with a radius of 10 cm, or set on the floor a quartz circle with a radius of 1 meter. Sign a series of right triangles ABC with A in the center of the circle, B on the circular arc and C on the 0-degree line. A shall pass degrees 10, 20, 30 up to 80. Table for each A value the length of BC and AC as well as the BC/AC ratio. Compare to the calculator's sin, cos and tan button.

Exercise 5. Construct a right triangle from abandoned dimensions. What's the minimum number of goals we can settle for? How many different types of right triangles are there? we can measure up to the unknown goals, but can we also count for the unknown goals?

Exercise 6. Construct a random triangle and divide it into two right triangles. Measure the parts in the right triangles and calculate them afterwards. Finally, enter the dimensions of the original triangle's parts, and check when measuring.

Exercise 7. Construct a known triangle and divide it into two right triangles. Measure the parts in the right triangles and calculate them afterwards.

Exercise 8. Set off a known SAA triangle. Measure and calculate the other parts.

Exercise 9. Set off a known SSA triangle. Measure and calculate the other parts.

Exercise 10. Set off a known rectangle. Measure and calculate the length and angles of the diagonal.

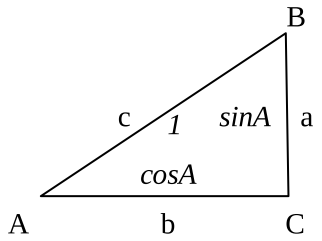
Exercise 11. Construct using the different known angle sizes of the seaweed, e.g. 27°, 42° and 133°.

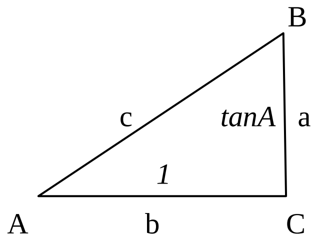
Exercise 12. Construct a rectangle and construct new rectangles with the same diagonal. How are they?

Exercise 13. Use a PC spreadsheet to set up the different types of triangular calculation.

GREEK DEFINITIONS: In a right triangle, the two short sides are called catheter, and the long hypotenuse.

ARABIC DEFINITIONS: Triangle ABC is right with C as the right angle. b is horizontal and a vertical. The floor side b is called the cosine side as seen from A. The wall side a is called the sine side or the tangent side as seen from the A. Sine and cosine sides are indicated as a percentage of the sloping wall c, or with c as a unit of measurement ("recounting" in c's: a = a/c*c = sinA*c). The tangent side is indicated as a percentage of the floor b.





sinA = a/c

cosA = b/c

tanA = a/b

The calculation problem

In a right triangle there are three unknowns. Calculation therefore requires three equations.

The Greeks knew only two, one angle- and one area equation (Pythagoras):

A+B = 90   a^2 + b^2 = c^2

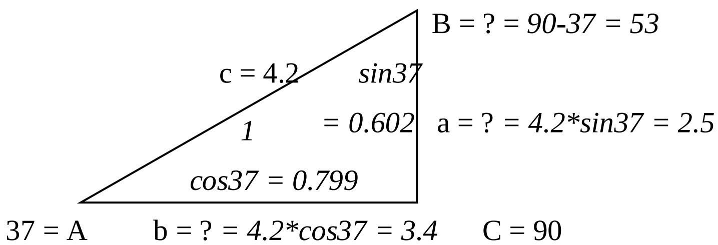
The Arabs knew three, one angle and two side equations:

A+B = 90   a = c*sinA   b = c*cosA

The following calculations use the two sides of the triangle: the outer side with the actual numbers and the inside with the percentages. Alternatively, we could use equation schemes, see the appendix.

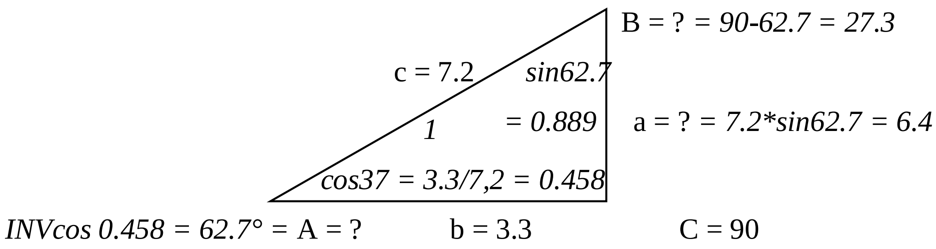
Example 1. *Given SAA, Find SSA.*

In triangle ABC is C = 90°, A = 37° and c = 4.2. Find a = ?, b = ? and B = ?



Example 2. *Given SSA, Find SAA.*

In triangle ABC, C = 90°, b = 3.3, and c = 7.2. Find a = ?, A = ? and B = ?



GE04 DIFFERENT APPLICATIONS

Problem 1: How is the height of something high, for example, a flagpole?

Problem 2: How to move a corner thing?

Problem 3: How are shortcuts calculated?

Problem 4: How does one thing move fastest from a point in one area to a point in another area when we move at different speeds in the two areas?

Problem 5: How are the different openings calculated at a door that is ajar?

Problem 6: How to build a path up a steep mountainside?

Problem 7: How steep can a ladder be set so as not to break glass or ice?

Problem 8: How are astronomical distances calculated?

Problem 9: How should a triangular bridge over a river be sized?

Problem 10: During a swing, when do we have to jump off to get the furthest away?

Exercise 1. Determine the height of a tall thing (a flagpole) in two different ways: the lightweight, where we can get all the way to the thing, and the hard one where we can't. Determine in the same way the width of a wide thing (a house wall or a "river"). If possible, check the calculations by measurement.

Exercise 2. Set off once with the width 2 meters, which forms a crack of 90 degrees. A stick should be turned around the corner without lifting. What is the maximum length of the stick? A box with a width of 1 meter must be turned around the corner without lifting. What is the maximum length of the box? If necessary, use the other objectives.

Exercise 3. Once with the width 3 meters forms a crack of 90 degrees and continues with a width of 4 meters. A stick should be turned around the corner without lifting. What is the maximum length of the stick? A box with a width of 2 meters must be turned around the corner without lifting. What is the maximum length of the box?

Exercise 4. A treasure can be found by first traveling 4 meters to the east, then 2. 5 meters to the south, and finally 3. 1 meter to the west. Find a shortcut to the treasure.

Exercise 5. A treasure can be found by first traveling 4 meters to the east in 32 degrees northerly direction, then 2. 5 meters to the south in 68 degrees westerly direction, and finally 3. 1 meter to the west in a 14-degree northerly direction. Find a shortcut to the treasure.

Exercise 6. A boat rows over a 50-meter-wide river at a speed of 20 meters/minute. The current in the river is 10 meters/minute. What angle should you praise for landing right across the face on the opposite side? How long does the trip take?

Exercise 7. Construct a straight line and two points on either side of the line. Go from one point to the other with steps equal to 1 shoe length. 1 second corresponds to 1 shoe length in one area and 2 shoe lengths in the other. Find the fastest route. Could the fastest route be figured out? If necessary, use a PC spreadsheet to calculate different routes. A rule applies to the approach and refraction angle at the point on the boundary line which the fastest route passes. Which?

Exercise 8. Open a door ajar. There are now three openings, one perpendicular to the wall, one parallel to the wall and one parallel to the door. How big are these openings? How much will 10 degrees extra increase these openings by?

Exercise 9. Tip a plate 30 degrees and sign up a way up that can rise no more than 20 degrees (a hairpin bend). Repeat the exercise with other degree numbers. How much does the gravitational pull of a car increase when the rise of the road increases 10 degrees?

Exercise 10. Place a heavy book on a scale. Tilt it to different positions. What happens to the weight? Could this result be calculated? How much is the pressure against a vertical hand supporting it?

Exercise 11. Construct and load a triangular bridge and control the pressure against the surface by placing the bridge on two scales. Can these numbers be calculated?

Exercise 12. Distances in space cannot be measured, but must be calculated. How is the radius of the earth calculated? How far is it to the moon? What is the radius of the moon? How far is the sun? What is the radius of the sun?

Exercise 13 (difficult). A person sits in a swing that is suspended in ropes that are 3 meters long. Pull out the swing so that it is in height 1 meter above the bottom point. What angle does this correspond to?

If the swing is released, the speed v can be calculated from the formula: $v^2 = 19.6*h$, where h is the distance from the maximum height in the outer position. The speed consists of a horizontal and a vertical part.

Where do we have to jump off to get the furthest away?

(After the jump, the horizontal part of the speed will be unchanged, while the vertical part will grow downwards by 9.8 m/s every second.)

DEFINITION 1. One incident angle is

DEFINITION 2. A reflective angle is

Rule 1. The fastest route between two points in two different areas with speed v1 and v2 will meet the wrestling law at the crack point between the two areas:

sin (incident angle)/sin (reflective angle) = v1/v2.

GE05 AREA, SURFACE AREA, COVERAGE

Basic problem: How is the size of an area designated and calculated?

Exercise 1. A piece of squared A4 paper covers a certain area. How can the extent, area, coverage, extent of this surface be measured?

Exercise 2. A piece of squared A4 paper is divided by a transverse diagonal. What is the area of a right triangle?

Exercise 3. A piece of squared A4 paper is divided by lines into three triangles. What is the area of an ordinary triangle? Does this rule also apply to blunt-angled triangles?

Exercise 4. Construct a triangle and determine its area.

Exercise 5. Construct a triangle and divide it into 2 parts with the same area. In 3 parts. In 4 parts.

Exercise 6. Same as 5, but now the divider must be a normal to the baseline.

Exercise 7. A height can be difficult to measure accurately. Can a triangle area be calculated solely from sides and angles?

Exercise 8. An angle can be difficult to measure accurately. Can the area of a triangle be calculated solely from sides?

Exercise 9. Find some other surface areas that are used elsewhere and at other times.

Exercise 10. Use a PC spreadsheet to set up the different types of area calculation.

DEFINITION 1. An area unit is a square (a tile) of 1x1. 1m x 1m = 1m^2

Rule 1. A rectangle with side lengths a and b has an area of A =

Rule 2. A right triangle with catheter a and b has an area of A =

Rule 3. A triangle with a height and a surface g has an area of A =

Rule 4. Triangle ABC area can be calculated using the sine formula: A = 1/2*a*b*sinC.

Rule 5. The area of Triangle ABC can be calculated using the Herons formula:

A^2 = s*(s–a)*(s–b)*(s–c) where s = 1/2*(a+b+c) = half the circumference.

GE06 SHAPE CHANGE 1: SCALING

Basic problem: How is magnification or shrinking of a triangle is termed and calculated?

Exercise 1. When we walk away from a triangle, it seems to resize, but not shape. Construct a triangle ABC (it may be right). Set triangle AB*C* where the side lengths are half the size. What's the angles? What applies to the sides? What is the area?

Exercise 2. Construct a triangle ABC and enlarge it by 20% to 120% (scaling percentage and factor) to triangle AB*C*. Draw through B and C lines parallel to b and c (parallel transversals). These lines cut a in B** and C**. How big are the triangles ABC, AB*C*, BB*B** and CC*C** relative to each other?

Exercise 3. Construct a triangle ABC. Reseal it, but now in the 1:2 ratio. What is the scaling percentage and factor?

Exercise 4. Construct a triangle on a floor. Draw the triangle of paper in the 1:10 ratio.

Exercise 5. Construct a triangle on paper. Draw the triangle of a floor in the 1:10 ratio.

Definition 1. In a triangle, all side lengths are made k times as large. The number k is then called scaling factor, and k-1 is called scaling percent.

Rule 1. Scaling a triangle preserves the size of the angles and the direction of the sides.

Rule 2. If two triangles are similar, one is a scaling of the other.

Rule 3. In the triangle ABC and AB*C* we know that AB* = k*AB (AB* "re-counted" in AB's: AB* = (AB*/AB)*AB = k*AB) and AC* = k*AC.

Triangle AB*C* will then be a triangle of ABC. BB* = (k-1)*AB (provided k>1).

GE07 CHANGE OF FORM 2: LAND CONSERVATION

Basic problem: How can a shape change shape without changing the area (property exchange)?

Exercise 1. Construct a triangle ABC. Construct triangles A*BC with the same area. How will the A* items lie?

Exercise 2. Construct a right triangle ABC. Set triangles A*B*C with the same area. How will points A* and B* lie?

Exercise 3. Set off a rectangle ABCD. Construct rectangles AB*C*D* with the same area. How will the b*, c* and d* points lie?

Exercise 4. Construct two rectangles with the same area, increasing the longest side by 30%.

Exercise 5. Construct two right triangles with the same area, increasing the height by 25%.

Exercise 6. Construct two triangles with the same area, reducing the height by 40%.

Exercise 7. Set off a rectangle ABCD. Construct a square AB*C*D* with the same area.

Exercise 8. Construct a right triangle ABC. Construct the area a^2, as well as a rectangle with the area a^2 if one side length is c. Set the area b^2, as well as a rectangle with the area b^2, if a side length is c. What is the total area of these two rectangles?

Definition 1. A rectangle with the sides b and c must be changed to a rectangle with the sides a and s to preserve the area: a*s = b*c or a/b = c/s.

Definition 2. A rectangle with the sides b and c must be changed to a square with the side s to preserve the area: s^2 = b*c or a/s = s/b. s is then called a medium proportional to a and b, or a geometric average of a and b.

Definition 3. In triangle ABC, the BC side is enlarged by the factor k to B*C. The line through B parallel to B*A cuts AC into A*. A*C is then called a k-incision for AC and BC.

Rule 1. A fourth proportional s to a, b and d is a d/a cut to a and b.

Rule 2. A triangle retains area by

Rule 3. A right triangle retains area at

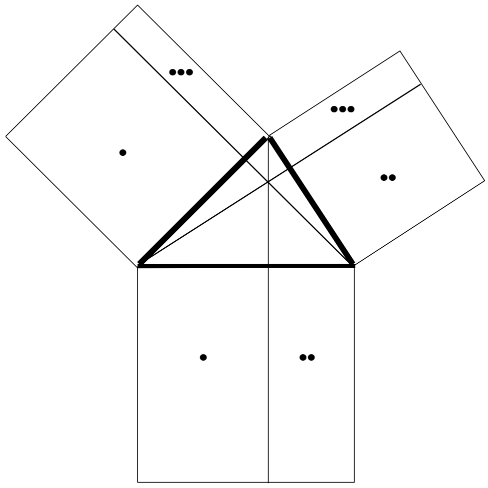
Rule 4. A rectangle retains area at

Rule 5. An mean proportional s to a and b can be constructed as the height of the semicircle arc above the a+b at the divider point.

GE08 OUTER SIDE SQUARES OF THE TRIANGLE, PYTHAGORAS

Basic problem: How can we make sure a triangle is right?

What applies to the outer side squares of a triangle?



Exercise 1. Construct a pointed triangle ABC with outer side squares. Set the three heights. The heights will divide the outer side squares into 3x2 parts. What is the case with these parts?

Exercise 2. Construct a right triangle ABC with outer side squares. Set the height. The height will divide one of the outer side squares into two parts. What is the case with these parts?

Exercise 3. Construct a blunt-angled triangle ABC with outer side squares. Set the three heights. The heights will divide the outer side squares into 3x2 parts. What is the case with these parts?

Exercise 4. Draw a triangle with heights and outer squares on cardboard. Cut out and road as a means of conviction.

Exercise 5. Use the land conservation technique as a means of conviction.

Exercise 6. Place a rectangle on a floor and use the diagonal to control the angles.

Exercise 7. Find some full numbers a, b, and c that meet the requirement a^2 + b^2 = c^2.

Exercise 8. Make of a string a closed loop with knots to check if an angle is right.

Exercise 9. Use a PC worksheet to set up diagonal calculation in a rectangle.

Definition 1. Triangle ABC's outer side squares are the squares a^2, b^2 and c^2.

Rule 1. In a triangle, the heights divide the outer squares of the triangle into one's "resettlement areas".

Rule 2. In triangle ABC is c^2 = a^2 + b^2 - 2*a*b*cosC

Rule 3. In the right triangle ABC (C right) is c^2 = a^2 + b^2 (Pythagoras' theorem)

Rule 4. In triangle ABC, C is right if c^2 = a^2 + b^2

GE09 NON-RIGHT TRIANGLE

Basic problem: How are the unknown parts calculated in a non-right triangle?

Exercise 1. Set off a known SSS triangle. Measure and calculate the three unknown angles.

Exercise 2. Set off a known SSA triangle. Measure and calculate the three unknown parts.

Exercise 3. Set off a known SAA triangle. Measure and calculate the three unknown parts.

Comment. An AAA triangle is only a AA triangle, as the last angle cannot be selected freely, but is determined by the other two. Thus, there is no information on a side, whereby the triangle becomes an SAA triangle.

Rule 1. In triangle ABC, the cosine relationships apply:

a^2 = b^2 + c^2 - 2*b*c*cos A

b^2 = a^2 + c^2 - 2*a*c*cos B

c^2 = a^2 + b^2 - 2*a*b*cos C,

Rule 2. In triangle ABC, the sinus relations apply:

a/sinA = b/sinB = c/sinC (watch out for the angles! why?)

GE10 OUTER BASE LENGTHS AND BASE ANGLES OF THE TRIANGLE

Basic problem: How can a triangle be measured from an outer base length? How can a triangle be re-established and calculated on the basis of external base dimensions?

Exercise 1. Construct a triangle ABC. Set off outside the triangle (possibly on the other side of the "river") an outer base line with known length KL. How can point A be determined from KL?

Exercise 2. Remove triangle ABC and restore it based on the outer base dimensions.

Exercise 3. How can triangle ABC's outer base dimensions (lengths or angles) form the basis for calculations of the triangle's parts, heights and area? Check by measurement.

Exercise 4. Use a PC spreadsheet to set up the calculation of the triangle's parts, heights, and area based on the outer base dimensions.

Definition 1. The base lengths of point A in relation to a base line KL are the lengths of the AK and AL line parts.

Definition 2. The base angles of point A in relation to a base line KL are the angles AKL and ALK.

Rule 1. The base angles can be calculated from the base lengths as follows:

AKL =

ALK =

Rule 2. The base lengths can be calculated from the base angles as follows:

h = height from K =

AK =

AL =

Rule 3. A right triangle ABC is considered from an outer baseline KL with length k.

Base lengths of the corners are referred to as:

KA = a1, KB = b1, KC = c1, LA = a2, LB = b2, LC = c2.

Base angles of the corners are referred to as:

AKL = A1, ALK = A2, BKL = B1, BLK = B2, CKL = C1, CLK = C2

The sides of the triangle can be calculated as follows:

a =

b =

c =

GE11 SQUARES

Basic problem: How are squares are designated and calculated?

Exercise 1. Construct different types of squares and name them.

Exercise 2. Find rules for a square's angles.

Exercise 3. Find rules for a square's diagonals.

Exercise 4. Find rules for a square area.

Exercise 5. Set a square and divide into two equal parts. Divide a square into three equal parts.

Exercise 6. Clip a square and find its center of gravity. Could we figure and make it to the center of gravity?

Definition 1. A trapeze is a square where a set of facing sides are parallel. A parallelogram is a square where both sets of facing sides are parallel. A rhombus is a parallelogram where all sides are equally long. A rectangle is a square where all angles are right. A square is a rectangle where all sides are equally long.

Rule 1. In a square, the angular sum

Rule 2. A trapeze with height h and parallel sides a and b has an area of A =

Rule 3. A parallelogram with height h and surface g has an area of A =

GE12 SPATIAL ANGULAR SHAPES, SURFACE AND VOLUME

Basic problem: How do describe spatial figures?

Exercise 1. Find different types of spatial shapes from everyday life and name them.

Exercise 2. Choose a box from everyday life. Measure the sides and diagonals. Calculate surface and volume, diagonal lengths and diagonal angles. If possible, check the volume by filling the box with water or sand, which can be poured into a cylinder glass that can measure volume.

Exercise 3. Construct a box according to given dimensions, partly of paper, partly of cardboard and partly on computer. If possible, check by calculating and measuring diagonal lengths.

Exercise 4. Choose an slanted cut-off box from everyday life, for example, a box that is cut off. Measure sides and angles. Calculate surface and volume. Measure and calculate diagonal lengths and diagonal angles.

Exercise 5. Set up a paper and floor tent. Measure or calculate side and diagonal lengths, angles, surface and volume.

Exercise 6. Construct a pyramid of paper and on the floor. Measure or calculate side and diagonal lengths, angles, surface and volume.

Exercise 7. A closed box with square bottoms must hold 1 liter. Describe different options in table or formula. Find the corresponding surface. Which box has the least surface. If necessary, use a PC spreadsheet.

Exercise 8. Same as exercise 7, but now the box is open at one end.

Exercise 9. Same as exercise 7, but now the box is open at both ends.

Exercise 10. A closed box with a square bottom shall have an outer surface of 1 m^2. Describe different options in table or formula. Find the corresponding volume. Which box has the largest volume. If necessary, use a PC spreadsheet.

Exercise 11. Same as exercise 10, but now the box is open at one end.

Exercise 12. Same as exercise 10, but now the box is open at both ends.

Exercise 13. Same as exercise 7-9, but now with a prism with an equilateral triangle as the base surface.

Exercise 14. Same as exercise 10-12, but now the prism has an equilateral triangle as the base surface.

Exercise 15. Same as exercise 7-9, but now with a pyramid with square surface instead of a box.

Exercise 16. Same as exercise 10-12, but now with a pyramid with square surface instead of a box.

Exercise 17. Same as exercise 7-9, but now with a pyramid with an equilateral triangular surface instead of a box.

Exercise 18. Same as exercise 10-12, but now with a pyramid with equilateral triangular surface instead of a box.

Exercise 19. Same as exercise 7-9, 13, 15 and 17, but now the material for the end surface is twice as expensive as aside. Minimum cost is requested.

Exercise 20. Where is the center of gravity of a box? How many degrees can the box be tilted before it topples over? Try and rain out.

Exercise 21. What is a polyhedron and what is a regular polyhedron?

Definition 1. A plane is

Definition 2. The angle between two planes is the angle between the normal of the plans.

Definition 3. A polyhedron or a multi-face is

Definition 4. A regular polyhedron is

Definition 5. A prism is

Definition 6. A fair prism is

Definition 7. A box is

Definition 8. A cube or cube is

Definition 9. A pyramid is

Definition 10. The volume or volume unit is a cube of 1x1x1. 1 liter = 1dm^3.

Rule 1. A box with side lengths a, b and c has the volume V =

Rule 2. A box of surface G and height h has the volume V =

Rule 3. A pyramid with a base G and height h has the volume V =

Rule 4. A box with side lengths a, b and c has the surface O =

Rule 5. A box with side lengths a, b and c has diagonal length D =

Rule 6. A roll with a side length a has diagonal length D =

Rule 7. The number of regular polyhedrons is

Rule 8. There are many rules for regular polyhedrons:

GE13 ROUND SHAPES: CIRCLES, CYLINDERS, SPHERES, ETC.

Basic problem: How are round plane and spatial shapes calculated and calculated?

Exercise 1. Find a series of circular things from everyday life. How can we find the center? Measure the circumference and diameter with a sewing thread. How many times can the diameter be around the circle?

Exercise 2. Draw a circle on paper. Cut it into narrow "Pizza pieces." If these are placed alternately in contrast, a rectangle-like figure will appear. What is the relationship between the two sides? What can the rectangle tell us about a circle's circumference and area.

Exercise 3. What is the circumference and area of a pie section and a circle section?

Exercise 4. What is the surface and volume of a sphere? Road the garbage of a mandarin and way then 1 cm^2 of the garbage. Way an apple and way behind a cube of 1 cm^3 carved from the apple.

Exercise 5. What is the surface and volume of a sphere? Way a plastic sphere. Cut a piece of 1 cm^2 and way it.

Exercise 6. What is the surface of a bullet? Take a mandarin or an orange. Make an equator cut and two on each other perpendicular polar cuts. This divides the surface into eight equal round triangles, each containing an equilateral triangle. Show that the approximate area of these triangles is A = 8*1.41*(1.5*r)^2/2 = 4* 3.17*r^2.

Exercise 7. What is the surface and volume of a sphere? Take a mandarin or an orange. Place three circular cuts: at the equator, at 30° and at 60° north latitude. The parts can be flattened like a circle and the chops can be flattened like circular rings. What is the total area of the two circular rings and the top? Arrow the mandarin in boats and place them alternately opposite. This produces something resembling part of a box. Assess the total volume of the boats.

Exercise 8. What is the surface of a cylinder? Make a cylinder out of an A4 paper. What is radius and height?

Exercise 9. What is the volume of a cylinder? Make a small cylinder of paper. What is radius and height? Fill the cylinder with sand and then pour the sand into a measuring glass.

Exercise 10. What is the surface and volume of a leg, a head, one, human body?

Exercise 11. Cut a slice out of a circle and fold it to form a fussy house (a cone). What is the surface and volume of a cone? Fill the cone with sand and then pour the sand into a measuring glass.

Exercise 12. Compare the volume between a cylinder, a hemisphere and a cone of the same height.

Exercise 13. Cut a triangular and square pyramid out of a piece of paper. What is the surface and volume of a pyramid? Fill the pyramid with sand and then pour the sand into a measuring glass.

Exercise 14. A closed cylinder must hold 1 liter. Describe different options in table or formula. Find the corresponding surface. Which cylinder has the least surface. If necessary, use a PC spreadsheet.

Exercise 15. Same as exercise 14, but now the cylinder is open at one end.

Exercise 16. Same as exercise 14, but now the cylinder is open at both ends.

Exercise 17. A closed cylinder shall have an outer surface of 1 m^2. Describe different options in table or formula. Find the corresponding volume. Which cylinder has the most volume. If necessary, use a PC spreadsheet.

Exercise 18. Same as exercise 17, but now the cylinder is open at one end.

Exercise 19. Same as exercise 17, but now the cylinder is open at both ends.

Exercise 20. Same as exercise 14-15, but now with a cone instead of a cylinder.

Exercise 21. Same as exercise 17-18, but now with a cone instead of a cylinder.

Exercise 22. Same as exercise 14-16 and 20, but now the material for the end surface is twice as expensive as aside. Minimum cost is requested.

Exercise 23 (difficult). A tube with diameter 1 m is filled by three equal tubes with which diameter?

Exercise 24. Place a bicycle lamp on the rim of a bicycle wheel. What is called the curve that the lamp will form if the bike ride is seen from the side (best seen in the dark)? This curve can also be produced on paper by letting a circle move along the edge of the paper. From the front, a circular motion will be a swing up and down. Is it reasonable for physics to describe the fluctuation u by a harmonic oscillation as u = R*sin(wt)? What does it say? R, w and t too?

Exercise 25. How can we make gear for a bike? When is a bike easy to tread? When will a pedal turn move the bike the most?

Exercise 26. Construct a triangle and place it in a circle. How is the center of the circle? Can we calculate the radius in this circle (the circumscribed circle of the triangle)?

Exercise 27. Construct a triangle and place a circle in it. How is the center of the circle? Can we calculate the radius in this circle (the inscribed circle of the triangle)?

Exercise 28. Under what conditions can a square be wrapped in a circle that touches all four corners? How is the center of the circle? Can we calculate the radius in this circle (the square's rewritten circle)?

Exercise 29. Under what conditions can a square wrap a circle that touches all four sides? How is the center of the circle? Can we calculate the radius in this circle (the square's inscribed circle)?

Exercise 30. Find some space targets that are used in other locations and at other times.

Exercise 31 (difficult). What can be understood by the outer piping circles of a triangle and how are they constructed? What can be said about radius and center?

Definition 1. A circle with radius r and center C is

Definition 2. A pie slice is

Definition 3. A circle section is

Definition 4. A sphere with radius r and center C is

Definition 5. A cylinder is

Definition 6. A cone is

Rule 1. In a circle, the circumference-diameter ratio is always $\pi = 3.1416$

Rule 2. A circle with radius r has the circumference O =

Rule 3. A circle with radius r has the area A =

Rule 4. From the circular arc, the diameter will always be seen at a right angle.

Rule 5. A sphere with radius r has the surface O =

Rule 6. A sphere with radius r has the volume V =

Rule 7. A cylinder with a radius r and height h has the volume V =

Rule 8. A cone with the base surface radius r and height h has the volume V =

Rule 9. A cylinder with a radius r and height h has the diagonal D =

GE14 GEOMETRY ON A SPHERE SURFACE

Basic problem: How to designate and calculate points, distances, angles, triangles, areas, etc. on a sphere surface?

Exercise 1. Look at a globe. How are points on the surface described (London, Paris, Rome, etc.)?

Exercise 2. What is the distance between two points and how is it (London-New York, Paris-Tokyo, etc.)?

Exercise 3. When are three points on a straight line (Oslo, Prague, Cairo)?

Exercise 4. Madrid-Paris-Bangkok makes up a triangle. What are the angles? What is the area?

Exercise 5. Can we count on the answers to the questions in exercise 2-4?

Definition 1. A grand circle goes through two diametrically opposite points. A pole circle or meridian circle passes through the two poles. An equator circle stands perpendicular to a pole circle midway between the poles.

Definition 2. On a sphere surface with center C and poles N and S, the equator circle and a standard pole circle are loaded. The intersections of the circles are called A and A*. The coordinates of a point P are determined as follows: through the point, a pole circle is placed that cuts the equator circle into P*. The width of the point is angle P*CP with one of the indications north or south. The length of the point is angle P*CA with one of the indications west or east.

Definition 3. The distance between two points P and Q means the smallest of the arc lengths of the grand circle through P and Q.

Definition 4. Three points A, B and C determine three large circle arcs AB, AC and BC. If these large circle arches do not coincide, they will delineate a spherical triangle ABC as well as another 7 spherical triangles.

Definition 5. The spherical exces e for a spherical triangle ABC is degree surplus above 180 degrees: e = A+B+C-180.

Rule 1. For a right spherical triangle ABC applies:

sinA = sina/sinc, cosA = tanb/tanc, tanA = tana/sinb

Rule 2. For a crooked spherical triangle ABC applies:

sina/sinA = sinb/sinB = sinc/sinC, cosa = cosb*cosc + sinb*sinc*cosA

Rule 3. On a sphere with radius r, a spherical triangle ABC has the area e*4πr^2/720

GE15 MOVING: TURNING, MIRRORING AND PARALLEL DISPLACEMENT

Basic problem: How to talk turning, mirroring and parallel displacement of a triangle?

Exercise 1. Set up a triangle. Make a copy of the triangle. With the copy, perform each of the operations turning, mirroring and parallel displacement. Then set up definitions for these operations.

Exercise 2. Set up a triangle. Make a copy of the triangle and place it randomly a distance from the original triangle. Describe how the two triangles can be brought to cover each other.

Exercise 3. Put a triangle on the floor. Shift the triangle 3 meters to the south and 2 meters to the north and turn it 40 degrees.

Definition 1. If triangle ABC is turned to triangle AB*C* the angle of rotation will be angle BAB*.

Definition 2. If triangle ABC is turned about AB to triangle ABC*, ABC* is said to be a reflection of ABC on the mirror axis AB.

Definition 3. A triangle ABC is moved to triangle A*B*C*. If the sides do not change direction, a parallel displacement is indicated, the size of which is indicated by the length AA* and if the direction is specified, for example, if the direction is indicated. at angle A*AC.

Rule 1. Any movement can be performed on one or more of the operations of turning, mirroring and parallel displacement.

GE16 BLUEPRINTS

Basic problem: How is a spatial shape transferred to a plane drawing that can be used to construct the shape?

Exercise 1. Build a character into LEGO bricks. Draw the shape as a blueprint with both front, side, top vision (FST drawing).

Exercise 2. Draw the shape from exercise 1 as a blueprint in skewed vision on ISO(metric) paper. From where should the shape be seen to be correctly enrolled?

Exercise 3. Build a shape out of bricks, some of which are round (hemispheres, cylinders, cones, etc.) and some triangular. Draw the shape as FST drawing. Also draw the figure on ISO paper.

Exercise 4. Make an FST drawing of a LEGO brick shape. Build the shape from the blueprint. Draw the shape on ISO paper.

Exercise 5. Make an FST drawing of a shape with round bricks. Build the shape from the blueprint. Draw the shape on ISO paper.

Exercise 6. Make a drawing on ISO paper of a LEGO brick shape. Build the shape from the drawing. Draw the shape as FST drawing.

Exercise 7. Make a drawing on ISO paper of a shape with round bricks. Build the shape from the drawing. Draw the shape as FST drawing.

Exercise 8. we can get work in a shipyard if we can read an ISO drawing of piping. Take a clip, copper wire or similar and bend it at 90-degree angles as a model of a piping. Sign on ISO paper and let others try to reconstruct the model based on the drawing.

Exercise 9. A kitchen is set on the wall of a room with cm-dimensions 250 x 250 x 250 (height x width x depth). At the bottom is a 25 x 250 x 50 socket. Upstairs on the left is a 150 x 50 x 50 cabinet, and on the right four 50 x 50 x 50 cabinets. Above these is with a clearance of 50 cm suspended four 50 x 50 x 25 wall cabinets. Make a blueprint of the kitchen. Make a drawing of the kitchen on ISO paper. (If necessary, use the more realistic numbers 30, 60, 180 and 300 cm.)

Exercise 10. Make an FST drawing of a house. Build the house in cardboard from the blueprint. Draw the house on ISO paper.

Exercise 11. As 10, but now the opposite.

DEFINITION 1. Place a cube so that it rests on the 4-side. The cube can be seen in three different ways: the 1-Side then indicates the front vision of the cube, the 2-side side vision and the 3-side top-vision of the cube.

DEFINITION 2. From an (isometric) skewed view of a spatial figure, all three visions are seen simultaneously. A shape is viewed from a point of view that is below the figure on the left. A skewed vision can be recorded on ISO(metric) paper, where one's distances are drawn the same. we are therefore disregarding the impact of perspective.

GE17 SHADOWS AND PROJECTION

Basic problem: How to calculate shadow shapes

Exercise 1. Put a book on a table under a lamp. Turn the book upwards in 10 degrees. Measure each time the angle of rotation and shadow length. Can the shadow length be calculated?

Exercise 2. Make a triangle of a thumb stick. Place a pocket lamp and triangle on a table so that the shade falls on a wall. Move the flashlight in a 20 cm spurt and measure each time the distance to the triangle and the parts of the shadow triangle. Can the shading triangle parts be calculated?

Exercise 3. Make a triangle of a thumb stick. Place a pocket lamp and triangle on a table so that the shade falls on a vertical plate at the end of the table. Turn the plate in a 10-degree spur. Measure each time the angle of rotation and the parts of the shadow triangle. Can the shading triangle parts be calculated?

GE18 PERSPECTIVE DRAWING

Basic problem: How does a spatial shape transfer to a plane drawing that shows what's being seen?

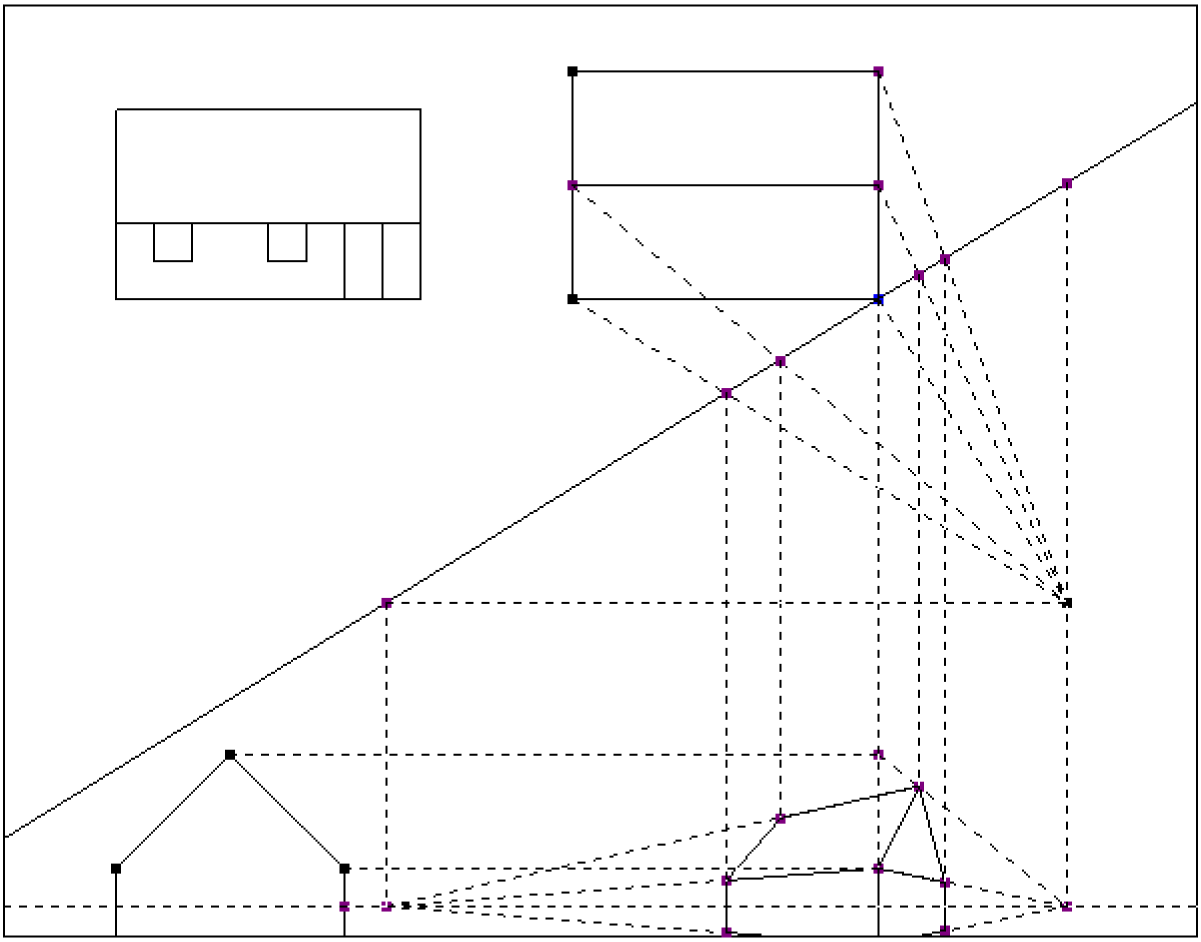
Exercise 1. Look along a surface on which there are parallel lines, such as lines on a brick wall. Move the eye in a vertical direction first up, then down. Which lines are horizontal and which fall and which increase? Move the eye in a horizontal direction first towards, then away from the surface. How does the steepness of the surface's lines change? What are the terms "horizontal line" and "disappearance point of parallel lines"?

Exercise 2. View a box-shaped thing through a window and copy the boxes' lines onto a piece of transparent plastic laid on the window. Mark the position of the eye on the plastic. Then copy the drawing onto a sheet of paper laid on top of the plastic. Sign horizontal line and vanishing points.

Exercise 3. The same exercise as exercise 2, this time, just used Albrecht Dürer's approach: Consider a box-shaped thing through a grid with horizontal and vertical lines, and copy the box lines onto a piece of A4 paper with the same number of horizontal and vertical lines as the grid. Mark the location of the eye. Sign horizontal line and vanishing points.

DEFINITION 1. Two points P and Q cast the shadows P' and Q' on a line l.

Front view Top view



Side view Perspective

Exercise 4. A house has meters 3 x 12 x 9 (height x width x depth). On top of the house there is a normal saddle roof that is 4 m high in the middle. Make an FTS drawing of the house. Use Top vision and side view to make a perspective drawing of the house seen in height 1 m from a position located 3 meters to the right and 6 meters in front of the house.

Exercise 5. Draw a tile floor seen from an eye level of 1.5 m when the horizontal distance to the tile floor is 1 m.

Exercise 6. Draw the shapes from the working drawing chapter as a perspective drawing.

Exercise 7. Draw the kitchen from the chapter on the drawing as a perspective drawing from a position that is 1 meter inside and 1 meter up in the kitchen and 1.25 meters from the wall of the closet.

Exercise 8. Draw a perspective drawing of a grid of 10 equally tall vertical bars with the same distance between them. Leave the first rod 10 cm high and let the horizontal line pass through the center of the rod. Set up two series of measurements from the drawing: one that measures the length of the rods and one that measures the distance from the vanishing point to the rods. What can be said about the two series of figures? Can the grid be perceived as a fractal?

Exercise 9. Draw a box from the supermarket in perspective (e.g. washing powder).

Exercise 10. Draw a house in perspective.

Exercise 11. Explore how we can draw in perspective on a PC.

NOTE. A perspective drawing takes into account the fact that a fixed length view decreases with increasing distance. A spatial figure is said to be drawn in perspective if all the points of the shapes are construct on a drawing plane perpendicular to the direction of vision using a point projection from the pupil of the eye (or a photographic film).

DEFINITION 1. A drawing plane is a plane perpendicular to the eye's direction of vision. A horizontal line at eye level is called the horizon line. The point on the horizon next to the eye is called the eye point. All points in the drawing plan are assumed to have the same distance from the eye.

DEFINITION 2. Parallel lines perpendicular to the direction of vision are drawn parallel. Otherwise, they are drawn so that they run together in the vanishing point of the lines on the horizon line.

COMMENT. Parallel planes: Subjects in the drawing plan are drawn from distance. Subjects in plans parallel to the drawing plan are drawn from distance, albeit on a different scale, determined by comparing top vision and side vision.

Rule 1. If a tile is drawn in perspective, its center point can be found as the intersection of the diagonals.

GE19 PARABOLAS AND PARABOLOIDS

Basic problem: Why are signals being transmitted in all directions being collected in the same direction?

Exercise 1. Set a line and a point outside the line. Select all points that are the same distance to the point and line. This shape is called a parable. The normal to the line through the point is called the axis of the parable.

Exercise 2. Construct a parable. Send a light ray (any laser light) in parallel to the axis. Examine the ray corridor after reflection on the side of the parable. Exercise can be performed by drawing or using light rays reflected in mirrors set up as keys in the reflection points.

Exercise 3. Try to prove that axis parallel rays pass the focal point of the parabola.

Exercise 4. Cut a parable in wood and use this to make a dish in clay or else. Place tin foil in the dish and try to use the dish as the sender and recipient of light.

Exercise 5. Borrow a car light from an auto-scrapper. Is it a dish?

Exercise 6. For a level clip in a cone, the following shapes can appear: Circle, ellipse, parable, and hyperbola (the so-called cone sections). How should the clips be laid in the different cases?

DEFINITION 1. A parable consists of all the points that have the same distance to a straight line (guideline) and a point (focal point). The normal to guide the guideline through the focal point is called the axis of the parable. A Paraboloid is produced by rotating a parable 360 degrees on its axis.

DEFINITION 2. An ellipsis consists of all the points whose distances to two given points (focal points) constitute a constant sum.

DEFINITION 3. A hyperbola consists of all the points whose distances to two given points (focal points) represent a constant differential.

Rule 1. At a parable, axis parallel rays will gather at the focal point, and rays emitted from the focal point will move axis parallels.

Rule 2. In the case of an ellipse, rays emitted from the focal point

Rule 1. At a hyperbola, rays emitted from the focal point

GE20 LIGHT BEND IN WATER AND IN GLASSES

Basic problem: How to calculate the bending of light rays when passing through water or glass?

Exercise 1. Fill a box-shaped glass with water. Allow a ray of light (any laser ray) to fall slanted onto the side of the glass. Mark with pins the ray passage through the glass and out on the other side. Measure the different angles and compare with the law of wrestling from physics.

Exercise 2. Let a light ray (possibly laser ray) fall slanted onto the side of a solid transparent glass box. Mark with pins the ray passage through the glass and out on the other side. Measure the different angles and compare with the law of wrestling from physics.

Exercise 3. Let a ray of light (possibly laser ray) fall slanted onto the side of a triangular transparent glass prism. Mark with pins the ray passage through the glass and out on the other side. Measure the different angles and compare with the law of wrestling from physics.

Exercise 4. Fill a cylinder-shaped glass with water. Allow a ray of light (any laser ray) to fall slanted onto the side of the glass. Mark with pins the ray passage through the glass and out on the other side. Measure the different angles and compare with the angle of a rainbow.

Exercise 5. Allow a ray of light (any laser ray) to fall perpendicular to a lens. Mark with pins the ray passage through the glass and out on the other side. Perform the test five times when the ray hits the lens in the middle, halfway and all the way out to both sides. Measure the different angles and compare with the law of wrestling from physics.

DEFINITION 1. A ray of light hits a wall. The angle between the normal and incoming ray of the wall is called the angle of approach. The angle between the normal and outgoing ray of the wall is called the outage angle. The angle between the normal of the wall and the broken ray is called the refraction angle.

Rule 1. On reflection, the incident ray angle = reflected ray angle.

Rule 2. In the case of refraction, sin(incident ray angle)/sin(reflected ray angle) = refractive index.

GE21 REPEATING FIGURES, FRACTALS

Basic problem: What aggregated figure comes from by figure repetition?

Exercise 1. Construct a square. Construct a new square whose side length is the diagonal of the previous square. Repeat this process many times.

Exercise 2. Construct a square. Construct a new square whose diagonal is the side length of the previous square. Repeat this process many times.

Exercise 3. Like 1 and 2, but now with a rectangle instead.

Exercise 4. Construct a right triangle ABC. Set a new right triangle one-angled with the first that has the short catheter as hypotenuse. Repeat this process.

Exercise 5. Construct a right triangle ABC. Set a new right triangle one-angled with the first that has the long catheter as hypotenuse. Repeat this process.

Exercise 6. Construct a right triangle ABC. Construct a new right triangle one-angled with the first that has the hypotenuse as the long catheter. Repeat this process.

Exercise 7. Construct a right triangle ABC. Construct a new right triangle one-angled with the first that has the hypotenuse as the short catheter. Repeat this process.

Exercise 8. Divide a right triangle using the height of the hypotenuse. What is the scaling factor? Repeat this process. Reverse the process.

Exercise 9. A horizontal line divides into two lines, each forming an angle of 30 degrees with the original line and half the length. Repeat this process many times.

Exercise 10. Make a number row starting with 0 and 1, with the next indent being the sum of the previous two. What are the properties of these Fibonacci numbers?

Exercise 11. Then divide a line AB with the point C/ AC = AC/BC (elevation, golden cut). What is the AC/CB scaling factor?

Exercise 12. Construct a square of the line AB. Let P be the golden cut in AB. Repeat the construction, but now from AP.

Exercise 13. Make a spiral with ray lengths $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, etc.

Exercise 14. Halves a square with side length 1 dm. Halves one part.

Exercise 15. Find some fractals yourself, e.g. in the program GeomeTricks.

Exercise 16. Find some fractals in nature.

DEFINITION 1. C is the golden cut in the line AB if AB/AC = AC/BC, i.e. if AC is mean proportional between AB and BC.

GE22 GEOMETRY OF AN OATMEAL BOX

Basic problem: Which geometry is hiding in an oatmeal box?

Suggestions for questions:

1. What is the surface and volume of an oatmeal box?

2. What is the smallest assault containing the given volume?

3. What is the largest volume that can be contained in the given surface?

4. What is the length and angles of the box's four diagonals?

5. Where is the center of gravity of the box? How much can the box be tilted without tipping over? (Full, half-filled)

6. Make a cylinder, a sphere, a cone, a pyramid, a tetrahedron with the same volume. What is the surface?

7. Make a cylinder, a sphere, a cone, a pyramid, a tetrahedron with the same surface. What's the volume?

8. Tilt the box so it looks like a steep mountain wall. Install a "hairpins" way up the box. What is the gravitational acceleration in this position?

9. Tilt the box to a specific position. How much is the load on the substrate reduced?

10. Draw the box, partly as blueprints, partly in perspective.

11. Draw the side of the box in scaled-down versions so that the wide side becomes diagonal on the wide side of the new box. What is the scaling factor? Repeat this process.

12. What is the smallest corner around the box?

13. If we can travel twice as fast on the wide side as on the narrow side, what is the fastest journey between two given points on the two sides?

14. Place 2-3 boxes on the dining table as islands in a large sea, and divide the sea between the islands.

GE23 NEWSPAPER GEOMETRY

Basic problem: Which geometry is hiding on a newspaper side?

Suggestions for questions:

1. What are the goals on a newspaper side? What are the relationships between these objectives? What is understood by Al, A2, A3, A4, A5, A6, etc. Paper?

2. Measure and calculate the diagonal (at least two ways) and its angles.

3. The diagonal divides the side into two triangles. Measure and calculate their height and elevation angles.

4. Calculate the area of the triangles in three different ways.

5. The height divides the triangle into two new triangles. How are these compared to the original? What are the scaling factors?

6. These new triangles can again be divided into new triangles. Repeat this split a few times and enjoy the resulting fractals.

7. As 2-6, but now with the newspaper folded once.

8. Calculate the right edge of the newspaper side from the lower left corner A when neither of the two center lines may be crossed.

9. A cracked corridor can be illustrated by three newspaper sides arranged appropriately in relation to each other. What is the length of the longest rod (bending ruler) that can travel around the corner? At what angle does the rod bump against both walls. Show that the angle can be calculated by the equation tanv^3 = a/b, where a and b are the two walking widths.

10. Draw the cheapest route between the two opposite corners when it is twice as expensive on one half. Show that sini/sinb = 1/2 where in and b is the angle of approach and refraction at the center line.

11. Fold a newspaper and open it 30 degrees. Sign a way up the newspaper that rises 20 degrees.

12. In two different ways, a newspaper side can be rolled around and form a cylinder. The surface is the same, is the volume also the same? What is the largest volume that can be formed by a newspaper side with the same area?

13. Fold the newspaper side and repeat the previous exercise. Will the volume be halved?

14. Fold the short side towards the long. Fold the remaining part to form a triangle. There will now be three triangles. What are their scaling factors?

GE24 Supplements: Equation Schemas

For calculations based on formulas, it is possible to use an equation schema that says:

What to figure out	Which equation to use
Which numbers are used	How the equation is reshaped

Example. In triangle ABC is C = 90°, A = 37° and c = 4.2. Find a and b and B.

a = ?	sinA = a/c
A = 37	c*sinA = a
c = 4.2	4.2*sin37 = a
	2.53 = a

b = ?	cosA = b/c
A = 37	c*cosA = a
c = 4.2	4.2*cos37 = a
	3.35 = a

B = ?	A+B = 90
A = 37	B = 90-A
	B = 90-37
	B = 53

Routine tasks

Tasks **Answers**

	A	B	C	A	B	C		A	B	C	A	B	C
1			3.917	33.3		90		2.151	3.274			56.7	
2			6.519	42.4		90		4.396	4.814			47.6	
3	2.534			23.8		90			5.745	6.279		66.2	
4	3.772			21.4		90			9.625	10.338		68.6	
5	2.707		4.628			90			3.754		35.8	54.2	
6	3.883		6.265			90			4.917		38.3	51.7	
7	7.502	3.611				90				8.326	64.3	25.7	
8	8.588	8.093				90				11.801	46.7	43.3	

Non-right triangles **Answers**

	A	B	C	A	B	C		A	B	C	A	B	C
9	1.075			57.2		72.4			0.985	1.219		50.4	
10	2.212			42.5		59.6			3.201	2.824		77.9	
11		4.104		66.9		72.6		5.812		6.029		40.5	
12		4.915		21.5		86.4		1.893		5.155		72.1	
13			2.165	35.1		68.6		1.337	2.259			76.3	
14			3.041	38.4		76.2		1.945	2.847			65.4	
15	1.748	3.562				68.6				3.346	29.1	82.3	
16	1.433	4.346				88.4				4.538	18.4	73.2	
17		3.078	4.892	60.8				4.326				38.4	80.8
18		3.938	3.706	59.3				3.787				63.4	57.3
19	4.298		5.027		42.9				3.477		57.3		79.8
20	4.861		4.439		81.8				6.097		52.1		46.1