SELFEXPLANATORY LEARNING MATERIAL TO IMPROVE LEARNER'S MASTERY OF MANY

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I. CHILDREN SHOW MASTERY OF MANY BEFORE SCHOOL

In an isolated covid-19 household we may ask: How to create simple material supporting the children in improving their mastery of Many?

This workshop-design is based on the observation that when asked 'How old next time?', a 3year old will say 4 showing 4 fingers; but, held together two by two, protests by saying 'That is not 4. That is 2 2s', thus rejecting the predication 'four' by insisting on describing what exists, bundles of 2s and 2 of them. Meeting Many, children develop a number-language with full sentences including a subject and a verb and a predicate as in the word-language, as well as 2-dimensional bundle-numbers with units, neglected by the school's 1-dimensional line-names making some children count-over by saying 'twenty-ten'. So, the goal of the workshop is to inquire into the mastery of Many children bring to school to see what kind of mathematics occur if allowing the children to develop their already existing quantitative competence under proper guidance (Tarp 2018, 2020). In the workshop you will need 12 sticks, 12 cubes, a pegboard with rubber bands, squared paper and a pencil.

II. BRIDGE OUTSIDE EXISTENCE TO INSIDE ESSENCE

E01. Many become icons.

Pushing sticks away, transform many OUTSIDE ones into one INSIDE many-icon with as many sticks or strokes as it represents. Repeat with cubes.

Sticks and a folding ruler show that digits are, not symbols as the alphabet, but sloppy writings of icons having in them as many sticks as they represent: five sticks in the 5-icon, etc. A looking glass finding nothing when looking for more will iconize zero. If used as bundle-size, ten has no icon but is reported as 1 bundle or 1B or 1B0.

E02. Flexible bundle-counting roots negative numbers.

Count ten fingers in 5s writing 6 in three different ways. Then count in 4s, 3s and 2s.

Including bundles, ten fingers may be bundle-counted in fives as '0Bundle1, 0B2, 0B3, 0B4, 1B0, 1B1' that also may be counted as 0B6 or 2B less 4 or 2B-4 with an overload or an underload, thus writing 6 in three ways as a 'flexible bundle-number':

T = 6 = 0B6 = 1B1 = 2B-4

Using flexible bundle-numbers, ten fingers may also be counted in 5s as '1B-4, 1B-3, etc.'

Counting on from ten, we meet 'Viking-counting' where eleven is 'one-left', twelve is 'two-left', and thirteen is 'three-ten', while 'three-twenty' becomes twenty-three.

Counting in scores (twenties) from forty, the Danish Viking-descendants still count: half-three-scores, three-scores, half-four-scores, four-scores, and half-five-scores for ninety. Unable to understand the half-notion, the French instead counts over when expressing 87 as '4 scores and 1 ten and 7.'

E03. Flexible bundle-counting roots polynomials.

Bundle-counting ten fingers in 3s, nine becomes 3B or 1BB, 1bundle-bundle, called hundred when using ten-bundles. And ten becomes 3B1 3s or 1BB0B1 3s, leading on to the general number-formula or polynomial T =

ten = 1*B^2 + 0*B + 1*1 3s, showing the four ways to unite numbers (the Arabic meaning of Algebra): on-top addition, multiplication, power and next-to stack-addition called integration, all with reverse splitting operations: subtraction, division, factor-finding (root), factor-counting (logarithm), and differentiation.

Counting ten fingers in 2s, eight becomes 1BBB, called thousand when using ten-bundles.

Bundle-counting thus allows meeting power before the other operations which allows replacing the place-value-system with bundle-writing: T = 345 = 3BB4B5.

E04. Recounting as flexible bundle numbers will ease traditional calculations.

T = 53 = 5B3 = 4B13 = 6B-7 tens

65 + 27 = 6B5 + 2B7 = 8B12 = 9B2 = 92

65 – 27 = 6B5 – 2B7 = 4B-2 = 3B8 = 38

7* 48 = 7* 4B8 = 28B56 = 33B6 = 336

336 /7 = 33B6 /7 = 28B56 /7 = 4B8 = 48

E05. Counting creates icons.

Count 7 cubes in 2s by 3 times pushing away 2s with a phone iconized as a division sign, predicted by a calculator as '7/2 = 3.some'. Stack the bundles by a lift iconized as a cross, 3x2. To look for unbundled, pull away the stack with a rope iconized as a subtraction sign, predicted by a calculator as '7-3x2 = 1'. So T = 7 = 3 2s & 1. So, operations predict.

E06. Placing the unbundled roots decimals, negative numbers and fractions.

Counting 7 cubes in 3s gives 2 3s & 1 as predicted: T = 7 = (7/3) = 2.some; 7-2x3= 1.

Placing the unbundled next-to the stack roots decimals and negative numbers:

T = 7 = 2.1 3s = 3.-2 3s

Placing the unbundled instead on-top of the stack counted in bundles roots fractions:

T = 7 = 2 1/3 3s

Counting in tens, T = 68 = 6.8 tens = 7.-2 tens = 6 8/10 tens.

E07. OUTSIDE bundle-counting with icons as units is predicted INSIDE by a recount-formula T = (T/B)*B (from T, T/B times, push away Bs) coming from recounting 8 in 2s by 8/2 times pushing away 2s as predicted on a calculator as T = 8 = (8/2)*2, thus using a full number-language sentence with a subject, a verb and a predicate.

Recounting 5 in 2s thus is predicted by: 5/2= 2.some, and 5 – 2*2 = 1, so 5 = 2.1 2s.

This recount 'proportionality' formula occurs all over mathematics and science: when relating proportional quantities as y = c*x; in trigonometry as sine and cosine and tangent, a = (a/c)*c = sinA*c; in coordinate geometry as line gradients, Δy = (Δy/Δx)*Δx = c*Δx; in calculus as the derivative, dy = (dy/dx)*dx = y'*dx; speed in science: Δs = (Δs/Δt)*Δt = v*Δt.

E08. Recount from tens to icons

OUTSIDE, to answer '40 = ? 5s' we use a pegboard or a squared paper to transform the stack 4.0 tens to 8.0 5s. So decreasing the base will increase the height. INSIDE, we formulate an equation to be solved by recounting 40 in 5s:

u*5 = 40 = (40/5)*5, so

u = 40/5 giving 40 = 8*5 = 8 5s

Notice that recounting gives the equation solution rule 'move to opposite side with opposite calculation sign'.

E09. Recount from icons to tens

OUTSIDE, to answer '4 7s = ? tens' we use a pegboard or a squared paper to transform the stack 4 7s to 2.8 tens. So, increasing the base will decrease the height.

INSIDE: oops, with no calculator ten-button we can't use the recount-formula? Oh, we just multiply, thus creating multiplication tables.

Using flexible bundle-numbers on a pegboard or a squared paper we see that

T = 4 7s = 4*7 = (B-6)*(B-3) = 10B-6B-3B - - 6 3s = 1B + 18 = 28, making - - to +.

E10. Double-counting in two physical units.

Double-counting in two physical units gives a 'per-number' as 2m per 3sec or 2m/3sec or 2/3 m/sec. To answer the question 'T = 6m = ?sec' we just recount 6 in the per-number:

T = 6m = (6/2)*2m = (6/2)*3sec = 9sec.

Double-counting in the same unit, per-numbers become fractions: 2m per 3m = 2 per 3 = 2/3, and 2 per 100 = 2/100 = 2%.

To answer the question '20 per hundred is ? per 400' we just recount 400 in 100s: T = 400 = (400/100)*100 giving (400/100)*20 = 80.

To answer the question '20 per 400 is what per hundred?' we just recount 100 in 400s: T = 100 = (100/400)*400 giving (100/400)*20 = 5.

STEM formulas contain per-numbers:

meter = (meter/sec)*sec = velocity*sec,

kg = (kg/cubic-meter)*cubic-meter = density*cubic-meter;

force = (force/square-meter)*square-meter = pressure*square-meter;

energy = (energy/sec)*sec = Watt*sec;

energy = (energy/kg)*kg = heat * kg.

Lego-bricks: number = (number/meter)* meter = density*meter.

E11. Mutual double-counting the sides in a stack with base b and height a, halved by its diagonal c, creates per-numbers:

a = (a/c)*c = sinA*c

b = (b/c)*c = cosA*c

a = (a/b)*b = tanA*b

$\pi \approx$ n*sin(180/n)

Draw a vertical tangent to a circle with radius 5. With a protractor, mark the intersection points on the tangent for angles from 10 to 80. Compare the per-number intersection/5 with tangent of the angle on a calculator.

E12. On squared paper a point has an out-number x and an up-number y, A(x,y). The per-number Δy/Δx allows moving on a line.

With A(2,5) and B(4,6), the line per-number is Δy/Δx = (6-5)/(4-2) = ½. Changing position to C(8,y) gives Δy = (Δy/Δx)* Δx = ½*(8-2) = 3, and y = 5+3 = 8, giving C(8,8).

E13. Next-to addition: If T1 = 2 3s and T2 = 4 5s, what is T1+T2 when added next-to as 8s? Here we see that next-to addition OUTSIDE means adding by areas where multiplication precedes addition. INSIDE, the recount-formula predicts the result. Next-to addition is called integral calculus.

E14. Reversed next-to addition: If T1 = 2 3s and T2 add next-to as T = 4 7s, what is T2? Here we see that when finding the answer by removing the initial stack and recounting the rest in 3s, subtraction precedes division, which is natural as reversed integration, also called differential calculus.

E15. On-top addition: If T1 = 2 3s and T2 = 4 5s, what is T1+T2 when added on-top as 3s; and as 5s? Here we see that on-top addition means changing units by using the recount-formula. Thus, on-top addition may apply proportionality; an overload is removed by recounting in the same unit.

E16. Reversed on-top addition: If T1 = 2 3s and T2 as some 5s add to T = 4 5s, what is T2? Here we see that when finding the answer by removing the initial stack and recounting the rest in 5s, subtraction precedes division, again called differential calculus. An underload is removed by recounting.

E17. Adding tens: If T1 = 23 and T2 = 48, what is T1+T2 when added as tens? Recounting removes an overload: T1+T2 = 23 + 48 = 2B3 + 4B8 = 6B11 = 7B1 = 71.

E18. Subtracting tens: If T1 = 23 and T2 add to T = 71, what is T2? Here, recounting removes an underload: T2 = 71 – 23 = 7B1 – 2B3 = 5B-2 = 4B8 = 48; or T2 = 956 – 487 = 9BB5B6 – 4BB8B7 = 5BB-3B-1 = 4BB7B-1 =

4BB6B9 = 469. Since T = 19 = 2.-1 tens, T2 = 19 -(-1) = 2.-1 tens take away -1 = 2 tens = 20 = 19+ 1, so -(-1) = + 1.

E19. Multiplying tens: What is 7 43s recounted in tens? Here we see that also multiplication may create overloads:

T = 7*43 = 7*4B3 = 28B21 = 30B1 = 301

T = 27*43 = 2B7*4B3 = 8BB+ 6B+ 28B+ 21 = 8BB34B21 = 8BB36B1 = 11BB6B1 = 1161, solved geometrically in a 2x2 stack.

E20. Dividing tens: What is 348 recounted in 6s? Here we see that recounting a total with overload often eases division:

T = 348 /6 = 34B8 /6 = 30B48 /6 = 5B8 = 58;

T = 349 /6 = 34B9 /6 = 30B49 /6 = (30B48 + 1) /6 = 58 + 1/6.

E21. Adding per-numbers: 2kg at 3$/kg + 4kg at 5$/kg = 6kg at what? Here we see that the unit-numbers 2 and 4 add directly whereas the per-numbers 3 and 5 add by areas since they must first transform to unit-numbers by multiplication, creating the areas. Here, the per-numbers are piecewise constant. Later, asking 2 seconds of 4m/s increasing constantly to 5m/s leads to finding the area in a 'locally constant' (continuous) situation defining local constancy by epsilon and delta.

E22. Subtracting per-numbers: 2kg of 3$/kg + 4kg of what = 6kg of 5$/kg? Here we see that unit-numbers 6 and 2 subtract directly whereas the per-numbers 5 and 3 subtract by areas since they must first transform into unit-number by multiplication, creating the areas. In a 'locally constant' situation, subtracting per-numbers is called differential calculus.

III. Grand theory holds conflicting conceptions on concepts

Within philosophy, Platonism and Existentialism discuss if concepts are examples of abstractions or abstractions from examples. Within psychology, Vygotsky and Piaget discuss if concepts are constructions mediated socially or experienced individually. Within sociology, the agent-structure debate is about establishing inclusion by accepting the agent's own concepts or establishing exclusion by insisting on teaching and learning institutionalized concepts.

IV. DISCUSSION

The physical fact Many makes children bring flexible bundle-numbers to school containing core mathematics as proportionality, calculus, solving equations, and modeling by number-language sentences with a subject, a verb and a predicate. Of course, a curriculum with counting before adding is contrary to the present tradition, and calls for huge funding for new textbooks and for extensive in-service training. However, it can be researched outside the tradition in special education, and when educating migrants and refugees. Likewise, applying grand theory in mathematics education is uncommon, but with education as a social 'colonization' of human brains, sociological warnings should be observed. Quality education, the fourth of the United Nations Sustainable Development Goals, thus should develop the child's existing mastery of Many.

References

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