Developing the Child's own Mastery of Many

Workshop by
Allan.Tarp@
MATHeCADEMY.net
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Children show mastery of many before school

- In an isolated covid-19 household we may ask:
- How to create simple material supporting the children in improving their mastery of Many?
- This workshop is based on the observation that when asked ‘How old next time?’, a 3year old will say 4 showing 4 fingers; but, held together two by two, protests by saying ‘That is not 4. That is 2 2s’,
- The child thus rejects the judgement ‘four’ by insisting on describing what exists, bundles of 2s and 2 of them.

Goal: Bridge the outside and the inside

- So, the goal of the workshop is to explore into the mastery of Many children bring to school to see what kind of mathematics occurs if allowing the children to develop their already existing quantitative competence under proper guidance (Tarp 2018, 2020).

- In the workshop you will need 12 sticks, 12 cubes, a pegboard with rubber bands, squared paper and a pencil.

In Mathematics Education, the goal is

Mastery of Mathematics?

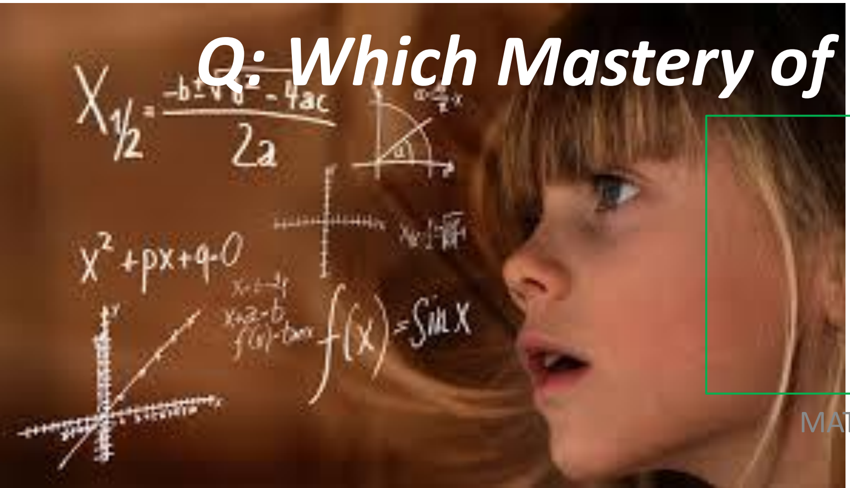
Or is this but a means

to the real end-goal:

Mastery of Many

- so we ask

Which of the 3 { *Present SET-centric* / *Pre SET-centric* / *Post SET-centric*



Q: Which Mastery of Many does a child have before school?

Ans: Allowed to keep and develop their own flexible bundle-numbers,
within ½ a year they begin to master
digital numbers, negative numbers, decimals, fractions, functions,
solving equations, proportionality, calculus, trigonometry

Is Mathematics WellDefined?

No, three Versions: MetaMatics, MatheMatism, ManyMath

This is true	Always	Never	Sometimes
2 + 3 = 5	Only with the same unit; 2weeks + 3days = 17days		**x** (MatheMatsim)
2 x 3 = 6	**x**	2x3 is 2 **3s** that can always be recounted as 6 **1s**	(ManyMath)
$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$	  1 of 2 apples + 2 of 3 apples gives 3 of 5 apples, and not 7 of 6		**x** (ManyMath)
$\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$	Only if taken of the same total *Fractions are not numbers, but operators, needing numbers to become numbers*		**x** (MatheMatsim)
C1: a **FUNCTION** is	**For example** 2+*x*, but not 2+3 i.e. a name for a calculation with an unspecified number		**(1750-1900)** (ManyMath)
C2:	**An example of** a SET-relation where first component identity implies second component identity		**(after 1900)** (MetaMatics)

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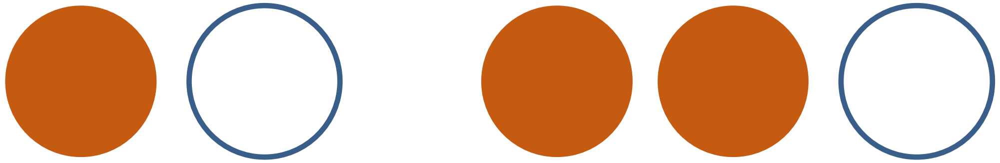
Adding without units creates MatheMatism *true INSIDE but seldom OUTSIDE*

The Teacher	**The Students** (the fraction paradox)
What is 1/2 + 2/3?	Well, 1 of 2 + 2 of 3 gives (1+2) of (2+3), or 3 of 5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But if the browns are 1/2 of 2 cakes, and 2/3 of 3 cakes, then they are 1+2 of 2+3 cakes, i.e. 3/5 of 5 cakes! How can the browns be 7 cakes out of 6 cakes?
INSIDE this classroom 1/2 + 2/3 **IS** 7/6 !	

Without units, fractions & digits are operators, needing numbers to become numbers.

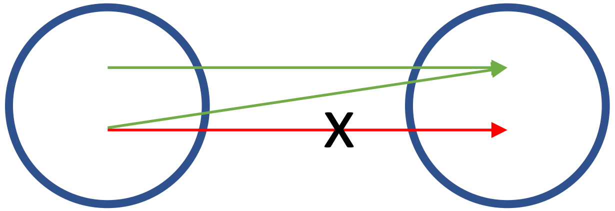
2+3 IS 5? No, 2weeks + 3days is 17days; and 2m + 3cm = 203cm.

2x3 IS 6? Yes, since 3 is the unit, and 2 **3s** can be recounted to 6 1s.

3 kinds of math, **pre**-, **present** & **post**-setcentric mathematics, defining a ‘function’ differently

Pre-setcentric math: a function is a CALCULATION with both specified and unspecified numbers, e.g. **2+*u***.

Present setcentric math: a function is a SUB**SET** OF **SET**-PRODUCT where first-component identity implies second-component identity.



Post-setcentric math: a function is a NUMBER-LANGUAGE SENTENCE, e.g. **T = 2+*u***, linking an OUTSIDE existence to an INSIDE chosen essence.

Post-setcentric math: math through its use, as with the other language in our 2 language houses

The **WORD-language** assigns words in sentences with The **NUMBER-language** assigns numbers instead with	• a subject • a verb • a predicate

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

	WORD-language	NUMBER-language
META-language, grammar	*'is' is a verb*	*'x' is an operation*
INSIDE Language	This is a chair	T = 3x4
OUTSIDE World		

Mathematics

Mother tongue

The **Communicative Turn** in language ed.

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: "A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us."
Likewise, Widdowson adopts a "communicative approach to the teaching of language" allowing more students to learn a language through its use for communication about outside things and actions.

Could mathematics also have its Communicative turn?

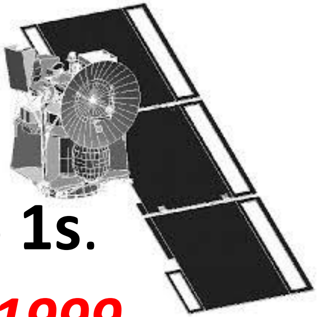
	WORD language	NUMBER language
(META-language, grammar)		
Inside Language	This is a chair	T = 3 4s
Outside world	Mother tongue 	 Mathematics

Defining MetaMatism = MetaMatics+MatheMatism

MetaMatics is defining a concept, not as a ~~BottomUp abstraction from many examples~~ but as a TopDown example of one abstraction, derived from the meta-physical abstraction **SET**, made meaningless by self-reference as shown by Russell's version of the liar paradox: M belongs, only if it does not, to the set of sets not belonging to itself:

$$\boxed{\text{With } \mathbf{M} = \{\mathbf{A} \mid \mathbf{A} \notin \mathbf{A}\}: \quad \mathbf{M} \in \mathbf{M} \Leftrightarrow \mathbf{M} \notin \mathbf{M}}$$

MatheMatism is a statement that is correct inside, but seldom outside a classroom , as e.g. adding numbers without units as 2+3 = 5, where e.g. 2**w**+3**d**=17**d**. In contrast to 2x3 = 6 saying that 2 **3s** can be recounted as 6 **1s**.



Neglecting English and metric units made NASA's Mars Climate Orbiter CRASH in 1999.

Education? Two different kinds

The 1700 Enlightenment Century rooted education, but in different forms in its two republics, in North America in 1776 and in France in 1789.

- In North America, education enlightens children about their OUTSIDE world, and enlightens teenagers about their INSIDE individual talent, to be uncovered and developed through self-chosen ½year **BLOCKS** with teachers teaching only one subject in the teacher's own classroom.
- To protect its republic from its German speaking neighbors, France was forced to create institutions controlled by a strong central administration with public servants trained at elite schools with forced multi-year **LINES**, later copied by the German Bildung-education (and by the rest of Europe).

3x2 different kinds of math education

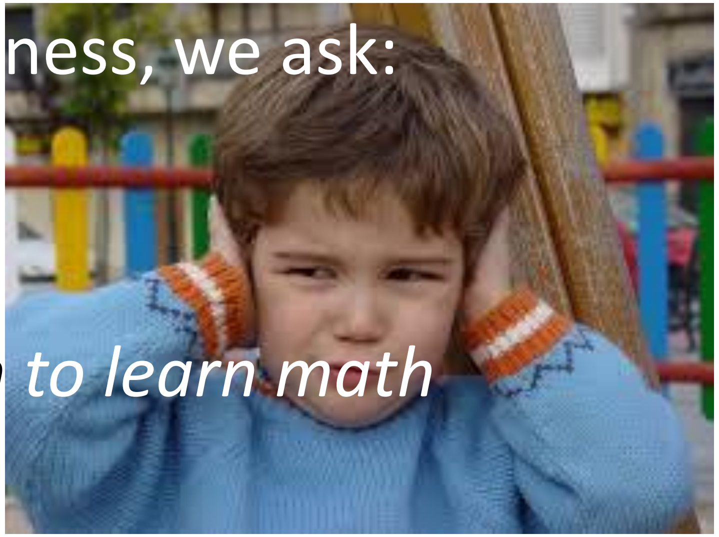
Mathematics in	**self-chosen ½year BLOCKS**	**forced multi-year LINES**
Pre-SETcentric	North America	UK Commonwealth
Present SETcentric	-	Continental Europe
Post-SETcentric	MATHeCADEMY.net	

Why teach children if they already know?

With education curing the diagnose un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*



Core Questions:

- What Mastery does children develop when adapting to Many?
- What could be a Question-guided <u>Child-Grounded-Curriculum</u> in Quantitative Competence?

Philosophizing the low success of 50 years of mathematics education research

Mathematics also needs a <u>COMMUNICATIVE TURN</u> where

- instead of learned **INSIDE-INSIDE** through its grammar, it is learned **OUTSIDE-INSIDE** as a INSIDE number-language communicating about OUTSIDE things and actions, thus learned through its use, and not before its use
- instead of learning about numbers, students learn how to number, and how to communicate about Many in full sentences containing:

  1) an **OUTSIDE** <u>subject</u>, 2) a linking <u>verb</u>, and 3) an **INSIDE** <u>predicate</u>: **T = 2x3**

So, maybe we need an **OUTSIDE-INSIDE post-SETcentric mathematics** to replace the present **INSIDE-INSIDE meta-matism** by asking:

What kind of mathematics grows from the Mastery of Many that children develop through use, and before school?

Pablo Picasso: It took me four years to paint like Raphael, but a lifetime to paint like a child

Ask a 3year old: How Old Next Time?

The answer is 4, showing 4 fingers

But, reacts strongly to 4 fingers held together 2 by 2:

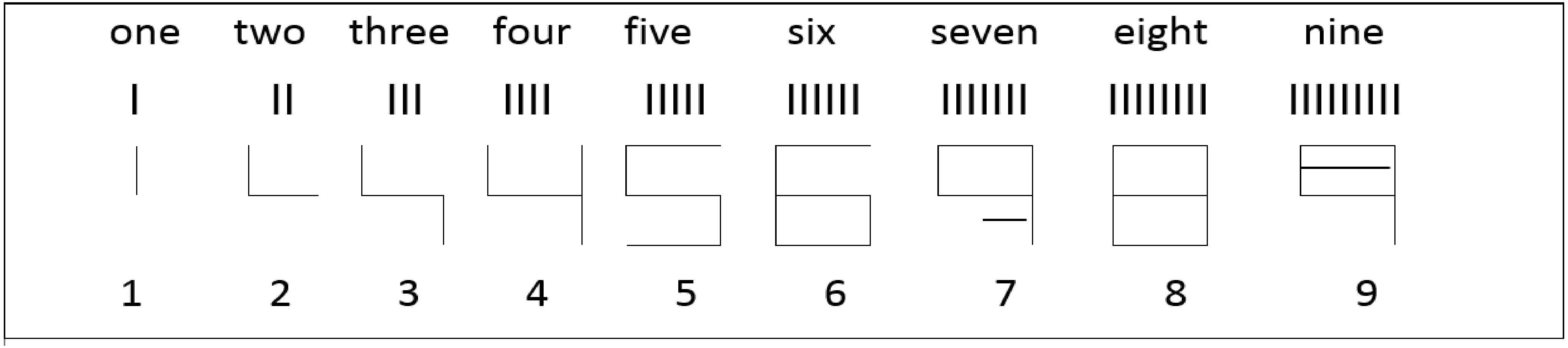
“That is not four, that is two twos”

- Observation 01: Inside, the child sees what exists outside, and with units: bundles of **2s**, and 2 of them.
- Observation 02: The child uses a full number-language sentence as in the word-language with a SUBJECT, a VERB, and a PREDICATE:

  ”That is two twos”, shortened to ”T = 2 **2s**”.

Creating icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.



So, let us explore the math jungle growing from the child own 'flexible bundle-numbers'

- What is flexible numbers
- What is flexible operations
- What is flexible functions
- What is flexible equations
- What is flexible linearity
- What is flexible calculus
- What is flexible trigonometry

Counting sequences

"How to count fingers?"

Using **5s** as the bundle-size, fingers can be counted as

"0**B**1, 0**B**2, 0**B**3, 0**B**4, 0**B**5 – sorry, **Bundle**"

and the rest can be counted in as

"**Bundle**&1, **B**&2, 2**B** less2, 2**B**-1, 2**B, 1left, 2left** (a-leven, twe- leven)".

Follow-up activities could be counting the fingers in **3s** and **4s** and **7s**:

T = ten = 1**B**3 **7s** = 2**B**2 **4s** = 3**B**1 **3s** = 1**BB**1 **3s**.

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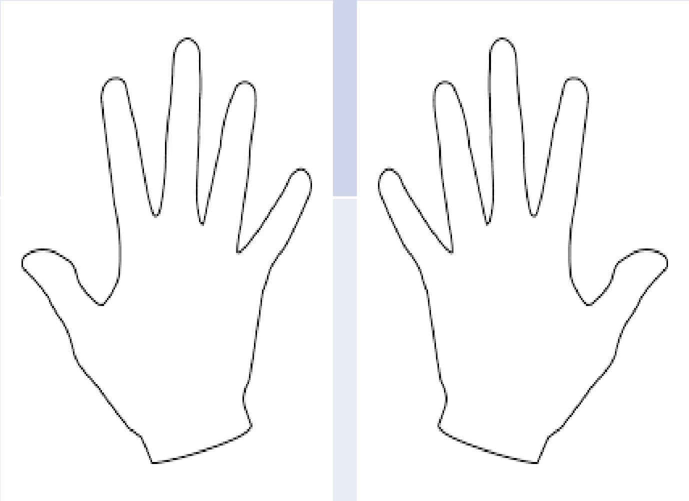
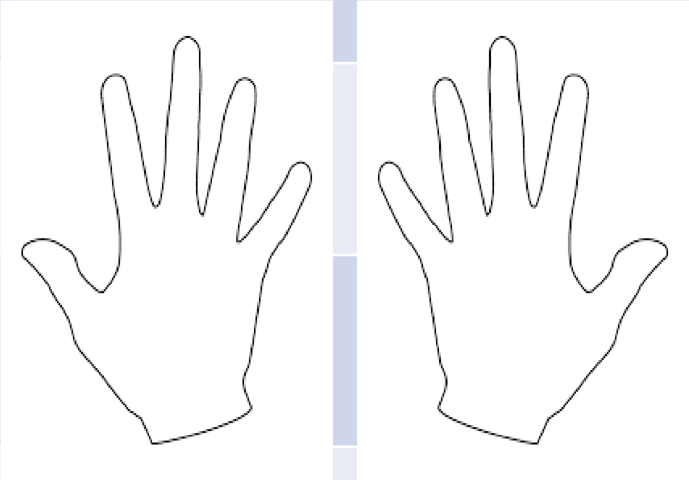
Counting sequences with flexible bundle-numbers
Ten fingers counted in threes

Outside	Action	Inside
	Count in 3s	1, 2, ~~3~~ no 1Bundle, 1B1, 1B2, ~~1B3~~, no 2Bundle, 2B1, 2B2, ~~2B3~~, no 3Bundle 3B1
	Oops 3B = 1 bundle-bundle	1, 2, 1Bundle, 1B1, 1B2, 2Bundle, 2B1, 2B2, ~~3Bundle~~, no 1BundleBundle 1BB1

Counting sequences with flexible bundle-numbers
Ten fingers counted in fives

Outside	Action	Inside
	Count in 5s	1, 2, 3, 4, ~~5~~ no 1Bundle, 1B1, 1B2, 1B3, 1B4, ~~1B5~~ no 2B
	BundleCount in 5s	0B1, 0B2, …,~~0B5~~ 1B0, 1B1, 1B2, …,~~1B5~~ 2B0
	Underload	1B less 4, 1B-3, 1B-2, 1B-1, 1B, 2B-4, …, 2B
	Overload	0B1, 0B2, …, 0B5, 0B6, 0B7, 0B8, 0B9, 0B10

Bundle-counting in icon-units

"How to count by bundling?"

Five fingers can be bundle-counted in pairs or triplets, allowing both an OVERLOAD and an UNDERLOAD; and reported in a number-language sentence with a subject & a verb & a predicate as e.g. T = 2 **3s.**

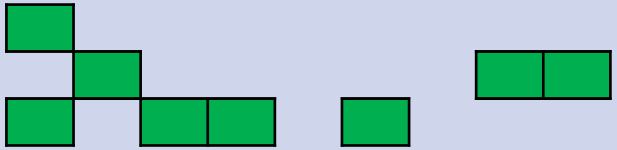
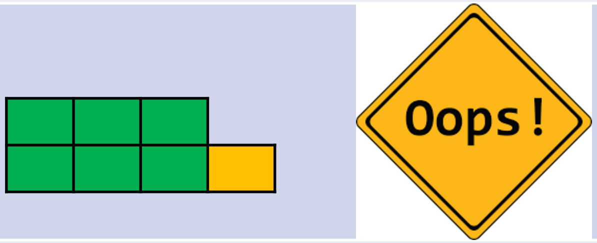
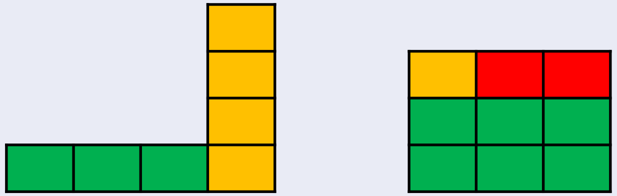
I I I I I • ǂ I I I • ǂ ǂ I • ǂ ǂ ǂ • ǂ ǂ I

T = 5 = 1**Bundle**3 **2s** = 2**B**1 **2s** = 3**B**-1 **2s** = 1**BB**1 **2s**

T = 5 = 1.3 **2s** = 2.1 **2s** = 3.-1 **2s** = 10.1 **2s**

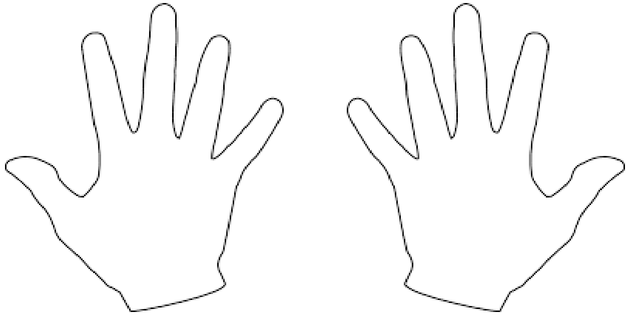
Likewise, if counting in **ten**-bundles:
T = 57 = 5**B**7 = 4**B**17 = 6**B**-3 **tens**

Outside Blocks & Inside Flexible BundleNumbers

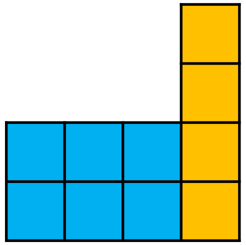
Outside	Actions	Inside
	*Count in 1s*	Total T
		T = 7
	*Re-count in 3s* *Oops, UNBUNDLED!?*	T = 2**B**1 **3s** T = 2.1 **3s**
	*Re-count with overload & underload*	T = 1**B**4 **3s** T = 3**B**-2 **3s** *over-load & under-load*

<u>Observation 03</u>. Decimal numbers and negative numbers seem natural: they just count for the unbundled

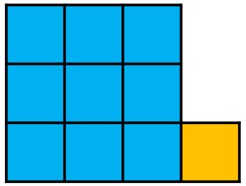
BundleCounting ten blocks in **3s**



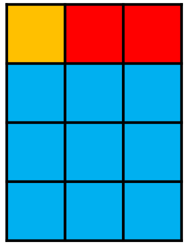
Outside	Inside
Two hands bundle-counted in **3s**:	But 3 **Bundles**, is 1 **Bundle-of-Bundles.** So T = 9 = 1**BB 3s** or T = ten = 3**B**1 **3s** = 1**BB**1 **3s** or T = ten = 1**BB**0**B**1 **3s** or T = ten = 101 **3s** T = ten = 1***B**^2 + 0***B** + 1 **3s**



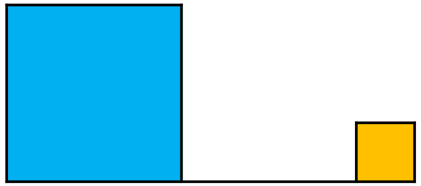
T = 2**B**4
over-load



T = 3**B**1
normal



T = 4**B**-2
under-load



T = 1**BB** 0**B** 1 = 101 **3s**

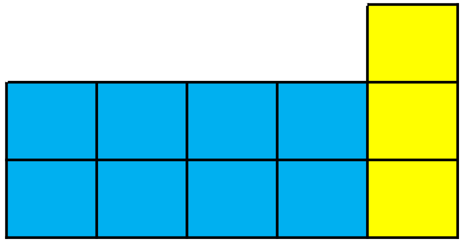
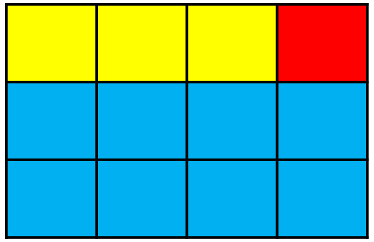
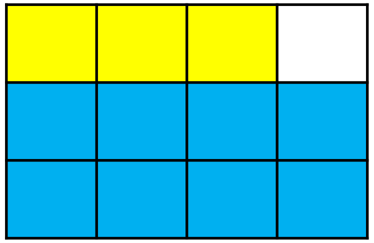
Unbundled as decimals or negatives or fractions

0.3 **4s** or 1.-1 **4s** or 3/4 **4s**

"Where to put the unbundled singles?"

When counting by bundling, the unbundled singles can be placed

NextTo the block — **OnTop** of the block

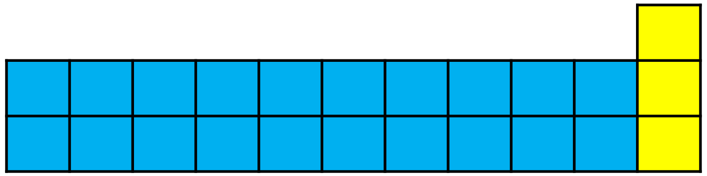
counted as a block of **1s**	counted as a bundle	counted in bundles
		
T = 2B3 **4s** = 2.3 **4s**	T = 3B-1 **4s** = 3.-1 **4s**	T = 2 3/4 **4s**
A decimal number	*A negative number*	*A fraction*

Counting in tens

"Where to put the unbundled singles with **tens**?"

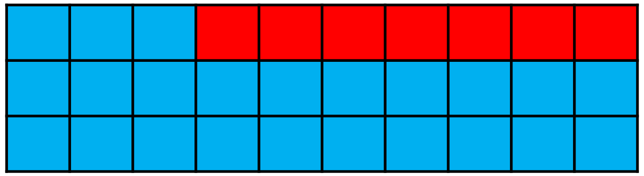
Counting in tens, an outside Total of 2 **tens** & 3 can be described inside as T = 23 if leaving out the unit and the decimal point,

- or as:



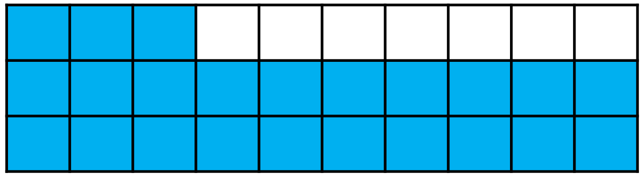
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

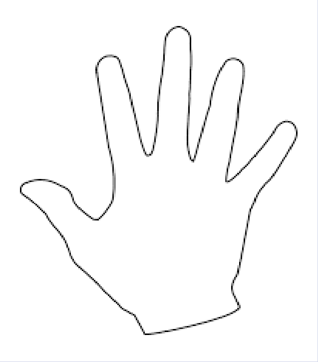
T = 3B-7 **tens**



T = 2 3/10 **tens**

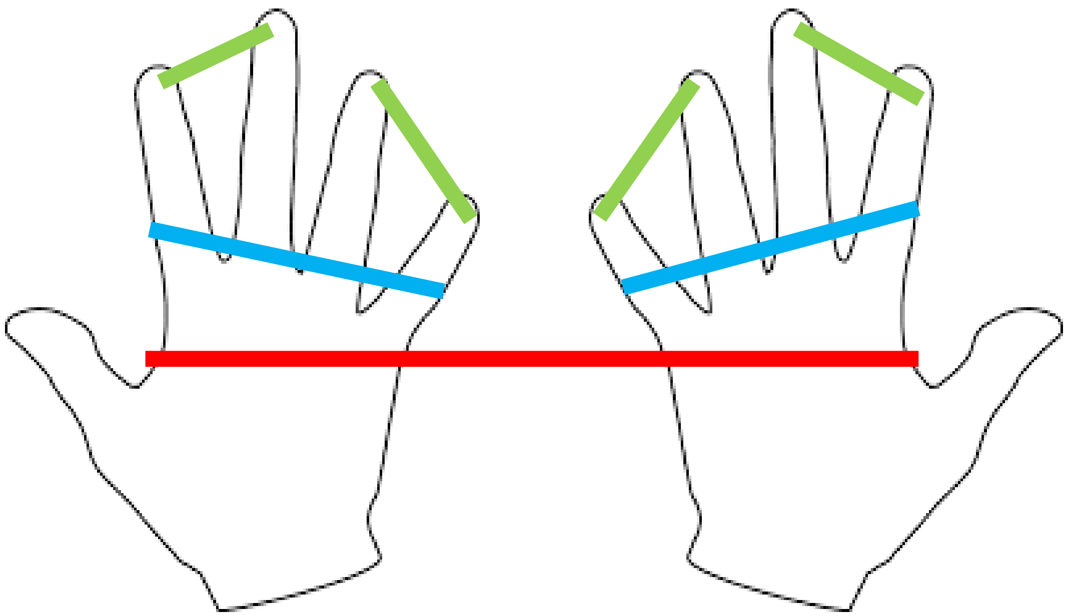
T = 2 3/10 B **tens**

Counting fingers in 2s with flexible bundle-numbers: Bye-bye to place-values

Outside	Inside
I I I I I II I I I II II I II II II 	T = 5 T = 1B3 overload T = 2B1 normal T = 3B-1 (less 1) underload
Seven ten four Three hundred seven ten four Counting formula (polynomial)	T = 74 = 7B4 = 6B14 = 8B-6 T = 374 = 3BB 7B 4 = 2BB 15B 24 T = 3 B^2 + 7 B + 4

BundleCounting Fingers in **2s**

1	1	1
2	1**B**0	10
3	1**B**1	11
4	1**BB**00	100
5	1**BB**01	101
6	1**BB**1**B**0	110
7	1**BB**1**B**1	111
8	1**BBB**000	1000
9	1**BBB**001	1001
Ten	1**BBB**01**B**0	1010



This can be shown with Lego bricks having different colors:

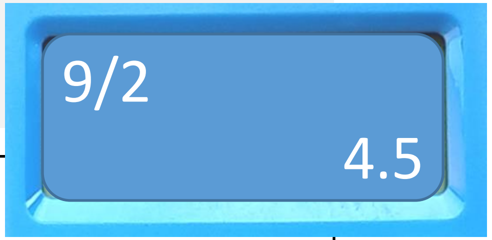
a green 2-brick is **B**

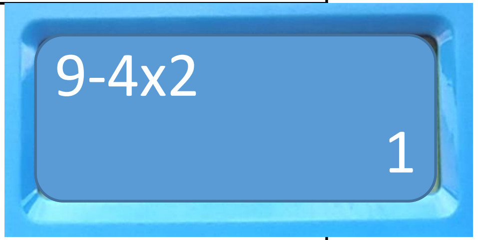
a blue 4-brick is **BB**

a red 8-brick is **BBB**

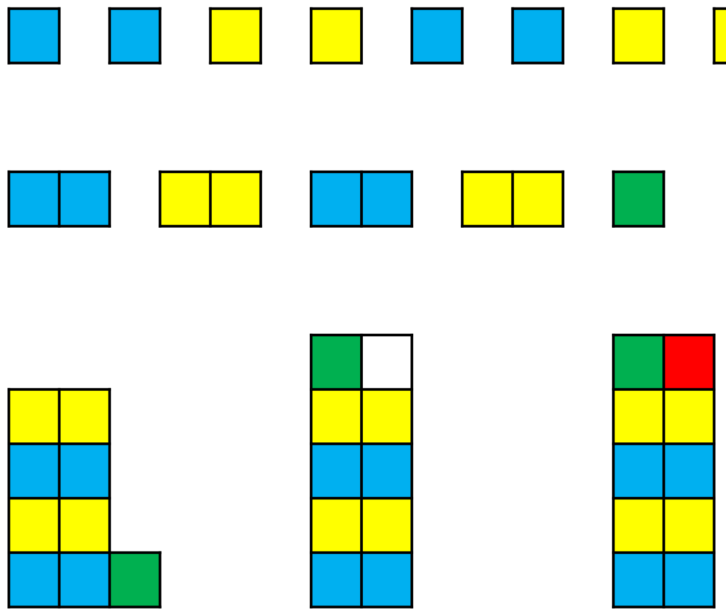
BundleCounting a Total of 9 in **2s**

9/2	**4.some**
9 – 4x2	**1**





Outside World



9

bundled in ***2s*** *with 1 unbundled*

stacked as 4x2 with 1 unbundled

4**B**1 | 4½**B** | 5**B**-1

2s

placed next-to or on-top

Inside Brain

From 9, 9/2 times push away 2.
From 9, pull away 4 **2s**, leaving 1.
Prediction by the recount-formula:

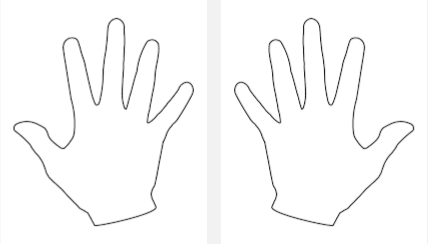
T = 9 = **4B1 2s**

The unbundled can be placed

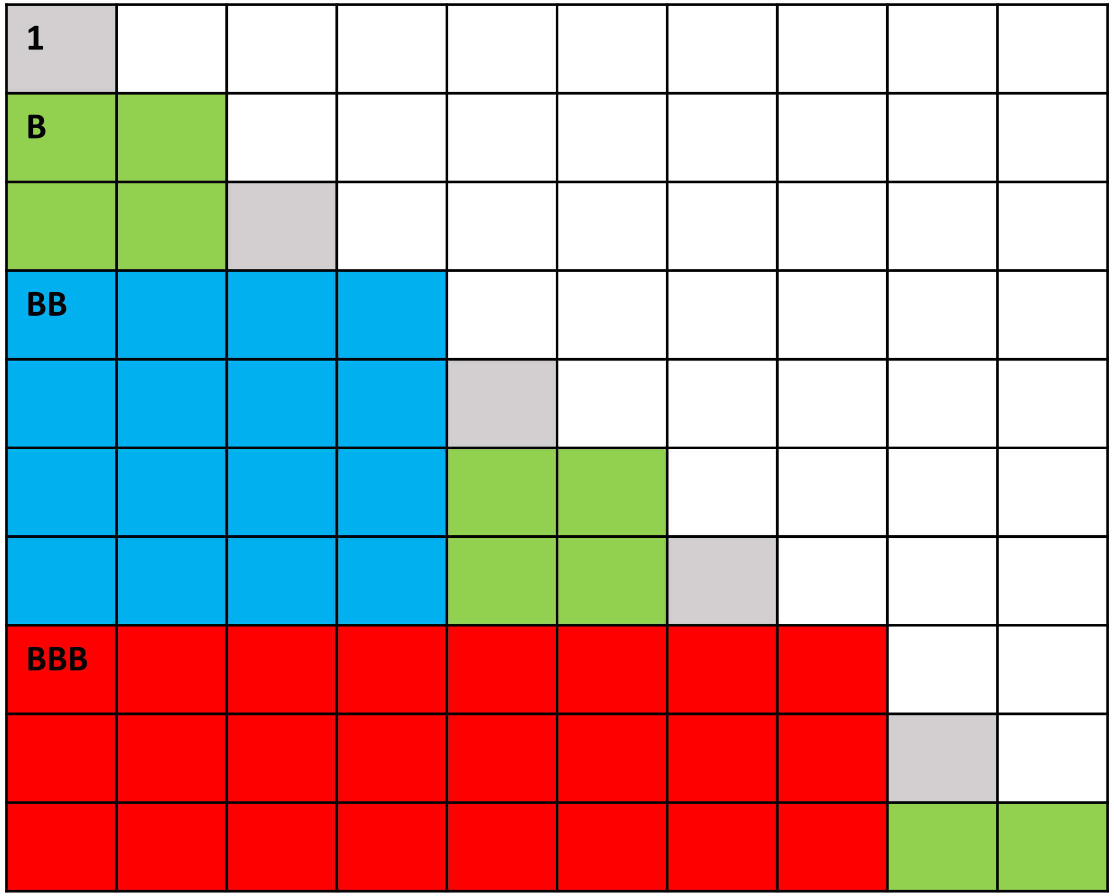
- next-to the stack iconized by a dot named a decimal point; 4.1 **2s**; or on-top of the stack
- counted in bundles as 1 = (1/2) x 2 giving 4½**B 2s**,
- counting what is missing in a full bundle, 5**B**-1 **2s.**

This de-models decimals, fractions & negatives.

BundleCounting Fingers in **2s**



1	1	1
2	1**B**0	10
3	1**B**1	11
4	1**BB**00	100
5	1**BB**01	101
6	1**BB**1**B**0	110
7	1**BB**1**B**1	111
8	1**BBB**000	1000
9	1**BBB**001	1001
Ten	1**BBB**01**B**0	1010



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Bundling-counting table

1BB0B0	1BB0B1	1BB0B2	1BB0B3	1BB0B4	1BB0B5	1BB0B6	1BB0B7	1BB0B8	1BB0B9	~~1BB0B10~~
~~10B0~~	~~10B1~~	~~10B2~~	~~10B3~~	~~10B4~~	~~10B5~~	~~10B6~~	~~10B7~~	~~10B8~~	~~10B9~~	~~10B10~~
9B0	9B1	9B2	9B3	9B4	9B5	9B6	9B7	9B8	9B9	~~9B10~~
8B0	8B1	8B2	8B3	8B4	8B5	8B6	8B7	8B8	8B9	~~8B10~~
7B0	7B1	7B2	7B3	7B4	7B5	7B6	7B7	7B8	7B9	~~7B10~~
6B0	6B1	6B2	6B3	6B4	6B5	6B6	6B7	6B8	6B9	~~6B10~~
5B0	5B1	5B2	5B3	5B4	5B5	5B6	5B7	5B8	5B9	~~5B10~~
4B0	4B1	4B2	4B3	4B4	4B5	4B6	4B7	4B8	4B9	~~4B10~~
3B0	3B1	3B2	3B3	3B4	3B5	3B6	3B7	3B8	3B9	~~3B10~~
2B0	2B1	2B2	2B3	2B4	2B5	2B6	2B7	2B8	2B9	~~2B10~~
1B0	1B1	1B2	1B3	1B4	1B5	1B6	1B7	1B8	1B9	~~1B10~~
0B0	0B1	0B2	0B3	0B4	0B5	0B6	0B7	0B8	0B9	~~0B10~~

Flexible bundle-counting roots negative numbers

- Including bundles, ten fingers may be bundle-counted in fives as ‘0Bundle1, 0B2, 0B3, 0B4, 1B0, 1B1’ that also may be counted as 0B6 or 2B less 4 or 2B-4 with an overload or an underload, thus writing 6 in three ways as a ‘flexible bundle-number’:
- T = 6 = 0B6 = 1B1 = 2B-4
- Using flexible bundle-numbers, ten fingers may also be counted in 5s as ‘1B-4, 1B-3, etc.’
- Counting on from ten, we meet ‘Viking-counting’ where eleven is ‘one-left’, twelve is ‘two-left’, and thirteen is ‘three-ten’, while ‘three-twenty’ becomes twenty-three.
- Counting in scores (twenties) from forty, the Danish Viking-descendants still count: half-three-scores, three-scores, half-four-scores, four-scores, and half-five-scores for ninety. Unable to understand the half-notion, the French instead counts over when expressing 87 as ‘4 scores and 1 ten and 7.’

Flexible BundleNumbers Ease Operations
Counting in tens, T = 78 = 7**B**8 = 6**B**18 = 8**B**-2

Overload	Underload	Overload	Overload
6 5 + 2 7	6 5 - 2 7	7 x 48	336 /7
6**B**5 + 2**B**7	6**B**5 - 2**B**7	7 x 4**B**8	33**B**6 /7
8**B**12	4**B**-2	28**B**56	28**B**56 /7
9**B**2	3**B**8	33**B**6	4**B**8
92	38	336	48

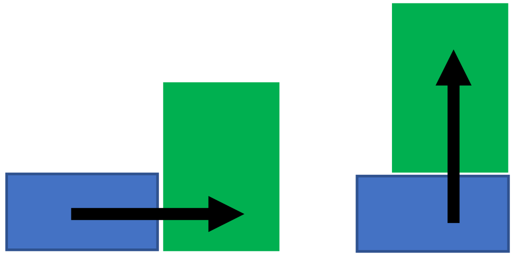
No need to carry!

Divide & Multiply & Subtract & Add may be 'de-modeled' as Icons also

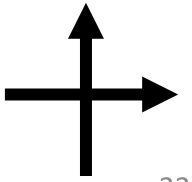
- From 9 **PUSH** away **4s** we write 9/4 iconized by a broom, called *division*.

- 2 times **LIFTING** the **4s** to a stack we write 2x4 iconized by a lift called *multiplication*.

- From 9 **PULL** away 2 **4s**' to find un-bundled we write 9 – 2x4 iconized by a rope, called *subtraction*.



- **UNITING** next-to or on-top we write A+C iconizing the two directions, called *addition*.



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BundleNumbers can Shift Units and create a RecountFormula

$8 = (8/2)x2$

$T = (T/B)xB$

Bundle-counting 8 in 2s: 8 = ? **2s**

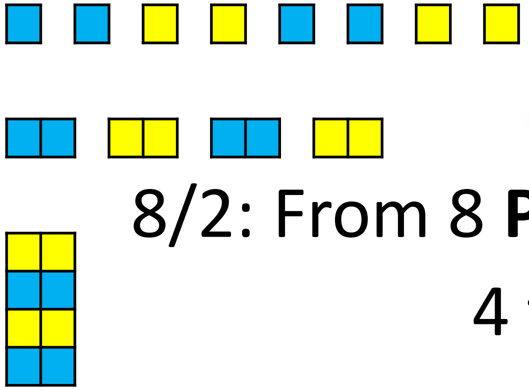
Fingers: 8 = 4 **2s**

Blocks bundle-counted: 8 = 8/2 **2s**

Bundles stacked: 4 **2s** = 4x2

Bundle-counting: 8 = (8/2)x2

Recount-Formula: **T = (T/B)xB**



8/2

4

8/2: From 8 **PUSH** away 2

4 times **LIFT** 2

*u*x2 = 8 = (8/2)x2

u **= 8/2**

OPPOSITE side & sign

Equations Proportionality

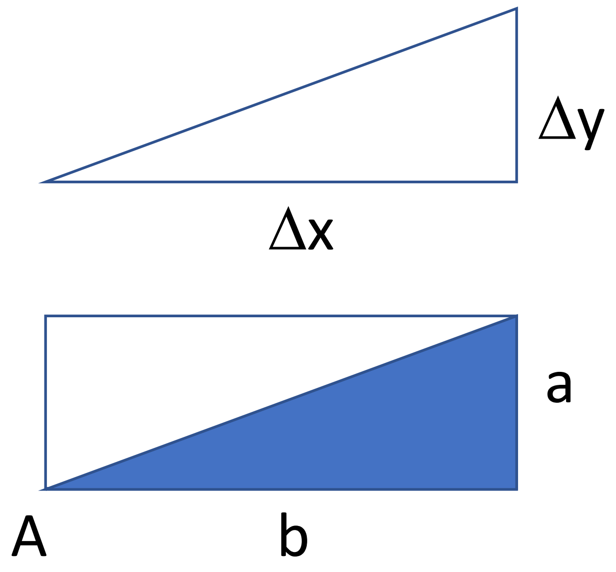
Shifting unit	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
STEM	meter = (**meter/sec**) * sec = **speed** * sec

Observation 04: ReCounting in BundleNumbers contains Core Mathematics & STEM

The **ReCount formula** is the core of math & science

T = (T/B)*B expresses proportionality when changing unit, and is all over:

Proportionality	y = **c** * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y’** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
Science	meter = (**meter/second**) * second = **speed** * second



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Counting is predicted by the ReCount formula

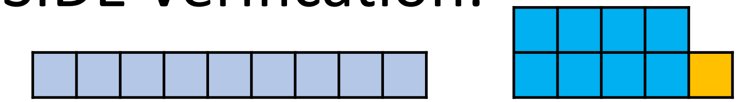
T = (T/B) x B	From a total **T**, **T/B** times, **B** is pushed away

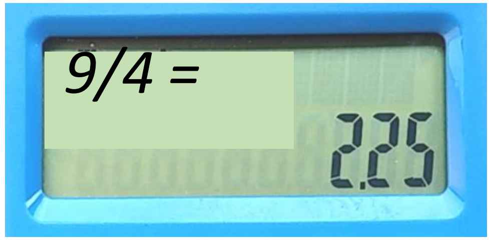
A formula is an INSIDE **prediction**, making the number-language a language for prediction.

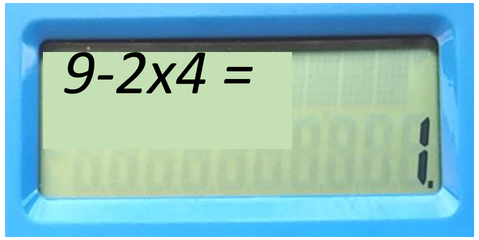
INSIDE Prediction: ReCounting 9 in **4s** gives 2**B**1 **4s:**

9/4	**2.some**
9 – 2x4	**1**

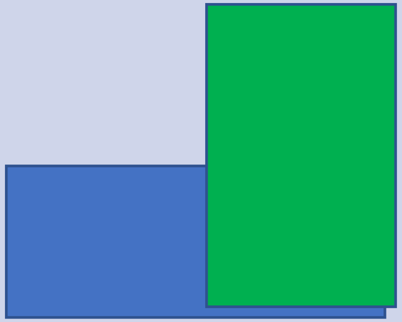
OUTSIDE Verification:







Recount from tens to icons: 40 = ? **5s**

outside		inside
		INSIDE, we formulate an equation to be solved by recounting 40 in **5s**: u*5 = 40 = (40/5)*5, so u = 40/5 = *8* *giving 40 = 8*5 = 8* ***5s*** Notice: recounting gives the equation solution rule 'move to opposite side with opposite calculation sign'.
we use a pegboard or a squared paper to transform the stack 4.0 tens to 8.0 5s. So decreasing the base will increase the height.		u+3 = 7 u*3 = 7 u^3 = 7 3^u = 7 df/dx = x^2 u = 7-3 u = 7/3 u = 3√7 u = log3(7) f = ∫x^2*dx

Recounting from icons to tens (multiplication) 3 **7s** = ? **tens**

"How to change unit from icons to tens?"

Asking 'T = 3 **7s** = ? **tens**', the recount-formula cannot be used since the calculator has no ten-button. However, it gives the answer directly by using multiplication alone: T = 3 **7s** = 3x7 = 21 = 2.1 **tens**, only it leaves out the unit and the decimal point.

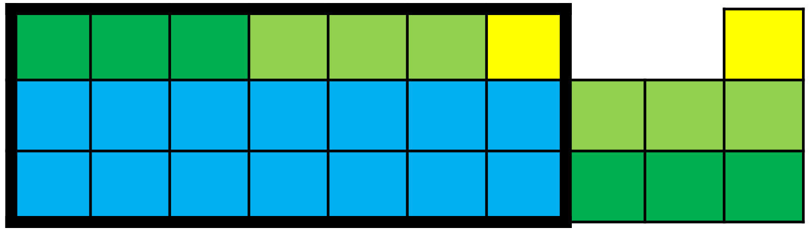
Alternatively, we may use 'less-numbers', so 7 = **ten** less 3

T = 3x7 = 3 x (**ten** less 3) = 3 x **ten** less 3x3 = 3**ten** less 9 = 2**ten**1 = 21,

or with 9 = **ten** less 1:

T = 3**ten** less (**ten** less1) = 2**ten** lessless 1 = 2**ten** & 1 = 21.

showing that 'lessless' cancel out, so that - - is +



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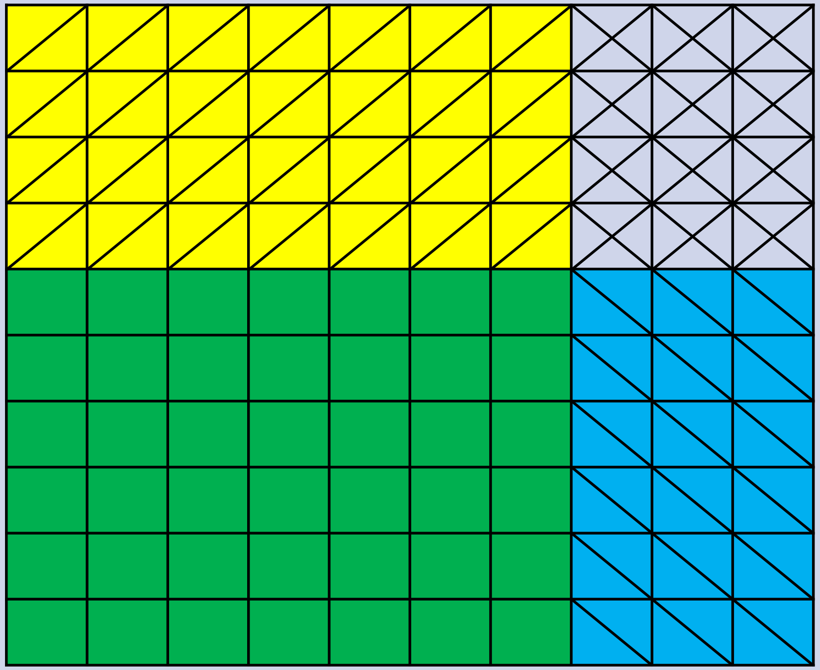
Recounting from icons to tens 6 **7s** = ? tens

Outside

4

6
B-4



7 B-3 3

Inside

T = 6 **7s**

= 6*7

= (B-4)*(B-3)

= 10B - 4B - 3B - - 4 3s

= 3B + 1B2

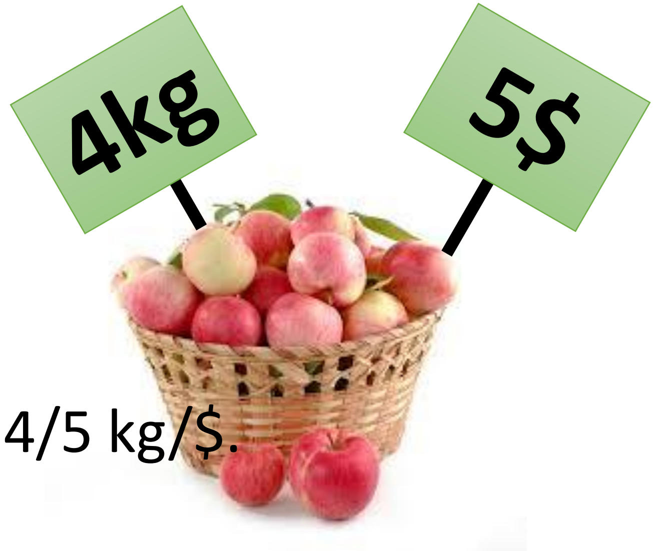
= 4B2

= 42

So- - is +

since it is pulled away twice

ReCounting in two Units creates PerNumbers & Proportionality



ReCounting in kg & $, we get a PerNumber 4kg **per** 5$ = 4kg/5$ = 4/5 kg/$.

With like units, per-numbers become fractions: 4$/5$ = 4/5, and 4$/100$ = 4/100 = 4%.

With 4kg linked to 5$, we simply recount in the per-number.

(Or we recount the units directly. Or we equate the per-numbers. Or we use the before 1900 'Rule of 3' (regula de tri) alternating the units, and, from behind, first multiply, then divide.)

Questions:

12kg = ?$	**20$ = ?kg**
12kg = (12/4) x 4kg = (12/4) x 5$ = 15$	20$ = (20/5) x 5$ = (20/5) x 4kg = 16kg
$ = ($/kg) x kg = 5/4 x 12 = 15	kg = (kg/$) x $ = 4/5 x 20 = 16
u/12 = 5/4, so *u* = 5/4 x 12 = 15	*u*/20 = 4/5, so *u* = 4/5 x 20 = 16
If 4kg is 5$, then 12kg is ?$; answer: 12x5/4 = 15	If 5$ is 4kg, then 20$ is ?kg; answer: 20x4/5 = 16

Proportionality shows the flexibility of ‘School Math’ I

Proportionality, **Q1**: “2kg costs 5$, what does 7kg cost”; **Q2**: “What does 12$ buy?”

➔1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: ‘2kg cost 5$, 7kg cost ?$’. Multiply-then-divide gives the $-number 7x5/2 = 17.5.

Q2: ‘5$ buys 2kg, 12$ buys ?kg’. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

➔2) Find the unit

Q1: 1kg costs 5/2$, so 7kg cost 7x(5/2) = 17.5$. **Q2**: 1$ buys 2/5kg, so 12$ buys 12x(2/5) = 4.8kg

➔3) Cross multiplication

Q1: 2/5 = 7/*u*, so 2**u* = 7*5, *u* = (7*5)/2 = 17.5. **Q2**: 2/5 = *u*/12, so 5**u* = 12*2, *u* = (12*2)/5 = 4.8

➔4) ‘Re-counting’ in the ‘per-number’ 2kg/5$ coming from ‘double-counting’ the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5$ = 17.5$; **Q2**: T = 12$ = (12/5)x5$ = (12/5)x2kg = 4.8kg.

Proportionality shows the flexibility of 'School Math' II

➔ 5) <u>Modeling</u> with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive \$-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ • $(c*2)*½ = 5*½$ • $c*(2*½) = 5/2$ • $c*1 = 5/2$ • $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$\$.
- With 12\$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

Double-counting gives per-numbers in STEM multiplication formulas I

STEM typically contains multiplication formulas with per-numbers coming from double-counting.

Examples:

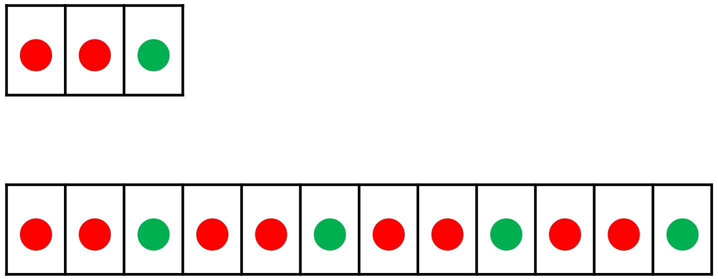
- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **velocity** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

Double-counting gives per-numbers in STEM multiplication formulas II

Extra STEM examples:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec;
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = **(energy/charge)** x **(charge/sec)** or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

With like Units, PerNumbers become Fractions, both Operators Needing Numbers to Become Numbers

Outside trial	Inside prediction
In a box filled with 2 red per 3 apples, re-counting reds and apples gives the FRACTION **2/3** reds/apples. How many red apples among 12 apples? 	Q: ? red in 12 apples. A: Recount 12 in **3s** (the per-number) $T = 12\ a = (12/3) \times 3a$ gives $(12/3) \times 2r = 8$ red apples Or, we equal the per-numbers: $u/12 = 2/3$; so $u = 2/3 \times 12 = 8$ *Moving 12 to opposite side with opposite sign*

ReCounting Sides in a Block gives Trigonometry

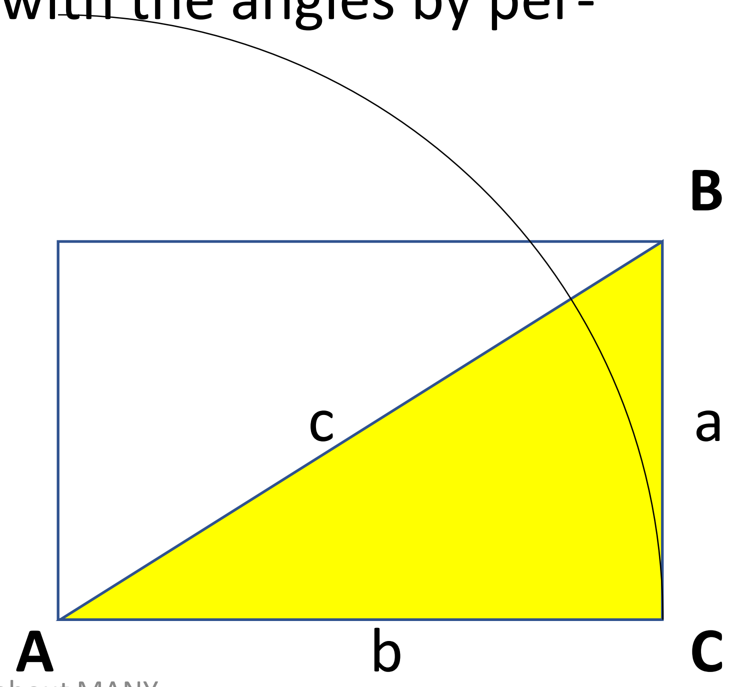
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a block halved by its diagonal thus having three sides: the base b, the height a and the diagonal c connected by the Pythagoras formula. And connected with the angles by per-number formulas re-counting the sides pairwise.

$\mathbf{A} + \mathbf{B} + \mathbf{C} = 180$

$a*a + b*b = c*c$ (the Pythagoras formula)

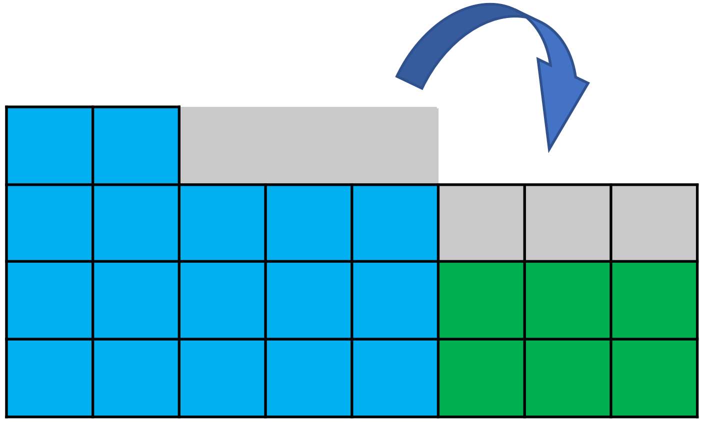
$a = (a/c) \times c = \sin\mathbf{A} \times c$; $\tan\mathbf{A} = a/b = \Delta y/\Delta x$ = gradient

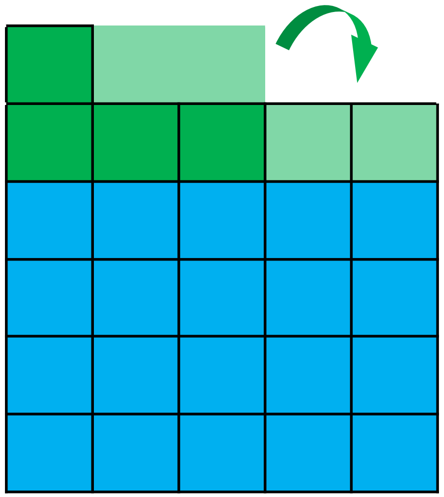
Circle: circum./diam. = $\boldsymbol{\pi = n*\tan(180/n)}$ for n large



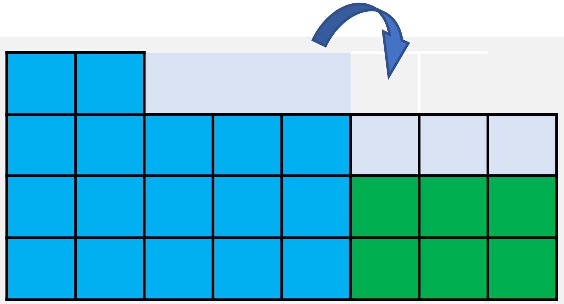
Once Counted & Recounted, Totals may Add

BUT: NextTo	or OnTop
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = ReCounting = Proportionality*





Next-to addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added next-to as **8s**?"

<u>Outside</u>, next-to addition geometrically means adding areas.
Next-to addition is also called integral calculus.

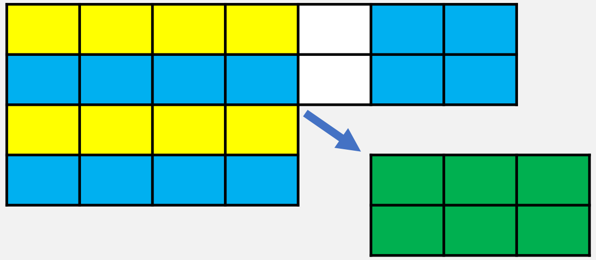
<u>Inside</u>, the recount formula algebraically predicts the result.
Here multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/8) x 8 = 3.2 **8s**

(4x5 + 2x3)/8	**3.some**
(4x5 + 2x3) – 3x8	**2**

Reversed next-to addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

Outside, we remove the initial block T1 and recount the rest in **4s**.

Thus reversed next-to addition geometrically finds area-differences.

Reversed next-to addition is also called differential calculus.

Inside, the recount formula algebraically predicts the result.

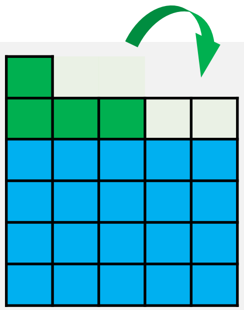
Here subtraction precedes division; which is natural as reversed integration.

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

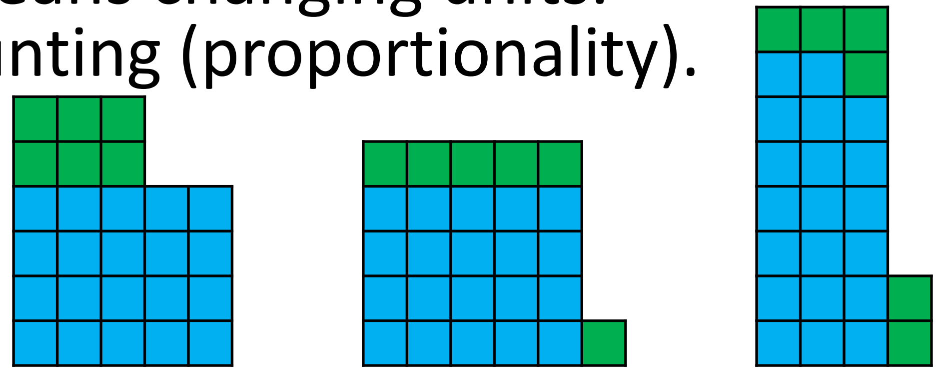
(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

On-top addition



"With T1 = 4 **5s** and T2 = 2 **3s**, what is T1+T2 when added on-top?"

Outside, on-top addition geometrically means changing units.
On-top addition thus often involves recounting (proportionality).



T = 4 **5s** + 2 **3s** = 4 **5s** + 1.1 **5s** = 5.1 **5s**

T = 4 **5s** + 2 **3s** = 6.2 **3s** + 2 **3s** = 8.2 **3s**

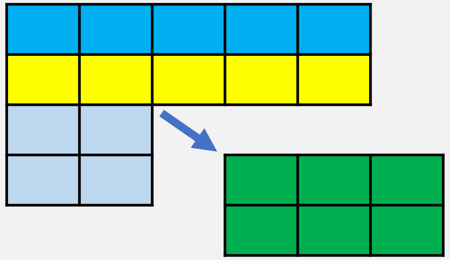
Inside, the recount formula algebraically predicts the result.
Here again, multiplication precedes addition.

T = (T/**B**) x **B**

= ((4x5 + 2x3)/5) x 5 = 5.1 **5s**

(4x5 + 2x3)/5	**5.some**
(4x5 + 2x3) – 5x5	**1**

Reversed on-top addition



"T1 = 2 **3s** and how many **5s** (T2) add on-top as 4 **5s**?"

<u>Outside</u>, we remove the initial block T1 and recount the rest in **5s**.

Thus reversed next-to addition geometrically finds area-differences.

Reversed on-top addition is also called differential calculus.

<u>Inside</u>, the recount formula algebraically predicts the result.

Here again, subtraction precedes division.

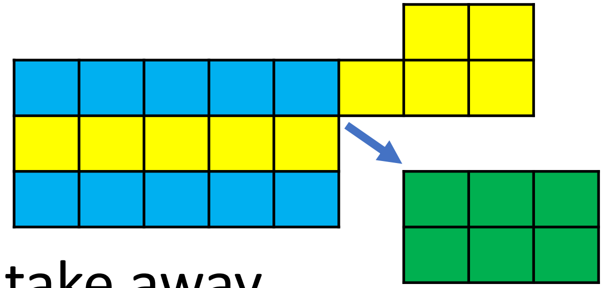
T2 = (T2/**B**) x **B**

= ((4x5 – 2x3)/5) x 5 = 2.4 **5s**

(4x5 – 2x3)/5	**2.some**
(4x5 – 2x3) – 2x5	**4**

Reversed Addition = Solving Equations

OppoSite Side with OppoSite Sign		**NextTo**
2 x ? = 8 = (8/2) x 2	**2 + ? = 8** = (8-2) + 2	2 **3s** + ? **5s** = 3.2 **8s**
? = 8/2 *Solved by ReCounting*	? = 8-2 *Solved by ReStacking*	? = (3.2 **8s** – 2 **3s**)/5 *Solved by differentiation: (T-T1)/5 = ΔT/5*



Hymn to Equations

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

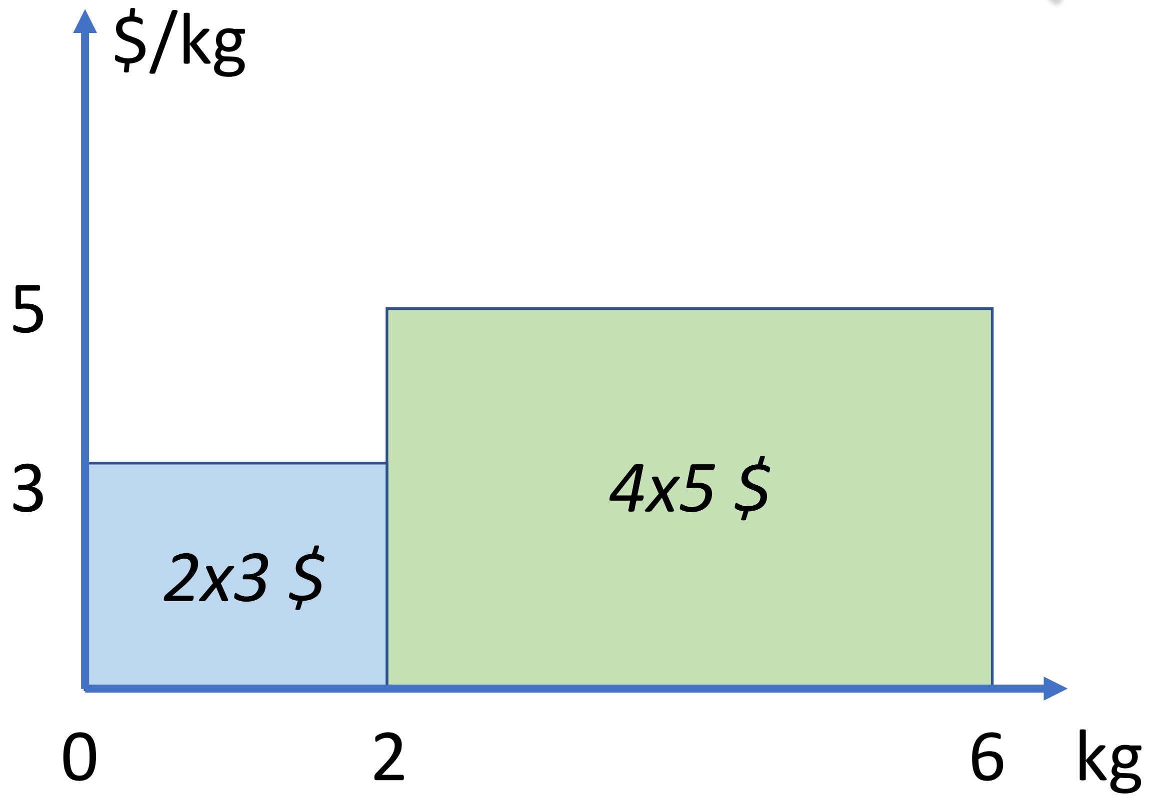
We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Adding PerNumbers as Areas (Integral Calculus)

"2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **? $/kg**?"

	2 kg	at	**3 $/kg**
+	4 kg	at	**5 $/kg**
(2+4)	kg	at	**? $/kg**

- Unit-numbers add on-top.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number graph.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.



Subtracting PerNumbers (Differentiation)

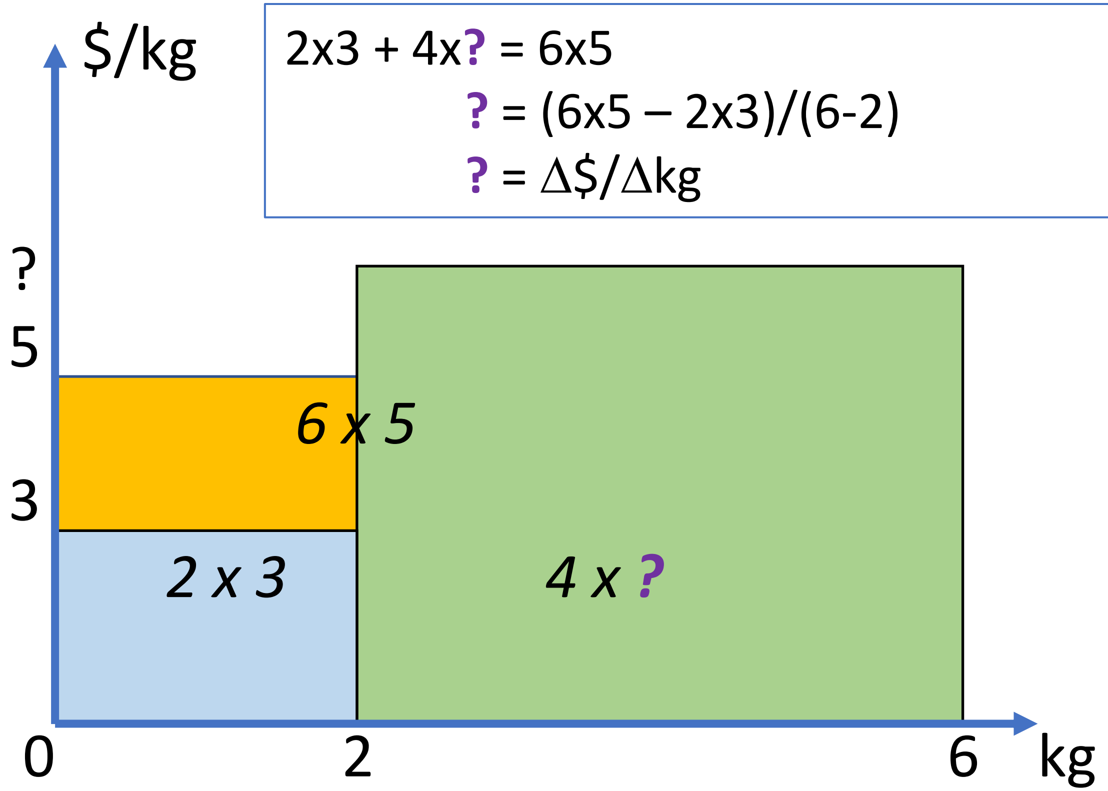
"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

	2 kg	at	3 $/kg
+	4 kg	at	? $/kg
	6 kg	at	5 $/kg

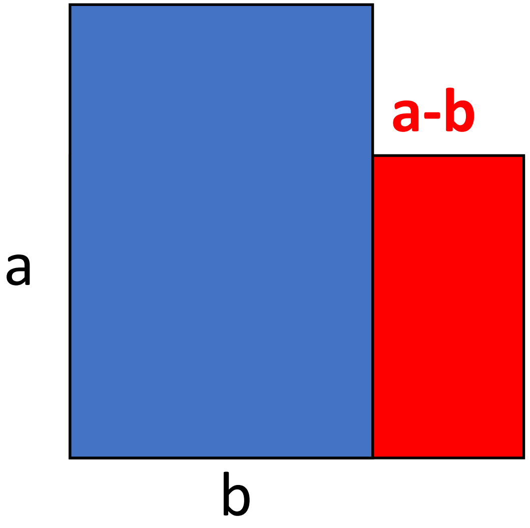
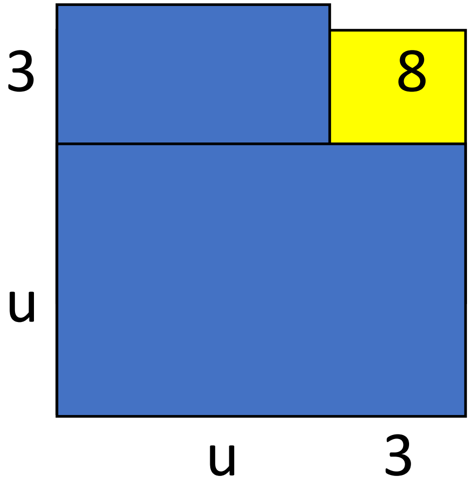
Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) comes before division.

2x3 + 4x? = 6x5
? = (6x5 – 2x3)/(6-2)
? = $\Delta$$/Δkg



Geometry & Algebra, Hand in Hand

Quadratic Rule with 2 Cards	**Quadratic Equations with 3 Cards**
	**$u^2 + 6u + 8 = 0$** 
Corner = $(a-b)^2 = a^2 - 2$ cards $+ b^2$ So $(a-b)^2 = a^2 - 2 \times a \times b + b^2$	$(u+3)^2 = u^2 + 6u + 8 + 1$ $(u+3)^2 = 0 + 1$ $u = -3 \pm 1$ **$\underline{u = -4 \ \& \ u = -2}$**

Four Ways to Unite and Split a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3*$\mathbf{B}$^2 + 4*$\mathbf{B}$ + 5 (a polynomial) shows the 4 ways to unite: +, *, ^, next-to block-addition (integration). Addition and multiplication unite changing and constant unit-numbers. Integration and power unite changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite. ● The 4 uniting operations each has a reverse splitting operation: Addition has subtraction (–), and multiplication has division (/). Power has factor-finding (root, √) and factor-counting (logarithm, log). Integration has per-number finding (differentiation dT/dn = T'). Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / *split into*	**changing**	**constant**
Unit-numbers *m, s, $, kg*	**T = a + n** *T – a = n*	**T = a * n** *T/n = a*
Per-numbers *m/s, $/kg, m/(100m) = %*	**T = ∫ a dn** *dT/dn = a*	**T = a^n** $\log_a T = n$, $\sqrt[n]{T} = a$

Adding or Subtracting Unspecified Numbers

“Only add like units, so how to add $T = 4ab^2 + 6abc$?”

Here units come from folding (factoring):

a factor-filter

T1	2	2	*a*	*b*	*b*
T2	2	3	*a*	*b*	*c*
unit	**2**		***a***	***b***	
T1 left		2			*b*
T2 left		3			*c*

T = $4ab^2 + 6abc$ = T1 + T2

= 2 x 2 x a x b x b + 2 x 3 x a x b x c

= 2 x b x (2 x a x b) + 3 x c x (2 x a x b)

= (2b+3c) x **2*ab***

= 2b+3c **2*ab*s**

What is 3/5 of 2/4

What is 3 per 5 of 2 per 4?

Since we are going to recount in both 5s and 4s, we might take 20 as the total.

2per4 of 20 is what?

Well, recounting 20 in 4s we get 20 = (20/4)*4 giving (20/4)*2 = 10

3per 5 of ten is what?

Well, recounting 10 in 5s we get 10 = (10/5)*5 giving (10/5)*3 = 6

So 3 per 5 of 2 per 4 of 20 gives 6 per 20.

*Test: 6 = (6/20)*20 = 6 per 20 of 20*

So 3 per 5 of 2 per 4 is 6 per 20

Discussion: What is the Difference?

		Flexible many-math	**Traditional math**
Digits	4	Icon with four strokes	Symbol
Numbers	456	Three numberings, 4**BB** 5**B** 6	One number
Division	8/2	8 counted in **2s**	8 split in 2
Multiplication	6 x 7	6 **7s** or 4**B**2 **tens**	42
Addition	2+3	2 **4s** + 3 **5s** = 2**B**3 **9s** 2 **4s** + 3 **5s** = 4**B**1 **5s**	2+3 = 5
Equations	3 x *u* = 12	Opposite side & sign *u* x 3 = 12 = (12/3) x 3 *u* = 12/3 = 4	Neutralize (3 x *u*) x 1/3 = 12 x 1/3 (*u* x 3) x 1/3 = 4 *u* x (3 x 1/3) = 4 *u* x 1 = 4, so *u* = 4
Fractions	2/3	Per-numbers, i.e. operators, needing numbers to become numbers: 1/2 of 2 + 2/3 of 3 **IS** 3/5 of 5	Numbers 1/2 + 2/3 **IS** 7/6

Is ManyMatics Different I

Same Question	**Many-Math**	**Trad. Mathe-Matics**
Digits	Icons, different from letters	Symbols like letters
Natural numbers	2.3 tens	23
Fractions	Per-numbers needing a number to produce a number	Rational numbers
Per-numbers	Double-counting	Not accepted
Operations	Icons for the counting process	Mappings from a set-product to a set
Order of operations	/, x, - , +	+, - , x, /

Is ManyMatics Different II

Addition	On-top and next-to	Only on-top
Integration	Preschool: Next-to addition Middle school: Adding piece-wise constant per-numbers High school: Adding locally constant per-numbers	Last year in high school, for the few
A formula	A stand-by calculation with numbers and letters	An example of a function that is an example of a relation in a set-product where first component identity implies second component identity
Algebra	Re-unite constant and variable unit-numbers and per-numbers	A search for patterns

Is ManyMatics Different III

The root of Mathematics	The physical fact Many	The metaphysical invention Set
A concept	An abstraction from examples	An example of an abstraction (MetaMatics)
How correct is 2+3 = 5 and 2x3 = 6	2x3 = 6 is correct by nature since 2 3s can be recounted as 6 1s. 2+3 = 5 is true in a library but not in a laboratory: 2w+3d = 17d, etc.	Both correct by nature (MatheMatism)
An equation	A reversed operation	An example of an equivalence relation between two number-names

Is Solving Equations Different?

ManyMatics

2 + u = 5 = (5-2) + 2	Solved by re-stacking 5
u = 5-2 = 3	Test: 2 + 3 = 5 OK

2 x u = 5 = (5/2) x 2	Solved by re-bundling 5
u = 5/2 = 2½	Test: 2 x 3 = 6 OK

MatheMatics

2 + u = 5	Addition has 0 as its neutral element, and 2 has -2 as its inverse element
↕ (2 + u) + (-2) = 5 + (-2)	Adding 2's inverse element to both number-names
↕ (u + 2) + (-2) = 3	Applying the commutative law to u + 2, 3 is the short number-name for 5+(-2)
↕ u + (2 + (-2)) = 3	Applying the associative law
↕ u + 0 = 3	Applying the definition of an inverse element
↕ u = 3	Applying the definition of a neutral element. *With arrows a test is not needed.*

Recounting looks like Dienes MultiBase Blocks

- “Dienes’ name is synonymous with the Multi-base blocks (also known as Dienes blocks) which he invented for the teaching of place value.
- He also is the inventor of Algebraic materials and logic blocks, which sowed the seeds of contemporary uses of manipulative materials in mathematics instruction.
- Dienes’ place is unique in the field of mathematics education because of his theories on how mathematical structures can be taught from the early grades onwards using multiple embodiments through manipulatives, games, stories and dance.”

(http://www.zoltandienes.com/about/)

Dienes on Numbers and MultiBase Blocks

“The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. This is why our system of numbering, introduced in the middle ages by Arabs, is called the place value system. My contention has been, that in order to fully understand how the system works, we have to understand the concept of power. (..) In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..) Educators today use the “multibase blocks”, but most of them only use the base ten, yet they call the set “multibase”. These educators miss the point of the material entirely.”

(What is a base?, http://www.zoltandienes.com/academic-articles/)

ManyMatics turns MetaMatics upside down

Dienes teaches the 1D place value system with 3D, 4D, etc. blocks to illustrate the importance of the power concept.

- ManyMatics teaches decimal numbers with units and stays with 2D to illustrate the importance of the block concept and adding areas.

Dienes wants to bring examples of abstractions to the classroom

- ManyMatics wants to build abstractions from concrete examples

Dienes teaches top-down 'MetaMatics' derived from the concept Set

- ManyMatics is a bottom-up natural science about the physical fact Many; and sees Set as a meaningless concept because of Russell's set-paradox.

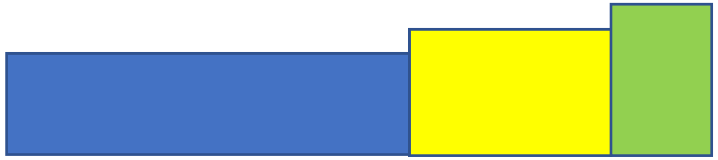
Conclusion I

What Mastery of Many does the child have already?

- Children typically see Many as blocks with a number af bundles, and use flexible numbers with units and with over- or underloads

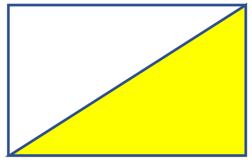
In ManyMath, BLOCKS are fundamental:

- in numbers: 456 = three blocks



- in algebra: adding blocks next-to or on-top

- in geometry: recounting half-blocks



Conclusion II

We may ask: Can children discover/invent mathematics themselves and obtain a concrete exemplified understanding?

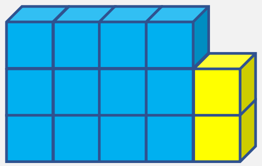
Based upon our observations, the answer is YES, if we

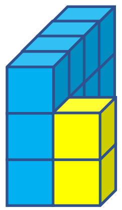
- de-model digits as icons with as many sticks as they represent
- use the flexible bundle-numbers children develop when adapting to Many
- de-model operations as means for bundle-counting 8 as 8/2 **2s**, leading directly to the re-count formula $T = (T/B) \times B$, used to change units, and to
- solve equations as 'How many 2s in 8?' by recounting 8 in 2s
- use re-counting to construct per-numbers, fractions and trigonometry
- add both next-to and on-top, making calculus be addition of per-numbers

$u \times 2 = 8 = 8/2 \times 2$

so $u = 8/2$

The child's own flexible math curriculum





1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving **OUTSIDE** geometrical multi-blocks, & (when turned to hide the units behind) **INSIDE** algebraic bundle-numbers.

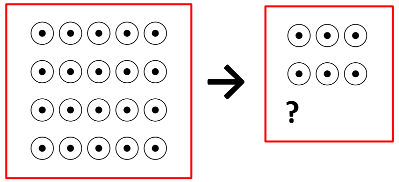
3) Operations are **INSIDE** icons, showing the 3 **OUTSIDE** counting steps: PUSHING & LIFTING bundles & PULLING stacks to find the unbundled ones.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** *(from T, T/B times, Bs can be taken away)*

Question: T = 4 ***5s*** *= ?* ***3s*** ● *Answer: T = 4* ***5s*** *= 6B2* ***3s*** ● *Prediction:*

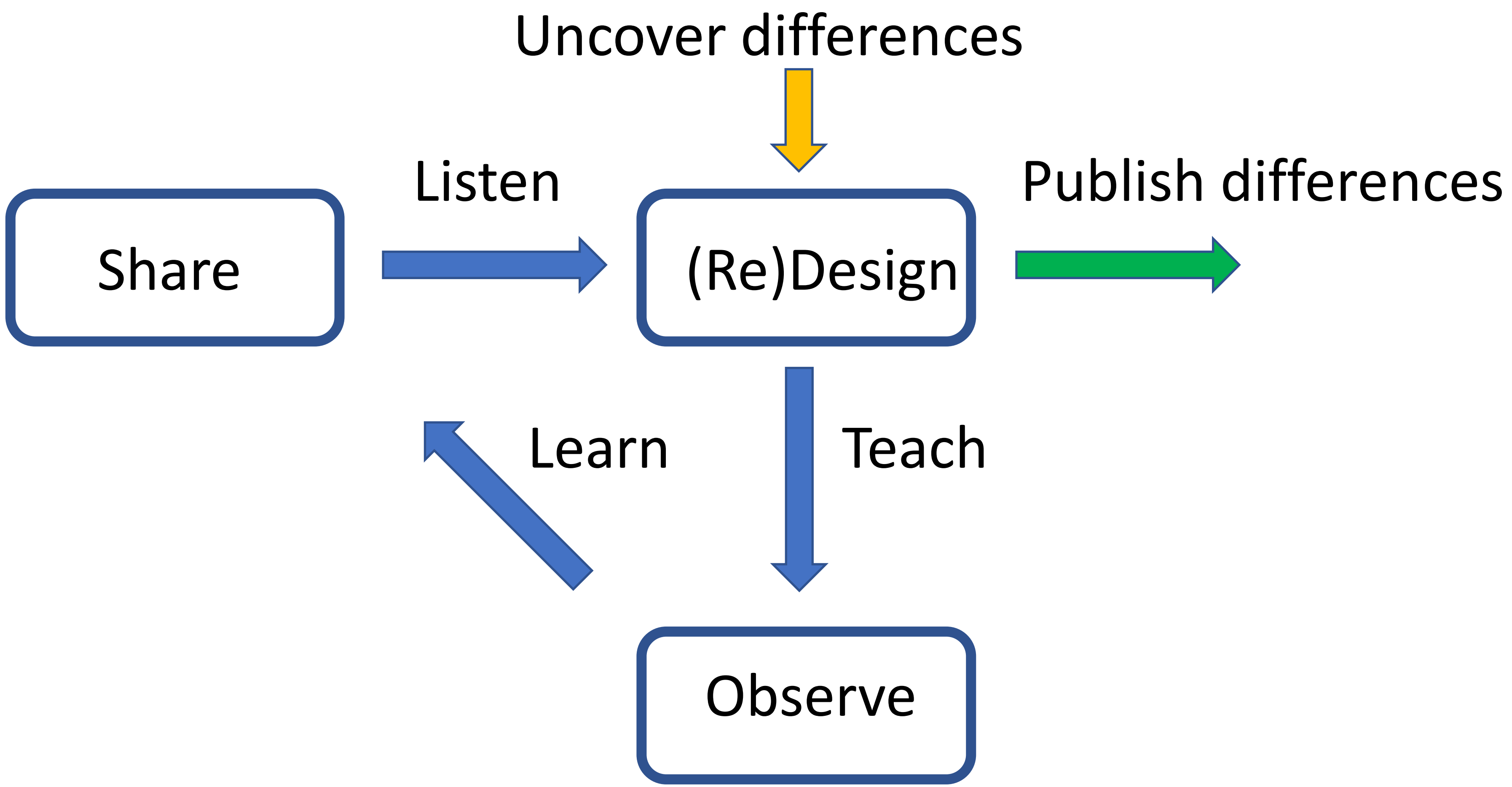
4x5/3	6.some
4x5 – 6x3	2



MatheMatics: Unmask Yourself, Please

- In Greek you mean ‘knowledge’. You were chosen as a common label for 4 activities: Music, Astronomy, Geometry & Arithmetic. Later only 2 activities remained: Geometry and Algebra
- Then Set transformed you from a natural science about the physical fact Many to a metaphysical subject, MetaMatism, combining MetaMatics and MatheMatism
- So please, unmask your true identity, and tell us how you would like to be presented in education:
- MetaMatism for the few - or ManyMatics for the many.

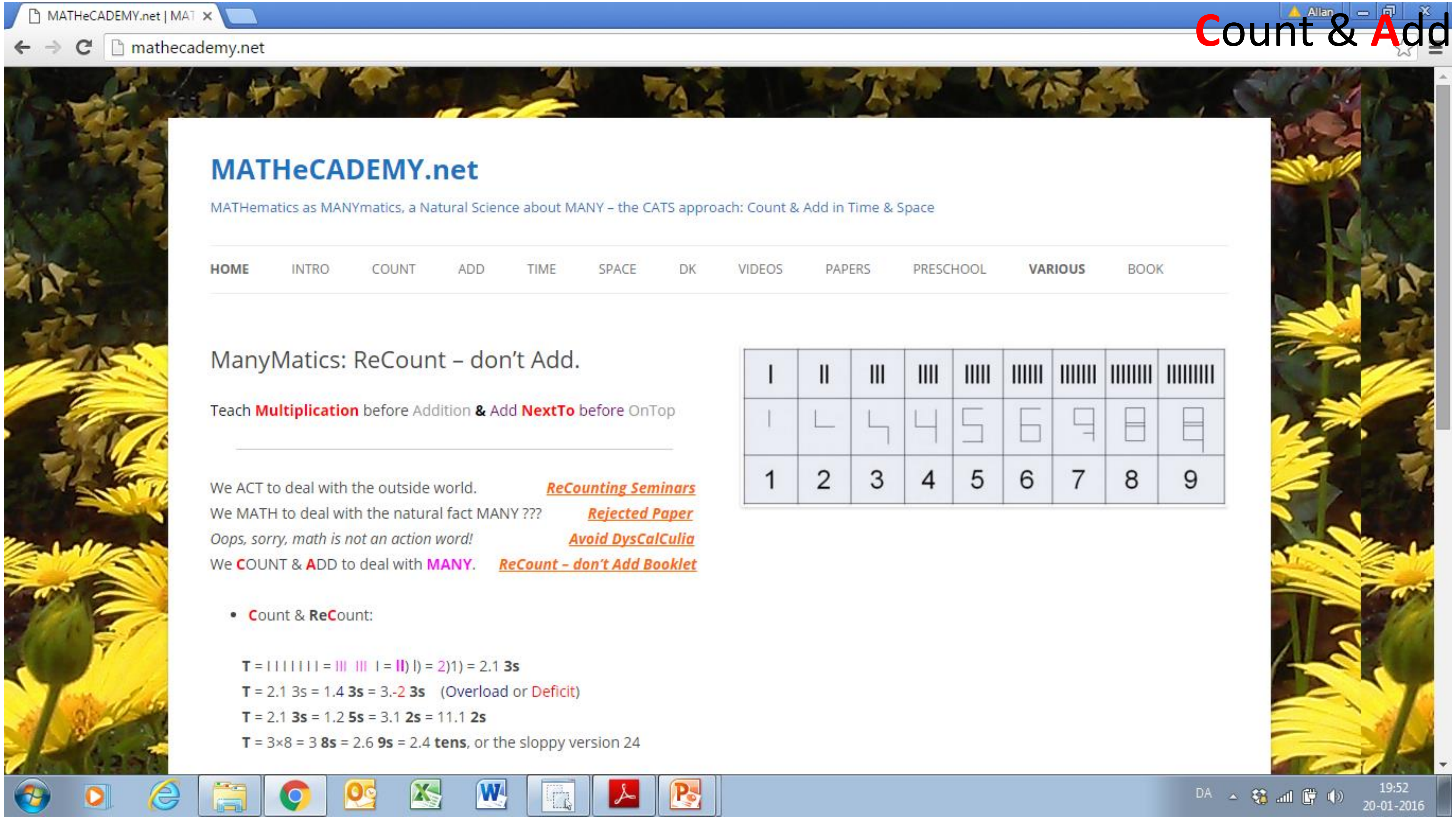
Difference-research finds differences making a difference
Action Learning & Action Research



MATHeCADEMY.net

Inspiration from MATHeCADEMY.net

Teaches Teachers to Teach
MATHEmatics as MANYmatics,
a Natural Science about MANY.
The CATS method: To learn Math
Count & Add in Time & Space



8 MicroCurricula for Action Learning & Research

C1. Create Icons

C2. Count in Icons (Rational Numbers)

C3. ReCount in the Same Icon (Negative Numbers)

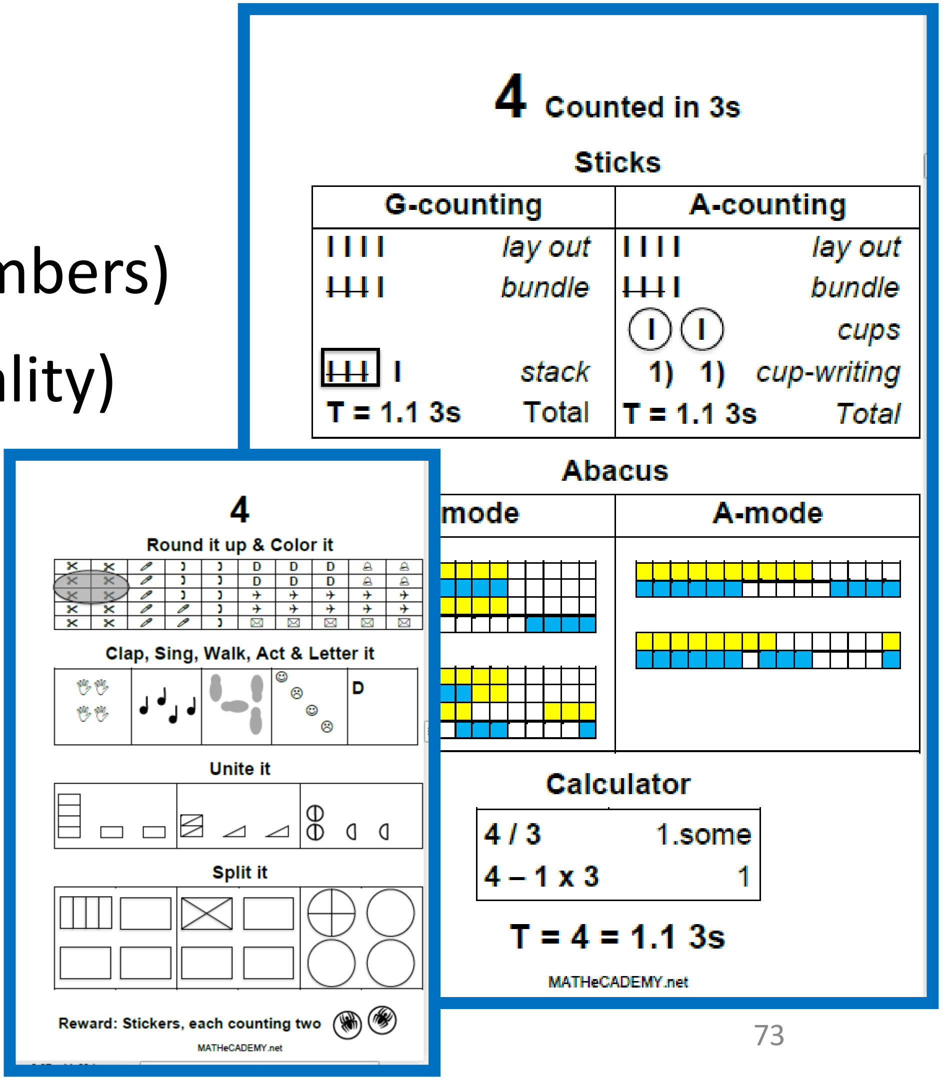
C4. ReCount in a Different Icon (Proportionality)

A1. Add OnTop (Proportionality)

A2. Add NextTo (Integrate)

A3. Reverse Adding OnTop (Solve Equations)

A4. Reverse Adding NextTo (Differentiate)



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ReCount – don’t Add Booklet, free to Download

ReCount

don’t Add

MatheMatics as ManyMatics
for NewComers & LateComers & Migrants
to Avoid DysCalCulia

The Direct Way to Core Mathematics:
Proportionality & Fractions & Calculus & Solving Equations

Allan.Tarp
MATHeCADEMY.net

Contents

03. ReCounting in Icons

Q?		Do	Calculator
9 in 5s	Line	T = I I I I I I I I I	
	Count	1, 2, 3, 4, B, 1B1, 1B2, 1B3, **1B4**	
	Bundle	T = ~~IIIII~~ I I I I	9/5 1.some
	Stack		9 – 1*5 4
	Cup	T = 1)4	
	Answer	T = 9 = 1.4 5s	
9 in 4s	Line	T = I I I I I I I I I	
	Count	1, 2, 3, B, 1B1, 1B2, 1B3, 2B, **2B1**	
	Bundle	T = ~~IIII~~ ~~IIII~~ I	9/4 2.some
	Cup	T = 2)1	9 – 2*4 1
	Stack		
	Answer	T = 9 = 2.1 4s	
9 in 3s	Line		
	Count		
	Bundle		9/
	Cup		9 –
	Stack		
	Answer		
8 in 4s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		
	Answer		
8 in 3s	Line		
	Count		
	Bundle		8
	Cup		8
	Stack		
	Answer		

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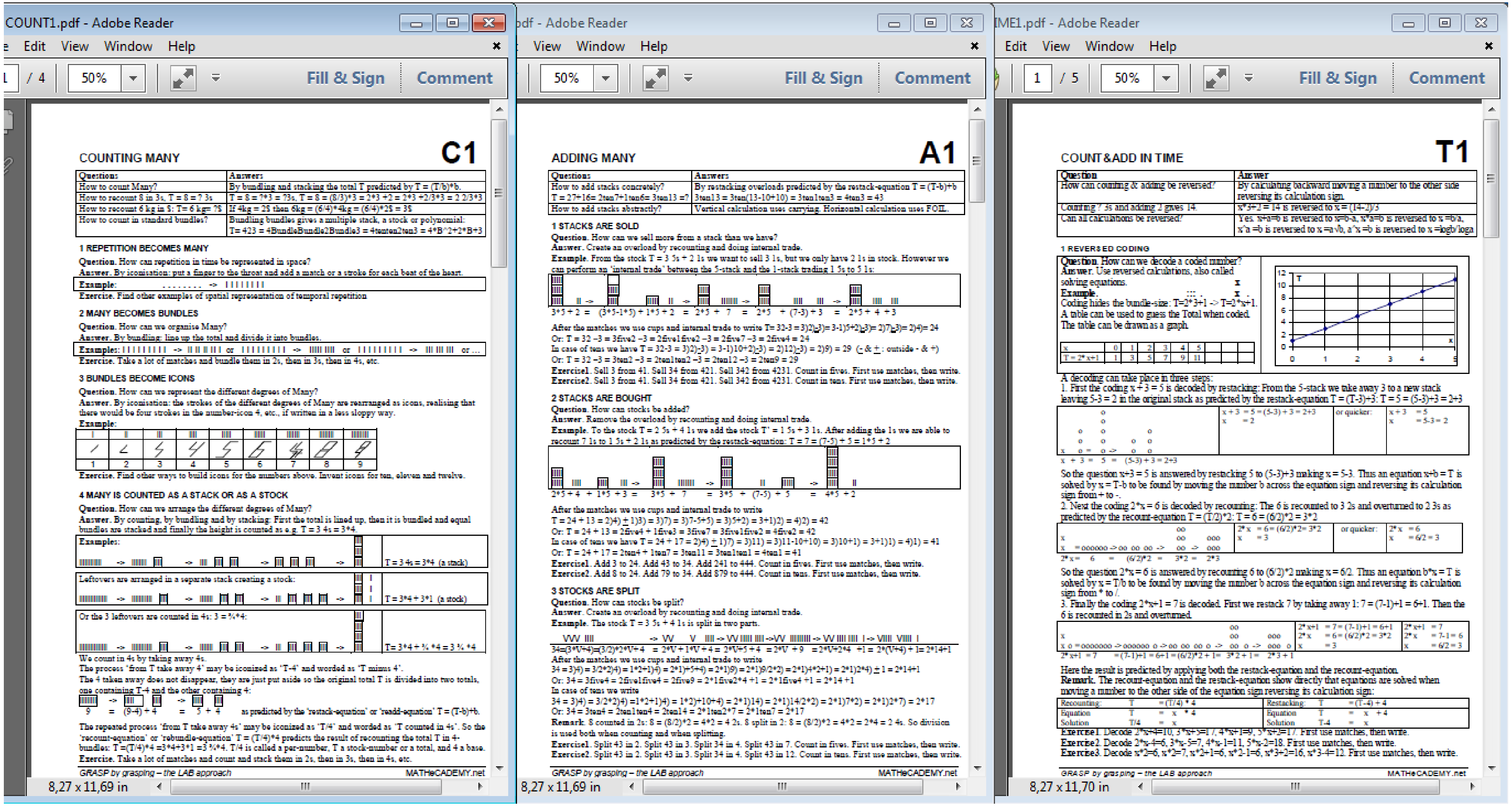
Teacher Training in CATS ManyMatics



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Question Guided Teacher Education

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Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

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1day Skype Seminar: To avoid Math Dislike,
ReCount in flexible BundleNumbers

Action Learning on the child's own 2D NumberLanguage as observed when showing 4 fingers together 2 by 2 makes a 3-year-old child say 'No, that is not 4, that is 2 2s.'

09-11. Listening and Discussing: Good & Bad & Evil MatheMatics
Bad MatheMatism is true inside but rarely outside classrooms.
Evil MetaMatics presents a concept TopDown as an example instead of BottomUp as an abstraction.
Good ManyMatics, a natural science Many mastering Many by ReCounting & adding OnTop/NextTo.
2D Bundle-Numbers with units as a hidden alternative to the traditional 1D Line Numbers without
Adding 1D Line Numbers without units may create Math Dislike.

11-13. Skype Conference. Lunch.

13-15. Doing: Trying out the 'ReCount – don't Add' booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in ReCounting and NextTo Addition.

15-16. Coffee. Skype Conference.

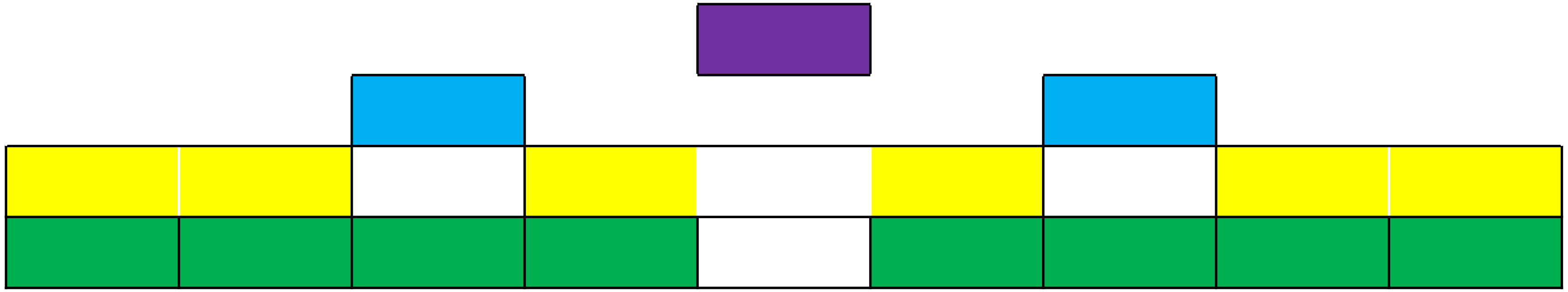
PYRAMIDeDUCATION

To learn MATH: Count&Add MANY
Always ask Many, not the Instructor
MATHeCADEMY.net - a VIRUSeCADEMY

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

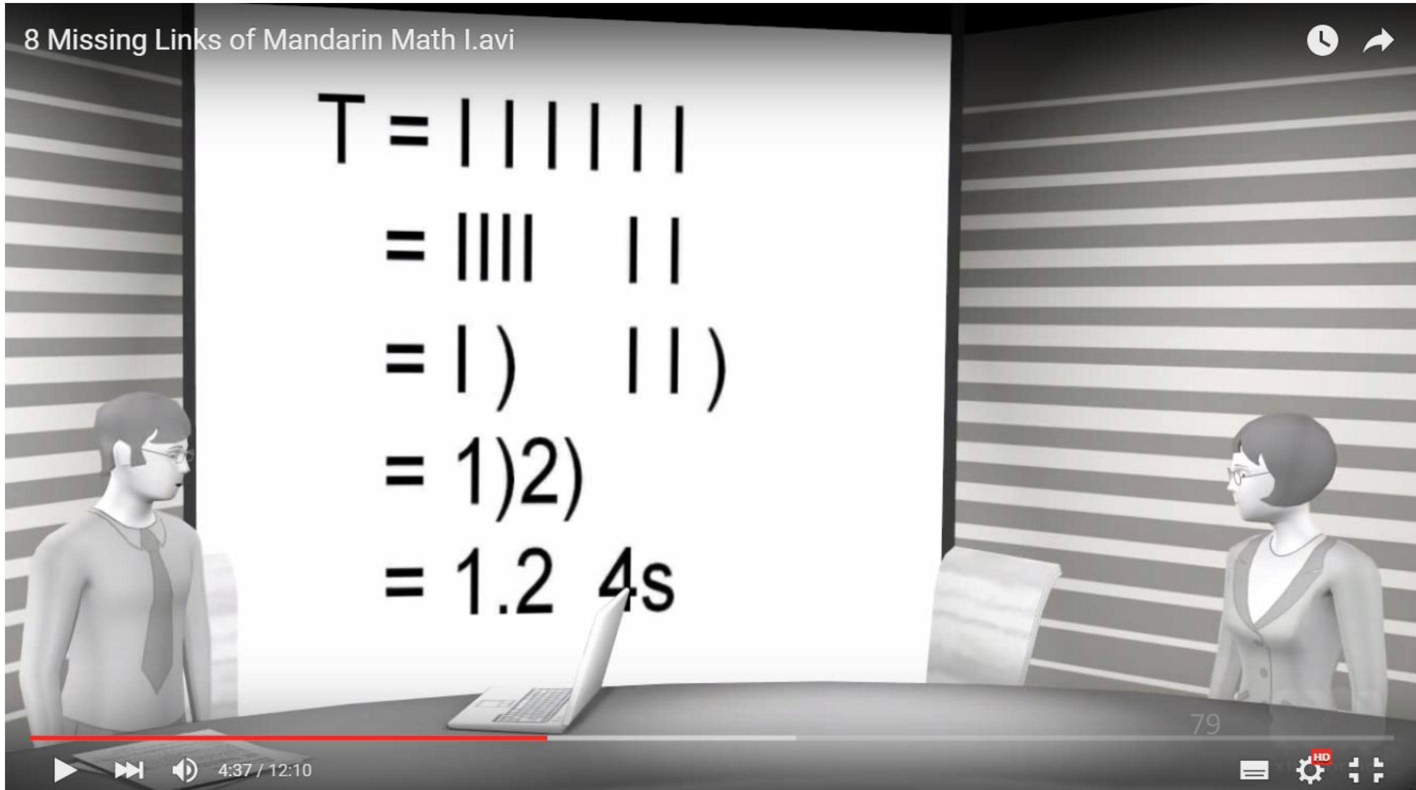
- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting the Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams



Watch MrAlTarp YouTube Videos

- Postmodern Mathematics Debate
- CupCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



Seminar on

the Childs own Number Language

ReCount - don't Add

From MatheMatism to ManyMatics

Thank You

for Your Time

Allan.Tarp@*MATHeCADEMY.net*

Free Uni Franchise

Theoretical Background

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City University of Education Journal of Science 17*(3), 453-466.