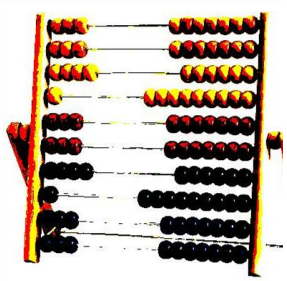

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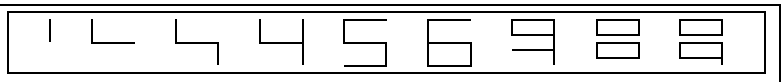
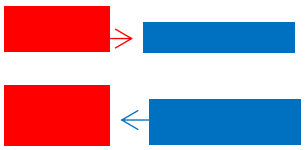
Count before you Add

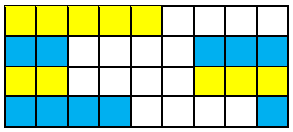
MatheMatics as ManyMath

a Natural Science about MANY

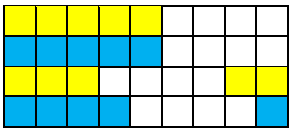
Cure Math Dislike with Kid's own **BundleNumbers** with **Units:** 2 **3s**

Count in ***Icons*** in ***Bundles***	 T = IIII = 4 = 4 T = 7 = ~~III~~ IIII = ~~III III~~ I = ~~III III~~ ~~II~~ = 1B4 **3s** or 2B1 **3s** or 3B-2 **3s**
ReCount in *same* ***Unit*** in *new* ***Unit***	**ReBundle** to create Overload or Underload T = 7 = IIIIIIII = 1B4 **3s** = 2B1 **3s** = 3B-2 **3s** T = 7 = 2B1 **3s** = 1B3 **4s** = 1B2 **5s** = 3B1 **2s** = 1BB1B1 **2s** = 11B1 **2s**
ReCounting Predicted by a ***Recount-Formula***	Push ● Lift ● Pull ● Unite: **/ X − +** 8 = ? **2s.** 8 count in **2s** 8/2 times, stack as 8/2 **2s.** **So** 8 = (8/2)x2 **T = (T/B) x B = T/B** Q: 2 **4S** = ? **5S** 2x4/5 1.some 2x4 – 1x5 3 A: 2 **4s** = 1B3 **5s**
ReCount in ***Tens*** from ***Tens***	3 **7s** = ? **tens** Answer: 3x7 = 21 = 2B1 **tens** ? **7s** = 3 **tens** Answer: (30/7) x 7 = 4B2 **7s** 
DoubleCount in *PerNumbers* in *PerFive, 3/5* in *PerHundred, %*	With 4$ per 5kg or 4/5 $/kg, T = 20kg = (20/5) x 5kg = (20/5) x 4$ = 16$ 3/**5** = 3$/**5**$ of 200$ = ?$. 200$ = (200/5) x **5**$ gives (200/5) x 3$ = 120$ 70**%** = 70$/100$ of 300$ = ?$. 300$ = (300/100) x 100$ = gives (300/100) x 70$ = 210$
Add *NextTo* *OnTop*	T = 2 **3s** + 4 **5s** = 3B2 **8s**  *Integration* T = 2 **3s** + 4 **5s** = 1B1 **5s** + 4 **5s** = 5B1 **5s**  *Proportionality*
Multiply, Divide *BundleWriting*	**7 x** 63 = **7 x** 6B3 = 42B21 = 44B1 = 441 245 **/7** = 24B5 **/7** = 21B35 **/7** = 3B5 = 35

Abacus in 2 modes: T = 7 = 2B1 **3s**



Geometry-mode



Algebra-mode

MrAlTarp

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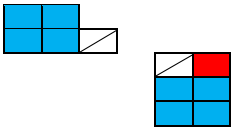
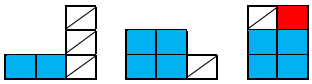
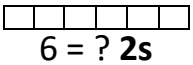
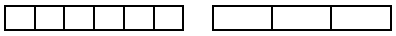
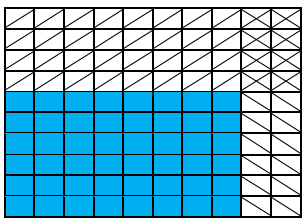
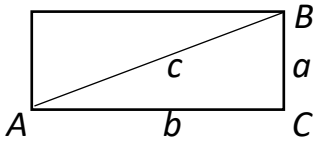
MATHeCADEMY.net	*Teaching Teachers to Teach MatheMatics as ManyMath* PYRAMIDeDUCATION **CATS**: **C**ount & **A**dd in **T**ime & **S**pace

Flexible BundleNumbers

Develops when **Kids** adapt to **Many**

Outside & Inside Math

Digits as ICONS I II III IIII IIIII	ㄣ 4 5	3 4 5
Operations as ICONS	Push ● Lift ● Pull ● Unite	/ X – +
Count Fingers in 5s using BundleCounting & BundleNumbers	I●●●● II●●● III●● IIII● 卌 卌 I●●●●	T = 0***B***1 = 1***B***-4 **5s** T = 0***B***2 = 1***B***-3 **5s** T = 0***B***3 = 1***B***-2 **5s** T = 0***B***4 = 1***B***-1 **5s** T = 1***B***0 = 1***B***0 **5s** T = 1***B***1 = 2***B***-4 **5s**
Unbundled creates Decimals & Fractions & Negative Numbers IIIII ➔ 卌 卌 I		T = 5 = 2***B***1 **2s** = 2.1 **2s** T = 2 1/2 **3s** T = 3***B***-1 2s = 3.-1 **2s** T = 1***BB*** 0***B***1 (T = *p*x^2 + q*x + r*)
ReCount in Same Unit creates Flexible Numbers ▌▌▌▌▌III ➔ 53	5: 卌III 卌卌I 卌卌 	T = 1***B***3 **2s** *Overload* T = 2***B***1 **2s** *Standard* T = 3***B***-1 **2s** *Underload* T = 53 = 5***B***3 = 4***B***13 = 6***B***-7 **tens**
Flexible BundleNumbers ease Operations	65 + 27 = ? = 65 – 27 = ? = 7 x 48 = ? = 336 /7 = ? =	6***B***5 + 2***B***7 = 8***B***12 = 9***B***2 = 92 6***B***5 – 2***B***7 = 4***B***-2 = 3***B***8 = 38 7 x 4***B***8 = 28***B***56 = 33***B***6 = 336 33***B***6 /7 = 28***B***56 /7 = 4***B***8 = 48
ReCount in New Unit  6 = ? **2s** *ReCount-Formula:*	 6 = (6/2) x 2 T = (T/***B***) x ***B***	T = 5 = (5/2) x 2 = ? = 2***B***1 **2s** 5/2 2.some 5 - 2*2 1
ReCount: Tens to Icons ▌▌▌IIIII = ? **7s**	3***B***5 **tens** = *u**7	*u**7 = 35 = (35/7)*7 so *u* = 35/7
ReCount: Icons to Tens 6 **8s** = ? **tens** 		T = 6 **8s** = 6 x 8 = (***B***-4) x (***B***-2) = ***BB*** - 4***B*** - 2***B*** - - 8 = 10***B*** - 6***B*** + 8 = 4***B***8 = 4.8 **tens** = 48
DoubleCount gives PerNumbers	2$ per 3kg = 2$/3kg	T = 6$ = (6/2) x 2$ = (6/2) x 3kg = 9kg
Like Units: Fractions 5% of 40	5$/100$ of 40$	T = 40$ = (40/100) x 100$ gives (40/100) x 5$ = 2$
DoubleCount a Block halved by its Diagonal		*a* = (*a*/*c*)**c* = sin *A***c* *a* = (*a*/*b*)**b* = tan *A***b* π = *n**tan(180/*n*) *for n large* *c*c* = *a*a* + *b*b*

Flexible Bundle-Numbers Respect & Develop Kids Own Math

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01. Meeting Many inspires transforming five ones into one five-icon containing five strokes or sticks. Likewise, with the other digits from one to nine, also containing as many strokes or sticks as they represent if written less sloppy. Icon-building may be illustrated with a folding ruler. Transforming five ones to one fives allows using five as a unit when counting a total *T* by bundling and stacking, to be reported in a full number-language sentence with a subject, a verb and a predicate, e.g. *T* = 2 5s.

02. Icons thus inspires 'bundle-counting' and 'bundle-writing' where a total *T* of 5 1s is recounted in 2s as *T* = 1*B*3 2s = 2*B*1 2s = 3*B*-1 2s, i.e. with or without an overload, or with an underload rooting negative numbers. The unbundled 1 can be placed next to the bundles separated by a decimal point, or on-top of the bundles counted in bundles, thus rooting fractions, *T* = 5 = 2*B*1 2s = 2.1 2s = 2 ½ 2s. Recounting in the same unit to create or remove over- or underloads eases operations. Example: *T* = 336 = 33*B*6 = 28*B*56 = 35*B*-14, so 336/7 = 4*B*8 = 5*B*-2 = 48.

03. Bundle-counting makes operations icons also. First a division-broom pushes away the bundles, then a multiplication-lift creates a stack, to be pulled away by a subtraction-rope to look for unbundles singles separated by the stack by an addition-cross. A calculator uses a 'recount formula', *T* = (*T/B*)**B*, to predict that 'from *T*, *T/B* times, *B*s can be taken away'. This recount formula occurs all over mathematics and science: when relating proportional quantities as $y = c*x$; in trigonometry as sine and cosine and tangent, e.g. $a = (a/c)*c = \sin A*c$; in coordinate geometry as line gradients, $\Delta y = (\Delta y/\Delta x)*\Delta x = c*\Delta x$; and in calculus as the derivative, $dy = (dy/dx)*dx = y'*dx$.

04. Recounting in a different unit is called proportionality. Asking '3 4s = ? 5s', sticks say 2*B*2 5s. Entering '3*4/5' we ask a calculator 'from 3 4s we take away 5s'. The answer '2.some' predicts that the singles come by taking away 2 5s, thus asking '3*4 – 2*5'. The answer '2' predicts that 3 4s can be recounted in 5s as 2*B*2 5s or 2.2 5s.

05. Recounting from tens to icons by asking '35 = ? 7s' is called an equation *u**7 = 35. It is easily solved by recounting 35 in 7s: *u**7 = 35 = (35/7)*7. So *u* = 35/7, showing that equations are solved by moving to opposite side with opposite calculation sign.

06. Recounting to tens by asking '2 7s = ? tens' is eased by using underloads: *T* = 2*7 = 2*(*B*-3) = 20-6 = 14; and 6*8 = (*B*-4)*(*B*-2) = *BB* - 4*B* - 2*B* -- 8 = 100 – 60 + 8 = 48.

07. Double-counting a quantity in units gives a 'per-number' as e.g. 2$ per 3kg, or 2$/3kg. To answer the question '*T* = 6$ = ?kg', we recount 6 in 2s since the per-number is 2$/3kg: *T* = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. Double-counting in the same unit creates fractions and percent: 2$/3$ = 2/3, and 2$/100$ = 2/100 = 2%.

08. Next-to addition geometrically means adding by areas, so multiplication precedes addition. Next-to addition is also called integral calculus, or differential if reversed.

09. On-top addition means using the recount-formula to get like units. Changing units is also called proportionality, or solving equations if reversed.

References

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City University of Education Journal of Science 17*(3), 453-466.