WORKSHOP EXERCISES IN FLEXIBLE BUNDLE-NUMBERS

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E01. Pushing sticks away, transform many OUTSIDE ones into one INSIDE many-icon with as many strokes as it represents. Repeat with cubes transforming 3 1s to 1 3s.

E02. Bundle-count ten fingers in 5s writing 6 in three different ways. Then count in 4s, 3s and 2s: Using 'flexible BundleNumbers', T = 6 = 0**B**6 = 1**B**1 = 2**B**-4 **5s** (overload, standard, underload). And 0**B**1 = 1**B**-4, 0**B**2 = 1**B**-3, … **5s**

E03. Bundle-count ten fingers in 3s using bundle-bundles. Then in 2s. T = ten = 1**BB**0**B**1 = 101 **3s**.

Write traditional numbers as flexible BundleNumbers: T = 53 = 5**B**3 = 4**B**13 = 6**B**-7 tens

E04.
Flexible BundleNumbers ease Operations

65 + 27 = ? =	6**B**5 + 2**B**7 = 8**B**12 = 9**B**2 = 92
65 – 27 = ? =	6**B**5 – 2**B**7 = 4**B**-2 = 3**B**8 = 38
7* 48 = ? =	7* 4**B**8 = 28**B**56 = 33**B**6 = 336
336 /7 = ? =	33**B**6 /7 = 28**B**56 /7 = 4**B**8 = 48

E05. With cubes, transform the three OUTSIDE parts of a counting process, PUSH & LIFT & PULL, into three INSIDE operation-icons: division / & multiplication x & subtraction -.

Five counted in **2s**: ׀ ׀ ׀ ׀ ׀ (push away **2s**) ׀׀ ׀׀ ׀ (lift to stack) $\frac{\text{II}}{\text{II}}$ ׀ (pull to find unbundles ones) $\frac{\text{II}}{\text{II}}$ ׀.

E06. Counting 7 cubes in 3s gives 2 3s & 1 as predicted: T = 7 = (7/3) = 2.some; 7-2x3= 1.

Placing the unbundled next-to the stack roots decimals and negative numbers:	T = 7 = 2.1 **3s** = 3.-2 **3s**
Placing the unbundled instead on-top of the stack counted in bundles roots fractions:	T = 7 = 2 1/3 **3s**

Recount traditional numbers: T = 68 = 6.8 tens = 7.-2 tens = 6 8/10 tens

E07. OUTSIDE bundle-counting with icons as units is predicted INSIDE by a **recount-formula T = (T/B)*B** (from T, T/B times, push away Bs) coming from recounting 8 in 2s by 8/2 times pushing away 2s as predicted on a calculator as T = 8 = (8/2)*2, thus using a full number-language sentence with a subject, a verb and a predicate.

OUTSIDE: T = ׀ ׀ ׀ ׀ ׀; T counted in **2s**: ׀׀ ׀׀ ׀; T – 2x2: = ׀׀ ׀׀ ׀ ; INSIDE:

5/2	2. some
5 – 2x2	1

E08. Recount from tens to icons (decreasing the base will increase the height)

OUTSIDE, to answer the question '40 = ? **5s'**, on squared paper transform the stack 4.0 **tens** to **5s**.

INSIDE, formulate an equation to be solved by recounting 40 in **5s**:

*u**5 = 40 = (40/5)*5, so *u* = 40/5.

Notice that recounting gives the solution rule 'move to opposite side with opposite calculation sign'.

E09. Recount from icons to tens (increasing the base will decrease the height)

OUTSIDE, to answer '3 **7s** = ? **tens'**, on squared paper or a pegboard change the stack 3 **7s** to **tens**.

INSIDE: oops, with no ten-button on a calculator we can't use the recount-formula? Oh, we just multiply! Use flexible bundle-numbers on a pegboard or a squared paper we see that

T = 4 7s = 4*7 = (B-6)*(B-3) = 10B-6B-3B - - 6 3s = 1B + 18 = 28, making - - to +.

E10. DoubleCounting in two physical units

DoubleCounting in two physical units gives a 'per-number' as e.g. 2m per 3sec, or 2m/3sec.

To answer the question ‘T = 6m = ?sec’, we just recount 6 in the per-number: T = 6m = (6/2)*2m = (6/2)*3sec = 9sec. Answer the question ‘T = 12sec = ? m’.

Find formulas with per-numbers in science and mathematics.

E11. Mutual double-counting the sides in an axb stack halved by its diagonal c creates trigonometry: a = (a/b)*b = tanA*b, etc

Draw a vertical tangent to a circle with radius r. With a protractor, mark the intersection points on the tangent for angles from 10 to 80. Compare the per-number intersection/radius with tangent of the angle on a calculator.

A 12x12 square ABCD has AB on the ground and is inclined 20 degrees. From B, a straight road is to be constructed intersecting the borderline AD in the point E, inclined 5 degrees. Find the length DE. (Hint: Show that if DE = 2, then the incline of the road is 3.2 degrees).

E12. On squared paper a point has an out-number x and an up-number y, A(x,y). The per-number $\Delta y/\Delta x$ allows moving on a line.

With A(2,5) and B(4,6), the line per-number is $\Delta y/\Delta x = (6-5)/(4-2) = ½$. Changing position to C(8,y) gives $\Delta y = (\Delta y/\Delta x)* \Delta x = ½*(8-2) = 3$, and y = 5+3 = 8, giving C(8,8).

E13. Next-to addition: If T1 = 2 3s and T2 = 4 5s, what is T1+T2 when added next-to as 8s?

E14. Reversed next-to addition: If T1 = 2 3s and T2 add next-to as T = 4 7s, what is T2?

E15. On-top addition: If T1 = 2 3s and T2 = 4 5s, what is T1+T2 when added on-top as 3s; and as 5s?

E16. Reversed on-top addition: If T1 = 2 3s and T2 as some 5s add to T = 4 5s, what is T2?

E17. E19. Multiplying tens: What is 27 43s recounted in tens? T = 27*43 = 2B7*4B3 = 8BB+6B+28B+21 = 8BB34B21 = 8BB36B1 = 11BB6B1 = 1161

E18. Adding per-numbers: 2kg at 3$/kg + 4kg at 5$/kg = 6kg at what?

E19. Subtracting per-numbers: 2kg of 3$/kg + 4kg of what = 6kg of 5$/kg?

E20. Solving STEM proportionality heating problems with recounting

With a heater giving 20 J in 30 sec, what does 40 sec give, and how many seconds is needed for 50J?

With 40 Joules melting 5kg, what will 60 Joules melt and what will 7 kg need?

With 3 degrees needs 50 Joules, what does 7 degrees need; and what does 70 Joules give?

With 4 deg. in 20kg needing 50 Joules, what does 9 deg. in 30 kg need? What does 70 Joules give in 40 kg?

1**BB**0	1**BB**1	1**BB**2	1**BB**3	1**BB**4	1**BB**5	1**BB**6	1**BB**7	1**BB**8	1**BB**9	~~1**BB**10~~
~~10**B**0~~	~~10**B**1~~	~~10**B**2~~	~~10**B**3~~	~~10**B**4~~	~~10**B**5~~	~~10**B**6~~	~~10**B**7~~	~~10**B**8~~	~~10**B**9~~	~~10**B**10~~
9**B**0	9**B**1	9**B**2	9**B**3	9**B**4	9**B**5	9**B**6	9**B**7	9**B**8	9**B**9	~~9**B**10~~
8**B**0	8**B**1	8**B**2	8**B**3	8**B**4	8**B**5	8**B**6	8**B**7	8**B**8	8**B**9	~~8**B**10~~
7**B**0	7**B**1	7**B**2	7**B**3	7**B**4	7**B**5	7**B**6	7**B**7	7**B**8	7**B**9	~~7**B**10~~
6**B**0	6**B**1	6**B**2	6**B**3	6**B**4	6**B**5	6**B**6	6**B**7	6**B**8	6**B**9	~~6**B**10~~
5**B**0	5**B**1	5**B**2	5**B**3	5**B**4	5**B**5	5**B**6	5**B**7	5**B**8	5**B**9	~~5**B**10~~
4**B**0	4**B**1	4**B**2	4**B**3	4**B**4	4**B**5	4**B**6	4**B**7	4**B**8	4**B**9	~~4**B**10~~
3**B**0	3**B**1	3**B**2	3**B**3	3**B**4	3**B**5	3**B**6	3**B**7	3**B**8	3**B**9	~~3**B**10~~
2**B**0	2**B**1	2**B**2	2**B**3	2**B**4	2**B**5	2**B**6	2**B**7	2**B**8	2**B**9	~~2**B**10~~
1**B**0	1**B**1	1**B**2	1**B**3	1**B**4	1**B**5	1**B**6	1**B**7	1**B**8	1**B**9	~~1**B**10~~
0**B**0	0**B**1	0**B**2	0**B**3	0**B**4	0**B**5	0**B**6	0**B**7	0**B**8	0**B**9	~~0**B**10~~